Theory of Mechanism Design
Assignment 3
Due: 5, October, 2017.

1. Consider a model with three alternatives $\{a, b, c\}$ and one agent. The type of the agent is a vector in $\mathbb{R}^3$. If t is a type of the agent, then for any $a' \in \{a, b, c\}$, let $t(a')$ denote the value of the agent for alternative a' in type t.

   Let $T = \mathbb{R}^3$ be the type space of the agent and $f : T \to \{a, b, c\}$ be an onto implementable allocation rule. Suppose $f(t) = a$, where $t(a) = 3, t(b) = 4, t(c) = 2$.

   Further, let
$$\inf_{s \in T : f(s) = a} [s(a) - s(b)] = -1.$$

   Find the alternative chosen by f at the following two types:

   (a) Let t^1 be a type such that $t^1(a) = 3, t^1(b) = 0, t^1(c) = 1$. What is $f(t^1)$?.

   (b) Let t^2 be a type such that $t^2(a) = 1, t^2(b) = 3, t^2(c) = \alpha$. Find a value of α for which $f(t_2) = b$?

2. A seller has two units of an object to sell to a single agent. He can either decide not to allocate any object or allocate one unit or allocate both the units to the agent. The type of the agent is captured by its marginal value for the units - $t = (t_1, t_2)$, where t_1 is the value for unit 1 and t_2 is the marginal value for the second unit, i.e., if he gets both the units then his value is $t_1 + t_2$. Assume that the value for zero units is zero.

   The agent can have three possible types $T = \{\theta^0, \theta^1, \theta^2\}$, where $\theta^0 = (55, 15), \theta^1 = (60, 25), \theta^2 = (40, 35)$. Consider the allocation rule $f : T \to \{0, 1, 2\}$ (it decides how many units to allocate):
$$f(\theta^0) = 0, f(\theta^1) = 1, f(\theta^2) = 2.$$

   Is f implementable? Explain your answer.

3. Consider the *binary* public good provision problem discussed in the class for quasilinear type space. Extend the pivotal mechanism when type space is classical (not necessarily quasilinear).

4. Consider the Vickrey mechanism for classical type space discussed in the class. The only information that such a mechanism considers is the willingness to pay of an agent at zero transfer level. Can you generalize the Vickrey mechanism (a la Groves mechanism), where you consider for every agent i, an arbitrary function $h_i : \mathcal{U}^{n-1} \to \mathbb{R}$, where $\mathcal{U}$ is

the classical type space, and take into account willingness to pay of agent i at $h_i(u_{-i})$ for every u_{-i}?