Game Theory - Midterm Examination

Date: September 14, 2015

Total marks: **42**

Duration: 2:00 PM to 5:00 PM

Note: Answer all questions clearly using pen. Please avoid unnecessary discussions.

1. Ten players are playing the following game. Each player writes down, on a peice of paper, an integer in $\{1, \ldots, 100\}$, alongside his identity (name). A *target integer* is the highest integer less than or equal to $\frac{2}{3}$ of the average of all the integers submitted. The winners of the game are all the players who submitted the target integer. Winners equally share a prize of 1000 (assume prize money equals payoff).

   - Describe this as a strategic form game. (**2 marks**)
   - What are the strictly dominated strategies of each player. (**3 marks**)
   - Compute the set of (correlated) rationalizable strategies in this game. (**4 marks**)
   - Find a pure strategy Nash equilibrium of this game. Is this a unique pure strategy Nash equilibrium? (**3 marks**)

2. Consider a two player zero-sum game Γ. Suppose one of the pure strategies of Player 1 is removed from the game Γ and denote by Γ' the new two player zero-sum game. Show that the Nash equilibrium payoff of Player 1 in Γ is greater than or equal to his Nash equilibrium payoff in Γ'. (**5 marks**)

   Is this result true for any finite two player game? Either prove your answer or provide a counter example. (**5 marks**)

3. A Nash equilibrium s^* in a finite strategic form game $\Gamma = (N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N})$ is a **strict Nash equilibrium** if for every $i \in N$, for every $s_i \in S_i$,

   $$u_i(s_i^*, s_{-i}^*) > u_i(s_i, s_{-i}^*).$$

   Prove that if the process of iterative elimination of strictly dominated strategies results in a unique strategy profile s^*, then s^* is a strict Nash equilibrium. (**5 marks**)

4. Consider the two player strategic form game in Table 1.

   (a) Draw the best response maps of both the players and conclude whether certain strategies are not played in any (mixed strategy) Nash equilibrium. (**6 marks**)

	ℓ	r
T	$(5,1)$	$(0,2)$
M	$(1,3)$	$(4,1)$
B	$(4,1)$	$(2,3)$

Table 1: A strategic form game

(b) Use the best response maps to compute all the (mixed strategy) Nash equilibria of this game. (**4 marks**)

5. Suppose there are n firms and each firm j chooses a price p_j. The demand function for firm i is given by

$$D_i(p_i, p_{-i}) = a_i - b_i p_i + g_i(\sum_{j \neq i} p_j),$$

where $b_i > 0$ for all i and for all i, $g_i : \mathbb{R}_+ \to \mathbb{R}_+$ is some increasing function of sum of prices of other players.

The payoff of a firm is its revenue - its price times the quantity demanded.

Suppose prices are chosen from a compact interval in $\mathbb{R}_+$. Can you argue that this game has a Nash equilibrium in pure strategies? Clearly state any result you use to argue this existence. (**5 marks**)