

GAME THEORY - ASSIGNMENT 6

Due date: **October 30, 2019.**

1. King Solomon is faced with two women, Elizabeth and Mary, who both claim to be the mother of an infant. King Solomon does not know the true mother. If the true mother gets the child, then she gets a utility of 100. On the other hand, the woman who is not the true mother only gets a utility of 50 if given the child. By not getting the child, both women get zero utility.

King Solomon sets up the following game.

Step 1. He will ask Elizabeth whether the child is hers. If she answers negatively, the child will be given to Mary. If she answers affirmatively, the king will continue to the next step.

Step 2. He will ask Mary whether the child is hers. If she answers negatively, the child will be given to Elizabeth. If she answers affirmatively, the king will ask Mary to pay 75 and Elizabeth to pay 10, and give the child to Mary. Utility from money is linear, i.e., paying p gives a utility of $-p$.

Since King Solomon does not know the true mother, there are two extensive form games possible - denote them as Γ_M (where Mary is the true mother) and Γ_E (where Elizabeth is the true mother).

- (a) Describe Γ_M and Γ_E .
 - (b) Argue that there is a unique subgame perfect equilibrium of each of the games where the true mother gets the infant.
2. A firm goes to a bank for loan. The firm is one of two types: (a) a honest (H) type or (b) a cheat (C) type. The probability that the firm is of type C is $p = \frac{2}{3}$ and of type H is $1 - p$. Bank does not know the type of the firm. The bank can either *approve* or *reject* the loan request of the firm. If the loan request is approved, then the firm can either *default* the loan or *repay* the loan.

If the bank rejects the loan request, then both the bank and the firm receive a payoff of 10 each. If the bank approves the loan request and the firm repays the loan, then the bank receives a payoff of 40 and the firm receives a payoff of 60. If the bank approves the loan request and the firm defaults, then the bank has a **loss** of 100 (i.e., payoff is -100). On the other hand, if the firm defaults, his payoff is zero if he is of type H and 150 if he is of type C .

- Describe this as an extensive form game of incomplete information (a graphical representation describing all information sets is good enough).
- Describe a perfect Bayesian equilibrium of this game.
- Verify if the perfect Bayesian equilibrium is a sequential equilibrium.

3. Consider the extensive form game in Figure 1.

- Find all pure strategy Nash equilibria and subgame perfect Nash equilibria of this game.
- Suppose Player 2 can observe the move of Player 1 with probability $p \in (0, 1)$, i.e., if Player 1 plays L , Player 2 will observe L with probability p and R with probability $(1-p)$ and if Player 1 plays R , Player 2 will observe R with probability p and R with probability $(1-p)$. So, Player 2 observes L or R but does not know if it is the correct one.
 - Describe this as an extensive form game of incomplete information - a game tree representation with information sets is sufficient.
 - Find a perfect Bayesian equilibrium of this game for $p = \frac{1}{2}$.

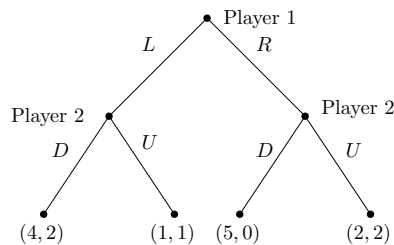


Figure 1: Extensive form game