Theory of Mechanism Design

Assignment on multidimensional mechanism design

1. Consider the single agent problem with a finite set of alternatives A. Let $T = \mathbb{R}^{|A|}$ be the type space (i.e., every vector in $\mathbb{R}^{|A|}$ belongs to the type space). Let $f : T \to A$ be an onto deterministic allocation rule. Suppose f is monotone, i.e., for every $s, t \in T$, we have $s(f(s)) - s(f(t)) \geq t(f(s)) - t(f(t))$. For every $a \in A$, define $T_a^f := \{t \in T : f(t) = a\}$ and $d^f(a, b) := \inf_{t \in T_a^f} [t(a) - t(b)]$.

   Show the following.

   (a) For every $a, b \in A$, we have $d^f(a, b) + d^f(b, a) = 0$.

   (b) f is implementable (this will prove that in $T = \mathbb{R}^{|A|}$ with deterministic f, monotonicity implies cycle monotonicity).

2. Consider a problem with n agents. Let $T = \mathbb{R}^{|A|}$ be the type space of each agent and let $f : T^n \to A$ be a deterministic allocation rule. Suppose f is monotone.

   We denote a type profile by $\mathbf{t} \equiv (t_1, \ldots, t_n)$. We say f satisfies **positive association of differences (PAD)** if for every $\mathbf{t}$ with $f(\mathbf{t}) = a$ and for every $\mathbf{s}$ satisfying

$$s_i(a) - t_i(a) > s_i(b) - t_i(b) \qquad \forall\, b \neq a,\ \forall\, i \in N,$$

   we have $f(\mathbf{s}) = a$. Show that f satisfies PAD. [1]

3. Consider a problem with n agents. Let $T \subseteq \mathbb{R}^{|A|}$ be *any* type space. Let $f^e : T \to A$ be an **efficient** allocation rule, i.e., for every $\mathbf{t} \in T^n$, we have

$$f^e(\mathbf{t}) \in \arg\max_{a \in A} \Big[\sum_{i \in N} t_i(a) \Big].$$

   Show that f^e satisfies cycle monotonicity (**do not** use the fact that f^e is implementable using Groves payments).

[1] Roberts (1979) showed in his work that every f satisfying PAD must be an affine maximizer if $|A| \geq 3$. We know that *almost* every affine maximizer is implementable.