Employment targeting in a frictional labor market

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Abstract

Purpose – Governments in both developing and developed economies play an active role in labor markets in the form of providing both formal public sector jobs and employment through public workfare programs. The authors refer to this as employment targeting. The purpose of the paper is to consider different labor market effects of employment targeting in a stylized model of a developing economy. In the context of a simple search and matching friction model, the authors show that the propensity for the public sector to target more employment can increase the unemployment rate in the economy and lead to an increase in the size of the informal sector.

Design/methodology/approach – The model is an application of a search and matching model of labor market frictions, where agents have heterogeneous abilities. The authors introduce a public sector alongside the private sector in the economy. Wage in the private sector is determined through Nash bargaining, whereas the public sector wage is exogenously fixed. In this setup, the public sector hiring rate influences private sector job creation and hence the overall employment rate of the economy. As an extension, the authors model the informal sector coupled with the other two sectors. This resembles developing economies. Then, the authors check the overall labor market effects of employment targeting through public sector intervention.

Findings – In the context of a simple search and matching friction model with heterogeneous agents, the authors show that the propensity for the public sector to target more employment can increase the unemployment rate in the economy and lead to an increase in the size of the informal sector. Employment targeting can, therefore, have perverse effects on labor market outcomes. The authors also find that it is possible that the private sector wage falls as a result of an increase in the public sector hiring rate, which leads to more job creation in the private sector.

Originality/value – What is less understood in the literature is the impact of employment targeting on the size of the informal sector in developing economies. The authors fill this gap and show that public sector intervention can have perverse effects on overall job creation and the size of the informal sector. Moreover, a decrease in the private sector wage due to a rise in public sector hiring reverses the consensus findings in the search and matching literature which show that an increase in public sector employment disincentivizes private sector vacancy postings.

Keywords Fiscal policy, Informal sector, Employment targeting, Search and matching frictions, Labor markets, Indian economy

Paper type Research paper

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1. Introduction
Governments in both developed and developing economies play an active role in labor markets to meet their growth and development objectives. In the case of India, the twin phenomena of jobless growth and the growing casualization of the workforce have led to a vibrant debate about the role of government policy in stimulating employment (Kapoor, 2017; Abraham, 2017). One particular intervention takes the form of the public sector being the provider of jobs. We refer to this as employment targeting. For instance, public workfare programs are among the most common forms of anti-poverty programs in developing countries. In Latin America, Africa and Asia, employment guarantee schemes have been at the center of an employment-oriented approach to anti-poverty policy-making (Basu et al., 2009). In India, the National Rural Employment Guarantee Scheme (NREGS), the flagship workfare government scheme employs several million people as a means to alleviate seasonal distress in employment. In the USA, the Works Projects Administration, started in 1935, was initiated in response to the Great Depression and hired unemployed workers directly.

In each of these cases, however, the general equilibrium effects of policies that target employment remains a key research question. More specifically, do employment guarantee schemes crowd out private sector employment by leading to an increase in wages? Muralidharan et al. (2018) show that a public employment guarantee, by improving the outside option for workers, puts upward pressure on labor markets and this drives up wages. Basu et al. (2009) develop a formal model of an employment guarantee scheme and show that such schemes introduce contestability in labor hiring and raise the reservation wage. Gomes (2015) characterizes a government’s acyclical wage policy that protects workers from business cycle fluctuations. He argues that very high public sector wages can create disincentives to private players for posting vacancies and can reduce overall employment. In this context, he proposes an optimum level of the public sector wage that maximizes welfare.

We build a simple model of a developing country labor market characterized by search and matching frictions and an informal sector. In the baseline model, with no informal sector, we show that publicly provided employment, depending on whether it increases or reduces private sector wages, either reduces or fosters formal employment, respectively. Our results are more general than the standard findings in the search and matching literature – typified by papers such as Gomes (2015) – which show that an increase in public sector employment disincentivizes private sector vacancy postings. Intuitively, in our model, a fall in the private sector wage means that the return to a vacancy posted by the firm is higher. This leads to more firms entering the market, which increases the number of vacancies created and, subsequently, a rise in the probability of a successful match, and hence unemployment falls.

Conversely, if a rise in the public sector hiring rate leads to a rise in the private sector wage, then unemployment can rise. This happens because a rise in the private sector wage makes the return to a vacancy posted by a firm fall. This makes firms leave the market, which leads to a fall in the number of vacancies and a fall in the probability of a successful match. If the decline in the probability of a successful match is larger than the increase in the public sector hiring rate, unemployment (in the net) rises. Hence, publicly funded employment programs may “crowd-out” private sector formal employment. This case may have some empirical support because in the context of NREGS, many authors show that the wage rate went up in rural India after the introduction of the scheme (Berg et al., 2012; Gulati et al., 2014; Muralidharan et al., 2018).
One aspect of publicly provided employment is the impact of such targeting on the size of the informal sector in developing economies. Although there are a handful of papers that use a search and matching framework to study informal labor markets (Albrecht et al., 2009; Castillo et al., 2010; Maarek, 2012; Charlot et al., 2013), none of these papers focus on the effects of employment targeting on the dynamics of informal sector employment. We assume that labor is divided into two categories: informal and formal. As before, within the formal sector, there is a public sector and a private sector, although the private sector is assumed to operate in both the informal and formal sectors. If they operate in the informal sector, they pay a training cost once they are matched with a worker. After receiving the training, the productivity of all matched workers is the same. In this environment, we show that public sector intervention in the labor market can lead to an increase in the size of the informal sector. Because the informal sector is characterized by a high firing rate and lower unemployment benefits, employment targeting can have a perverse effect on labor market outcomes.

2. The model
The economy is composed of three infinitely lived agents: firms, agents or workers and the government. Heterogeneous individuals are uniformly distributed according to their abilities. Each individual’s ability is indexed as $i \in (0, 1)$, where 0 is the lowest ability and 1 is the highest ability. Because agents do not have any other distinguishing feature, they are indexed as $i$. Firms present in the economy produce a single final good that is consumed by agents. We call a private firm’s production unit as the “private sector”, denoted by $P$. The government’s production unit is termed as the “public sector”, denoted by $G$. Unemployed agents are denoted by $U$. Agents are risk-neutral and their utility comes only from consuming the final good.

Each agent has one unit of labor endowment, which he supplies inelastically in each point of time. However, the labor market is characterized by frictions. Private sector firms and agents face search and matching frictions before commencing production activity. Unemployed agents search for jobs irrespective of their abilities and can search for both private sector and public sector jobs. Vacant firms looking for workers post a vacancy by paying a vacancy posting cost, $d > 0$. Private sector firms and job seekers are matched according to a Pissarides-style matching function: $m = m(u, v)$, where $u$ is the number of unemployed, and $v$ is the number of vacant firms (Pissarides, 2000). The function, $m$, is homogeneous of degree one, concave and increasing in each of its arguments. Hence, $m/u = m(1, \theta)$, (where $\theta = v/u$) denotes the job finding rate, while $m/v = m(\theta^{-1}, 1)$ is the vacancy matching rate[1]. Production starts in the private sector once a firm and a worker are matched. Production follows a constant returns to scale technology in the economy; i.e. the $i^{th}$ ability agent produces $i$ units of output. Firms get to know about their workers’ ability once they are matched.

Unemployed agents get an amount, $b > 0$, which is an unemployment benefit from the government[2]. Workers who are employed in the private sector get a per period wage, $w$, according to their ability. The firing rate in the private sector is given by $\lambda > 0$. The rate at which an unemployed agent finds a public sector job is given by $\gamma > 0$. The parameter $\gamma$ can be considered as the hiring rate of the public sector, or a policy parameter for the government, although it is also a rate for matching agents with the public sector. Note that $(m(1, \theta))$ is endogenously determined in equilibrium. We assume that the government pays a fixed wage to its employees, $\bar{w}$, irrespective of their ability. The firing rate in the public sector is given by $\lambda$. Therefore, in a small time span, $\Delta t$, an unemployed agent can get a public sector job with a probability, $\gamma \cdot \Delta t$, while a public sector worker can be fired with
the probability, \( \tilde{\lambda} \). Similarly, a private sector job match can break with probability, \( \lambda \Delta t \), within the period, \( \Delta t \). \( r \) is the discount rate in the economy. Finally, we assume that a job seeker cannot get a net surplus from a public sector job and a private sector job simultaneously. All the public/private job creation and job destruction rates follow a Poisson process as in Pissarides (2000).

We formalize the public sector’s employment policy by the policy-tuple \( \{\bar{w}, b, \gamma\} \) and call this the employment targeting policy of the government. Our main focus in this paper, however, is on the parameter, \( \gamma \), the public sector hiring rate, and its effect on unemployment and informalization. In this paper, we focus just on characterizing the steady state.

2.1 Steady state

Let \( V^i_j \) denote the infinite income stream of the \( i^{th} \) worker, where the state \( j = P, G, U \). This implies that:

\[
rv^p_i = w_i + \lambda \left( V^i_U - V^i_P \right)
\]  

This implies that the flow value of a private sector job (or a filled vacancy), \( rV^p_i \), equals the wage from the private sector job (\( w_i \)) plus the expected net surplus from being unemployed if the private sector job is destroyed \( \left( \lambda \left( V^i_U - V^i_P \right) \right) \). Analogously, the flow value of being employed in the public sector is given by:

\[
rv^g_i = \bar{w} + \tilde{\lambda} \left( V^i_U - V^i_G \right)
\]  

And lends it to a similar interpretation to equation (1), except now, the wage in the public sector is given by \( \bar{w} \), with the job destruction rate in the public sector given by \( \tilde{\lambda} \). The flow value of being unemployed is given by:

\[
rv^u_i = b + m(1, \theta) \left( V^i_P - V^i_U \right) + \gamma \left( V^i_G - V^i_U \right)
\]  

which equates the flow value of being unemployed, \( rV^u_i \), to the level of the unemployment benefit, \( b \), plus the net surplus from finding a job in either the private sector or the public sector. Because workers cannot work in both sectors simultaneously, there is no net surplus associated with joint employment in both sectors[3].

Subtracting equation (3) from equation (1) yields:

\[
(r + \lambda + m(1, \theta)) \left( V^i_P - V^i_U \right) = w_i - \gamma \left( V^i_G - V^i_U \right) - b
\]  

Likewise, subtracting equation (3) from equation (2), and solving for \( V^i_G - V^i_U \) yields:

\[
V^i_G - V^i_U = \frac{1}{r + \tilde{\lambda} + [\bar{w} - b - m(1, \theta)]} \left( V^i_P - V^i_U \right)
\]  

Equation (5) gives the net surplus of being employed in the public sector relative to the net surplus of being employed in the private sector. Likewise, substituting equation (5) into equation (4) and manipulating terms yields:
Equation (6) expresses the net return of a productive matching to a worker. After a productive matching, workers receive $V^i_p$, but at the cost of sacrificing $V^i_U$.

We denote the value functions of infinitely lived private firms as $J^i_P$ and $J^i_V$, where $P$ stands for a productive matching and $V$ stands for a vacancy, respectively. The flow value of a productively matched private firm is given by:

$$rf^i_P = (i - w_i) + \lambda (J^i_V - J^i_P)$$

And for a firm with a vacancy,

$$rf^i_V = -d + m(\theta^{-1}, 1) \left( E\left(J^i_P\right) - J^i_V \right)$$

The first term on the right-hand side (RHS) of equation (7), $(i - w_i)$, is the per period return to the firm from a successful match. The second term, $\lambda (J^i_V - J^i_P)$, is the expected surplus to the firm from moving from a state of being matched to being vacant. Similarly in equation (8), when a firm has a vacancy, it pays a vacancy cost, $d$. The second term, $(E(J^i_P) - J^i_V)$, corresponds to the expected surplus accruing to the firm from a state of having a vacancy to being matched. Crucially, equation (8) contains the term $E(J^i_P)$. A vacant firm ex ante does not know a worker’s ability prior to a successful match and therefore does not know the exact return before the firm gets matched with a worker. Instead, vacant firms use the information about expected returns from a filled job, $E(J^i_P)$, to take a vacancy posting decision.

In equilibrium, firms enter and exit freely in the market such that:

$$J^i_V = 0$$

Equation (8) therefore implies that:

$$E\left(J^i_P\right) = \frac{d}{m(\theta^{-1}, 1)}$$

Likewise, substituting $J^i_V = 0$ into equation (7) and solving for $J^i_P$, yields:

$$J^i_P = \frac{i - w_i}{\lambda + r}$$

Which is increasing in the ability of the $i^{th}$ worker. Notice that for a private sector firm, the net return from a productive matching is given by $(J^i_P - J^i_V)$. 

Equation (6)
2.2 Wage bargaining

The Nash bargaining solution is the \( w_i \) that satisfies:

\[
\begin{align*}
  w_i &= \arg\max_{w_i} \left( V_p - V_U^i \right)^\beta \left( f_p^i - f_U^i \right)^{1-\beta} 
\end{align*}
\]

(12)

where \( \beta \in (0, 1) \) represents the worker’s bargaining power. It is imperative to understand the effect of heterogeneous agents in the bargaining process. Because each individual has a unique ability, his corresponding wage is also unique. This has an important implication in wage bargaining. If the workers were homogeneous, then one individual could not affect the wage rate that is available outside one’s particular job match, because there would be a large number of similar agents participating in the labor market. One agent would be too small to affect the rest of the market. However, in the present setup with heterogeneous ability, this argument does not hold. A matched worker knows that, ceteris paribus, any wage decision in a particular matching is going to replicate in all possible productive matchings because each agent is unique in their ability, \( i \). In other words, a change in \( w_i \) also changes the agent’s outside option, \( V_U^i \). This implies that \( \frac{\partial V_U^i}{\partial w_i} \neq 0 \).

The first-order maximization condition is given by:

\[
\beta \left[ \frac{\partial V_P^i}{\partial w_i} - \frac{\partial V_U^i}{\partial w_i} \right] J_p^i + (1 - \beta) \left[ V_P^i - V_U^i \right] \frac{\partial f_p^i}{\partial w_i} = 0.
\]

(13)

To obtain an expression for \( \frac{\partial V_P^i}{\partial w_i} - \frac{\partial V_U^i}{\partial w_i} \), we differentiate equation (6) to get:

\[
\frac{\partial V_P^i}{\partial w_i} - \frac{\partial V_U^i}{\partial w_i} = \frac{r + \bar{\lambda} + \gamma}{(r + \lambda)(r + \bar{\lambda} + \gamma) + m(1, \theta)(r + \bar{\lambda})}
\]

(14)

Substituting equation (14) and \( \frac{\partial f_p^i}{\partial w_i} \) from equation (11) and putting these into equation (13), we obtain an expression for \( w_i \):

\[
w_i = \left[ i \beta + b(1 - \beta) \right] + \frac{(\bar{w} - b)(1 - \beta)}{m(1, \theta)} \left[ \lambda - \gamma + \frac{\gamma m(1, \theta)}{r + \bar{\lambda} + \gamma} \right]
\]

(15)

Note that \( w_i \) is increasing in the ability of the \( i^{th} \) worker, although as our focus is on employment targeting, we would like to know how an increase in \( \gamma \), the hiring rate of the public sector, affects the optimal wage. To see this, recall equation (13). Using equations (1), (7) and (9), we can rewrite equation (13) as:

\[
(1 - \beta) \left[ V_P^i - V_U^i \right] = \beta \left( 1 - r \frac{\partial V_U^i}{\partial w_i} \right) \left( \frac{i - w_i}{\lambda + r} \right)
\]

\[
w_i - r V_U^i = \frac{\beta}{1 - \beta} \left( i - w_i \right) \left( 1 - r \frac{\partial V_U^i}{\partial w_i} \right)
\]
Using equations (1), (2) and (3), it is easy to show that
$$r \frac{\partial V_i}{\partial w_i} = \frac{m(1, \theta)}{1 + m(1, \theta) + \gamma}.$$ Using this, and after a few algebraic manipulations, we obtain:

$$w_i = r V_i + \left( i - r V_i \right) \left[ \frac{\beta \frac{1+\gamma}{1+\gamma+m(1, \theta)}}{(1 - \beta) + \beta \frac{1+\gamma}{1+\gamma+m(1, \theta)}} \right]$$  \hspace{1cm} (16)

The first term on the RHS, $r V_i$, is the minimum compensation a worker requires to give up the search (Pissarides, 2000). On top of this, the worker requires a fraction of the rent, or net surplus, that a productive match generates. It can be shown that if $\gamma$ increases, then both $r V_i$ (because a public sector job serves as an outside option for a private sector worker) and the square bracketed term on the RHS increase. However, due to an increase in $r V_i$, the term, $i - r V_i$, is falling, or the surplus itself is less. Because the proportionate share of the surplus accruing to the worker is more (because of the monopoly power of the $i^{th}$ worker), the effect of the fall in net surplus pulls the wage down and gets amplified. This means that an increase in $\gamma$ creates an ambiguous effect on the wage.

2.3 Equilibrium
Recall that agents are distributed uniformly over the interval $[0,1]$. Therefore, from equation (11), we have:

$$E(J_i) = \int_0^1 f_i di = \int_0^1 \frac{i - w_i}{\lambda + r} di.$$  \hspace{1cm} (17)

Substitute out for $w_i$ in equation (17) using equation (15). Solving the integration makes equation (17) free of $i$ and $w_i$. The only remaining endogenous variable in equation (17) is $\theta$. Hence:

$$E(J_i) = \frac{1}{2(\lambda + r)} - \frac{1}{(\lambda + r)} \left[ b(1 - \beta) + \frac{\beta}{2} \right] - \frac{(\overline{w} - b)(1 - \beta)}{m(1, \theta)} \left[ \lambda - \gamma + \frac{\gamma m(1, \theta)}{r + \lambda + \gamma} \right]$$  \hspace{1cm} (18)

Equating equations (10) and (18) implies:

$$\frac{d}{m(\theta^{-1}, 1)} + \frac{(\overline{w} - b)(1 - \beta)}{m(1, \theta)(\lambda + r)} (\lambda - \gamma) = \frac{1}{2(\lambda + r)} - \frac{1}{(\lambda + r)} \left[ b(1 - \beta) + \frac{\beta}{2} \right]$$

$$- \frac{(\overline{w} - b)(1 - \beta) \gamma}{r + \lambda + \gamma}$$  \hspace{1cm} (19)

Which implicitly solves for the value of $\theta^*$. 
Steady state unemployment happens when the flow out of unemployment equals the flow into unemployment, i.e., \( u[m(1, \theta^*) + \gamma] = (1 - u)(\lambda + \bar{\lambda}) \). This implies the steady state unemployment rate, \( u^* \), is given by:

\[
u^* = \frac{(\lambda + \bar{\lambda})}{m(1, \theta^*) + \gamma + \lambda + \bar{\lambda}}
\]

Notice, \textit{ceteris paribus}, as the firing rates (\( \lambda \) and/or \( \bar{\lambda} \)) increase, \( u^* \) increases. Also, as the public sector hiring rate (\( \gamma \)) increases, \( u^* \) falls. However, as \( u^* \) is an endogenous variable, when the government changes \( \gamma \) (which is a policy parameter to the government), \( \theta^* \) also changes, and this provides an additional channel through which \( u^* \) gets affected. This requires a comparative statics exercise to obtain the impact of employment targeting on the overall level of unemployment.

2.4 Comparative statics
To obtain this, we totally differentiate both sides of equation (19) with respect to \( \gamma \) to obtain:

\[
\frac{d\theta^*}{d\gamma} = \left[ \frac{(\bar{\omega} - b)(1 - \beta)}{(r + \bar{\lambda} + \gamma)^2} \right] \left[ \frac{m(1, \theta^*)}{(d - \varepsilon_m(1, \theta^*)) - (\bar{\omega} - b)(1 - \beta)\left[1 + (\lambda - \gamma)\frac{\varepsilon_m(1, \theta^*)}{\theta^*}\right]} \right]
\]

where \( \varepsilon_m(1, \theta^*) \) is the elasticity of the matching function with respect to \( \theta^* \), i.e. \( \frac{\partial m(1, \theta^*)}{\partial \theta^*} \). The condition for \( \frac{d\theta^*}{d\gamma} > 0 \) is given by:

\[
\frac{\theta^*}{\lambda - \gamma} > \varepsilon_m(1, \theta^*) \left[ \frac{d - \varepsilon_m(1, \theta^*)}{(\bar{\omega} - b)(1 - \beta) - 1} \right]^{-1}
\]

We can interpret the above condition more precisely if we consider the class of matching functions with constant elasticity[4]. In this case, the RHS of equation (22) will be a constant in terms of \( d, \bar{\omega}, b, \beta \) and \( \varepsilon_m \), which we denote by \( \kappa \). Equation (22) can be written as:

\[
\theta^* + \kappa(\gamma - \lambda) > 0
\]

Figures 1 and 2 below show that if the equilibrium value of \( \theta^* \), which we denote by \( \theta^* \), lies to the RHS or above (respectively) of the line given in equation (23), then \( \frac{d\theta^*}{d\gamma} > 0 \). Conversely, if \( \theta^* \) lies to the left or below, then \( \frac{d\theta^*}{d\gamma} < 0 \). This leads to our first proposition.

\( P1. \) Consider a value \( \gamma \) such that the equilibrium value of \( \theta^*(=\frac{\theta}{\gamma}) \) lies above the straight line, \( \theta^* + \kappa(\gamma - \lambda) = 0 \). Employment targeting, or an increase in hiring by the public sector (increase in \( \gamma \)), increases \( \theta^* \), or reduces equilibrium unemployment, \( u^* \). If \( \theta^* \) lies below the straight line, then an increase in \( \gamma \) leads to a fall in \( \theta^* \), or an increase in equilibrium unemployment, \( u^* \), if \( \varepsilon_m(1, \theta^*) \) is sufficiently large.
The intuition behind \( P1 \) is as follows. Recall that the impact of \( \gamma \) on \( w_i \) is ambiguous. Suppose a rise in \( \gamma \) increases \( w_i \), then the return from a vacant post for a firm falls. Hence, firms start leaving the market and the number of vacancies, \( v \), falls, as in equilibrium, \( J_v = 0 \). This leads to a fall in \( m(1, \theta) \). If the fall in \( m(1, \theta) \) is large enough to off-set the rise in \( \gamma \), then from equation (20), \( u^* \) can rise. On the other hand, if a rise in \( \gamma \) makes \( w_i \) fall, then the return from vacancies rise, and more firms enter the market and more vacancies are created. Both \( \gamma \) and \( m(1, \theta) \) increase, and \( u^* \) falls. Equation (23) is the sufficiency condition for the fall in \( u^* \).

There is an important corollary to \( P1 \), which relates to the case when \( \frac{w - b}{b} \rightarrow 0 \). In this case, the public sector wage is so low that it is close to the per-period unemployment benefit, \( b \). It is easily seen from equation (19) that the equation is independent of \( \gamma \). This implies that changes in \( \gamma \) have no impact on \( \theta \), or on the rate of getting a private sector job and a private sector wage. This implies that an increase in \( \gamma \) unambiguously reduces \( u^* \). Intuitively, \( \frac{w - b}{b} \) is the net surplus from working in the public sector relative to being unemployed. As the net surplus falls, the outside option (the public sector job) facing a worker in the bargaining process to determine his wage is negligible. This is true for a firm
too. So, the private sector offers more vacancies. There is more matching. And this leads to lower unemployment.

3. Informal sector
In this section we extend the baseline model above to include an informal sector. Our main goal is to derive conditions under which employment targeting by the public sector can lead to an increase in the size of the informal sector. We assume that labor is divided into two categories: formal and informal. In the formal sector, individual ability is uniformly distributed over \([i^*, 1]\), whereas in the informal sector, individual ability is distributed over \([0, i^*]\). The ability level \(i^*\) corresponds to the ability of the pivotal worker who is indifferent between working in the informal and formal sectors. We later determine \(i^*\) endogenously in equilibrium. Once the worker size of the formal and informal sectors is decided, unemployed workers cannot search in both sectors. As before, within the formal sector, there is a public sector and a private sector, and their characterization remains the same.

Private sector firms operate in both the informal and formal sectors (e.g. textiles or leather goods). If they operate in the informal sector, they pay a training cost, \(c\), once they are matched with a worker. After receiving the training, the productivity of all matched workers (in the informal sector) becomes the same, and workers get a wage corresponding to their new productivity[6]. Hence, the heterogeneity in ability of the worker is not reflected in the wage that they receive in the informal sector. We assume that the firing rate is higher in the informal sector than in the formal sector. For simplicity, we assume that the firing rate of the informal sector is 1 (Charlot and Decreuse, 2005). Therefore, each episode of wage bargaining in the informal sector corresponds to a fresh matching, and the wage is affected by the training cost in each period. Firms post vacancies unless the returns to posting vacancies become zero. When the returns from posting a vacancy become zero, there is no incentive for firms to enter into the market. In the informal sector, firms and job seekers match through the typical matching function used in the previous section.

3.1 Labor market in the informal sector
Let \(V_U^l\) denote the value function corresponding to the infinite income stream of an unemployed worker in the informal sector \((l)\). The value function does not include the subscript \(i\) that corresponds to individual ability; as mentioned before, workers get a homogenous return. Similarly, \(V_E^l\) is the value function corresponding to the infinite income stream of an employed worker in the informal sector. The flow values are given by:

\[
rV_U^l = b_l + m(1, \theta_l)\left(V_E^l - V_U^l\right)
\]  
(24)

And as the rate of job break is 1 in the informal sector, this implies:

\[
rV_E^l = w_l + \left(V_U^l - V_E^l\right)
\]  
(25)

where \(\theta_l\) is the market tightness in the informal sector, and \(w_l\) is the wage rate in the informal sector. Equations (24) and (25) show the annuity value of being unemployed and employed in the informal sector, respectively. These are equated to the sum of the per period return (\(b_l\) in case of being unemployed, \(w_l\) in case of being employed) and the per period expected surplus obtained when moving from the current state (either being unemployed, \(V_U^l\), or being employed, \(V_E^l\)).
Let $J^I_E$ be the value function of matched firm, while $J^V_I$ denotes the value function of a vacant firm in the informal sector, i.e.:

$$rJ^I_E = (p - w_I - c) + (J^V_I - J^I_E)$$ (26)

And:

$$rJ^V_I = -d + m \left( \theta^{-1}_I, 1 \right) \left( J^I_E - J^V_I \right)$$ (27)

where $p > 0$ is the constant productivity from a productive matching in the informal sector. After a productive matching, firms pay the wage, $w_I$, and the training cost, $c$.

As before, in equilibrium, $J^V_I = 0$ due to the free entry condition. The wage in the informal sector, like the private sector wage, is determined by Nash bargaining. However, the difference relative to the previous section is that in case of the informal sector, an individual’s differential ability is not reflected in their productivity. Hence, the wage in the informal sector is the same for all workers. For the same reason, in this bargaining problem, the assumption that one individual worker’s decision cannot change the outside option is a valid one[7].

3.2 Wage bargaining

The Nash bargaining solution is the $w_I$ that satisfies:

$$w_I = \arg\max_{w_I} \left( V^I_E - V^I_U \right)^\beta \left( J^I_E - J^V_I \right)^{1-\beta}.$$ (28)

The maximization exercise yields:

$$\left( V^I_E - V^I_U \right) = \beta \left( V^I_E - V^I_U + J^I_E \right)$$ (29)

Which implies:

$$w_I - rV^I_U = \beta (p - c) - \beta rV^I_U$$

Or:

$$w_I = \beta (p - c) + (1 - \beta) rV^I_U.$$ (30)

Equation (29) can be also be written as:

$$V^I_E - V^I_U = \frac{\beta}{1 - \beta} J^I_E$$ (31)

Substituting $(V^I_E - V^I_U)$ in equation (31) into equation (24), we obtain:

$$rV^I_U = b_I + m(1, \theta_I) \frac{\beta}{1 - \beta} J^I_E.$$ (32)
As the free entry condition requires that \( J_I^V = 0 \), from equation (27), we obtain:
\[
J_I^E = \frac{d}{m(\theta_I^{-1}, 1)}
\]  
(33)

Substituting the value of \( J_I^E \) from equation (33) into equation (32) yields:
\[
rV_I^U = b_I + \frac{\beta}{1 - \beta} \theta_I d
\]  
(34)

Putting this back into equation (30) yields:
\[
w_I = (1 - \beta)b_I + \beta(p - c + \theta_I d)
\]  
(35)

Hence, the optimal wage in the informal sector is a positive function of labor market tightness in the informal sector, \( \theta_I \). What is noteworthy is that for a given \( \theta_I \), a rise in the training cost leads to a fall in the informal sector wage. This is because a rise in training costs reduces the surplus accruing to the informal sector firm, which responds by reducing its wage rate.

From equation (26), setting \( J_I^V = 0 \) implies \( J_I^E = \frac{(p - w_I - c)}{1 + r} \). Setting this equal to the value of \( J_I^E \) in equation (33) implies:
\[
\frac{(p - w_I - c)}{1 + r} = \frac{d}{m(\theta_I^{-1}, 1)}
\]  
(36)

Equation (36) depicts a negative relationship between \( \theta_I \) and \( w_I \). On the other hand, equation (35) depicts a positive relationship between \( \theta_I \) and \( w_I \). Figure 3 below depicts the two equations. Their intersection yields the equilibrium values of \( w_I \) and \( \theta_I \). An interesting implication is that as the training costs facing informal sector firms increase, as shown in Figure 4, both curves shift. In particular, equation (36) shifts down/out, while equation (35) shifts in. Hence, both \( w_I^* \) and \( \theta_I^* \) fall. Intuitively, as \( c \) increases, effective output from a productive matching, \( p - c \), falls in the informal sector. Because both firms and workers share their returns from the surplus, \( p - c \), both their returns fall. Hence, facing \( J_I^V < 0 \), firms exit

![Figure 3. Solution of \( \theta_I^* \) and \( w_I^* \)](image)
the market, to ensure that $f_Y = 0$ in equilibrium. As a result, both $\theta_I^*$ and $w_I^*$ decrease. If the training cost becomes smaller, then both $w_I$ and $\theta_I$ go up. This makes the informal sector more lucrative to both workers and firms.

3.3 The formal sector

Individuals from $[i^*, 1]$ work in the formal sector. As mentioned in the previous section, the wage in the formal sector is an increasing function of an individual’s ability (see equation (16)). Because the return from the informal sector is independent of the ability of the worker (i.e. fixed), an individual with higher ability is incentivized to work harder in the formal sector. In essence, the formal sector here is not different from the previous section, apart from the fact that the formal sector corresponds to individuals with ability distributed over $[i^*, 1]$. As a result, equation (17) becomes:

$$E\left(f_p\right) = \int_{i^*}^{1} \frac{f_p}{1-i^*} \, di = \int_{i^*}^{1} \frac{i-w_i}{(\lambda+r)(1-i^*)} \, di$$

Recall that the expression for $w_i$ in the formal sector is given by equation (15). We proceed in steps. First:

$$\int_{i^*}^{1} w_i \, di = \frac{\beta}{2} (1-i^*) + (1-i^*) \left[ b(1-\beta) + \frac{(\bar{w} - b)(1-\beta)}{m(1, \theta)} \left[ \lambda - \gamma + \frac{\gamma m(1, \theta)}{r+\lambda+\gamma} \right] \right]$$

Therefore:

$$\int_{i^*}^{1} (i-w_i) \, di = \frac{1-\beta}{2} (1-i^*)^2 - (1-i^*) \left[ b(1-\beta) + \frac{(\bar{w} - b)(1-\beta)}{m(1, \theta)} \left[ \lambda - \gamma + \frac{\gamma m(1, \theta)}{r+\lambda+\gamma} \right] \right]$$

Substituting the value of $\int_{i^*}^{1} (i-w_i) \, di$ above into equation (37) and simplifying yields:
Equating the value of $E(f_p) = \frac{d}{m(\theta^{-1},1)}$ from (10) with the expression given above in equation (38), we obtain:

$$\frac{d}{m(\theta^{-1},1)} = \frac{1}{(\lambda + r)} \left[ \frac{1 - \beta}{2} (1 + i^*) \right]
- \left[ b(1 - \beta) + \frac{1 - \beta}{m(1, \theta)} \left( \lambda - \gamma + \frac{\gamma m(1, \theta)}{r + \lambda + \gamma} \right) \right]$$

Or:

$$\frac{d}{m(\theta^{-1},1)} + \frac{1 - \beta}{m(1, \theta)(\lambda + r)} \left( \lambda - \gamma + \frac{\gamma m(1, \theta)}{r + \lambda + \gamma} \right)$$

$$= \frac{1}{(\lambda + r)} \left[ \frac{1 - \beta}{2} (1 + i^*) - b(1 - \beta) - \frac{1 - \beta}{m(1, \theta)} \right]$$

Equation (39) depicts the equilibrium relationship between $\theta$ and $i^*$ which guarantees a firm’s free entry and exit. Here, $\theta$ and $i^*$ are positively related, as long as $\gamma > \lambda$. If $i^*$ increases, to clear the labor market, more firms enter and increase the number of vacancies. This is because a firm’s entry decision is based on the expected return from a filled post. Because $i^*$ increases, and the upper bound of ability is 1, the average productivity in the formal sector must rise. In other words, more able individuals are left, and therefore, average productivity must be higher.

Because we have two endogenous variables ($\theta$ and $i^*$), we need another equation to pin down both variables. We turn to this in the next section.

### 3.4 Equivalence of formal and informal sectors

In the previous subsection, we assumed the existence of an interior solution where the workforce could be partitioned between the formal and informal sectors. Therefore, there must be a marginal worker who is indifferent between joining the informal and formal sectors. We denote the marginal worker as $i^*$. As the ability of every individual in the population is indexed by $i$, the marginal worker’s ability is indexed by $i^*$. Therefore, the flow value of search for a job in the formal sector for the marginal worker is $r V^i_U$. Likewise, in the informal sector, it is given by $r V^i_U$. As the individual with $i^*$ ability is indifferent between joining both the informal sector and formal sectors, it follows that:

$$V^i_U = V^i_U$$

(40)
Using equation (3) and equation (5), we can determine \( rV_U^* \) as a function of \((V_P^* - V_U^*)\):

\[
rV_U^* = b + \frac{(\bar{w} - b)\gamma}{r + \lambda + \gamma} + \frac{m(1, \theta)(r + \bar{\lambda})}{r + \lambda + \gamma}(V_P^* - V_U^*). \tag{41}
\]

Wage determination in the formal sector is determined from:

\[
(V_P^* - V_U^*) = \frac{\beta}{1 - \beta} (i^* - w_r) \left( \frac{r + \bar{\lambda} + \gamma}{(r + \lambda)(r + \bar{\lambda} + \gamma) + m(1, \theta)(r + \bar{\lambda})} \right). \tag{42}
\]

We now have \((V_P^* - V_U^*)\) in terms of \((i^* - w_r)\). Equation (15) already solves for the optimal \( w_0 \), and therefore \( w_r \). So we can get an expression for \((i^* - w_r)\). Using equation (15), equation (41) and equation (42), \( rV_U^* \) is determined by:

\[
rV_U^* = b + \frac{(\bar{w} - b)\gamma}{r + \lambda + \gamma} + \beta \frac{(i^* - w_r)}{1 - \beta} \left[ \frac{r + \bar{\lambda} + \gamma}{(r + \lambda)(r + \bar{\lambda} + \gamma) + m(1, \theta)(r + \bar{\lambda})} \right] + \frac{(\bar{w} - b)(\gamma - \lambda)}{m(1, \theta)} \frac{\lambda}{(r + \lambda + \gamma)} \tag{43}
\]

Equation (34) determines \( V_U^* \). Therefore, both the RHS and left-hand side in the equivalence equation, equation (40), are now a function of \( \theta \) and \( \overline{\theta} \). Using equations (34) and (43), we obtain:

\[
1 + \frac{(r + \lambda)(r + \bar{\lambda} + \gamma)}{m(1, \theta)(r + \lambda)} \left[ (i^* - b) + \frac{(\gamma - \lambda)(\bar{w} - b)}{m(1, \theta)} \right] + \frac{\gamma(\bar{w} - b)}{(r + \lambda + \gamma)} \left[ 1 + \frac{\beta}{1 - \beta} \theta d \right] = \frac{\beta}{1 - \beta} \theta d \tag{44}
\]

3.5 Equilibrium

Equations (39) and (44) denote the labor market equilibrium and equivalence equations, respectively. The solutions of these two equations solve for \( i^* \) and \( \theta \) endogenously. However, equation (44) depicts an ambiguous relationship between \( \theta \) and \( \overline{\theta} \). This makes the impact of employment targeting by the public sector unclear[8].

3.6 Comparative statics

We focus on an analytical special case to find whether employment targeting can have an impact on the composition of the workforce between the informal and formal sectors. Later,
we consider a numerical example that shows that our result is more general. We consider the special case where \((\bar{w} - b) \to 0\). Note that \(\theta_f\) has already been solved in equations (35) and (36). Equation (44) now shows a negative relationship between \(i^*\) and \(\theta\). Equation (39) has a positive intercept in the \(i^*\) and \(\theta\) plane, for \((\bar{w} - b) \to 0\). This ensures an interior equilibrium for \(i^*\) and \(\theta\), as shown in Figure 5. In Figure 5, if the government decides to increase its hiring rate (increase \(\gamma\)), or target a higher employment rate (when \((\bar{w} - b) \to 0\)), equation (39) remains unchanged, but equation (44) shifts upward. In this case, market tightness in the formal sector and the size of the informal sector – \(i^*\) and \(\theta^*\) respectively, both rise. In terms of Figure 5, \(\theta^*\) moves to \(\theta_f^*\) and \(i^*\) becomes \(i_f^*\). This is because an increase in the market tightness of the formal sector results in an increase in the rate of obtaining a job in the formal sector. We summarize this result in terms of the following proposition:

**Proposition 2.** Suppose \((\bar{w} - b) \to 0\). Then an increase in \(\gamma\), or more public sector hiring, increases market tightness in the formal sector (\(\theta^*\)) and the size of the informal sector (\(i^*\)).

The intuition is as follows. When \((\bar{w} - b) \to 0\), the per-period (net) return to public sector employment tends to zero. If the public sector expands, the marginal job seeker, \(i^*\), who was originally getting the same return as if he was in the informal sector finds it detrimental to stay in the formal sector, as staying in this sector is not remunerative. However, once \(i^*\) increases, \(\theta^*\) starts increasing to clear the market because the average productivity in the formal sector is higher, and more firms enter into the market. This creates more vacancies, which means \(\theta^*\) increases. Hence, as \(\gamma\) increases, provided that \((\bar{w} - b) \to 0\), both \(\theta^*\) and \(i^*\) increase. Thus, the size of the informal sector increases.

There is an interesting implication with training costs. As \(c\) increases, the opposite happens (the size of the informal sector falls). This is because \(\theta_f^*\) falls (see Figure 4) and this shifts equation (44) backwards, although equation (39) remains unchanged. As \(\theta_f^*\) falls, staying in the informal sector becomes less remunerative because the rate of getting a job is lower. So \(i^*\) falls. To clear the labor market, \(\theta^*\) also falls. Conversely, a fall in \(c\) leads to a rise in \(i^*\) and \(\theta^*\). A rise in \(\theta^*\) happens because the informal sector becomes more lucrative, and a rise in \(\theta^*\) stems from an increase in the average productivity of formal sector workers due to a rise in \(i^*\).
If the training did not exist in the informal sector, then the ability-related uncertainty would re-appear (as in the case for the formal private sector). Additionally, the informal sector has a high job break up rate, which is costly for the firm. Hence, unless the productivity of all matched workers is not equalized, firms would not have an incentive to produce in the informal sector.

3.7 A numerical example

The assumption of \((\bar{w} - b) \to 0\) is a special case. What happens if \((\bar{w} - b)\) is sufficiently small but non-zero? We show that the results of P2 go through, at least locally, using a set of arbitrary parameters and some parameters borrowed from the literature that allow for a sufficiently small \(\bar{w} - b > 0\). We utilize a matching function of the Cobb–Douglas form: \(aw^a b^{1-a1}\). Table I summarizes the parameter values and sources. Figures 6 and 7 characterize the equilibrium in the informal and formal markets, respectively. Figure 8 examines the effect of change in \(\gamma\) on labor market outcomes.

Figure 6, generated using equations (35) and (36), shows an interior solution corresponding to the parameters for the informal sector where \(\bar{w} > b\). We assume \(\gamma = 0.5\) in the baseline case. The numerical solution of \(\theta_I\) is 0.24. This number says that of all the job

<table>
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<th>Values</th>
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</tr>
<tr>
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<tr>
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<tr>
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<td>Gomes (2015)</td>
</tr>
</tbody>
</table>

Table I. Parameter values

Figure 6. Equilibrium in the informal sector
seekers in the informal sector, at most only 24 per cent of them can be matched with vacancies in the informal sector.

Figure 7, generated using equations (39) and (44), characterizes equilibrium in the formal market. For $\gamma = 0.5$, the solutions for $\theta^*$ and $i^*$ are shown to approximately be $\theta^* = 5.62$ and $i^* = 0.469$ (approx). This means that 46.9 per cent of the population works in the informal sector, and the remaining part (53.1 per cent) in the formal sector.

Now, as a counterfactual exercise, suppose we increase the government hiring rate, $\gamma$, to 0.8. Figure 7 shows that for a small but non-zero $(\bar{w} - b)$, a higher $\gamma$ leads to an increase in both $\theta^*, i^*$ consistent with $P2$. As $i^*$ increases to 0.471 (approx), the size of the informal sector increases by about 2 per cent. This increases $\theta^*$, which means compared with the earlier case, the number of vacancies posted by firms per job seeker is increased to 8.81 (approx) from 5.62 (approx), as the return from posting a vacancy in the formal sector has increased.
How does a change in the bargaining power of workers in the informal sector change the measure of informal sector employment? Suppose we assume that in equation (28) the bargaining power of workers is given to be 0.25 (which is less than the formal sector’s bargaining power of workers \( \beta = 0.35 \)). Under this assumption, we get \( \theta^*_f = 0.35 \). Correspondingly, \( \theta^* \) and \( i^* \) become 3.144 and 0.4462, respectively, where \( \gamma = 0.5 \). The decline in bargaining power of workers in the informal sector leads to a fall in the size of the informal sector. This happens because a fall in the bargaining power leads to a fall in the wage of the informal sector. Workers leave the informal sector (thus, \( i^* \) falls). The drop in the wage makes the informal sector attractive to firms and more firms enter in the informal sector. Thus, \( \theta^*_f \) increases. As the size of informal sector falls, the average ability of the formal sector falls. Therefore, the return from a vacancy in the formal sector goes down. Thus, firms leave the formal sector and \( \theta^* \) falls.

4. Conclusion and policy implications
Many governments, as part of their growth and development objectives, play an active role in labor markets. Such interventions come in the form of setting a minimum wage, providing unemployment benefits and directly hiring workers. We refer to this as employment targeting. In the context of a simple search and matching friction model with heterogeneous agents, we show that the propensity for the public sector to target more employment can increase the unemployment rate in the economy and lead to an increase in the size of the informal sector. Employment targeting can therefore have perverse effects on labor market outcomes. We also find it is possible that the private sector wage falls as a result of an increase in the public sector hiring rate, which leads to more job creation in the private sector. This reverses the consensus findings in the search and matching literature which show that an increase in public sector employment disincentivizes private sector vacancy postings.

While we have analyzed the impact of public sector hiring on welfare, for future work, we hope to endogenize the public sector hiring rate that maximizes social welfare as in Gomes (2015). There are a few considerations here. Our model allows the government to intervene in the job market. The effect on welfare comes from two channels when the government increases the hiring rate. First, an increase in public sector matching creates more production in the public sector, which increases social welfare. However, as our model shows that the overall unemployment rate may rise due to the crowding out of private sector jobs, the effect on welfare is ambiguous. Additionally, it is worth noting that the fall in private sector job creation leads to a fall in the social dead weight cost of posting a vacancy. Therefore, the net effect of an increase in the public sector on social welfare is heavily dependent on the elasticity of the matching function (Hosios, 1990).

Notes
1. \( m(1, \theta) \ast \Delta t \) and \( m(\theta^{-1}, 1) \ast \Delta t \) are the transition probabilities from being unemployed to being employed and from a vacant to a filled post, respectively, in the private sector, in a very small time interval \( \Delta t \).
2. Unemployment benefits can be interpreted as either a government transfer to the unemployed (as in Pissarides, 2000) or any imputed real return from leisure or home activity. The government in our model takes part in production through the public sector. The public sector wage, however, does not result from any explicit optimization exercise and is set exogenously. We assume,
however, that the entire wage bill is financed from public sector production. This allows us to incorporate a public sector without the government having to set taxes.

3. As equation (3) suggests, given that the surplus values \((V_p - V_i)\) and \((V_o - V_i)\) are positive, searching for both private sector and public sector jobs gives more utility than searching for jobs in one sector.

4. For example, the most commonly used matching function in the literature has a Cobb–Douglas form (homogeneous of degree one) which has a constant elasticity property.

5. In the previous section, individual ability was uniformly distributed over \([0,1]\).

6. This productivity equalizing training resembles those in “routine” jobs where the worker does not have the scope to showcase his/her ability, but firms can get rid of the uncertainty surrounding a worker’s ability.

7. This is a commonly made assumption in the literature on Pissarides-type search and matching. However, in the case of the formal private sector wage bargaining problem in the previous section, this assumption was not valid.

8. It is important to note that in our model, once the marginal agent is determined in equilibrium, agents who decide to stay in the formal sector do not look for jobs in the informal sector and vice-versa. Our setup therefore segregates the workforce into two distinct parts and gives a clean measure of the size of the informal sector in the economy. For instance, in the real world, it is often observed that those at the lower end of the ability continuum are stuck with routine informal jobs and never move to ability-driven formal jobs. However, there are other possible ways to characterize the equilibrium. For example, agents can be allowed to look for jobs in both the formal and informal sectors simultaneously. Or, even after getting matched with a firm in any particular sector, agents could search for jobs in other sectors. This latter approach is a canonical example of “on-the-job-search”. While this is relevant, we hope to explore this in future research.

References
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**Further reading**

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