

# Voting, Wealth Heterogeneity, and Endogenous Labor Supply

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We examine the link between voting outcomes, wealth heterogeneity, and endogenous labor–leisure choice in the majority-voting–endogenous-growth frameworks of Alesina and Rodrik (1994) and Das and Ghate (2004). We augment these frameworks to incorporate leisure-dependent utility and allow households to vote on factor-specific income taxes. When agents vote on factor-specific taxes, we show that the asymptotic convergence of factor holdings does not imply unanimity over the growth-maximizing tax policy in the steady state. Unanimity over growth-maximizing policies holds only when agents vote on a general income tax, and when agents vote on factor-specific taxes but labor is exogenous.

*Keywords:* factor income taxation, endogenous distribution, median-voter theorem, heterogeneous agents, endogenous labor supply

*JEL classification:* E 62, O 40

## 1. Introduction

We examine the implications of an endogenous labor–leisure choice on the equilibrium tax rate in a heterogeneous-agent endogenous-growth framework of the Barro (1990) type. We allow for voting over factor-specific income taxes and examine the links between voting outcomes and wealth heterogeneity when labor is endogenous. The theoretical model augments the frameworks of Das and Ghate (2004), and Alesina and Rodrik (1994). We extend these frameworks by incorporating leisure-dependent utility and allowing for voting over factor income taxes. As in the Das–Ghate and Alesina–Rodrik models, the equilibrium tax rate is determined under major-

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ity voting, and redistribution occurs through the tax rate. Income inequality is defined in terms of the functional distribution of income. Majority voting determines the extent of redistribution and thus a relationship between inequality and growth in a simple way.

Our analysis makes two contributions. First, we characterize the dynamics of wealth inequality and the steady-state distribution of factor holdings. We show that the steady-state dynamics of wealth inequality are unaffected by the underlying factor-specific tax system: there is complete convergence of factor holdings in the steady state, as in Stiglitz (1969). Our second contribution constitutes the main result of the paper: While there is unanimity over the tax rate in the steady state, convergence in factor holdings does not imply unanimity over the growth-maximizing tax policy. In the steady state, the equilibrium capital income tax rate is less than the growth-maximizing tax rate, while the equilibrium labor income tax rate is greater than the growth-maximizing tax rate. Both outcomes lead to lower steady-state growth. Our general result is to show that unanimity over the growth-maximizing tax rate depends crucially on how the labor supply varies with respect to individual factor taxes. We identify the intuition behind these results.

Our research is motivated by a large body of literature that analyzes redistributive policies and economic growth in an endogenous-growth framework.<sup>1</sup> Our framework is similar to the majority-voting–endogenous-growth models developed in Alesina–Rodrik and Das–Ghate.<sup>2</sup> However, neither of these models endogenizes the labor supply. We show that endogenizing the labor supply through leisure-dependent utility has significant implications for the unanimity results obtained in Das–Ghate, Ghate (2005), and Alesina–Rodrik.<sup>3</sup>

- 1 There is ample evidence supporting the empirical validity of an AK-type endogenous-growth model. For instance, Li (2002) conducted a number of time-series and panel-data tests employing more extensive data sets and a broader definition of investment. Li (2002) finds that both the time-series and the panel evidence for a large number of OECD countries accords with the implications of the AK model. Similarly, using annual data for 98 countries from 1960 to 1998, Bond, Leblebicioglu, and Schiantarelli (2004) find that an increase in the share of investment predicts a higher growth rate of output per worker, both in the short run and in the steady state. This evidence is consistent with the main implications of AK-type endogenous-growth models.
- 2 Ghate (2005) is also similar to these papers, but extends the unanimity results in Das–Ghate to the case of a general income tax. Ghate (2005) shows that when voting on a general income tax, unanimity occurs in both the short run and the long run. In Das–Ghate, unanimity holds only in the long run.
- 3 Bertola (1993) also analyzes the growth and distributional effects of fiscal policy in the context of a simple endogenous-growth model with externalities. While Bertola's setup also leads to a monotonic positive relation between capital subsidy rates and growth, in the current framework the growth rate is hump-shaped with respect to factor-specific taxes, as in Alesina–Rodrik and Das–Ghate.

Endogenizing the labor supply has an additional motivation that we think is important: to *fully* endogenize the dynamics of wealth inequality in the frameworks of Das–Ghate, Ghate (2005), and Alesina–Rodrik. In Alesina–Rodrik the distribution of wealth remains constant and is pinned down by the initial distribution of capital. While this allows Alesina–Rodrik to explain the growth effects of different after-tax wealth distributions, they cannot account for how growth influences the distribution of wealth, as this always remains constant. Das–Ghate endogenize the dynamics of wealth inequality in Alesina–Rodrik, although in Das–Ghate the steady-state distribution of factor composition ratios is pinned down by the exogenous distribution of skill. Hence, in both Das–Ghate and Ghate (2005), the equilibrium factor holdings remain exogenous. In the model proposed here, agents are different only in their capital holdings (not skill), and they value leisure. This fully endogenizes the dynamics of wealth inequality, both in and outside the steady state. The model is therefore more general.

Incorporating leisure-dependent utility is consistent with a large literature that studies the growth effects of endogenous labor supply (see de Hek, 1998, 2006; Eriksson, 1996; Ladron-de-Guevara et al., 1997, 1999; Ortigueira, 1998, 2000; and Turnovsky, 1999, 2000). An important feature of these models is that they employ an infinite-horizon representative-agent framework. One of the focuses of this literature is to study how leisure-dependent utility induces multiple equilibria.<sup>4</sup> In contrast, we show that multiple equilibria do not arise when leisure-dependent utility is incorporated into the frameworks of Das–Ghate and Alesina–Rodrik. This is because the marginal-benefit and marginal-cost schedules of higher factor-specific taxes are monotonically decreasing and increasing, respectively. This guarantees a unique equilibrium tax rate.

Another focus of the literature on the growth effects of endogenous labor supply – while maintaining the representative-agent framework – is to consider the consequences of endogenizing the labor supply for fiscal policy. For instance, Orteguerra (1998) studies the effect of labor and capital income

4 For instance, de Hek (1998) constructs a one-sector aggregative growth model where both consumption and leisure enter as arguments into the utility function. De Hek (1998) shows that either multiple steady states or nonmonotone (cyclical) behavior obtains. Similarly, de Hek (2006) constructs a model similar to Rebelo (1991) augmented with an endogenous labor–leisure choice. De Hek (2006) shows that these features lead to multiple balanced growth paths. Ladron-de-Guevara et al. (1999) present an endogenous-growth model with unqualified leisure in the utility function (i.e., leisure not adjusted by the stock of human capital). Orteguerra (2000) also examines the dynamic implications of qualified leisure. These authors also show that multiple growth paths may arise. Finally, Ladron-de-Guevara et al. (1997) show that there can be multiple balanced growth paths in an endogenous-growth model with human capital if leisure is endogenously determined.

taxation on the transitional dynamics to the balanced growth path, using a two-sector framework. Orteguerra (1998) shows that distortionary taxes may exert a nonnegligible influence on equilibrium behavior, both along the transitional dynamics and along the balanced growth path. Turnovsky (2000) shows that endogenizing the labor supply leads to fundamental changes in the equilibrium tax structure of the AK growth model. Turnovsky (1999) examines the equilibrium structure of a small open economy and shows that the introduction of an elastic labor supply leads to a *less* (rather than more) potent role of distortionary taxes in influencing growth.<sup>5</sup> In our model, agents are assumed to be heterogeneous with respect to wealth holdings, to have single-period lives, and to have a one-sided bequest motive. Households enjoy utility from leisure and care about the future capital stock. They inherit bequests and pay taxes on their inherited income. Because of the finite-lifetime assumption plus the diminishing marginal utility from bequests, this introduces transitional dynamics.<sup>6</sup> Further, the assumed heterogeneity generates a mapping between household-specific wealth holdings and the households' preferred tax rates. Convergence to the representative agent's wealth holding occurs in the steady state – irrespective of the initial distribution of wealth – as the factor-holding ratios of all agents converge to that of the representative agent.

## 2. The Model with Capital Income Taxes

We now formalize the model. We first allow for voting on the capital income tax rate. In the next section, we allow for voting on the labor income tax rate, and finally, a general income tax. In each case we analyze the dynamics of wealth inequality, and solve for the equilibrium tax rate under majority voting and compare it with the growth-maximizing tax rate. We solve the household's problem with labor supplied endogenously.

The population, or number of households,  $N$ , is given. Each household is differentiated on the basis of its capital holdings,  $K_h$ , whose distribution is assumed to be continuous on a finite support,  $R_+$ . The distribution of  $K_h$  is skewed to the right. This implies that the median household's capital holdings are less than the mean household's. The aggregate stock of capital is given by  $K = \sum_1^N K_h$ . Capital is the only accumulable factor in the model.<sup>7</sup>

5 Eriksson (1996) shows that unlike the standard optimal-growth model, preferences over consumption and leisure can affect the steady-state growth rate (although not the rate of time preference). However, Eriksson (1996) does not analyze fiscal policy.

6 In contrast, in the Turnovsky (1999, 2000) AK framework, the economy will always lie on its balanced growth path.

7 The current setup differs from that of Das-Ghate and Ghate (2005) in that in those papers agents are heterogeneous with respect to skill, not capital holdings.

A single good is produced in the economy according to a Cobb–Douglas production technology, given by

$$Y_t = K_t^a (G_t H_t)^{1-a}, \quad (1)$$

where  $Y_t$  is the aggregate output at time period  $t$ ,  $K_t$  is the aggregate capital stock in the economy,  $H_t$  is the aggregate labor supply in each period, and  $G_t$  is a public infrastructure input, which is the source of labor augmentation. We assume that  $G$  is a pure public good as in Barro (1990).<sup>8</sup> Following the endogenous-growth literature, we interpret  $K$  as both physical and human capital. Hence  $a \in [0, 1]$  is the private return to physical capital as well as human capital. We require the regularity condition,  $a > \frac{1}{2}$ , to ensure that the return to capital is positive in equilibrium.<sup>9</sup>

We assume that the public infrastructure input,  $G$ , is financed by a specific tax,  $\tau_k \in [0, 1]$ , on capital income in each period. This specification is more empirically plausible than, and departs from, both Alesina–Rodrik and Das–Ghate, who assume that infrastructure is financed by a tax on the capital stock, or wealth. The government budget constraint is balanced in each period and given by  $G_t = \tau_{kt} r_t K_t$ , where  $r_t$  is the competitive rate of return to capital. Given (1), we have that the rental rate to capital,  $r_t$ , and the wage rate,  $w_t$ , are given by  $r_t = \phi(\tau_{kt}) H_t^{(1-a)/a}$  and  $w_t = \xi(\tau_{kt}) H_t^{(1-2a)/a} K_t$ , respectively, where  $\phi(\tau_{kt}) = a^{1/a} \tau_{kt}^{(1-a)/a}$  and  $\xi(\tau_{kt}) = (1-a) a^{(1-a)/a} \tau_{kt}^{(1-a)/a}$ . This allows us to write the after-tax rental–wage ratio as

$$\frac{r_t}{w_t} = \frac{a H_t}{(1-a) K_t}. \quad (2)$$

Without any loss of generality, we assume that capital depreciates fully in each period.

Following Aghion and Bolton (1997), agents are assumed to live for a single period. In each period, households are also endowed with a single unit of time, which they allocate optimally between labor and leisure. The tax rate is known before households make their consumption, bequest, and labor supply decisions. Households decide their labor supply choices at the beginning of each period, after which production occurs. Once production occurs, households make their consumption and bequest decisions, and then die. Hence, at time  $t$ , the  $h$ th household derives utility over consumption  $C_{ht}$ ,

8 See Barro (1995, p. 153) for a discussion on the definition of  $G$ . We assume a Cobb–Douglas production structure primarily for analytical tractability. However, recent empirical evidence casts doubt on the Cobb–Douglas specification (see Bentolila and Saint-Paul, 2003, and Duffy and Papageorgiu, 2000).

9 With a narrower interpretation of  $K$  as physical capital, it would be empirically implausible to assume that  $a > \frac{1}{2}$ , but it is not so when capital is interpreted more broadly as we do here. Further, according to Barro and Sala-i-Martin (1995, p. 38), even a value  $\alpha = 0.75$  is quite reasonable. See Das–Ghate for details.

a bequest  $K_{ht+1}$ , and leisure  $1 - H_{ht}$ , where  $H_{ht}$  is the amount of labor supplied by the  $h$ th household in time period  $t$ . The utility function  $U : \mathfrak{N}_+^3 \rightarrow R_+$  satisfies the standard properties, and is assumed to be Cobb–Douglas for tractability.

The household’s problem is the maximization

$$\text{Max}_{C_{ht}, K_{ht+1}, H_{ht}} C_{ht}^\alpha K_{ht+1}^\beta (1 - H_{ht})^{1-\alpha-\beta} \tag{3}$$

subject to

$$C_{ht} + K_{ht+1} \leq w_t H_{ht} + r_t(1 - \tau_{kt})K_{ht}, \tag{4}$$

where  $\alpha \in (0, 1), \beta \in (0, 1)$ , and  $\alpha + \beta \leq 1$ .<sup>10</sup>

The optimization exercise implies the following household decision rules:

$$C_{ht} = \frac{\alpha}{\beta} K_{ht+1}, \tag{5}$$

$$K_{ht+1} = \frac{\beta}{\alpha + \beta} \{w_t H_{ht} + r_t(1 - \tau_{kt})K_{ht}\}, \tag{6}$$

and

$$H_{ht} = (\alpha + \beta) - (1 - \alpha - \beta) \frac{r_t(1 - \tau_{kt})}{w_t} K_{ht}. \tag{7}$$

Equation (5) governs optimal consumption. Equation (6) is the household capital accumulation equation. What is new in relation to Das–Gbate is equation (7), which is the household labor supply equation. This is increasing in the tax rate on capital income. Intuitively, a higher tax rate raises the infrastructure,  $G$ , which increases the rewards from working. This induces households to supply more labor. Noting that  $\sum_1^N H_{ht} = H_t$ , using (2), and rearranging equation (7) leads to an expression for the aggregate labor supply determined endogenously as a function of the tax rate,

$$H_t = H(\tau_{kt}) = \frac{N(\alpha + \beta)(1 - a)}{(1 - a) + a(1 - \alpha - \beta)(1 - \tau_{kt})}. \tag{8}$$

Let  $\delta(\tau_{kt}) = (1 - a) + a(1 - \alpha - \beta)(1 - \tau_{kt})$ . It is easy to verify that  $H'(\tau_{kt}) > 0 \forall \tau_{kt} \in [0, 1]$ . Similarly, the aggregate capital accumulation equation is

$$K_{t+1} = \frac{\beta}{\alpha + \beta} \left\{ \xi(\tau_{kt}) H_t^{\frac{1-a}{a}} + \phi(\tau_{kt})(1 - \tau_{kt}) H_t^{\frac{1-a}{a}} \right\} K_t. \tag{9}$$

<sup>10</sup> A more general approach would be to allow factor-specific and flat-rate taxes to represent benchmark cases of a more continuous tax system: i.e., write equation (4) as  $C_{ht} + K_{ht+1} \leq (1 - \tau_{wt})w_t H_{ht} + r_t(1 - \tau_{kt})K_{ht}$ , where  $\tau_{wt} \in [0, 1]$  denotes the tax on labor income, while  $\tau_{kt} \in [0, 1]$  denotes the tax rate on capital income. This way of formulating the consumer budget constraint would allow all three cases:  $\tau_{wt} = 0, \tau_{kt} = 0$ , and a flat income tax rate,  $\tau_{wt} = \tau_{kt} = \tau_t$ . However, it is well known that the median-voter theorem holds only if voting occurs on a single policy, and second, the model below cannot be solved analytically under a more complex tax system in which optimal growth is implemented as the outcome of a voting process with differentiated nonzero tax rates on both factors of production.

Equations (8) and (9) express the aggregate decision rules for labor and capital, respectively.

Next, as in Das–Ghate and Ghate (2005), we define the economy growth rate as  $g_{t+1} = K_{t+1}/K_t$ , which is given by<sup>11</sup>

$$g_{t+1} = \text{constant} \cdot \{(1 - a) + a(1 - \tau_{kt})\}(\tau_{kt}H_t)^{\frac{1-a}{a}}, \tag{10}$$

where the constant is  $[(\alpha + \beta)/\beta]a^{(1-a)/a}$ . Equations (10) and (8) determine the long-run endogenous growth rate of the economy. The growth–tax curve takes the well-known inverted U-shape, as in Barro (1990), which leads to a unique growth-maximizing tax rate. We denote this as  $\tau_{ek}^g$ . It can be shown that the exogenous growth-maximizing tax rate,  $\tau_x^g$ , is given by  $\tau_{xk}^g = \frac{1-a}{a}$ , where  $\tau_{xk}^g$  denotes the growth-maximizing tax rate when  $\alpha + \beta = 1$ .<sup>12</sup> This allows us to provide a sufficient condition for the existence of a unique growth-maximizing tax rate under endogenous labor–leisure choice ( $\alpha + \beta < 1$ ) and compare it with the growth-maximizing tax rate under exogenous labor–leisure choice ( $\alpha + \beta = 1$ ).

**Proposition 1** Suppose  $\alpha + \beta < 1$ . There exists a unique growth-maximizing tax rate under endogenous labor–leisure choice,  $\tau_{ek}^g$ , which is greater than the growth-maximizing rate under exogenous labor–leisure choice, i.e.,

$$\tau_{ek}^g > \tau_{xk}^g = \frac{1 - a}{a}, \tag{11}$$

if and only if  $2a - 1 > a(1 - \alpha - \beta)(1 - a)$ .

**Proof.** See appendix 6.1. ■

As shown in the appendix, the growth-maximizing tax rate under  $\alpha + \beta < 1$ ,  $\tau_{ek}^g$ , is obtained from differentiating equation (10) with respect to  $\tau_{kt}$ . After manipulating this expression, this leads to a constant growth-maximizing tax rate, which is determined by

$$\underbrace{(1 - a)\{a\tau_{ek}^g - (1 - a)\}}_{\text{MC}} = \underbrace{(1 - \alpha - \beta)a[(1 - a)(1 - a\tau_{ek}^g) - a(1 - \tau)a\tau_{ek}^g]}_{\text{MB}}. \tag{12}$$

11 More accurately,  $g_{t+1}$  refers to the growth factor or gross growth rate. Since our results would not change if we used the growth rate  $K_{t+1}/K_t - 1$ , we use these terms interchangeably. To obtain an expression for  $g_{t+1}$ , we substitute out the expression for  $H_{ht}$  [using (7) in (6)], aggregate across households, and simplify. This yields  $K_{t+1} = \beta\{Nw_t + r_t(1 - \tau_{kt})K_t\}$ . From equation (2), the wage rate can be expressed as  $w_t = (1 - a)K_t r_t / aH_t$ . Using this expression for  $w_t$  and the expression for the rental rate, and substituting out the expression for  $H_t$  from (8) into the above expression for  $K_{t+1}$  leads to equation (10).

12 To see this, note that by Euler’s theorem,  $Y_t = \frac{\partial Y}{\partial K}K_t + \frac{\partial Y}{\partial H}H_t = r_tK_t + w_t$ , where we normalize  $H$  to 1. This implies  $w_t + r_t(1 - \tau_{kt})K_t = w_t + r_tK_t - \tau_{kt}r_tK_t = Y_t - \tau_{kt}r_tK_t$ . Substituting out for  $Y_t$  and differentiating with respect to the tax rate yields the desired result.

**Figure 1**  
*The Growth Maximizing Tax Rate*

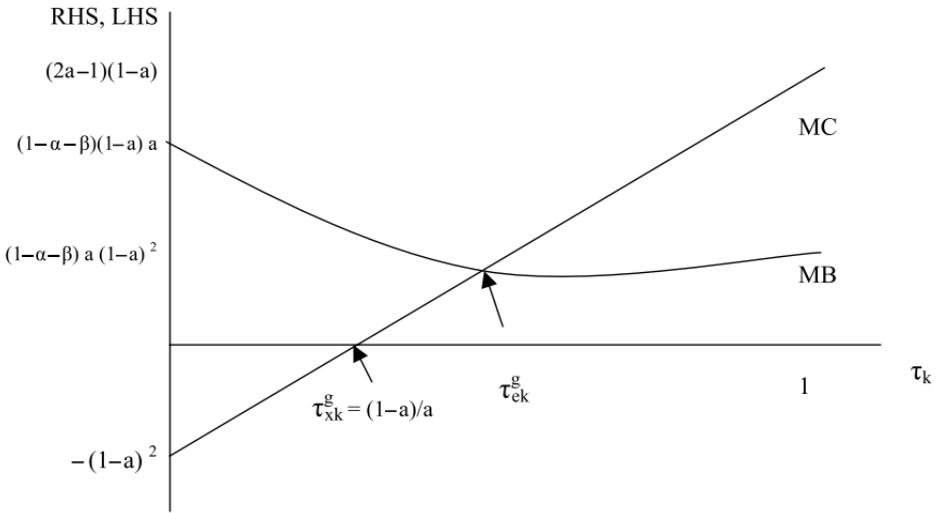


Figure 1 plots the marginal-cost and -benefit schedules corresponding to the growth-maximizing tax rate under  $\alpha + \beta < 1$  and  $\alpha + \beta = 1$  based on equation (12).  $\tau_{ek}^g$  is determined by the intersection of these two schedules. As  $\alpha + \beta \rightarrow 1$ , the marginal benefit of higher taxes falls for each value of the tax rate. This leads to a reduction in the growth-maximizing tax rate. When  $\alpha + \beta = 1$ , the marginal-benefit schedule intersects the marginal-cost schedule at  $\tau_{xk}^g = \frac{1-a}{a}$ : in this case, the marginal benefit is a horizontal line and equal to zero for all feasible values of the tax rate. Intuitively, when labor is endogenous, the tax rate maximizes the net return to capital *as well as* the aggregate labor supply. Under exogenous labor supply, aggregate labor is invariant with respect to the tax rate. Hence, the growth-maximizing tax rate is greater when labor-leisure choice is endogenous. This proves the existence of a unique growth-maximizing tax rate.

**2.1. The Dynamics of Wealth Inequality**

We first consider the case where  $\alpha + \beta < 1$ , and derive the transitional dynamics governing the law of motion of household capital holdings as in Das-Ghate and Ghate (2005). We then characterize the equilibrium tax rate under majority voting. For any household  $h$ , let  $\eta_{ht} = K_{ht}/K_t$ ,  $\eta_h \in [0, 1]$ , denote the relative capital holdings of the  $h$ th household relative to the aggregate capital stock.<sup>13</sup> The dynamic law of motion of household specific

<sup>13</sup> When  $\eta_h = 1$ , then the  $h$ th household owns the entire capital stock.

capital holdings is given by<sup>14</sup>

$$\eta_{ht+1} = \eta_{ht} \left\{ 1 + \frac{\xi(\tau_{kt}) \left[ \frac{H_{ht}/H_t}{\eta_{ht}} - 1 \right]}{\xi(\tau_{kt}) + \phi(\tau_{kt})(1 - \tau_{kt})} \right\}. \tag{13}$$

Equation (13) is the index of inequality in the model and governs the transitional dynamics of relative capital holdings of the  $h$ th household. It is easy to verify from equation (13) that the transition to the steady state is monotone and there is a unique stable steady state. This gives the following result.

**Proposition 2** In the steady state, the factor-holding ratios of agents converge to a mass point that is independent of the initial distribution of capital, i.e.,

$$\frac{H_h}{H} = \eta_h = \frac{1}{N} \quad \forall h. \tag{14}$$

This holds for all feasible values of the tax rate.

**Proof.** See appendix 6.1. ■

The important implication of proposition 2 is that the asymptotic dynamics of wealth inequality under leisure-dependent utility is independent of the capital income tax rate. Incorporating leisure-dependent utility does not change the unanimity results in Das–Ghate and Ghate (2005). In the steady state every agent is a representative agent and there is complete equality in relative factor holdings. Here the fraction of hours worked by households is also pinned by their relative capital holdings in the steady state. Each agent works the same fraction of hours in the steady state.

To obtain the equilibrium tax rate, after several manipulations, the household indirect utility function  $V_{hkt}$  can be written as follows:

$$V_{hkt} = \text{constant} + \log \left\{ 1 + aN(\alpha + \beta) \frac{1 - \tau_{hkt}}{\delta(\tau_{hkt})} \eta_{ht} \right\} + (\alpha + \beta) \log(w_t). \tag{15}$$

The optimal tax rate,  $\tau_{hkt}$ , for the  $h$ th household is obtained from the household’s first-order condition with respect to (15), and is determined by the following first-order condition:<sup>15</sup>

$$\underbrace{\frac{(1 - a)[(1 - a) + a(1 - \alpha - \beta)]}{a\tau_{hkt}}}_{\text{MB}} = \underbrace{\frac{aN(1 - a)\eta_{ht}}{\{(1 - a) + a[(1 - \alpha - \beta) + (\alpha + \beta)N\eta_{ht}](1 - \tau_{hkt})\}}}_{\text{MC}} + a(1 - \alpha - \beta). \tag{16}$$

14 We divide equation (6) by equation (9) and simplify to get equation (13).

15 Technical details are available in the appendix. Throughout the paper, we assume that individuals care not only about how their optimal choices affect individual labor supply, but

Two aspects deserve mention. First, from (16) it is easily verified that as  $\eta_h$  (relative capital holdings) increases, the optimal tax rate of households,  $\tau_{hkt}$ , falls, as in Das–Ghate and Ghate (2005). Intuitively, the RHS of equation (16) corresponds to the marginal-cost schedule of a rise in the tax rate facing households. The first term on the RHS of equation (16) is increasing in  $\eta_h$ . Hence, a higher  $\eta_h$  pushes the marginal cost up for each tax rate. This reduces the household’s preferred tax rate. This is intuitive: the more capital-rich households are, the more they care about their net capital income, and the less their preferred tax on capital. Second, equation (16) allows us to rank households in terms of their capital holdings and preferred tax rates. For capital-rich households (relative to the mean),  $\eta_h > \frac{1}{N}$ . This implies their preferred tax on capital will be less than that of a capital-poor household whose capital holdings are less than the average,  $\eta_h < \frac{1}{N}$ . This is because the marginal cost for an increase in the tax rate is higher for the capital-rich households. Hence, their preferred tax on capital is less than that of a capital-poor household.

Using proposition 2, we substitute  $\eta_h = \frac{1}{N}$  into (16) to get the preferred tax rate of all households in the steady state:

$$\underbrace{\frac{(1 - a)[(1 - a) + a(1 - \alpha - \beta)]}{a\tau_{kh}}}_{MB} = \underbrace{\frac{a(1 - a)}{1 - a\tau_{kh}} + a(1 - \alpha - \beta)}_{MC}, h = \frac{1}{N}. \tag{17}$$

The equilibrium tax is constant. Finally, setting  $h = m$  in (17) yields the equilibrium tax rate under majority voting in the steady state, which is the preferred tax of the median voter. Let us denote this as  $\tau_{mk}$ . We compare  $\tau_{mk}$  with  $\tau_{ek}^g$ , the latter determined by equation (10), which we rewrite as

$$\underbrace{\frac{(1 - a)[(1 - a) + a(1 - \alpha - \beta)]}{a\tau_k}}_{MB} = \underbrace{\frac{a(1 - a)}{1 - a\tau_k} + \frac{a^2(1 - \alpha - \beta)(1 - \tau_k)}{1 - a\tau_k}}_{MC}. \tag{18}$$

The LHSs of both (18) and (17) present the marginal-benefit schedule from higher taxes. These are identical. The difference lies in the marginal-cost schedules. In particular,  $a(1 - \tau)/(1 - a\tau_k) < 1$  for all  $\tau_k \in [0, 1]$ . Hence, the marginal cost of a rise in the tax rate is greater for households in the steady state *for each level of the tax rate*. Thus, for higher values of the tax rate, the optimal tax of households in the steady state – as well as the median

the aggregate  $H$  as well. It is sufficient to note that for any given values of  $K_t$  and  $K_{ht}$  the indirect utility function is single-peaked with respect to  $\tau_{kt}$ . By the median-voter theorem, this implies that the median household’s preferred tax rate is the equilibrium tax rate in the economy. As is well known, this is a sufficient condition for the median-voter theorem to hold.

household’s preferred tax rate – *is less than* the growth-maximizing tax rate. This allows us to state our main result for this section:

**Proposition 3** Let  $\alpha + \beta < 1$ . While there is complete factor-holding convergence in the steady state, the equilibrium capital income tax rate is strictly less than the growth-maximizing tax rate: i.e.,  $\tau_{mk} < \tau_{ek}^g$ .

Proposition 3 implies that factor-holding convergence is not affected by voting on capital income taxes and incorporating leisure-dependent utility. However, in direct contrast to Das–Ghate and Ghate (2005), the equilibrium tax rate is lower than the growth-maximizing tax rate. This happens for a specific reason. Since agents work less because they value leisure, they choose to tax themselves less, as depicted in equation (8). This leads to a lower equilibrium tax rate under majority voting and lower steady-state growth. In Das–Ghate and Ghate (2005), both tax rates are the same, while the inequality is reversed in Alesina–Rodrik: i.e., the tax rate chosen in a political equilibrium is *greater* than the growth-maximizing tax rate. What is common to the current paper, Das–Ghate, and Ghate (2005) is that initial inequality is not preserved under leisure-dependent utility.<sup>16</sup>

## 2.2. Dynamics of Wealth Inequality when $\alpha + \beta = 1$

As in Das–Ghate and Ghate (2005), the two tax rates coincide in the steady state when labor is exogenous. To see this, and following the same steps as before, the relative capital holdings of households (under exogenous labor) evolve according to

$$\eta_{ht+1} = \eta_{ht} \left\{ 1 + \frac{\xi(\tau_{kt}) \left[ \frac{1}{\eta_{ht}} - 1 \right]}{\xi(\tau_{kt}) + \phi(\tau_{kt})(1 - \tau_{kt})} \right\}. \tag{19}$$

This implies that  $\eta_{ht} = 1 \forall h$  in the steady state. There is complete equality in the steady state. The indirect utility function of households is given by

$$V_{ht} = \text{constant} + \log \left\{ 1 + \frac{a}{1-a}(1 - \tau_{kt})H_t\eta_{ht} \right\} + (\alpha + \beta) \log(w_t). \tag{20}$$

Since agents take  $H$  as given, the first-order condition is given by

$$\frac{\frac{a}{1-a}\eta_{ht}H_t}{1 + \frac{a}{1-a}\eta_{ht}H_t(1 - \tau_{kt})} = (\alpha + \beta) \frac{1 - a}{a} \tau_{kt}. \tag{21}$$

<sup>16</sup> In contrast, in Alesina–Rodrik, factor holdings are constant and initial inequality is preserved in the steady state. Further, lower growth obtains for a different reason from that in Alesina–Rodrik. Here unanimity holds, and slower growth comes together with valued leisure. In Alesina–Rodrik, slower growth comes from conflicting choices over the tax rate, with a capital-poor median voter prevailing.

Setting  $\alpha + \beta = 1$  implies that the optimal tax of the  $h$ th household is given by

$$\tau_{hkt} = (1 - a) \left\{ 1 + \frac{1 - a}{a\eta_{ht}} \right\}. \tag{22}$$

The optimal tax rate is decreasing in the relative capital holdings of the  $h$ th household. Setting  $h = m$  and  $\eta_{mt} = 1$  in this expression implies that  $\tau_{mkx} = \frac{1-a}{a}$ , which is the median household’s preferred tax rate. Note that this is identical to the growth-maximizing tax rate,  $\tau_{xk}^g$ , derived before.

### 3. The Model with a Labor Income Tax

The results in the previous section relied crucially on the household labor supply curve described by equation (8), in which labor supply is an increasing function of the tax rate. This leads to a lower equilibrium tax rate characterized by proposition 3. The case of a linear capital income tax may not be realistic for many real-world economies.<sup>17</sup> We now extend the model and consider the dynamics of wealth inequality when there is voting on a tax on labor income.<sup>18</sup>

The setup of the model is identical to the model where  $G$  is financed by capital income taxation. The only difference is that we assume that the public infrastructure input,  $G$ , is financed by a specific tax,  $\tau_{wt} \in [0, 1]$ , on labor income in each period. The government budget constraint is balanced in each period, and given by  $G_t = \tau_{wt}w_tH_t$ , where  $w_t$  is the competitive wage rate.<sup>19</sup>

Households maximize (3) subject to (4), which yields the optimal consumption equation,  $C_{ht} = \frac{\alpha}{\beta}K_{ht+1}$ , the optimal capital accumulation equation,  $K_{ht+1} = \frac{\beta}{\alpha+\beta}\{w_tH_{ht}(1 - \tau_{wt}) + r_tK_{ht}\}$ , and the agent’s optimal labor supply equation,

$$H_{ht} = (\alpha + \beta) - (1 - \alpha - \beta) \frac{r_t}{w_t(1 - \tau_{wt})}. \tag{24}$$

Equation (24) is the optimal labor supply equation. Importantly, *and opposite to the case of capital income taxation*, household labor supply is

17 This is true especially in Europe, where there is a growing tendency to tax labor incomes more heavily than capital incomes. See Mendoza and Tesar (2003) and Quadrini (2005).

18 The derivations of this section are detailed in appendix 6.2.

19 Given (1), we can express the rental rate to capital,  $r_t$ , and the wage rate  $w_t$  by  $r_t = \nu(\tau_{wt})H_t^{(1-a)/a}$ , where  $\nu(\tau_{wt}) = a\tau_{wt}^{(1-a)/a}$ , and  $w_t = \varphi(\tau_{wt})H_t^{(1-2a)/a}K_t$ , where  $\varphi(\tau_{wt}) = (1 - a)^{1/a}\tau_{wt}^{(1-a)/a}$ . The rental-rate–wage ratio is

$$\frac{r_t}{w_t(1 - \tau_{wt})} = \frac{aH_t}{(1 - a)^{1/a}K_t(1 - \tau_{wt})}. \tag{23}$$

decreasing in  $\tau_{wt}$ . This has an important implication for the steady-state tax rate determined under majority voting. Aggregating the household capital accumulation equation above yields the aggregate capital accumulation equation:

$$K_{t+1} = \frac{\beta}{\alpha + \beta} \{ \varphi(\tau_{wt})(1 - \tau_{wt}) + \nu(\tau_{wt}) \} K_t H_t^{\frac{1-\alpha}{a}}. \quad (25)$$

Similarly, aggregating across households in (24) yields the aggregate labor supply equation:

$$H_t = \frac{N(\alpha + \beta)(1 - a)^{\frac{1}{a}}(1 - \tau_{wt})}{(1 - a)^{1/a}(1 - \tau_{wt}) + (1 - \alpha - \beta)a}. \quad (26)$$

The aggregate labor supply  $H_t$  is also decreasing in the tax rate.<sup>20</sup>

The remaining analysis is similar to the case of voting on capital income taxes. The constant growth-maximizing tax rate is obtained by solving for  $\tau_{ew}^g$  from  $\partial g_{t+1} / \partial \tau_{wt} = 0$ . From the first-order condition,<sup>21</sup>

$$\frac{1 - a}{a\tau_{ew}^g} = \frac{(1 - a)^{\frac{1}{a}}}{(1 - a)^{\frac{1}{a}}(1 - \tau_{ew}^g) + a} + \frac{(1 - a)(1 - \alpha - \beta)}{\epsilon(\tau_w)(1 - \tau_{ew}^g)}, \quad (28)$$

where  $\tau_{ew}^g \in [0, 1]$  denotes the growth-maximizing tax rate under  $\alpha + \beta < 1$ . It is easily verified from equation (28) that this equation defines a unique growth-maximizing tax rate.

The dynamics of wealth inequality are given by

$$\eta_{ht+1} = \eta_{ht} \left\{ 1 + \frac{\varphi(1 - \tau_{wt}) \left[ \frac{H_{ht}/H_t}{\eta_{ht}} - 1 \right]}{\varphi(1 - \tau_{wt}) + \nu(\tau_{wt})} \right\}. \quad (29)$$

There is complete factor convergence in the steady state.<sup>22</sup> This is consistent with the results derived when there is voting on capital income taxation.

The equilibrium tax rate is obtained by substituting the individual decision rules of households back into their utility functions. After simplifying, we

<sup>20</sup> Specifically,  $H'(\tau_{wt}) = -\frac{N(\alpha+\beta)(1-a)^{1/a}(1-\alpha-\beta)a}{(1-a)^{1/a}(1-\tau_{wt})+(1-\alpha-\beta)a} < 0 \quad \forall \tau_{wt}, 0 < \tau_{wt} < 1$ .

<sup>21</sup> Substituting equation (24) into the household-capital accumulation equation and aggregating across households yields  $\sum K_{ht+1} = K_{t+1} = \beta \{ Nw_t(1 - \tau_{wt}) + r_t K_t \}$ . Substituting out for the wage rate  $w$ , the rental rate on capital,  $r$ , and aggregate labor  $H$  and simplifying yields

$$g_{t+1} = \text{constant} \cdot \left\{ (1 - a)^{\frac{1}{a}}(1 - \tau_{wt}) + a \right\} (\tau_{wt} H_t)^{\frac{1-\alpha}{a}}. \quad (27)$$

<sup>22</sup> Setting  $\eta_{ht+1} = \eta_{ht} = \eta_h$  in (29) yields  $\eta_h = H_h/H = 1/N$ .

obtain<sup>23</sup>

$$W_{ht} = \text{constant} + \log \left\{ 1 + \frac{a\eta_{ht}N(\alpha + \beta)(1 - a)^{\frac{1}{a}}}{(1 - a)^{\frac{1}{a}}\epsilon(\tau_{wt})} \right\} + (\alpha + \beta) \log[w_t(1 - \tau_{wt})]. \tag{30}$$

We show in appendix 6.1 that the optimal tax rate for the *h*th household,  $\tau_{hwt}$ , is determined by the first-order condition

$$\frac{a\eta_{ht}N(\alpha + \beta)(1 - a)^{\frac{1}{a}}/\epsilon(\tau_{hwt})}{(1 - a)^{\frac{1}{a}}(1 - \tau_{hwt}) + a[(1 - \alpha - \beta) + \eta_{ht}(\alpha + \beta)N]} + (\alpha + \beta) \left[ \frac{1 - a}{a\tau_{hwt}} + \frac{1 - 2a}{a} \cdot \frac{H'}{H} \right] = (\alpha + \beta) \frac{1}{1 - \tau_{hwt}}. \tag{31}$$

Importantly, the optimal tax rate is decreasing in the relative capital holdings of the *h*th household. Since equation (29) implies factor-holding convergence, setting  $\eta_h = \frac{1}{N}$  in (31) yields a constant equilibrium tax rate for households in the steady state:

$$\frac{1 - a}{a} \frac{1}{\tau_w} = \frac{(1 - a)^{\frac{1}{a}}(1 - \tau_w)}{\epsilon(\tau_w)} \left\{ \frac{(1 - a)^{\frac{1}{a}}}{(1 - a)^{\frac{1}{a}}(1 - \tau_w) + a} \right\} + \frac{(1 - a)(1 - \alpha - \beta)}{\epsilon(\tau_w)(1 - \tau_w)}. \tag{32}$$

We denote the constant steady-state equilibrium tax rate – the preferred tax rate of the median voter – that solves (32) as  $\tau_{mw}$ . The LHS of (32) corresponds to the marginal benefit of an increase in labor income taxes. The marginal benefit of an increase in taxes tends to infinity as  $\tau_w$  tends to zero and converges to  $\frac{1-a}{a}$  as  $\tau_w$  tends to 1.<sup>24</sup> Hence, the marginal-benefit curve is declining in the tax rate. The RHS of equation (32) corresponds to the marginal cost (MC) of an increase in labor income taxes. Since the marginal-cost curve at  $\tau_w = 0$  is above zero, and the marginal-cost curve at  $\tau_w = 1$  approaches infinity, there exists a unique equilibrium tax rate.

A comparison of equations (32) and (28) determines the main result. We summarize this in terms of the following proposition.

**Proposition 4** Let  $\alpha + \beta < 1$ . While there is complete factor-holding convergence in the steady state, the equilibrium labor income tax rate is greater than the growth-maximizing tax rate: i.e.,  $\tau_{wm} > \tau_{ew}^g$ .

23 Define  $\epsilon(\tau_{wt}) = (1 - a)^{1/a}(1 - \tau_{wt}) + (1 - \alpha - \beta)a$ .

24 The derivative of the marginal-benefit schedule with respect to  $\tau_w$  is

$$-\frac{1 - a}{a} \frac{1}{\tau_w^2} < 0 \quad \forall \tau_w \in [0, 1].$$

**Proof.** The proof involves a simple comparison of the marginal-cost schedules of (32) and (28), since the marginal-benefit schedules are identical. Since  $(1 - a)^{1/a}(1 - \tau_w)/\epsilon(\tau_w) < 1 \forall \tau_w \in [0, 1]$ , the marginal-cost curve – for a rise in each tax rate – is lower than the marginal-cost schedule for the growth-maximizing tax rate. Hence, the equilibrium tax rate under majority voting exceeds the growth-maximizing tax rate in the steady state. ■

Intuitively, and in contrast to the case of capital income taxation, since households value leisure, they work less, but choose to tax themselves more than with the growth-maximizing policy. This follows from equations (24) and (26). It leads to lower steady-state growth, even though there is complete factor-holding convergence in the steady state.

### 3.1. Dynamics of Wealth Inequality when $\alpha + \beta = 1$

When the labor-leisure choice is exogenous ( $\alpha + \beta = 1$ ), the steady tax rate coincides with the growth-maximizing tax rate. To see this, set  $\alpha + \beta = 1$  in (32) and (28). The marginal-benefit and marginal-cost schedules are identical, and given by

$$\frac{1 - a}{a\tau_w} = \frac{(1 - a)^{\frac{1}{a}}}{(1 - a)^{\frac{1}{a}}(1 - \tau_w) + a}, \tag{33}$$

which yields a closed-form solution for the constant growth-maximizing tax rate:

$$\tau_w^g = \frac{(1 - a)^{\frac{1}{a}} + a}{(1 - a)^{\frac{1-a}{a}}}. \tag{34}$$

In the steady state, the equilibrium tax rate under majority voting equals the growth-maximizing tax rate.

## 4. The Model with a General Income Tax

We now consider the case of a general flat income tax ( $\tau_k = \tau_w = \tau_y$ ). We assume that the public infrastructure input,  $G$ , is financed by a constant general income tax,  $\tau_y \in [0, 1]$ , as in Barro (1990) and Ghate (2005).<sup>25</sup> The government budget constraint is balanced in each period, and given by

$$G_t = \tau_{yt} Y_t. \tag{35}$$

The setup of the rest of the model follows Ghate (2005) and the two cases considered above.<sup>26</sup> It is easily shown that the preferred constant steady-state

<sup>25</sup> In Ghate (2005), the labor-leisure choice is exogenous. Here it is endogenous.

<sup>26</sup> The factor rewards are computed in the standard way and are given by  $r_t = \tau_{yt}^{(1-a)/a} H^{(1-a)/a}$  and  $w_t = \frac{1-a}{a} K_t \tau_{yt}^{(1-a)/a} H^{(1-2a)/a}$ , where  $r$  denotes the competitive re-

tax rate for each household is given by<sup>27</sup>

$$\tau_{hy} = 1 - a \quad \forall h. \tag{36}$$

This is also the median household’s preferred tax rate. Likewise, the constant growth-maximizing tax rate is given by  $\tau_y^g = 1 - a$ .<sup>28</sup> There is AK growth in the steady state. This generalizes the results of Gbate (2005) to the case of endogenous labor supply. It also extends the well-known Barro (1990) result that a growth-maximizing policy is always welfare-maximizing with identical individuals. In the current framework, because the marginal costs and benefits of higher taxes are exactly proportional (unlike the case of capital and labor income taxation), every household’s preferred tax rate coincides with the growth-maximizing policy even though agents value leisure.

### 5. Conclusion

To summarize, the contribution that this paper makes is to examine the implications of an endogenous labor–leisure choice and factor income taxation on the political and economic equilibrium in a heterogeneous-agent Barro (1990) endogenous-growth framework. We allow for voting on a tax on capital income, a tax on labor income, and a general income tax, with voting always on single issues.

This paper makes two contributions. We show that complete factor-holding convergence occurs in the steady state. Hence, wealth dynamics are independent of the underlying factor-specific tax system in the steady state. These results are consistent with Das–Gbate and Gbate (2005). While there is convergence in factor holdings, we show that the equilibrium tax rate diverges from the growth-maximizing tax rate when there is voting on capital and labor income taxes, and coincides with the growth-maximizing tax rate only when households vote on a general income tax. Importantly, the divergence between the equilibrium tax rate and the growth-maximizing tax rate depends crucially on how labor supply responds to capital income taxation, labor income taxation, and a general income tax.

Future work could allow for infrastructure funded by a nonlinear progressive tax system, with voting on the progressivity parameter. Also, in

turn to capital, while  $w$  denotes the return to labor. Both  $r$  and  $w$  are increasing in the tax rate, with the rental–wage ratio given by  $r_t/w_t = aH_t/(1 - a)K_t$ . Note that the rental–wage ratio here is identical to (2) and is independent of  $\tau_{yt}$ .

27 The indirect utility function,  $W_{ht}^y$ , is given by  $W_{ht}^y = \text{constant} + \log[1 + \frac{a}{1-a}(H_t/K_t)K_{ht}] + (\alpha + \beta) \log[w_t(1 - \tau_{hyt})]$ .

28 The equation for the gross growth factor is given by

$$g_{t+1} = \frac{K_{t+1}}{K_t} = \frac{\beta}{\alpha + \beta} \frac{1}{a} (1 - \tau_{yt}) \tau_{yt}^{\frac{1-a}{a}} H_t^{\frac{1-a}{a}}. \tag{37}$$

the current framework, the initial distribution of wealth does not matter for steady-state convergence. This is typical of the overlapping-generations setup used in this paper. One possible extension would be to allow for initial inequality to affect the transition path, with an initially more unequal economy taking longer to reach the steady state. Finally, we choose the current setup to keep the intertemporal wealth distribution tractable. Alternatively, we could allow agents to care about the utility of their children, as opposed to the level of capital that they bequeath. We leave this for future research.

## 6. Appendix

### 6.1. Proof of Propositions 1 and 2

Proof of Proposition 1. Log-differentiating (10) with respect to  $\tau_{kt}$  and rearranging yields the following first-order condition:

$$\frac{a}{(1-a) + a(1-\tau_{kt})} = \frac{1-a}{a\tau_{kt}} + \frac{(1-a)(1-\alpha-\beta)}{\delta(\tau_{kt})}. \tag{38}$$

Multiplying through by  $\delta(\tau_{kt})$  and simplifying yields

$$\underbrace{\frac{a\delta(\tau_{kt})}{1-a\tau_{kt}}}_{MC} = \underbrace{\frac{(1-a)[(1-a) + a(1-\alpha-\beta)]}{a\tau_{kt}}}_{MB}. \tag{39}$$

Substituting for  $\delta(\tau_{kt}) = (1-a) + a(1-\alpha-\beta)(1-\tau_{kt})$  above, this can be simplified to

$$\underbrace{(1-a)\{\tau_k - (1-a)\}}_{MC} = \underbrace{(1-\alpha-\beta)a[(1-a)(1-a\tau_k) - a(1-\tau) \tau_k]}_{MB}, \tag{40}$$

since equation (40) defines a constant tax rate  $\tau_k$ . Notice that changes in  $\alpha$  and  $\beta$  only lead to changes in the marginal-benefit schedule. Let  $\alpha + \beta < 1$ . To obtain figure 1, evaluating the LHS of (40) when  $\tau_k = \{0, 1\}$  implies  $LHS(0) = -(1-a)^2$  and  $LHS(1) = (2a-1)(1-a)$ , with the marginal-cost schedule increasing linearly in  $\tau$  and intersecting the  $x$ -axis at  $\tau_k = \frac{1-a}{a}$ . Evaluating the RHS of (40) when  $\tau_k = \{0, 1\}$  implies  $RHS(0) = (1-\alpha-\beta)a(1-a)$  and  $RHS(1) = (1-\alpha-\beta)a(1-a)^2$ , with the marginal-benefit schedule decreasing in  $\tau_k \forall \tau_k \in [0, 1]$ . The existence of a growth-maximizing tax rate occurs when  $LHS(1) > RHS(1)$ , or  $2a-1 > (1-\alpha-\beta)a(1-a)$ . Notice that when  $\tau_k = \frac{1-a}{a}$ , the marginal-benefit term is positive. Hence,  $\tau_k = \frac{1-a}{a}$  cannot be the growth-maximizing tax rate. Since the marginal benefit is falling, when  $\alpha + \beta < 1$  we have  $\tau_{ek}^g > \tau_{xk}^g$ . Note that in the current setup multiple equilibria will not arise. This is because the marginal-benefit schedule is monotonically

decreasing in the tax rate on capital income, while the marginal-cost schedule is monotonically increasing in the tax rate. The condition  $\alpha + \beta < 1$  in proposition 1 simply ensures that an equilibrium exists. ■

**Proof of Proposition 2.** Setting  $\eta_{ht+1} = \eta_{ht} = \eta_h$  in (13) implies that

$$\frac{H_{ht}}{H_t} = \eta_h \quad \forall h. \tag{41}$$

Dividing equation (7) by the expression for  $H_t$  in (8) and simplifying yields

$$\frac{H_{ht}}{H_t} = \frac{\delta(\tau_{kt})}{N(1-a)} - \frac{a(1-\alpha-\beta)}{1-a} \frac{K_{ht}}{K_t} (1-\tau_{kt}). \tag{42}$$

Since equation (13) implies that

$$\frac{H_{ht}/H_t}{\eta_{ht}} = 1 \tag{43}$$

in the steady state, dividing both sides of (42) by  $K_{ht}/K_t$ , setting  $(H_{ht}/H_t)/\eta_{ht} = 1$ , and simplifying yields the result. ■

Next, we derive the first-order condition of the  $h$ th household in equation (16). The  $h$ th agent’s indirect utility function is given by

$$V_{hkt} = \underbrace{\text{constant} + \log \left\{ 1 + aN(\alpha + \beta) \frac{1 - \tau_{hkt}}{\delta(\tau_{hkt})} \eta_{hkt} \right\}}_{\text{Term I}} + \underbrace{(\alpha + \beta) \log(w_t)}_{\text{Term II}}. \tag{44}$$

Evaluating term I and simplifying yields

$$\frac{\partial \text{Term I}}{\partial \tau_{hkt}} = \frac{-aN(\alpha + \beta)(1-a)\eta_{ht}}{\{a[(1-\alpha-\beta) + (\alpha + \beta)N\eta_{ht}](1-\tau_{hkt})\}} \frac{1}{\delta(\tau_{hkt})}. \tag{45}$$

Evaluating term II and simplifying yields

$$\frac{\partial \text{Term II}}{\partial \tau_{hkt}} = \frac{\xi'(\tau_{hkt})}{\xi(\tau_{hkt})} + \frac{1-2a}{a} \frac{H'(\tau_{hkt})}{H(\tau_{hkt})}. \tag{46}$$

Note that

$$\frac{\xi'(\tau_{hkt})}{\xi(\tau_{hkt})} = \frac{1-a}{a\tau_{hkt}}, \quad \text{while} \quad \frac{H'(\tau_{hkt})}{H(\tau_{hkt})} = \frac{a(1-\alpha-\beta)}{\delta(\tau_{hkt})}.$$

Substituting these expressions back and rearranging terms yields (16).

### 6.2. Dynamics of Wealth Inequality under a Labor Income Tax

To obtain equation (29), we substitute out the factor prices from the household capital accumulation equation. This yields

$$K_{ht+1} = \frac{\beta}{\alpha + \beta} \left\{ \frac{\varphi(\tau_{wt})H_t^{\frac{1-2a}{a}}(1-\tau_{wt})H_{ht}}{\eta_{ht}} + \nu(\tau_{hwt})H_t^{\frac{1-a}{a}} \right\}. \tag{47}$$

Dividing the above expression by equation (25) and simplifying yields equation (29), from which it follows that  $\eta_h = H_h/H = 1/N$ . To obtain (30), substitute out the expression for  $C_{ht}$  in (3) with the optimal consumption equation and equation (24). Note that  $1 - H_{ht} = K_{ht+1}(1 - \alpha - \beta)/\beta w_t(1 - \tau_{wt})$ , from (24). The indirect utility function is then given by

$$W_{ht} = \text{constant} + \log K_{ht+1} - (1 - \alpha - \beta)w_t - (1 - \alpha - \beta) \log(1 - \tau_{hwt}). \quad (48)$$

Substituting out

$$\frac{K_{ht+1}(1 - \alpha - \beta)}{\beta w_t(1 - \tau_{wt})} = \frac{\beta}{\alpha + \beta} \left\{ H_{ht} + \frac{r_t}{w_t(1 - \tau_{wt})} K_{ht} \right\}$$

in the above expression for  $W_{ht}$ , and then noting that from (23) and (24) we have

$$H_{ht} + \frac{a}{(1 - a)^{\frac{1}{a}}} \frac{H_t}{K_t(1 - \tau_{wt})} = \frac{(\alpha + \beta)aH_tK_{ht}}{(1 - a)^{\frac{1}{a}}K_t(1 - \tau_{wt})},$$

the indirect utility function of agents, given by equation (30), yields

$$W_{ht} = \text{constant} + \underbrace{\log \left\{ 1 + \frac{a\eta_{ht}N(\alpha + \beta)(1 - a)^{\frac{1}{a}}}{(1 - a)^{\frac{1}{a}}\epsilon(\tau_{hwt})} \right\}}_{\text{Term I}} + \underbrace{(\alpha + \beta) \log[w_t(1 - \tau_{hwt})]}_{\text{Term II}}.$$

We now derive (32).<sup>29</sup> Differentiating term I with respect to  $\tau_{hwt}$  and simplifying yields

$$\frac{\partial \text{Term I}}{\partial \tau_{hwt}} = \frac{\frac{a\eta_{ht}N(\alpha + \beta)(1 - a)^{\frac{1}{a}}}{\epsilon(\tau_{hwt})}}{\epsilon(\tau_{hwt}) + a\eta_{ht}N(\alpha + \beta)}. \quad (49)$$

Differentiating term II and simplifying yields

$$\frac{\partial \text{Term II}}{\partial \tau_{hwt}} = (\alpha + \beta) \left[ \frac{\varphi'(\tau_{hwt})}{\varphi(\tau_{hwt})} + \frac{1 - 2a}{a} \frac{H'(\tau_{hwt})}{H(\tau_{hwt})} - \frac{1}{1 - \tau_{hwt}} \right]. \quad (50)$$

Combining equations (49) and (50), setting the resulting expression equal to zero, and simplifying yields equation (31). Finally, note that

$$\frac{\varphi'(\tau_{hwt})}{\varphi(\tau_{hwt})} = \frac{1 - a}{a\tau_{hwt}} \quad \text{and} \quad \frac{H'(\tau_{hwt})}{H(\tau_{hwt})} = -\frac{(1 - \alpha - \beta)a}{\epsilon(\tau_{hwt})(1 - \tau_{hwt})}.$$

<sup>29</sup> It can be verified that single-peakedness holds, and therefore the first-order condition of the indirect utility function with respect to the tax rate is sufficient to determine the optimal tax rate under a majority-rule equilibrium (setting  $h = m$ ). Technical details are available from the author on request.

Substituting these expressions as well the steady-state equilibrium factor holdings ( $\eta_h = \frac{1}{N}$ ) into (31) yields the optimal tax rate for households in the steady state, determined by equation (32).

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