A Technical Appendix (To be published online)

A.1 Household's first order conditions

- The Lagrangian for the household problem is given by,

$$L_t = E_t \sum_{s=0}^{\infty} \beta^s [U(C_{t+s}) - \Phi(H_{t+s}) + V(D_{t+s}/P_{t+s}, D^a_{t+s}/P_{t+s}) - \tag{A.1}$$
$$\lambda_t (P_{t+s}C_{t+s} + P_{t+s}T_{t+s} + D_{t+s} + D^a_{t+s} - W_{t+s}H_{t+s} -$$
$$\left(1 + i^D_{t+s}\right) D_{t+s-1} - (1 + i^a)\, D^a_{t+s-1} - \Pi^k_{t+s} - \Pi^r_{t+s} - \Pi^b_{t+s})]$$

The household's optimal choices are given by

$$\frac{\partial L_t}{\partial C_t} = U'(C_t) - \lambda_t P_t = 0$$
$$\frac{\partial L_t}{\partial H_t} = \Phi'(H_t) - \lambda_t W_t = 0$$
$$\frac{\partial L_t}{\partial D_t} = \frac{V_1(D_t/P_t, D^a_t/P_t)}{P_t} - \lambda_t + \beta E_t \left\{\lambda_{t+1}\left(1 + i^D_{t+1}\right)\right\} = 0$$
$$\frac{\partial L_t}{\partial D^a_t} = \frac{V_2(D_t/P_t, D^a_t/P_t)}{P_t} - \lambda_t + \beta E_t \left\{\lambda_{t+1}\left(1 + i^a\right)\right\} = 0.$$

Substituting out the lagrange multipliers, one obtains the household's first order conditions (5) through (7).

A.2 Derivation of Tobin's q equation

Capital goods producing firms maximize

$$\max_{I_t} E_t \sum_{j=0}^{\infty} \Omega_{t,t+j} P_{t+j} \left[Q_{t+j} I_{t+j} - \left\{ 1 + S\left(\frac{I_{t+j}}{I_{t+j-1}}\right)\right\} I_{t+j} \right] \tag{A.2}$$
$$\text{s.t.} \quad K_t = (1-\delta_k)K_{t-1} + Z_{x,t} I_t \tag{A.3}$$

where $\Omega_{t,t+j}$ is the stochastic discount factor and $Z_{x,t}$ is an investment specific technology (IST) shock that follows an AR(1) process:

$$\ln Z_{xt} - \ln \bar{Z}_x = \rho_{z_x}\left(\ln Z_{xt-1} - \ln \bar{Z}_{zx}\right) + \xi^{zx}_t$$

where ξ^{zx}_t is an i.i.d shock.

The first order condition is

$$\frac{\partial\,(.)}{\partial I_t} = \Omega_{t,t} P_t Q_t - \Omega_{t,t} P_t \left\{1 + S\left(\frac{I_t}{I_{t-1}}\right)\right\} - \Omega_{t,t} P_t S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} + \Omega_{t,t+1} P_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 = 0. \tag{A.4}$$

Set $\Omega_{t,t} = 1$ in (A.4), and solve for Q_t,

$$0 = P_t Q_t - P_t \left\{ 1 + S\left(\frac{I_t}{I_{t-1}}\right) \right\} - P_t S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} + \Omega_{t,t+1} P_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2$$

$$P_t Q_t = P_t \left\{ 1 + S\left(\frac{I_t}{I_{t-1}}\right) \right\} + P_t S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} - \Omega_{t,t+1} P_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2$$

$$Q_t = \left\{ 1 + S\left(\frac{I_t}{I_{t-1}}\right) \right\} + S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} - \Omega_{t,t+1} \frac{P_{t+1}}{P_t} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{A.5}$$

Note that $\Omega_{t,t+1} = \beta E_t \left[U'\left(C_{t+1}\right)/U'\left(C_t\right)\right]\left[P_t/P_{t+1}\right]$. Substituting this in (A.5), we get equation (11).

A.3 Bank's first order conditions

The Lagrangian is given by

$$E_t \sum_{s=0}^{\infty} \Omega_{t,t+s} \left\{ \begin{array}{c} \left[\begin{array}{c} (1+i_t^L)L_{t+s-1} + (1+i^R)M_{t+s-1}^R - (\zeta - \alpha_q)(1+i_{t+s}^G)D_{t+s-1} \\ -(1+i^p - i_t^D)\chi_{t+s}(\widetilde{W_{t+s-1}} - M_{t+s-1}^R) - (1-\chi_{t+s})\widetilde{W_{t+s-1}} \\ +(1-\alpha_q)D_{t+s} - L_{t+s} - M_{t+s}^R \end{array} \right] \\ +\lambda_{t+s}\left[M_{t+s}^R - \alpha_r D_{t+s}\right] \end{array} \right\}$$

The first order condition with respect to M_t^R is given by[1]

$$(-1)\Omega_{t,t} + \Omega_{t,t}\lambda_t + E_t\Omega_{t,t+1}(1+i^R) + E_t\Omega_{t,t+1}(1+i^p)\int_{M_t^R}^{D_t} f(\widetilde{W_t})d\widetilde{W_t} = 0. \tag{A.6}$$

Setting $\Omega_{t,t} = 1$, the Euler equation for M_t^R is given by equation (21).

The Kuhn Tucker condition states that

$$\frac{M_t^R}{D_t} = \alpha_r \text{ if } \lambda_t > 0.$$

Assume that the reserve requirement is not binding, which implies that $\lambda_t = 0$ (i.e., banks hold excess reserves). Assuming a rectangular distribution for $\widetilde{W_t}$ over $[0, D_t]^2$, equation 21 reduces to

[1] Note that
$\frac{d}{dM_t^R}\left[\int_{M_t^R}^{D_t}\left[\widetilde{W_t} - M_t^R\right] f(\widetilde{W_t})dW_t - \int_0^{M_t^R} \widetilde{W_t} f(\widetilde{W_t})dW_t\right]$
$= -\int_{M_t^R}^{D_t} f(\widetilde{W_t})d\widetilde{W_t}$

[2] Since $\widetilde{W_t}$ follows a rectangular distribution, over $[0, D_t]$

$$\int_{M_t^R}^{D_t} f(\widetilde{W_t})d\widetilde{W_t} = \frac{D_t - M_t^R}{D_t} = 1 - \frac{M_t^R}{D_t}.$$

$$M_t^R : 1 = E_t\Omega_{t,t+1}\left[(1+i^R) + (1+i^p - i_{t+1}^D)(1-\frac{M_t^R}{D_t})\right]. \tag{A.7}$$

We solve $\frac{M_t^R}{D_t}$ as follows:

$$\frac{M_t^R}{D_t} = 1 - \frac{1-(1+i^R)E_t\Omega_{t,t+1}}{(1+i^p - i_{t+1}^D)E_t\Omega_{t,t+1}} \tag{A.8}$$

which is equation (22).

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B Technical Appendix

B.1 Short run equation system for the baseline model

The short run system has 19 endogenous variables:

$\Omega_{t,t+1}$ i_t^L K_t H_t Y_t C_t I_t d_t d_t^p x_t i_t^D i_t^G T_t π_t Q_t W_t/P_t G_t P_t/P_t^w A_t .

There are four interest rate parameters, i^R, i^a, i^G, i^D,and note that $i^a = i^G = i^D = \bar{i^s}$. The 19 equations are given by.

$$1.\ \Omega_{t,t+1} = \frac{\beta u'(c_{t+1})}{u'(c_t)}.(1+\pi_{t+1})^{-1} \tag{B.9}$$

$$2.\ U'(C_t) = V_1^{'}(d_t, d_t^a) + \beta E_t \left\{ U'(C_{t+1}) \left(1+i_{t+1}^D\right)(1+\pi_{t+1})^{-1} \right\} \tag{B.10}$$

$$3. U'(C_t) = V_2^{'}(d_t, d_t^a) + \beta E_t \left\{ U'(C_{t+1})(1+i^a)(1+\pi_{t+1})^{-1} \right\} \tag{B.11}$$

$$4.\ \Phi'(H_t) = (W_t/P_t)\, U'(C_t). \tag{B.12}$$

$$5. Q_t = 1+S\left(\frac{I_t}{I_{t-1}}\right)+S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} - \beta E_t \frac{U'(C_{t+1})}{U'(C_t)}\left[S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right]. \tag{B.13}$$

$$6.\ (1+i_t^L) = \frac{(P_t^W/P_t)MPK_t + (1-\delta_k)(1+\pi_t)Q_t}{Q_{t-1}} \tag{B.14}$$

$$7.\ \frac{W_t}{P_t} = (1-\alpha)\frac{P_t^W Y_t}{P_t H_t} \tag{B.15}$$

$$8. \frac{P_t}{P_t^W} = \frac{\varepsilon^Y}{\varepsilon^Y - 1}\left[\begin{array}{c} 1+\frac{\phi_p}{\varepsilon^Y-1}\left(\frac{1+\pi_t}{1+\pi}\right)\left\{\frac{(1+\pi_t)}{(1+\pi)}-1\right\} \\ -\Omega_{t,t+1}\frac{\phi_p}{\varepsilon^Y-1}\left\{\frac{y_{t+1}}{y_t}\frac{(1+\pi_{t+1})^2}{(1+\pi)}\left[\frac{(1+\pi_{t+1})}{(1+\pi)}-1\right]\right\} \end{array}\right]^{-1} \tag{B.16}$$

$$9.\ Y_t = AK_{t-1}^{\alpha} H_t^{1-\alpha} \tag{B.17}$$

$$10.\ C_t + I_t + G_t + \underbrace{\frac{\phi_p}{2}\left[\left\{\frac{1+\pi_t}{(1+\pi)}-1\right\}^2 y_t\right] + S\left(\frac{I_t}{I_{t-1}}\right)I_t}_{price\ adj\cos t} = A_t K_{t-1}^{\alpha} H_t^{1-\alpha} \tag{B.18}$$

$$11. G_t + \left(1+i_t^G\right)\frac{b_{t-1}}{1+\pi_t} + (1+i_t^R)\frac{x_{t-1}}{1+\pi_t} + (1+i^a)\frac{d_{t-1}^a}{1+\pi_t} = T_t + b_t + x_t + d_t^a + (1+i_t^p)d_t E \max\left(\frac{\widetilde{W}_t}{D_t} - \frac{M_t^R}{D_t}, 0\right) \tag{B.19}$$

$$12.\ \frac{x_t}{d_t} = 1 - \frac{1-(1+i^R)E_t\Omega_{t,t+1}}{(1+i^p)E_t\Omega_{t,t+1}} \tag{B.20}$$

$$13.\ L_t:\ 1 = E_t\Omega_{t,t+1}(1+i^L_{t+1}) \tag{B.21}$$

$$14.\ G_t - \bar{G} = \rho_G\left(G_{t-1} - \bar{G}\right) + \xi^G_t \tag{B.22}$$

$$15.\ A_t - \bar{A} = \rho_A\left(A_{t-1} - \bar{A}\right) + \xi^A_t \tag{B.23}$$

$$16.\ K_t = (1-\delta)K_{t-1} + I_t \tag{B.24}$$

$$17\ \frac{(1+\pi_t)(x_t/x_{t-1})}{1+\pi} = \left(\frac{(1+\pi_{t-1})(x_{t-1}/x_{t-2})}{1+\pi}\right)^{\rho_\mu} \tag{B.25}$$

$$18.\ 1+i^D_t = \zeta(1+i^G_t) \tag{B.26}$$

$$19.\ \frac{(1+i^G_t)}{(1+\overline{i^G})} = \left(\frac{(1+i^G_{t-1})}{(1+\overline{i^G})}\right)^{\rho_{iG}} \left[\left(\frac{1+\pi_{t-1}}{1+\pi}\right)^{\varphi_\pi}\left(\frac{Y_t}{Y}\right)^{\varphi y}\right]^{(1-\rho_{iG})} \tag{B.27}$$

B.2 Steady State

In this section, we solve for the steady state values of the endogenous variables. The steady state inflation rate is set as: $\pi_{t+1} = \pi_t = \overline{\pi}$, which pins down the steady state mark-up as:

$$\frac{P}{P^W} = \frac{\varepsilon^Y}{\varepsilon^Y - 1}. \tag{B.28}$$

Equation (16) in the steady state is given by,

$$1 + i^L = \left[\left(\frac{P^W}{P}\right)\frac{MPK}{Q} + 1 - \delta_K\right](1+\pi) \tag{B.29}$$

as $\frac{P_{t+1}}{P_t} = 1 + \pi_{t+1}$. Further, from equation (11) and (B.28) in the steady state, $Q = 1$ and $P^W = \frac{\varepsilon^Y - 1}{\varepsilon^Y}P$, respectively. Also, in the steady state, $Y^W = K^\alpha H^{1-\alpha}$ which implies that $MPK = \frac{\alpha Y^W}{K}$. Equation (B.29) thus reduces to,

$$1 + i^L = \left[\left(\frac{\varepsilon^Y - 1}{\varepsilon^Y}\right)\left(\frac{\alpha Y^W}{K}\right) + 1 - \delta_K\right](1+\pi) \tag{B.30}$$

Recalling that in the steady state, the stochastic discount factor is given by $\frac{\beta}{1+\pi}$, substituting this into the steady version of equation (23) yields, $1 + i^L = \frac{(1+\pi)}{\beta}$. From this expression, we can solve for the steady state capital-labor ratio, K/H, which is given by

$$\frac{K}{H} = \left\{ \alpha \left[\frac{\varepsilon^Y - 1}{\varepsilon^Y} \right] \left[\frac{1}{\frac{1}{\beta} - (1 - \delta_K)} \right] \right\}^{\frac{1}{1-\alpha}} \tag{B.31}$$

which we call Λ hereafter.

The national income identity is given by,

$$C + \delta_K K + G = K^\alpha H^{1-\alpha} \tag{B.32}$$

Assume the following functional forms: $\Phi(H_t) = H_t$, $U(C_t) = \ln(C_t)$ and $V(d_t, d_t^a) = \eta \ln d_t + (1 - \eta) \ln d_t^a$. Thus in steady state, $\Phi'(H) = 1$, $U'(C) = 1/C$, $V_1'(.,.) = \frac{\eta}{d}$ and $V_2'(.,.) = \frac{(1-\eta)}{d^a}$. Substituting for these values into equation (7), in the steady state we get

$$C = W/P. \tag{B.33}$$

Using the labor market condition, $W/P = (1 - \alpha) \left(\frac{\varepsilon^Y - 1}{\varepsilon^Y} \right) \left(\frac{K}{H} \right)^\alpha$. Therefore,

$$C = (1 - \alpha) \left(\frac{\varepsilon^Y - 1}{\varepsilon^Y} \right) (\Lambda)^\alpha . \tag{B.34}$$

Now, substituting $V_1'(.,.) = \frac{\eta}{d}$ in equation (5), we get,

$$\frac{1}{C} = V_1'(.,.) + \beta \frac{1}{C} \frac{(1 + i^D)}{(1 + \pi)} \tag{B.35}$$

which can be re-written as,

$$1 + i^D = \frac{1 + \pi - \eta \frac{C}{d} (1 + \pi)}{\beta} \tag{B.36}$$

Similarly substituting $V_2'(.,.) = \frac{(1-\eta)}{d^a}$. in equation (6) ,we get,

$$1 + i^a = \frac{1 + \pi - (1 - \eta) \frac{C}{d^a} (1 + \pi)}{\beta} \tag{B.37}$$

Since $\frac{K}{H} = \Lambda$, equation $(B.32)$ above thus reduces to,

$$C + G = \left[\Lambda^{-(1-\alpha)} - \delta_K \right] K \tag{B.38}$$

Recall, from equation (27) the government budget constraint is given by,

$$P_t G_t + \left(1 + i^G\right) B_{t-1} + (1 + i^R) M_{t-1}^R + (1 + i^a) D_{t-1}^a = P_t T_t + B_t + M_t^R + D_t^a + (1 + i^p) E \max(\widetilde{W}_t - M_t^R, 0) \tag{B.39}$$

Dividing throughout by P_t and noting that $\frac{P_t}{P_{t-1}} = 1 + \pi_t$, we get

$$G_t + \left(1 + i_t^G\right) \frac{b_{t-1}}{1+\pi_t} + (1+i_t^R)\frac{x_{t-1}}{1+\pi_t} + (1+i^a)\frac{d_{t-1}^a}{1+\pi_t} = T_t + b_t + x_t + d_t^a + (1+i_t^p)d_t E \max(\frac{\widetilde{W}_t}{D_t} - \frac{M_t^R}{D_t}, 0) \tag{B.40}$$

where $x_t = M_t^R/P_t$, $d_t = \frac{D_t}{P_t}$, and $b_t = B_t/P_t$.

In the steady state, the above equation becomes

$$G + \left(1 + i_t^G\right) \frac{b}{1+\pi} + (1+i_t^R)\frac{x}{1+\pi} + (1+i^a)\frac{d^a}{1+\pi} = T + b + x + d^a + (1+i_t^p)dE \max(\frac{\widetilde{W}_t}{D_t} - \frac{M_t^R}{D_t}, 0), \tag{B.41}$$

or,

$$G(1+\pi) + \left(i^G - \pi\right) b + (i^R - \pi)x + (i^a - \pi)d^a = T(1+\pi) + (1+i^p)dE \max(\frac{\widetilde{W}_t}{D_t} - \frac{M_t^R}{D_t}, 0)(1+\pi) \tag{B.42}$$

Dividing through the above expression by d, yields,

$$\frac{G(1+\pi)}{d} + \left(i^G - \pi\right) \alpha_q + (i^R - \pi)\frac{x}{d} + (i^a - \pi)\frac{d^a}{d} = \frac{T}{d} + (1+i^p)E \max(\frac{\widetilde{W}_t}{D_t} - \frac{M_t^R}{D_t}, 0)(1+\pi) \tag{B.43}$$

since $B/D = \alpha_q$ (which implies $b/d = \alpha_q$) Also, $\frac{x}{d} = \frac{M^R/P}{D/P}$.

We can substitute out for $\frac{d^a}{d}$ in the above equation (B.43) from equation $(B.36)$ and $(B.37)$ nothing that.

$$d\left[1 + \pi - \beta\left(1 + i^D\right)\right] = \eta C(1+\pi) \tag{B.44}$$
$$d^a\left[1 + \pi - \beta\left(1 + i^a\right)\right] = (1-\eta)\, C(1+\pi) \tag{B.45}$$

or,

$$\frac{d}{d^a} = \frac{\eta}{1-\eta}\left[\frac{1 + \pi - \beta\left(1 + i^a\right)}{1 + \pi - \beta\left(1 + i^D\right)}\right], \tag{B.46}$$

and

$$\frac{x}{d} = 1 - \frac{1 - (1+i^R)\frac{\beta}{1+\pi}}{(1+i^p)\frac{\beta}{1+\pi}}. \tag{B.47}$$

Finally, let us solve for $E \max(\frac{\widetilde{W}_t}{D_t} - \frac{M_t^R}{D_t}, 0)$ in the steady state. Assume $\frac{\widetilde{W}_t}{D_t} = Z_t$, and since D_t is given, Z_t follows an uniform distribution as $\widetilde{W}_t$ but between $[0, 1]$. Thus,

$$E_t \max(Z_t - \frac{M_t^R}{D_t}, 0) = \int_{M_t^R/D_t}^{1} \left(Z_t - \frac{M_t^R}{D_t}\right) h\left(Z_t\right) dZ_t$$

Since $h(Z_t) = 1$,

$$E_t \max(Z_t - \frac{M_t^R}{D_t}, 0) = \int_{M_t^R/D_t}^{1} \left(Z_t - \frac{M_t^R}{D_t} \right) dZ_t$$
$$= \left. \frac{Z_t^2}{2} \right|_{M_t^R/D_t}^{1} - \frac{M_t^R}{D_t} \; Z_t|_{M_t^R/D_t}^{1}$$
$$= \left(\frac{1 - \left(\frac{M_t^R}{D_t}\right)^2}{2} \right) - \frac{M_t^R}{D_t} \left(1 - \frac{M_t^R}{D_t} \right)$$
$$= \frac{1}{2} + \frac{1}{2} \left[\frac{M_t^R}{D_t} \right]^2 - \frac{M_t^R}{D_t}$$
$$= 0.5 \left(1 - \frac{M^R}{D} \right)^2 \tag{B.48}$$

where $\frac{M^R}{D}$ is given by (A.8) evaluated at the steady state.

Continuing from the above government budget constraint ($B.43$) we get

$$\frac{G(1+\pi)}{d} + \left(i^G - \pi\right) \alpha_q + (i^R - \pi)\frac{x}{d} + (i^a - \pi)\frac{d^a}{d} = \frac{T}{d} + (1+i^p) 0.5 \left(1 - \left\{ 1 - \frac{1 - (1+i^R)\frac{\beta}{1+\pi}}{(1+i^p)\frac{\beta}{1+\pi}} \right\} \right)^2 \tag{B.49}$$

From the above equation, we can solve for steady state lump-sum taxes, T.

We have 19 steady state equations, which can be written as a recursive system. These are:

1. $(1 + i^L) = (1 + \pi)/\beta$
2. $(1 + i^L) = \left[\left(\frac{\varepsilon^Y - 1}{\varepsilon^Y} \right) \alpha \left(\frac{K}{H} \right)^{\alpha - 1} + 1 - \delta_K \right] (1 + \pi)$
3. $W/P = (1 - \alpha) \left(\frac{\varepsilon^Y - 1}{\varepsilon^Y} \right) (\Lambda)^{\alpha}$ where $\Lambda = K/H$ solved from the preceding equation
4. $C = W/P$
5. $G = \bar{G}$
6. Using $C + G = \left[\Lambda^{-(1-\alpha)} - \delta_K \right] K$, and steady state G, Solve K
7. Using $K/H = \Lambda$, solve H
8. Using $d \left[1 + \pi - \beta \left(1 + i^D \right) \right] = \eta C (1 + \pi)$, and (5) above solve for d.
9. $d^a \left[1 + \pi - \beta \left(1 + i^a \right) \right] = (1 - \eta) C (1 + \pi)$, solve for d^a
10. $\frac{x}{d} = 1 - \frac{1 - (1 + i^R) \; \Omega}{(1 + i^p) \Omega}$
11. $\frac{P_t}{P_t^W} = \frac{\varepsilon^Y}{\varepsilon^Y - 1}$.
12. $I = \delta K$
13. $\pi =$Long run inflation target ($\bar{\pi}$) (Note that this is pinned down by the money supply rule (24))
14. T solved from the steady state government budget constraint
15. (Stochastic Discount Factor) $\Omega = \beta/(1 + \pi)$

16. $Y = AK^{\alpha}H^{1-\alpha}$
17. $A = \bar{A}$
18. $i^G = \overline{i^G}$
19. $1 + i^D = \zeta(1 + i^G)$

C Technical Appendix for the Extended Model

The present value Lagrangian is given by:

$$L_t^{'P} = E_t \sum_{s=0}^{\infty} \lambda_s^{'} [L_{t+s} + M_{t+s-1}^T + (1-\delta_k) P_{t+s} Q_{t+s} K_{t+s-1} + P_{t+s}^w Y_{t+s}^w - M_{t+s}^T$$
$$- W_{t+s}^{RT} H_{t+s}^{RT} - W_{t+s}^F H_{t+s}^F - (1 + i_{t+s}^L) L_{t+s-1} - Q_{t+s} P_{t+s} K_{t+s}]$$
$$+ \sum_{s=0}^{\infty} \mu_{t+s}^{'} \left[M_{t+s-1}^T - W_{t+s}^{RT} H_{t+s}^{RT} \right] + \sum_{t=0}^{\infty} \nu_{t+s}^{'} \left[L_{t+s} - P_{t+s} Q_{t+s} K_{t+s} \right].$$

where $\mu_t^{'}$, $\nu_t^{'}$ are respective lagrange multipliers.

The first order conditions are given by:

$$M_t^T : \ -\lambda_t^{'} + E_t(\lambda_{t+1}^{'} + \mu_{t+1}^{'}) = 0 \tag{C.50}$$

$$H_t^{RT} : \ \lambda_t^{'} \left[P_t^W MPH_t^{RT} - W_t^{RT} \right] - \mu_t^{'} W_t^{RT} = 0 \tag{C.51}$$

$$H_t^F : P_t^W MPH_t^F - W_t^F = 0 \tag{C.52}$$

$$K_t' : \ -\lambda_t^{'} Q_t P_t + E_t \lambda_{t+1}^{'} [P_{t+1}^w MPK_t + (1-\delta_k) P_{t+1} Q_{t+1}] - \nu_t^{'} Q_t P_t = 0 \tag{C.53}$$

$$L_t : \lambda_t^{'} - E_t \lambda_{t+1}^{'} (1 + i_{t+1}^L) + \nu_t^{'} = 0 \tag{C.54}$$

Since the borrowing constraint binds ($\nu_t^{'} > 0$), substitute out $\nu_t^{'}$ from (C.53) and (C.54) and verify that the basic return equation (16) holds meaning

$$1 + i_t^L = \left[\left(\frac{P_t^w}{P_t} \right) \frac{MPK_t}{Q_t} + 1 - \delta_k \right] \left[\frac{P_t Q_t}{P_{t-1} Q_{t-1}} \right]. \tag{C.55}$$

Effectively a binding borrowing constraint means that wholesalers virtually rent capital from banks as in Chari, Kehoe and McGrattan (1995).

Next rewrite (C.51) as:

$$\frac{\lambda_t^{'}}{\mu_t^{'}} = \frac{(W_t^{RT}/P_t)}{\left[(P_t^w/P_t) MPH_t^{RT} - \left(W_t^{RT}/P_t \right) \right]}$$

Using (C.50),

$$\frac{\lambda_t^{'}}{\lambda_{t-1}^{'} - \lambda_t^{'}} = \frac{\left(W_t^{RT}/P_t \right)}{\left[(P_t^w/P_t) MPH_t^{RT} - \left(W_t^{RT}/P_t \right) \right]}.$$

Equation (C.56) which can be rewritten as:

$$\frac{1}{\left[\left(\lambda_{t-1}^{'}/\lambda_t^{'} \right) - 1 \right]} = \frac{\left(W_t^{RT}/P_t \right)}{\left[(P_t^w/P_t) MPH_t^{RT} - \left(W_t^{RT}/P_t \right) \right]} \tag{C.56}$$

C.1 Specification of the discount factor $\lambda_t^{'}$

The wholesaler's discount factor $\lambda_t^{'}$ is given by the sequence of loan rates. In other words,

$$\lambda_t^{'} = \frac{1}{(1+i_0^L)}.\frac{1}{(1+i_1^L)}.\frac{1}{(1+i_2^L)}.......\frac{1}{(1+i_t^L)}$$

which means:

$$\lambda_t^{'}/\lambda_{t-1}^{'} = \frac{1}{(1+i_t^L)}$$

which after plugging into (C.56) yields,

$$\left(\frac{1}{i_t^L}\right) = \frac{\left(W_t^{RT}/P_t\right)}{\left[(P_t^w/P_t)MPH_t^{RT} - \left(W_t^{RT}/P_t\right)\right]} \tag{C.57}$$

The loan rate i_t^L can be pinned down by (16). Plugging this and rearranging (C.57) we get (31).

C.2 Steady state

Assume the same log utility for consumption.

It is straightforward to verify that the steady state real wage is:

$$\frac{W^{RT}}{P} = \frac{\beta}{1+\pi}.\frac{\varepsilon^Y - 1}{\varepsilon^Y}MPH^{RT} \tag{C.58}$$

In other words, now

$$W^{RT}/P = \frac{\beta}{1+\pi}(1-\alpha)\left(\frac{\varepsilon^Y-1}{\varepsilon^Y}\right)\left(K/(\phi.1+(1-\phi)H^F\right)^{\alpha}$$

$$\frac{G(1+\pi)}{d}+\left(i^G-\pi\right)\alpha_q+(i^R-\pi)\frac{x}{d}+(i^a-\pi)\frac{d^a}{d} = T+\frac{\pi}{1+\pi}m^T+(1+i^p)E\max(\frac{\widetilde{W}_t}{D_t}-\frac{M_t^R}{D_t},0) \tag{C.59}$$

where we have the new term $\frac{\pi}{1+\pi}m^T$ which is the inflation tax revenue from wholesaler's holding of real balance (where $m^T = M^T/P$).

The cash in advance constraint in (30) gives the steady state money demand function:

$$(W^{RT}/P)H^{RT} = \frac{m^T}{1+\pi}$$
$$=> m^T = (W^{RT}/P)(1+\pi)H^{RT}$$

Given that $H^{RT} = 1$,

$$m^T = (W^{RT}/P)(1+\pi)$$

The national income identity changes to:

$$(1-\phi)C^F + \phi C^{RT} + G = \left[\Lambda^{-(1-\alpha)} - \delta_K\right] K \tag{C.60}$$

$$=> (1-\phi)W^F/P + \phi W^{RT}/P + G = \left[\Lambda^{-(1-\alpha)} - \delta_K\right] K$$

From here we can solve K and then using $\Lambda = K/H$ solve H. To sum up: the steady state system thus changes to 22 equations (three extra variables m^T, $\frac{W^{RT}}{P}$, and C^{RT}).

1. $(1+i^L) = (1+\pi)/\beta$
2. $(1+i^L) = \left[\left(\frac{\varepsilon^Y-1}{\varepsilon^Y}\right)\alpha\left(\frac{K}{\phi H^{RT}+(1-\phi)H^F}\right)^{\alpha-1} + 1 - \delta_K\right](1+\pi)$
3. $W^{RT}/P = \frac{\beta}{1+\pi}(1-\alpha)\left(\frac{\varepsilon^Y-1}{\varepsilon^Y}\right)(\Lambda)^\alpha$ where $\Lambda = K/(\phi+(1-\phi)H^F)$ solved from the preceding equation
4. $W^F/P = (1-\alpha)\left(\frac{\varepsilon^Y-1}{\varepsilon^Y}\right)(\Lambda)^\alpha$

(Note there are two steady state real wages. Higher inflation depresses the RT real wage and creates more wage inequality)

5. $C^F = W^F/P$ from (7) given the assumption that utility function: $\ln C - H$
6. $C^{RT} = (W^{RT}/P)$ because RT consumers F.O.C dictates $H^{RT} = 1$
7. $m^T = (W^{RT}/P)(1+\pi)$ (from CIA)
8. $G = \bar{G}$
9. Using $\phi C^{RT} + (1-\phi)C^F + G = \left[\bar{A}\Lambda^{-(1-\alpha)} - \delta_K\right] K$, and steady state G, Solve K (Modified)
10. Using $\Lambda = K/(\phi + (1-\phi)H^F)$ solve H^F
11. Using $d\left[1+\pi-\beta\left(1+i^D\right)\right] = \eta C^F(1+\pi)$, and (5) above solve for d.
12. $d^p\left[1+\pi-\beta\left(1+i^a\right)\right] = (1-\eta)C^F(1+\pi)$, solve for d^a
13. $\frac{x}{d} = 1 - \frac{1-(1+i^R)\ \Omega}{(1+i^p)\Omega}$
14. $\frac{P_t}{P_t^W} = \frac{\varepsilon^Y}{\varepsilon^Y-1}$.
15. $I = \delta K$
16. $\pi =$ long run inflation target ($\bar{\pi}$) (Note that this is pinned down by the money supply rule (24))
17. T solved from the steady state government budget constraint (C.59) (Modified)
18. (Stochastic Discount Factor) $\Omega = \beta/(1+\pi)$
19. $Y = AK^\alpha H^{1-\alpha}$
20. $A = \bar{A}$
21. $i^G = \overline{i^G}$
22. $1 + i^D = \zeta(1+i^G)$

D Technical Appendix: IRF Plots of Model 2

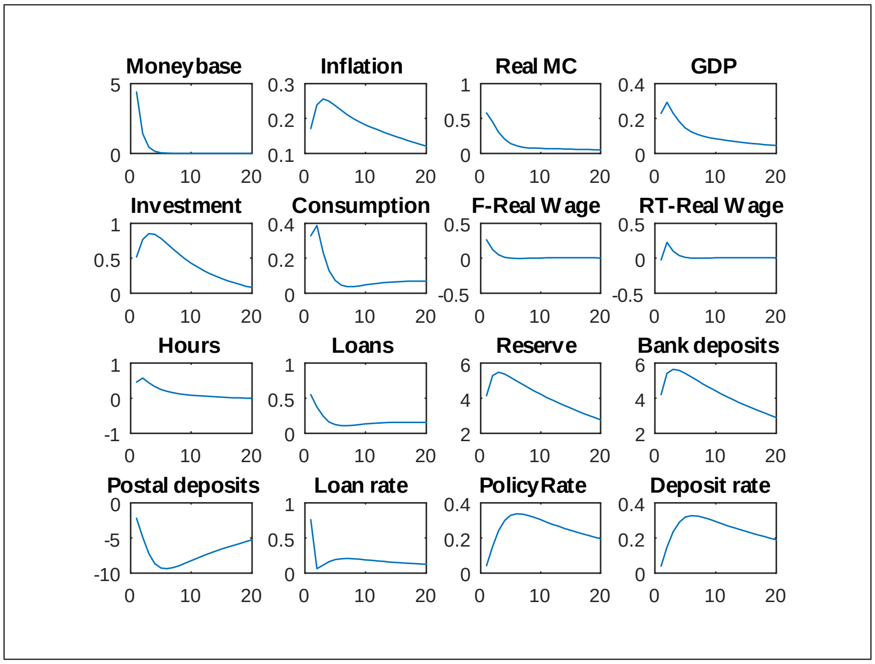

Figure 9: Impulse responses with respect to a positive money base shock

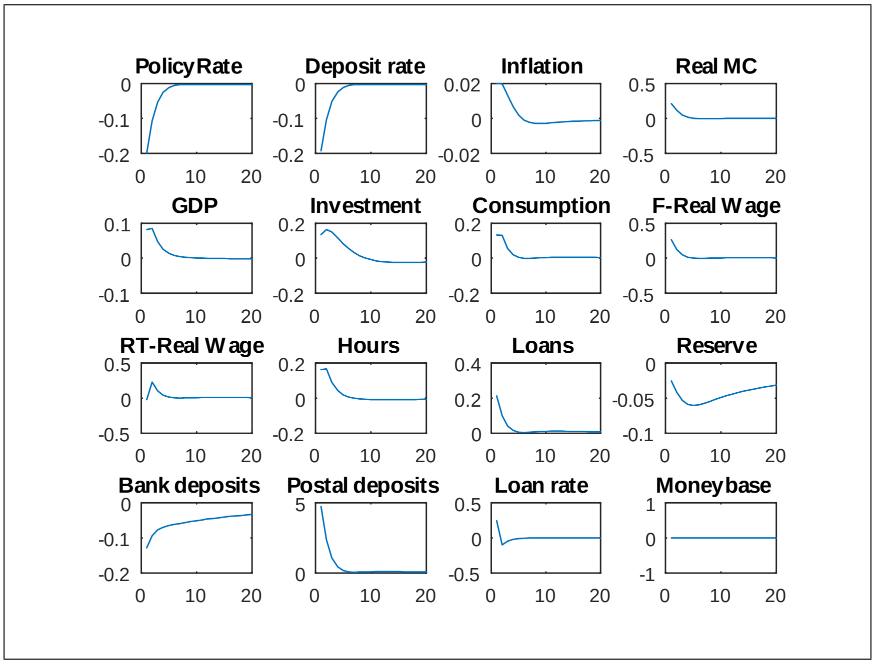

Figure 10: Impulse responses with respect to a negative policy rate shock

E Technical Appendix: Data Sources and Transformations

We use quarterly data of the macroeconomic and financial variables over the sample period of 1996: Q4 to 2016: Q4 both for the sign restricted Vector Autoregression (SRVAR) analysis as well as the model validation exercise (presented in Section 2 and Section 5, respectively). We choose this sample period of study as it offers a balanced sample for all the variables of our interest. The list of variables included in our analysis is as follows: real GDP, real consumption, real investment, index of total credit to non-financial sector, real commercial bank deposits, real postal deposits, the 91 days treasury bill rate, CPI inflation, the bank lending rate, and the growth rate of the monetary base. Except for Consumer Price Index (CPI) and index of total credit to non-financial sector, all data are taken from the database of the Reserve Bank of India (RBI). Consumer Price Index for all items and Index of total credit to non-financial sector data are taken from the database of Federal Reserve Bank of St. Louis.

The monetary variables like growth rate of reserve money, growth rate of currency and reserve money and 91 days treasury bill rate are taken at their levels. Choice of the monetary base is based on two definitions: one is reserve money (comparable for Model 1) and the other is sum of currency and reserve money (comparable for Model 2). Growth rates of the monetary base for both definitions are computed by taking the first difference of log-transformed data series. The prime lending rate of the State Bank of India (SBI) is used as a proxy measure for the bank lending rate since the SBI group plays a dominant role in the Indian banking system and their lending rate is followed by other competing banks. Index of total credit to non-financial sector is taken with base year 1951. GDP, private consumption, and private investment are measured in constant prices with base year 2011-2012. Commercial bank deposits and postal deposits are deflated by the Consumer Price Index to get the real bank deposit and postal deposit. The Consumer Price Index chosen for all commodities with base year 2010.

In order to avoid seasonality, inflation is computed year on year basis from Consumer Price Index (CPI). We also de-seasonalize log-transformed series of real GDP, real consumption, real investment, index of total credit to non-financial sector, real commercial bank deposits and real postal deposits. Then, we de-trend them using Baxter and King (1999) band pass filter to obtain the business cycle component of each series with the periodicity of fluctuations between 6 to 32 quarters. We validate the baseline Model 1 and 2 with the statistical properties of the business cycle frequency.