Project

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Indian Statistical Institute

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Random Numbers

- The word "random" means apparent absence of cause, planning, or design, "lack of method or system", or "accidental, haphazard."
- Statistically, it means inability to predict outcomes or to find any pattern in a series of outcomes.
- **A random number is a number generated by a process, whose outcome is unpredictable, and which cannot be subsequentially reliably reproduced.**

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- Such processes are almost unachievable in practice.
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Pseudo Random number

- Psedurandom sequences are outcomes of a deterministic causal process.
- Psedurandom processes can be used repeatedly to produce exactly the same numbers.
- Pseudorandom number generators are algorithms that use mathematical formulae or simply precalculated tables to produce sequences of numbers that appear random. For example, linear congruential method.

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Characteristics of Pseudorandom numbers

- PRNGs are efficient i.e. they can produce many numbers in a short time.
- They are deterministic in the sense that a given sequence of numbers can be reproduced at a later stage if the starting point in the sequence is known.
- PRNGs are typically also periodic, which means that the sequence will eventually repeat itself.

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History of Random Number Generators

- Rolling a die is a preliminary RNG which was being used in anciant times in the games of chance.
- The chinese were perhaps the earliest people to formalize odds and chance 3000 years ago.
- In the sixteenth century that Italian mathematicians began to formalize the odds associated with various games of chance, like Bingo.
- Brownian motion arises from the aggregated effect of the random collsions of many molecules with suspended objects. Robert Brown claimed that it is one of very few that truely can not be predicted or is truely random. But in 1905 Albert Einstein suggested that this is not really random.

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- **Wichmann-Hill RNG:**
  - Invented in 1982.
  - Periodicity: 6953607871644
- **Super-Duper:**
  - Invented in 1970.
  - Periodicity 4.6×10^{18}
  - Failed in MTUPLE test.
- **Marsaglia-Multicarry:**
  - Invented round 1985.
  - Periodicity: More than 2^{60}

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“Anyone who considers arithmetical methods of producing random digits is, ofcourse, in a state of sin.”

– Von Neumann

Von Neumann method

- John Von Neumann suggested the middle square method in about 1946.
- It is a computer based PRNG.
- Iterating Von Neumann's procedure produces a series of numbers generated by a deterministic process intended merely to imitate a random sequence.
- The procedure is:
  1. Take any integer number of n digits.
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Code for Von Neumann Algorithm

- Von Neumann Algorithm is carried out in two ways, viz, direct method and using bitwise shift operator.
- Direct method is implemented both in R and in C.
- Needless to say that Bitwise shift method is implemented using C.
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CODES

(in HTML)

Preliminary Analysis based on Graphs

- The occurence of the digits in the generated random numbers should be random.
- If it follows any pattern then we can say that the appearance of the digit is non-random.
- If the appearance of the digit falls in the particular sequence of the duration of successive occurences then the same conclusion holds.
- If the last time a particular digit appears is too small then we conclude that, that digit never comes later on and hence there exists a non-random pattern in its occurence.

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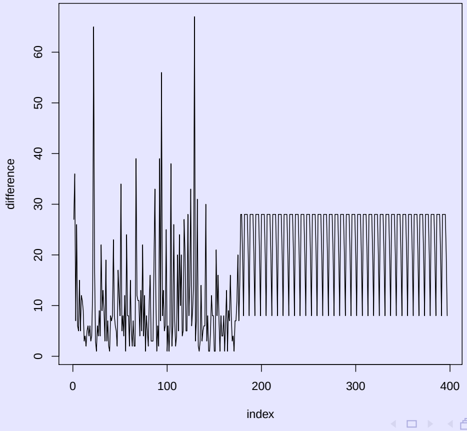
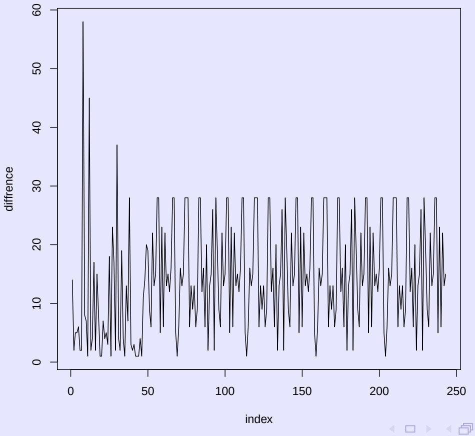
Duration of successive appearances of 4

- Graph 1



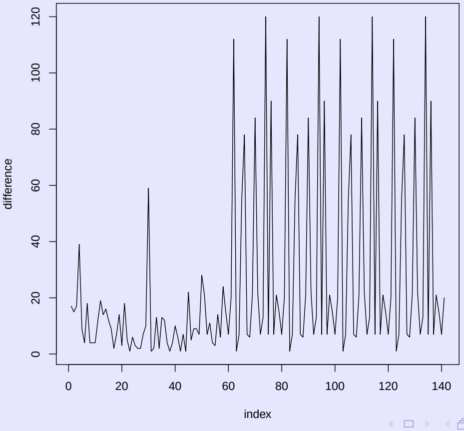
Duration of successive appearances of 2

- Graph 2



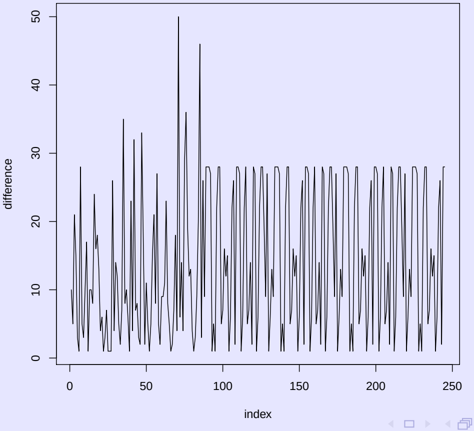
Duration of successive appearances of 3

- Graph 3



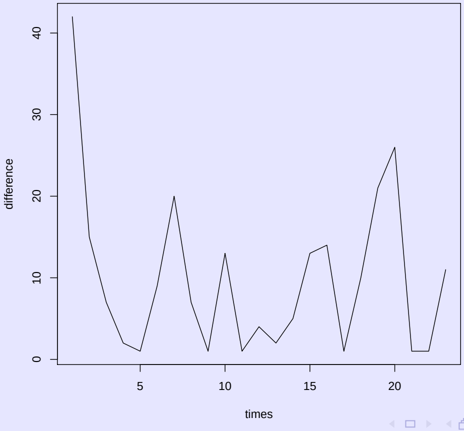
Duration of successive appearances of 4

- Graph 4



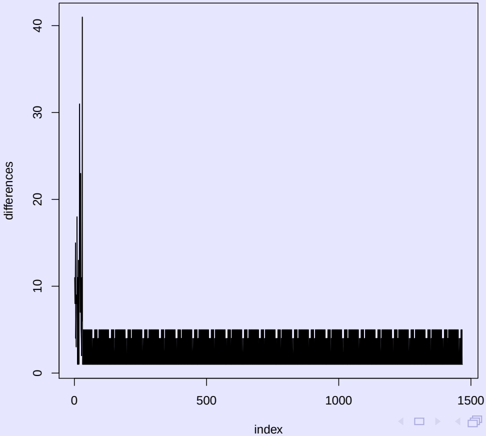
Duration of successive appearances of 4

- Graph 5



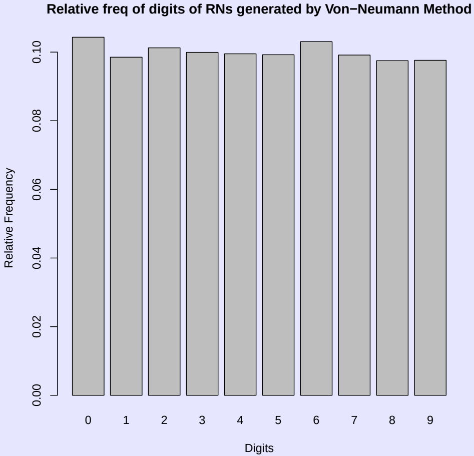
Duration of successive appearances of 0

- Graph 6



Relative Frequency of digits

- Graph 7



Observations

- In the first four graphs we see that the sequence of durations of successive occurances falls in a loop.
- In the graph 5 we see that the digit 4 has appeared very few times.
- Infact 4 appears at 249 th position for the last time, but there are total $3500(500 \times 7)$ random digits.
- In the graph 6 we see that the digit 0 is appearing very frequently though the sequence is not degenerated at 0.
- The last graph shows that in long run the digits are coming almost with equal probability.

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Bitwise Programming

- In Bitwise programming a random number between 0 and $2^n - 1$ is chosen.
- Then it is squared.
- Next, appropriate number of left and right bitwise shifts give the middle digits.
- Finally the output is given as a decimal number.

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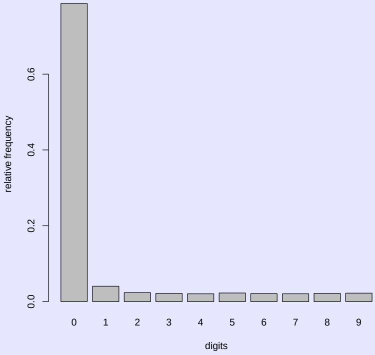

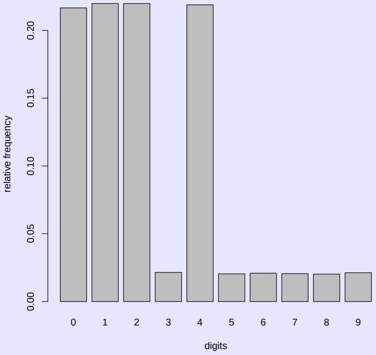

Analysis of Graphs

- The graph on the left side shows that the sequence of digits eventually converges to zero.
- Graph on the right side shows that the sequence falls in a loop.
- The loop consists of four digits $0, 1, 2, 4$
- We have observed that the loop is actually $24, 10, 24, 10, \ldots$(Initially we have taken the number of digits in the binary number is 22)

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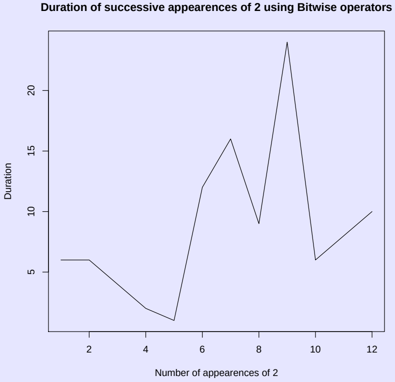

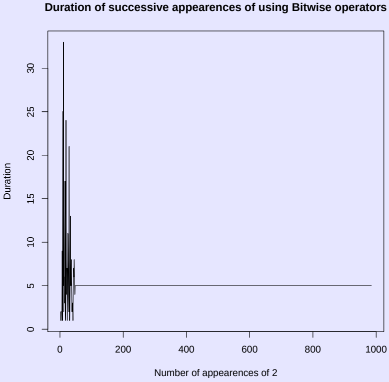

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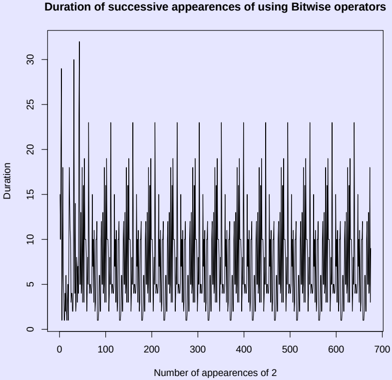

Conclusions based on the Graphs

- The proportion of occurences of digit 2 is approximately $\frac{15}{10000}$ which is much less than $\frac{1}{10}$.
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Comparison between Bitwise and Direct method

- Barplot of relative frequencies of digits in the sequence of random number generated is one way to compare these two ways of implementation of Von Neumann method.
- In case of direct method,the relative frequencies of all the digits are approximately same but in case of Bitwise method, in general, the digit 0 occur more often than other digits.
- Another way to do so is to plot the duration between the successive appearance of any digit.
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K1 [Choose Number of Iterations] Set $Y \leftarrow \lfloor X/10^9 \rfloor$, the most significant digit of X (We will execute steps K2 through K13 exactly $Y + 1$ times)

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- K5 [Multiply] Replace X by $(1001001001X)\mathbf{mod}10^{10}$
- K6 [ ] If $X < 100000000$, then set $X \leftarrow X + 9814055677$; otherwise set $X \leftarrow 10^{10} - X$
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- To solve this problem we used $\mathbf{C}$ language.
- This can also be solved in $\mathbf{R}$.
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CODES

(in HTML)

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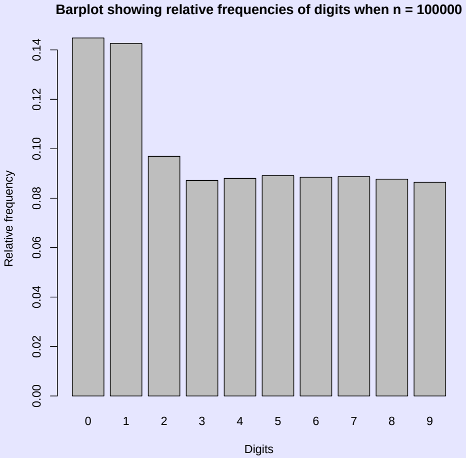

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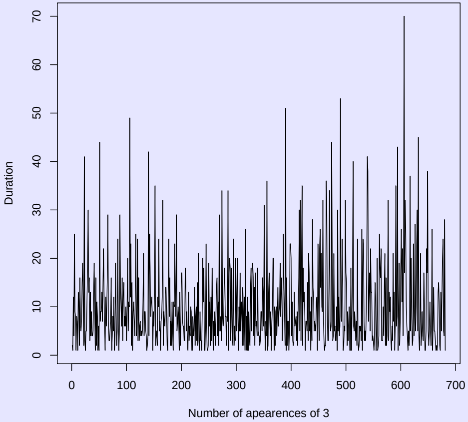

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- The generator is defined by the

$$X_{n+1} = (aX_n + c)\bmod m$$

Where,

1. $\{X_n\}$ = Sequence of pseudorandom values
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Choice of Multiplier 'a'

- When $a = 0$ $X_n = c \textbf{ mod } m$ which is obviously not random.
- When $a = 1$, $X_n = (X_0 + nc)\textbf{mod } m$ amd the sequence will not behave like a random sequence.
- So in general we take $a \geq 2$

Choice of Modulus 'm'

- First we should not take $m = 2$, then the sequence would at best have a form $0, 1, 0, 1, 0, 1, \ldots$
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- Since only m different values are possible, the period surely can not be larger than m.
- It can be proved that a linear congruential method has period length m iff
  - c is relatively prime to m,
  - $b = a - 1$ is a multiple of p, for every prime p dividing m.
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- Fast and Requires minimal memory.
- Proper choice of parameters gives 'good' sequence of PRNs.
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