Theory of Games - Problem Set I

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1. There are two firms, Entrant (denoted by E) and an Incumbent (denoted by I). Player E moves first and plays either IN or OUT (i.e. comes into the market or stays out). If he plays OUT, the game ends with payoffs 0 to E and 100 to I. Player I can observe what E has played. If E plays IN, player I can either play FIGHT (F) or ACCOMMODATE (A) and the game ends with payoffs (-10,0) and (40,50) respectively (the first payoff in brackets is E's payoff, and the second I's). Represent this situation as a game in normal form.

2. Consider a finite normal form game $\langle \{1, 2, ..n\}, S_1, ...S_n, \pi_1, ..., \pi_n \rangle$. We say that $s_i' \in S_i$ is weakly * dominated if there exists $s_i^* \in S_i$ such that $\pi_i(s_i^*, s_{-i}) \geq \pi_i(s_i', s_{-i})$ for all $s_{-i} \in S_{-i}$ with strict inequality holding for at least one $s_{-i} \in S_{-i}$.
(i) Show by means of an example that there exists beliefs for which s_i' is a best response.
(ii) Prove that s_i' can not be a best response to any belief which puts strictly positive probability on all $s_{-i} \in S_{-i}$.

3. Suppose that there are at least n ($n \geq 2$) bidders at an auction of a single indivisible object. Let $k \leq n$ be an integer. The k^{th} price auction is defined as follows: each player has a valuation v_i and bids b_i for the object. The object is given to the highest bidder at the k^{th} highest bid. For what values of k is bidding $b_i = v_i$ a weakly dominant strategy for player i. For a given k what strategies are never-best-responses?

4. There are three voters who have to elect one of three candidates a, b and c. Each voter votes for one of the candidates. The candidate who wins the most number of votes wins; in case of a tie, a is elected. Each voter has a utility function where having her best, second-best and worst outcomes elected gives her payoffs of 1, 0.5 and 0 respectively. Show that voting for one's best candidate is *not* a weakly dominant strategy. (Hint: You have to carefully specify the rankings of the three candidates for each voter)

5. There are three legislators 1, 2 and 3 who have to decide by voting, the percentage of the budget on education that will be spent on primary education, i.e they have to pick a point on the closed interval $[0, 1]$. Each legislator i votes for a point $x_i \in [0, 1]$. Given the three points x_1, x_2, x_3, the point $\phi(x_1, x_2, x_3) \in$

$[0,1]$ is chosen (ϕ is a mapping $\phi : [0,1]^3 \to [0,1]$). Legislator i's payoff is given by $\pi_i(x_1, x_2, x_3) = -(v_i - \phi(x_1, x_2, x_3))^2$. (You should think of v_i as i's "ideal point". Show that (i) if $\phi(x_1, x_2, x_3) = \frac{x_1+x_2+x_3}{3}$, then i does not have a weakly dominant strategy (ii) if $\phi(x_1, x_2, x_3) =$ median $\{x_1, x_2, x_3\}$, then $x_i = v_i$ is a weakly dominant strategy (iii) what happens if $\phi(x_1, x_2, x_3) = \max\{x_1, x_2, x_3\}$ and $\phi(x_1, x_2, x_3) = \min\{x_1, x_2, x_3\}$? (iv) (HARD) Suppose that ϕ satisfies the requirement that for all x_1, x_2, x_3 , $\phi(x_1, x_2, x_3)$ lies in the smallest interval containing x_1, x_2, x_3. Suppose that v_1, v_2 and v_3 are picked arbitrarily. Characterize the class of functions ϕ satisfying the property that $v_i = x_i$ is a weakly dominant strategy for all i.