Game Theory - Assignment 2

Due date: September 12, 2024

1. Consider the following two player game in Table 1. Draw the best response maps of the two players and use this to find out

   - any mixed strategy of each player which is never a best response.
   - the set of all (pure and mixed) Nash equilibria using this.

	a	b	c
A	$(1,0)$	$(3,0)$	$(2,1)$
B	$(3,1)$	$(0,1)$	$(1,2)$
C	$(2,1)$	$(1,6)$	$(0,2)$

Table 1: Two Player Game

2. Suppose Player i has a pure strategy s_i that is chosen with positive probability in each of his maxmin strategies. Prove that s_i is not weakly dominated by any other strategy (pure or mixed).

3. A finite square matrix $A = \{a_{ij}\}_{i,j \in T}$, where T is the set of rows/columns is called anti-symmetric if for every row i and column j, $a_{ij} + a_{ji} = 0$. Consider a two player zero sum game with T as the set of pure strategies for both the players. The utility of player 1 is $u_1(i,j) = a_{ij}$ for every $i, j \in T$. Find the payoff of any player in any (mixed strategy) Nash equilibrium of this zero-sum game.

4. A Nash equilibrium s^* in a finite strategic form game $\Gamma = (N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N})$ is a **strict Nash equilibrium** if for every $i \in N$, for every $s_i \in S_i \setminus \{s_i^*\}$,

$$u_i(s_i^*, s_{-i}^*) > u_i(s_i, s_{-i}^*).$$

   Prove that if the process of iterative elimination of strictly dominated strategies results in a unique strategy profile s^*, then s^* is a strict Nash equilibrium.

5. Suppose (σ_1, σ_2) and (σ_1', σ_2') are two (mixed strategy) Nash equilibria in the mixed extension of a two-player zero-sum game. Show that (σ_1, σ_2') and (σ_1', σ_2) are also Nash equilibria.