

Supplement: D-Efficient Composite-type Second Order Designs via Computer Search

The random search procedures used in the article are based on the following basic algorithms. The procedure $\text{Random}(S)$ used in these algorithms, when called with a finite set S as argument, selects an element of S randomly with equal probability.

1. RANDOMDESIGN generates a random design.

```

procedure  $\text{RANDOMDESIGN}(m, n)$ 
   $X \leftarrow$  new  $n \times m$  matrix
  for  $i \leftarrow 1, 2, \dots, n$  do
    for  $j \leftarrow 1, 2, \dots, m$  do
       $X_{ij} \leftarrow \text{Random}(\{-1, 0, 1\})$      $\triangleright$  Select one with equal probability
    end for
  end for
  return  $X$ 
end procedure

```

2. $\text{IMPROVEDESIGNGREEDY}$ improves a design by greedy search.

```

procedure  $\text{IMPROVEDESIGNGREEDY}(X)$ 
   $m \leftarrow$  number of rows of  $X$ 
   $n \leftarrow$  number of columns of  $X$ 
  for  $i \leftarrow 1, 2, \dots, n$  do
    for  $j \leftarrow 1, 2, \dots, m$  do
      for  $x \leftarrow -1, 0, 1$  do
         $E_{ijx} \leftarrow$   $D$ -efficiency when  $X_{ij}$  is replaced by  $x$ 
      end for
    end for
  end for
  if  $\max E_{ijx} > D$ -efficiency of  $X$  then
    return updated  $X$  corresponding to  $\max E_{ijx}$ 
  else
    return  $X$      $\triangleright$  Unchanged
  end if
end procedure

```

3. IMPROVEDSIGNRANDOM improves a design by random search.

```

procedure IMPROVEDSIGNRANDOM( $X$ )
   $Y \leftarrow X$ 
   $m \leftarrow$  number of rows of  $X$ 
   $n \leftarrow$  number of columns of  $X$ 
   $i \leftarrow \text{Random}(\{1, 2, \dots, n\})$ 
   $j \leftarrow \text{Random}(\{1, 2, \dots, m\})$ 
   $Y_{ij} \leftarrow \text{Random}(\{-1, 0, 1\} \setminus \{X_{ij}\})$   $\triangleright$  Replace  $Y_{ij}$  randomly
  if  $D$ -efficiency of  $Y > D$ -efficiency of  $X$  then
    return  $Y$   $\triangleright$  Updated
  else
    return  $X$   $\triangleright$  Unchanged
  end if
end procedure

```