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Menu-dependent preferences and revelation principle ☆

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Abstract

We extend the set of preferences to include menu-dependent preferences and characterize the domain in which the revelation principle holds. A weakening of the well-known contraction consistency is shown to define a subset of this domain. However, we show that minimax-regret preference can be outside the domain.

JEL classification: C72; D78

Keywords: Revelation principle; Menu-dependent preferences; Weak contraction consistency; Minimax regret

1. Introduction

The revelation principle is the foundation of mechanism design (see, for example, Dasgupta, Hammond and Maskin [3], Myerson [7]). Applied to environments with incomplete information, it states that for any Bayesian–Nash equilibrium of any mechanism there exists an outcome-equivalent Bayesian–Nash equilibrium of a direct mechanism in which all players report their types truthfully. Thus, the revelation principle greatly simplifies the search for optimal mechanisms; we need to search only in the set of incentive compatible direct mechanisms. In other

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words, in order to confirm whether a social choice function is partially implementable, we simply have to check whether it is incentive compatible.[1]

Bayesian–Nash equilibrium assumes that the players have expected-utility preferences, which are *menu independent*, i.e., preference over any two alternatives does not vary with the menu (or the choice set). However, several experiments have documented *menu dependence* of choice (e.g., Huber, Payne and Puto [5], Simonson [12], Simonson and Tversky [13]). In these experiments, subjects chose x from some menu $S \supseteq \{x, y\}$ but chose y when the menu was changed to $S' \supseteq \{x, y\}$. It is therefore pertinent to incorporate menu-dependent preferences in the theory of mechanism design.[2]

We consider a very general setup with incomplete information that admits both private and interdependent values. An alternative for a type of a player is an interim Anscombe–Aumann act (IAA act) that specifies a lottery over the set of outcomes for each realization of types of the other players. A menu is a set of IAA acts. For any menu, each type of a player has a preference relation over the IAA acts that belong to the menu. Moreover, these preferences can vary with the menu.

We characterize the domain of preferences in which the revelation principle holds. To see the impact of menu dependence on the revelation principle in the simplest possible way, consider a situation with $\{a, b, c\}$ as the set of outcomes and a single player with two types $\{t^1, t^2\}$. Type t^1 chooses a from any menu containing a whereas type t^2's choice is menu dependent, she chooses b from $\{a, b, c\}$ but a from $\{a, b\}$. Consider the indirect mechanism in which the player is asked to choose an outcome from $\{a, b, c\}$ and the chosen outcome is implemented. The menu available to both types in this indirect mechanism is $\{a, b, c\}$—these are the outcomes each type can generate by varying her message—and hence, the social choice function obtained in "equilibrium" is $f(t^1) = a$ and $f(t^2) = b$. In the direct mechanism corresponding to this social choice function, the player is asked to report her type, and outcomes a and b are implemented whenever the reported types are, respectively, t^1 and t^2. Thus, the menu available to both types in this direct mechanism is $\{a, b\}$. Truth-telling is not an "equilibrium" of this direct mechanism since t^2 prefers to report herself as t^1 in order to implement a. Thus, the revelation principle fails. This is because firstly, t^2's choice is menu dependent and secondly, in going from the indirect mechanism to the direct mechanism, the menu available to each type changes; in fact, the menu contracts as messages not used in the "equilibrium" of the indirect mechanism are removed. This contraction in the menu is of a specific kind: the contracted menu in the direct mechanism can be mapped onto by the set of types of the player as she can only vary her type report in the direct mechanism. Our characterization is closely tied to this contraction in menus.

As a corollary to our characterization, we show that "weak contraction consistency" (WCC) of the choice functions is a sufficient condition for the revelation principle. WCC requires that an optimal IAA act in a menu remains optimal in those subsets of the menu which contain the IAA act and can be mapped onto by the set of types of the player. This is weaker than the well-known

[1] Partial implementation ignores the problem of multiple equilibria. We leave the topic of full implementation with menu-dependent preferences for future research. The literature on full implementation has studied other departures from the standard framework; for example, Hurwicz [6], Eliaz [4] and Cabrales and Serrano [1].

[2] Sen [9,10] argues for incorporating menu dependence in standard rational choice theory. Also see [11] for a formal analysis.

contraction consistency, which requires that an optimal IAA act in a menu remains optimal in *any* subset of the menu containing the IAA act.[3]

Although WCC is a weak condition, we show that minimax-regret preference (Savage [8]) does not always generate choice functions that satisfy WCC[4]; the consequence is the failure of the revelation principle.

The rest of this note is organized as follows. We outline the model in Section 2. Section 3 presents the main results. We conclude in Section 4 by discussing the weakness/strength of WCC and the implications of our results for partial implementation.

2. Preliminaries

Let $N = \{1, \dots, n\}$ be the set of players. A type of player i, which is privately known, is denoted by t_i and the set of such types is T_i. Let $T = \prod_{i \in N} T_i$ be the type space and t be a type profile. t_{-i} is a type profile of all players other than i and T_{-i} is the set of such profiles.

A is the set of outcomes. ΔA denotes the set of probability measures on A. A (random) *social choice function* (SCF) is a function $f : T \to \Delta A$.

An *interim Anscombe–Aumann act* (IAA act) for player i is a function $f_i : T_{-i} \to \Delta A$. Let $\mathcal{F}_i$ be the set of all IAA acts for player i. A *menu* for player i, F_i, is a subset of $\mathcal{F}_i$.

Each type t_i of each player i has a preference relation $\succcurlyeq_{t_i}^{F_i}$ over each menu F_i.[5] Let $\succ_{t_i}^{F_i}$ and $\sim_{t_i}^{F_i}$ be, respectively, the strict preference and indifference relations derived from $\succcurlyeq_{t_i}^{F_i}$. Let $\succcurlyeq_{t_i} = (\succcurlyeq_{t_i}^{F_i})_{F_i \subseteq \mathcal{F}_i}$ and $\succcurlyeq_i = (\succcurlyeq_{t_i})_{t_i \in T_i}$.

We call $\mathcal{E} = [N, T, A, (\succcurlyeq_i)_{i \in N}]$ to be the *environment*, which is common knowledge.

The *choice function* $C_{\succcurlyeq_{t_i}}$ generated by $\succcurlyeq_{t_i}$ specifies the set of IAA acts that are optimal in any menu, i.e., for any $F_i \subseteq \mathcal{F}_i$, $C_{\succcurlyeq_{t_i}}(F_i) = \{f_i \in F_i \mid f_i \succcurlyeq_{t_i}^{F_i} f_i', \ \forall f_i' \in F_i\}$.

A *mechanism* $\Gamma = ((M_i)_{i \in N}, g)$ defines the set of messages M_i available to each player and the outcome $g : \prod_{i \in N} M_i \to \Delta A$ associated with each message profile. Let Σ_i be the set of strategies $\sigma_i : T_i \to M_i$ of player i. Then, $\sigma = (\sigma_i)_{i \in N}$ is a strategy profile. Note that $g(\sigma)$ is an SCF. Let $\Lambda^\Gamma = [\mathcal{E}, g, (\Sigma_i)_{i \in N}]$ be the game induced by the mechanism Γ in $\mathcal{E}$.

A *direct mechanism* is such that $M_i = T_i$, $\forall i \in N$. We identify a direct mechanism $((T_i)_{i \in N}, f)$ by its outcome function f, which is an SCF. Let Λ^f be the game induced by the direct mechanism f in $\mathcal{E}$. The strategy of player i in a direct mechanism is a function $\psi_i : T_i \to T_i$. Let ψ_i^* be the truthful strategy, i.e., $\psi_i^*(t_i) = t_i$, $\forall t_i \in T_i$.

2.1. Equilibrium

Consider Λ^Γ. Given a profile of the other players' strategies σ_{-i}, if any type of player i sends the message m_i, the outcome is an IAA act $g(m_i, \sigma_{-i})$. Hence, the menu available to each type of player i when others play σ_{-i}, denoted by $F_i(g, \sigma_{-i})$, is equal to $\{g(m_i, \sigma_{-i}) \mid m_i \in M_i\}$ since

[3] Contraction consistency, also known as Property α, was originally introduced by Chernoff [2].

[4] It is known that the choice function generated by minimax-regret preference relation does not satisfy contraction consistency. See [2] for an example.

[5] No assumptions are imposed on this preference relation. Hence, the choice set from a menu could be empty.

the latter is the set of IAA acts that she can induce by unilaterally varying her message. The following definition of equilibrium is then natural in this setup.

Definition 2.1. A strategy profile σ^* is an *equilibrium* of Λ^Γ if $\forall i \in N$ and $\forall t_i \in T_i$,

$$g\big(\sigma_i^*(t_i), \sigma_{-i}^*\big) \in C_{\succcurlyeq_{t_i}}\big(F_i\big(g, \sigma_{-i}^*\big)\big).$$

Thus, σ^* is an equilibrium of Λ^Γ if for each type t_i of each player i, $g(\sigma_i^*(t_i), \sigma_{-i}^*)$ is optimal in the menu $F_i(g, \sigma_{-i}^*)$ that is available to her when others play according to σ_{-i}^*.

2.2. Revelation principle

Fix an environment $\mathcal{E}$. The *revelation principle* states that for every mechanism Γ and for every equilibrium outcome $g(\sigma^*)$ of Λ^Γ, there exists a direct mechanism f which induces a game Λ^f with an outcome-equivalent equilibrium in which all players report their types truthfully.[6] That is, if $g(\sigma^*)$ is an equilibrium outcome of some mechanism Γ, then it must be that the truthful strategy profile $(\psi_i^*)_{i\in N}$ is an equilibrium of the direct mechanism $f = g(\sigma^*)$.

$(\psi_i^*)_{i\in N}$ is an equilibrium of the direct mechanism $f = g(\sigma^*)$ if and only if $\forall i \in N$ and $\forall t_i \in T_i$,

$$f\big(t_i, \psi_{-i}^*\big) = g\big(\sigma_i^*(t_i), \sigma_{-i}^*\big) \in C_{\succcurlyeq_{t_i}}\big(F_i\big(f, \psi_{-i}^*\big)\big),$$

where $F_i(f, \psi_{-i}^*) = \{f(t_i', \psi_{-i}^*) \mid t_i' \in T_i\} = \{g(\sigma_i^*(t_i'), \sigma_{-i}^*) \mid t_i' \in T_i\}$ is the menu available to type t_i of player i when other players play according to ψ_{-i}^* in the direct mechanism f.

Since $\sigma_i^*(t_i') \in M_i$ for all $t_i' \in T_i$, we have $F_i(f, \psi_{-i}^*) \subseteq F_i(g, \sigma_{-i}^*)$, i.e., the menu available to type t_i of player i contracts in going from the equilibrium σ^* of the mechanism Γ to the truthful equilibrium of the outcome-equivalent direct mechanism $f = g(\sigma^*)$. Furthermore, the contracted menu $F_i(f, \psi_{-i}^*)$ is such that it can be mapped onto by T_i. As we show below, consistency of choice with respect to this contraction in the menus is crucial for the success or failure of the revelation principle.

3. Main results

We now characterize the domain of preferences where the revelation principle holds.

For any SCF f, let f_{t_i} be the *IAA act assigned by f to t_i*, i.e., $f_{t_i}(t_{-i}) = f(t_i, t_{-i})$, $\forall t_{-i} \in T_{-i}$. Then the *menu assigned by f to player i* is $\{f_{t_i'} \mid t_i' \in T_i\}$.

The characterization says that the revelation principle holds if and only if the preferences are such that whenever there exist a type t_i, a menu F_i, a contracted menu $\hat{F}_i \subseteq F_i$ that can be mapped onto by T_i, and an IAA act f_i that is chosen by t_i in F_i but not in $\hat{F}_i$, then it must be that for any SCF f that assigns the menu $\hat{F}_i$ to player i and the IAA act f_i to t_i, there exists either (1) another type t_i' of player i for whom the IAA act assigned by f to her is not optimal in F_i or (2) another player j such that for any menu F_j that contains the menu assigned by f to player j, there exists a type t_j for whom the IAA act assigned by f to her is not optimal in F_j. Formally, we have the following theorem:

[6] Two SCFs f and $\hat{f}$ are outcome equivalent if and only if $f(t) = \hat{f}(t)$, $\forall t \in T$. Similarly, for any two IAA acts $f_i = \hat{f}_i$ if and only if $f_i(t_{-i}) = \hat{f}_i(t_{-i})$, $\forall t_{-i} \in T_{-i}$.

Theorem 3.1. *The revelation principle holds in an environment $\mathcal{E}$ if and only if $(\succcurlyeq_i)_{i\in N}$ is such that $\forall i \in N$, $\forall F_i \subseteq \mathcal{F}_i$, $\forall \hat{F}_i \subseteq F_i$ such that there exists a surjective map $h: T_i \to \hat{F}_i$, and $\forall f_i \in \hat{F}_i$ such that $f_i \in C_{\succcurlyeq_{t_i}}(F_i)$ but $f_i \notin C_{\succcurlyeq_{t_i}}(\hat{F}_i)$ for some $t_i \in T_i$, at least one of the following conditions is satisfied $\forall f$ such that $\{f_{t_i'} \mid t_i' \in T_i\} = \hat{F}_i$ and $f_{t_i} = f_i$:*

1. $\exists t_i' \neq t_i$ *such that* $f_{t_i'} \notin C_{\succcurlyeq_{t_i'}}(F_i)$ *or*
2. $\exists j \neq i$ *such that* $\forall F_j \supseteq \{f_{t_j'} \mid t_j' \in T_j\}$, $\exists t_j$ *such that* $f_{t_j} \notin C_{\succcurlyeq_{t_j}}(F_j)$.

Proof. We first show that the revelation principle holds if the condition is satisfied. Let σ^* be an equilibrium of some mechanism Γ. Thus, $g(\sigma_j^*(t_j), \sigma_{-j}^*) \in C_{\succcurlyeq_{t_j}}(F_j(g, \sigma_{-j}^*))$ for all t_j and all j. We need to show that for all t_j and all j, $g(\sigma_j^*(t_j), \sigma_{-j}^*) \in C_{\succcurlyeq_{t_j}}(F_j(f, \psi_{-j}^*))$, where $f = g(\sigma^*)$. Suppose not, i.e., there exists t_i such that $g(\sigma_i^*(t_i), \sigma_{-i}^*) \notin C_{\succcurlyeq_{t_i}}(F_i(f, \psi_{-i}^*))$. Now, $F_i(f, \psi_{-i}^*) \subseteq F_i(g, \sigma_{-i}^*)$, there exists a surjective map from T_i to $F_i(f, \psi_{-i}^*)$ and $g(\sigma_i^*(t_i), \sigma_{-i}^*) \in F_i(f, \psi_{-i}^*)$ is such that $g(\sigma_i^*(t_i), \sigma_{-i}^*) \in C_{\succcurlyeq_{t_i}}(F_i(g, \sigma_{-i}^*))$ but $g(\sigma_i^*(t_i), \sigma_{-i}^*) \notin C_{\succcurlyeq_{t_i}}(F_i(f, \psi_{-i}^*))$. Hence, at least one of the two conditions must hold for $f = g(\sigma^*)$ because $\{f_{t_i'} \mid t_i' \in T_i\} = F_i(f, \psi_{-i}^*)$ and $f_{t_i} = g(\sigma_i^*(t_i), \sigma_{-i}^*)$. But this is not the case since for all $t_i' \neq t_i$, $f_{t_i'} = g(\sigma_i^*(t_i'), \sigma_{-i}^*) \in C_{\succcurlyeq_{t_i'}}(F_i(g, \sigma_{-i}^*))$ whereas for all $j \neq i$, there exists $F_j(g, \sigma_{-j}^*)$ such that $\{f_{t_j'} \mid t_j' \in T_j\} = F_j(f, \psi_{-j}^*) \subseteq F_j(g, \sigma_{-j}^*)$ and $f_{t_j} = g(\sigma_j^*(t_j), \sigma_{-j}^*) \in C_{\succcurlyeq_{t_j}}(F_j(g, \sigma_{-j}^*))$ for all t_j.

Next we argue the necessity of the condition for the revelation principle. Suppose the condition is not satisfied, i.e., there exist an $i \in N$, a menu $F_i \subseteq \mathcal{F}_i$, an $\hat{F}_i \subseteq F_i$ such that there exists a surjective map $h: T_i \to \hat{F}_i$, an $f_i \in \hat{F}_i$ such that $f_i \in C_{\succcurlyeq_{t_i}}(F_i)$ but $f_i \notin C_{\succcurlyeq_{t_i}}(\hat{F}_i)$ for some $t_i \in T_i$ and an f such that $\{f_{t_i'} \mid t_i' \in T_i\} = \hat{F}_i$ and $f_{t_i} = f_i$, and the following hold:

1. for all $t_i' \neq t_i$, we have $f_{t_i'} \in C_{\succcurlyeq_{t_i'}}(F_i)$ and
2. for all $j \neq i$, there exists an $F_j \supseteq \{f_{t_j'} \mid t_j' \in T_j\}$ such that $f_{t_j} \in C_{\succcurlyeq_{t_j}}(F_j)$ for all t_j.

Pick any $a \in A$. Define mechanism Γ^* with $M_j = T_j \cup F_j \setminus \{f_{t_j'} \mid t_j' \in T_j\}$, $\forall j \in N$, and

$$g(m_1, \dots, m_n) = \begin{cases} f(t_1', \dots, t_n'), & \text{if } m_j = t_j',\ \forall j \in N, \\ f_j(t_{-j}'), & \text{if } m_k = t_k',\ \forall k \neq j, \\ & \text{and } m_j = f_j \in F_j \setminus \{f_{t_j'} \mid t_j' \in T_j\}, \\ a, & \text{otherwise.} \end{cases}$$

Consider σ^* such that $\sigma_j^*(t_j) = t_j$, $\forall t_j \in T_j$, $\forall j \in N$. Given σ_{-j}^*, any type t_j of player j can induce either any $f_{t_j'}$ by announcing t_j' or any $f_j \in F_j \setminus \{f_{t_j'} \mid t_j' \in T_j\}$ by announcing such f_j; but she cannot induce the outcome a. Thus, $F_j(g, \sigma_{-j}^*) = F_j$, $\forall j \in N$. But for all j and t_j, we have $g(\sigma_j^*(t_j), \sigma_{-j}^*) = f_{t_j} \in C_{\succcurlyeq_{t_j}}(F_j(g, \sigma_{-j}^*))$. Thus, σ^* is an equilibrium of Λ^{Γ^*}.

The equilibrium outcome is $g(\sigma^*) = f$. If the revelation principle holds, then $(\psi_j^*)_{j\in N}$ must be an equilibrium of Λ^f. Now, the menu $F_i(f, \psi_{-i}^*) = \{f_{t_i'} \mid t_i' \in T_i\} = \hat{F}_i$. However, $f(t_i, \psi_{-i}^*) = f_{t_i} = f_i \notin C_{\succcurlyeq_{t_i}}(F_i(f, \psi_{-i}^*))$. Thus, the revelation principle fails. $\square$

3.1. Weak contraction consistency

Definition 3.2. $C_{\succcurlyeq_{t_i}}$ satisfies *weak contraction consistency* (WCC) if $\forall F_i \subseteq \mathcal{F}_i$ and $\forall \hat{F}_i \subseteq F_i$ such that there exists a surjective map from T_i to $\hat{F}_i$, we have

$$f_i \in C_{\succcurlyeq_{t_i}}(F_i) \quad \text{and} \quad f_i \in \hat{F}_i \quad \Longrightarrow \quad f_i \in C_{\succcurlyeq_{t_i}}(\hat{F}_i).$$

Thus, WCC requires that if f_i is optimal for t_i in menu F_i, then f_i must remain optimal for her in those subsets $\hat{F}_i$ of F_i that contain f_i and which can be mapped onto by T_i.

It is straightforward to see that if $C_{\succcurlyeq_{t_i}}$ satisfies WCC for all t_i and all i, then the condition in Theorem 3.1 is vacuously satisfied and hence the revelation principle will hold in such an environment. We note this as a corollary (proof is omitted).

Corollary 3.3. *If $\mathcal{E}$ is such that $C_{\succcurlyeq_{t_i}}$ satisfies WCC for all $t_i \in T_i$ and $i \in N$, then the revelation principle holds.*

4. Discussion

4.1. Weakness/strength of WCC

WCC is a weak requirement on preferences as we show next.

Definition 4.1. $\succcurlyeq_{t_i}$ is a *menu-independent preference* (MIP) if there exists a binary relation $\tilde{\succcurlyeq}$ over $\mathcal{F}_i$ such that $\forall f_i, f_i' \in \mathcal{F}_i$ and $\forall F_i \subseteq \mathcal{F}_i$ with $f_i, f_i' \in F_i$, we have $f_i \succcurlyeq_{t_i}^{F_i} f_i' \Leftrightarrow f_i \tilde{\succcurlyeq} f_i'$.

Definition 4.2. $C_{\succcurlyeq_{t_i}}$ is a *menu-independent choice function* (MIC) if there exists a binary relation $\tilde{\succcurlyeq}$ over $\mathcal{F}_i$ such that $\forall F_i \subseteq \mathcal{F}_i$, we have $C_{\succcurlyeq_{t_i}}(F_i) = \{f_i \in F_i \mid f_i \tilde{\succcurlyeq} f_i',\ \forall f_i' \in F_i\}$.

Definition 4.3. $C_{\succcurlyeq_{t_i}}$ satisfies *contraction consistency* (CC) if $\forall F_i \subseteq \mathcal{F}_i$ and $\forall \hat{F}_i \subseteq F_i$, we have $f_i \in C_{\succcurlyeq_{t_i}}(F_i)$ and $f_i \in \hat{F}_i \Rightarrow f_i \in C_{\succcurlyeq_{t_i}}(\hat{F}_i)$.

The following observation follows immediately from the definitions (proof is omitted).

Observation 4.4. $\succcurlyeq_{t_i}$ is MIP $\Rightarrow C_{\succcurlyeq_{t_i}}$ is MIC $\Rightarrow C_{\succcurlyeq_{t_i}}$ satisfies CC $\Rightarrow C_{\succcurlyeq_{t_i}}$ satisfies WCC.

Moreover, as we show below, the converse of any of the above implications is not true.

Example 4.5. Suppose $N = \{1, 2\}$, $T_1 = \{t_1^1, t_1^2\}$, $T_2 = \{t_2\}$ and $A = \{a, b\}$. For any IAA act for player 1, f_1, let $a(f_1)$ be the probability of outcome a. Let $\tilde{\succcurlyeq}$ be a binary relation on $\mathcal{F}_1$ such that $f_1 \tilde{\succcurlyeq} f_1' \Leftrightarrow a(f_1) \geqslant a(f_1')$. Let menu $F_1' = \{f_1^1, f_1^2, f_1^3\}$ be such that $a(f_1^1) = 1$, $a(f_1^2) = \frac{1}{2}$ and $a(f_1^3) = 0$. Suppose $\succcurlyeq_{t_1^1}$ is such that for F_1', we have $f_1^1 \succ_{t_1^1}^{F_1'} f_1^3 \succ_{t_1^1}^{F_1'} f_1^2$ whereas for any menu $F_1 \neq F_1'$ and any $f_1, f_1' \in F_1$, we have $f_1 \succcurlyeq_{t_1^1}^{F_1} f_1' \Leftrightarrow a(f_1) \geqslant a(f_1')$. $\succcurlyeq_{t_1^1}$ is not MIP since in F_1', we have $f_1^3 \succ_{t_1^1}^{F_1'} f_1^2$ whereas in $F_1'' = \{f_1^2, f_1^3\}$, we have $f_1^2 \succ_{t_1^1}^{F_1''} f_1^3$. However, $C_{\succcurlyeq_{t_1^1}}$ is MIC since for any F_1, $C_{\succcurlyeq_{t_1^1}}(F_1) = \{f_1 \in F_1 \mid f_1 \tilde{\succcurlyeq} f_1',\ \forall f_1' \in F_1\}$.

Example 4.6. Reconsider the setup of the previous example. Now suppose $\succcurlyeq_{t_1^1}$ is such that for any $F_1 \neq \{f_1^1, f_1^2\}$, we have $C_{\succcurlyeq_{t_1^1}}(F_1) = \{f_1 \in F_1 \mid a(f_1) \geqslant a(f_1'),\ \forall f_1' \in F_1\}$ but $C_{\succcurlyeq_{t_1^1}}(\{f_1^1, f_1^2\}) = \{f_1^1, f_1^2\}$. $C_{\succcurlyeq_{t_1^1}}$ is not MIC; otherwise, there would exist a binary relation $\tilde{\succcurlyeq}$ on $\mathcal{F}_1$ such that $f_1^1 \tilde{\succ} f_1^2$ since $C_{\succcurlyeq_{t_1^1}}(\{f_1^1, f_1^2, f_1^3\}) = \{f_1^1\}$ and $C_{\succcurlyeq_{t_1^1}}(\{f_1^2, f_1^3\}) = \{f_1^2\}$, and at the same time $f_1^2 \tilde{\succcurlyeq} f_1^1$ since $C_{\succcurlyeq_{t_1^1}}(\{f_1^1, f_1^2\}) = \{f_1^1, f_1^2\}$. However, $C_{\succcurlyeq_{t_1^1}}$ clearly satisfies CC.

Example 4.7. Suppose player i has two types, $T_i = \{t_i^1, t_i^2\}$. For each t_i^k, there exists a function $u_{t_i^k} : \mathcal{F}_i \to \Re$ such that for any menu F_i and for all $f_i, f_i' \in F_i$, we have

$$f_i \succcurlyeq_{t_i^k}^{F_i} f_i' \quad \Longleftrightarrow \quad \left|u_{t_i^k}(f_i) - \mu_{t_i^k}(F_i)\right| \leqslant \left|u_{t_i^k}\big(f_i'\big) - \mu_{t_i^k}(F_i)\right|,$$

where $\mu_{t_i^k}(F_i) = \frac{1}{2}(\inf_{f_i'' \in F_i} u_{t_i^k}(f_i'') + \sup_{f_i'' \in F_i} u_{t_i^k}(f_i''))$. Intuitively, $\mu_{t_i^k}(F_i)$ is the "mid-point" of the menu F_i. The preference over any menu is such that the further away an IAA act is from the "mid-point" of the menu, the less it is liked by t_i^k. Hence, each type of player i displays "extremeness aversion" (see Simonson and Tversky [13]).

Consider any menu $\hat{F}_i$ that has at most two IAA acts. Clearly, $C_{\succcurlyeq_{t_i^k}}(\hat{F}_i) = \hat{F}_i$. Since player i has two types, it follows that $C_{\succcurlyeq_{t_i^k}}$ satisfies WCC.

However, $C_{\succcurlyeq_{t_i^k}}$ does not satisfy CC if there exists a menu $F_i = \{f_i^1, f_i^2, f_i^3, f_i^4\}$ such that $u_{t_i^k}(f_i^1) > u_{t_i^k}(f_i^2) > u_{t_i^k}(f_i^3) > u_{t_i^k}(f_i^4)$. In this menu, either $f_i^2 \in C_{\succcurlyeq_{t_i^k}}(F_i)$ or $f_i^3 \in C_{\succcurlyeq_{t_i^k}}(F_i)$. If $f_i^2 \in C_{\succcurlyeq_{t_i^k}}(F_i)$, then consider $\hat{F}_i' = \{f_i^2, f_i^3, f_i^4\}$. We have $C_{\succcurlyeq_{t_i^k}}(\hat{F}_i') = \{f_i^3\}$, which is a violation of CC. If $f_i^3 \in C_{\succcurlyeq_{t_i^k}}(F_i)$, then consider $\hat{F}_i'' = \{f_i^1, f_i^2, f_i^3\}$. We have $C_{\succcurlyeq_{t_i^k}}(\hat{F}_i'') = \{f_i^2\}$, which is again a violation of CC.

Thus, the revelation principle holds for a rich class of preferences that could be menu dependent but generate choice functions satisfying WCC. Nevertheless, as we show next, an important menu-dependent preference, minimax-regret, does not always generate choice functions that satisfy WCC, which causes the revelation principle to fail.

Example 4.8. According to minimax regret, each type of a player chooses the alternative that minimizes her maximum regret. Regret of choosing an alternative in a state is defined as the difference between the payoff that is obtained and the maximum payoff that could have been attained in that state. Maximum regret of choosing an alternative is the maximum of these differences over all states. Thus, $\succcurlyeq_i$ is a *minimax-regret preference* if there exists a $u_i : A \times T \to \Re$ such that $\forall t_i \in T_i$, $\forall f_i, f_i' \in \mathcal{F}_i$ and $\forall F_i \subseteq \mathcal{F}_i$ with $f_i, f_i' \in F$, we have

$$\begin{aligned} & f_i \succcurlyeq_{t_i}^{F_i} f_i' \\ & \quad \Longleftrightarrow \quad \sup_{t_{-i} \in T_{-i}} \left[\sup_{f_i'' \in F_i} \int_{a \in A} u_i(a, t_i, t_{-i})\, df_i''(t_{-i}) - \int_{a \in A} u_i(a, t_i, t_{-i})\, df_i(t_{-i}) \right] \\ & \qquad\qquad \leqslant \sup_{t_{-i} \in T_{-i}} \left[\sup_{f_i'' \in F} \int_{a \in A} u_i(a, t_i, t_{-i})\, df_i''(t_{-i}) - \int_{a \in A} u_i(a, t_i, t_{-i})\, df_i'(t_{-i}) \right]. \end{aligned}$$

Minimax-regret preference need not satisfy WCC. Consider the following example. Suppose $N = \{1,2\}$, $T_1 = \{t_1^1, t_1^2\}$, $T_2 = \{t_2^1, t_2^2\}$ and $A = \{a,b,c\}$.

Player 1 has the minimax-regret preference with $u_1(a, t_1^1, t_2) = 0$, $u_1(b, t_1^1, t_2) = 1$ and $u_1(c, t_1^1, t_2) = 2$, $\forall t_2 \in T_2$, and $u_1(a, t_1^2, t_2) = 0.5$, $u_1(b, t_1^2, t_2) = 1$ and $u_1(c, t_1^2, t_2) = 0$, $\forall t_2 \in T_2$.

Player 2 also has the minimax-regret preference with $u_2(a, t_1, t_2^1) = 1$, $u_2(b, t_1, t_2^1) = 0$ and $u_2(c, t_1, t_2^1) = 3$, $\forall t_1 \in T_1$, and $u_2(a, t_1, t_2^2) = 0$, $u_2(b, t_1, t_2^2) = 1$, $u_2(c, t_1, t_2^2) = 2$, $\forall t_1 \in T_1$.

Pick the menu $F_1 = \{f_1^1, f_1^2, f_1^3\}$ of IAA acts for player 1, where

$$f_1^1(t_2^1) = a, \quad f_1^1(t_2^2) = b; \qquad f_1^2(t_2^1) = b, \quad f_1^2(t_2^2) = b;$$
$$f_1^3(t_2^1) = a, \quad f_1^3(t_2^2) = c.$$

Consider type t_1^1 of player 1. $\forall f_1^k \in F_1$, we have

$$\max_{t_2 \in T_2}\Big[\max_{f_1'' \in F_1} u_1\big(f_1''(t_2), t_1^1, t_2\big) - u_1\big(f_1^k(t_2), t_1^1, t_2\big)\Big] = 1.$$

Therefore, $f_1^1 \succcurlyeq_{t_1^1}^{F_1} f_1^k$, $\forall f_1^k \in F_1$, and hence, $f_1^1 \in C_{\succcurlyeq_{t_1^1}}(F_1)$.

Now, consider the menu $\hat{F}_1 = \{f_1^1, f_1^2\} \subset F_1$, which clearly can be mapped onto by T_1. In $\hat{F}_1$, we have

$$\max_{t_2 \in T_2}\Big[\max_{f_1'' \in \hat{F}_1} u_1\big(f_1''(t_2), t_1^1, t_2\big) - u_1\big(f_1^1(t_2), t_1^1, t_2\big)\Big] = 1,$$

$$\max_{t_2 \in T_2}\Big[\max_{f_1'' \in \hat{F}_1} u_1\big(f_1''(t_2), t_1^1, t_2\big) - u_1\big(f_1^2(t_2), t_1^1, t_2\big)\Big] = 0.$$

Thus, $f_1^2 \succ_{t_1^1}^{\hat{F}_1} f_1^1$, and hence, $f_1^1 \notin C_{\succcurlyeq_{t_1^1}}(\hat{F}_1)$. This is a violation of WCC.

The revelation principle fails in this example. Consider the SCF f shown below:

	t_2^1	t_2^2
t_1^1	a	b
t_1^2	b	b

We have a menu F_1, a contracted menu $\hat{F}_1 \subseteq F_1$ that can be mapped onto by T_1, an IAA act f_1^1 that is optimal for t_1^1 in F_1 but not in $\hat{F}_1$, and SCF f such that $\{f_{t_1} \mid t_1 \in T_1\} = \hat{F}_1$ and $f_{t_1^1} = f_1^1$. However, none of the two conditions in Theorem 3.1 holds. For type t_1^2, we have $f_{t_1^2} = f_1^2 \in C_{\succcurlyeq_{t_1^2}}(F_1)$ since $u_1(b, t_1^2, t_2^1) > u_1(a, t_1^2, t_2^1)$ and $u_1(b, t_1^2, t_2^2) > u_1(c, t_1^2, t_2^2)$. On the other hand, consider the menu $F_2 = \{f_{t_2} \mid t_2 \in T_2\}$ for player 2. For type t_2^1, we have $f_{t_2^1} \in C_{\succcurlyeq_{t_2^1}}(F_2)$ since $u_2(a, t_1^1, t_2^1) > u_2(b, t_1^1, t_2^1)$, and for type t_2^2, we have $f_{t_2^2} \in C_{\succcurlyeq_{t_2^2}}(F_2)$ since $u_2(b, t_1^1, t_2^2) > u_2(a, t_1^1, t_2^2)$. Thus, the necessary condition for the revelation principle fails.

4.2. Partial implementation

An SCF f is *partially implementable* if $f = g(\sigma^*)$, where σ^* is an equilibrium of some mechanism. An SCF f is *incentive compatible* if $(\psi_i^*)_{i \in N}$ is an equilibrium of the direct mechanism f. If the revelation principle holds, then we can confirm whether an SCF is partially implementable or not by simply checking if it is incentive compatible or not. It follows from Corollary 3.3 that for the purpose of partial implementation, we can allow players with richer behavioral motivations than the standard expected-utility maximization.

On the other hand, the failure of the revelation principle for some menu-dependent preferences offers a new set of challenges for mechanism design. In such cases, even for partial implementation, we have to search in the set of indirect mechanisms. Nevertheless, as the following result shows, this search can be restricted to the set of all menu profiles.

Proposition 4.9. *f is partially implementable in $\mathcal{E}$ if and only if there exists a profile of menus $(F_1, \ldots, F_n)$ such that $\forall i \in N$, $F_i \supseteq \{f_{t_i'} \mid t_i' \in T_i\}$ and $f_{t_i} \in C_{\succcurlyeq_{t_i}}(F_i)$, $\forall t_i \in T_i$.*

Proof. If such a profile of menus exists, then we can partially implement f in equilibrium σ^* of mechanism Γ^* defined in the proof of Theorem 3.1. Whereas, if f is partially implementable in equilibrium $\hat{\sigma}$ of some Γ, then there exists such a profile of menus with $F_i = F_i(g, \hat{\sigma}_{-i})$. $\square$

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