

Auctions with Contingent Payments - an Overview.

Andrzej Skrzypacz*

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Abstract

I survey a literature on auctions with contingent payments, that is auctions in which payments are allowed to depend on an ex-post verifiable variable, such as revenues in oil lease auctions. Based on DeMarzo, Kremer and Skrzypacz (2005), I describe a partial ranking of auction revenues for auctions that differ in terms of contract forms, pricing rules and seller commitment and why the Revenue Equivalence Theorem does not apply even in an independent private values setup. I discuss models that incorporate adverse selection, moral hazard, competition between auctioneers, common values and the sale of multiple units.

1 Introduction

In some auctions the value of the good being auctioned is completely subjective. For example, in a charity auction for a dinner with a local celebrity, the value to bidders cannot be observed. In other situations while values/costs are objective, they cannot be easily verified. For example, in auctions selecting contractors to repair a highway, the costs of completing the project are hard to verify. On the other hand, in many commercial settings, the value of an asset/contract is at least partially observed. For example, in oil-lease auctions, if the winner explores the field, the government can measure revenue obtained from the exploration. It is a common practice around the world for the government selling the rights to drill for oil or natural gas to collect additional revenue in the form of royalties.

*Graduate School of Business, Stanford University (e-mail: skrz@stanford.edu). I thank Peter DeMarzo and Ilan Kremer for inspiration since the core of this paper is based on my joint work with them. I thank Simon Board for many discussions and comments and exposing me to this topic. I also thank Yeon-Koo Che, Will Cong, Alexander Gorbenko, Jinwoo Kim, Vladimir Vladimirov, Elmar Wolfstetter and two anonymous referees for comments and suggestions.

Auctions with contingent payments describe situations where either via a formal auction or informal negotiations a set of players compete to either purchase an asset or obtain a contract to deliver a service and the payments to the auctioneer/seller/procurer are at least partially contingent on future outcomes. Theoretical analysis of such auctions has received a lot of attention in recent years. This paper offers a selected survey of that literature. The core of this paper (including the benchmark model and its analysis presented in Sections 3 and 4) is based on DeMarzo, Kremer and Skrzypacz (2005), henceforth DKS.

The plan of the paper is as follows. I first describe some markets where auctions with contingent payments are common. Then I discuss the benchmark model of DKS with independent private values as in the setup of the revenue equivalence theorem. I explain why that theorem does not apply to auctions with contingent payments and why revenue in such auctions is higher than in cash auctions. Then I discuss the ranking of auctions with different types of contracts if the seller restricts bidders to a single-dimensional set of contracts. Next, I describe a model without seller commitment and explain why in that environment bidders would choose offers that "flat". Combining these two sections leads to a conjecture that sellers prefer auctions with "steep" contracts while buyers prefer auctions with "flat" contracts. In Section 5 I review papers that enrich the benchmark model with important real-life considerations (for example, moral hazard, entry of bidders into auctions, adverse selection of buyers, and private information of the seller). These considerations bring up new economic tradeoffs which change the predictions of the basic model.

This is not a comprehensive survey. There are some topics I do not cover at all. For example, in all of the paper I assume bidders and the seller are risk neutral which ignores the importance of contingent payments for risk sharing.

1.1 Auctions with Contingent Payments in Practice

It is useful to divide the practical examples into two categories: formal and informal auctions. The major difference between these two types is the level of commitment by the seller. In an informal auction, bidders choose what forms of payments to offer and then the seller selects the most attractive offer. Such informal auctions contain the elements of a signaling game because evaluating offers, the seller may infer bidder's private information, and hence the value of the offer, from the type of contract offered by that bidder. For example, an author of a book may be worried about the private information of a publisher who offers a higher-than-usual royalty rate but a lower-than-usual advance.

In a formal auction, the seller restricts bidders to use contracts from a pre-specified ordered

set and commits ex-ante to choose a bidder with the highest bid according to that order. The seller is committed to disqualify any offers outside this set. The seller also commits to an auction format, such as a first-price or a second-price auction.¹

The best known example in the economic literature of formal auctions with contingent payments is the oil and gas lease auctions. See for example Hendricks and Porter (1988), Hendricks, Pinkse, and Porter (2003) and Haile, Hendricks and Porter (2010). In the U.S., bidders typically compete in cash for the right to drill oil in some area and if they find oil, they pay $\frac{1}{6}$ th of revenues in royalties. There is a large variation across countries in the royalty rate used and even in the U.S. some fields are sold with different rates than $\frac{1}{6}$ (for example, $\frac{1}{8}$ has been employed in areas perceived as risky). The U.S. Mineral Management Service experimented in the 1970s with other auction formats, some of them allowing bidders to compete on royalty rates and others allowing the bidders to deduct estimated costs before royalties were computed. Additionally, the 1995 Deepwater Royalty Relief Act exempted bidders for deep water tracks from royalty payments on production up to a cap. From the point of economic analysis, auctions to allocate a limited number of business licenses (for example, electronic gambling machines), are similar to the royalty auctions because future profits from the use of these licenses are usually taxed at some pre-specified rate. In both cases the level of royalties/taxes affects bidding and overall revenue in these auctions.

In procurement auctions, it is common to use incentive contracts that speculate a pre-specified cost-sharing rule for cost over-runs. For example, as described in McAfee and McMillan (1988), in the 1980s and 1970s, the U.S. military used auctions with such incentive contracts with cost-sharing parameter varying between 0.5 and 0.9, with 0.8 being typical.

Another example of formal auctions is auctions to select a lead-plaintiff in class-action suits and to determine a formula for setting legal fees (see Fish 2001 and 2002). In some build-operate-transfer highway construction contracts seller revenue/cost depend on the terms offered by the bidders.² When the FCC ran auctions for wireless spectrum licenses with preferences for small businesses (for example, FCC Auctions 5 and 10); winners did not have to pay immediately but in installments. Formally, their bids were debt obligations and the debt was to a large extent secured by the licenses won. Indeed, many of the winners defaulted on the payments later and the licenses were re-auctioned (the most prominent example is

¹The seller commits to no renegotiation of the terms of the awarded contract (renegotiations sometimes do happen in the oil and gas lease contracts). Also, the seller commits not to contract with losing or non-participating bidders, a problem discussed in Ding and Wolfstetter (2011).

²Moreover, in such auctions the government commonly offers guarantees to the winners and renegotiations are common (see Engel, Fischer and Galetovic 1997 and 2003), making these auctions to be somewhat between the extremes of formal and informal auctions.

NextWave, see Zheng 2001 and Board 2007b).

Finally, in many online advertising auctions bidders pay per click or per conversion. If the seller and bidders are equally informed about the click or conversion rates, as assumed in the main papers on this topic, then the information linkage discussed in this paper is not present. Symmetric information about click rates seems to be a good assumption for publisher/audience/advertiser combinations with large volumes that allow for a quick and precise estimation of these click rates. It is probably less good assumption in low-volume sub-markets.

Informal auctions with contingent payments are common in the private sector. The most studied example is corporate takeovers and intercorporate asset sales (see, among others, Martin 1996, Betton, Eckbo and Thorburn 2008 on the forms of payments, as well as references on stapled finance in Povel and Singh 2010). Firms bidding to acquire another company commonly mix cash and equity in their offers and raise debt backed at least partially by the asset being transacted. Another example in financial markets is entrepreneurs who participate in informal auctions with contingent payments at different stages of the growth of their companies to obtain venture capital/angel funding (see Kaplan and Stromberg 2003 and discussion in Kogan and Morgan 2010).

In the arts/entertainment industries, authors selling publishing rights (Caves 2003) or experienced actors in motion pictures (Chisholm 1997) commonly obtain a contract with some revenue sharing and advance. McMillan (1991) describes that bidders for broadcast rights to Olympic Games used revenue sharing offers.

2 The Model

We start with the benchmark model of auctions with contingent payments presented in DKS. There is a seller and N bidders (all risk-neutral). Bidders are ex-ante symmetric and have independent private values.³ The seller runs an auction for a project that requires the winner to make an up-front investment $X > 0$.⁴ If bidder i wins the project, it generates verifiable revenue/cashflow Z_i . Before the auction nobody knows Z_i 's. Each bidder has private information about his expected cashflow, z_i . The estimates/types, z_i , are distributed independently and symmetrically according to some distribution $f(z_i)$ over a range $[\underline{z}, \bar{z}]$,

³See DKS for discussion of affiliated private values. See Abhishek, Hajek and Williams (2012) for the analysis of second-price auctions with risk-averse bidders.

⁴For example, exploring an oil field requires costly preparation work and operation of a rig. X may be a required cash investment or it may represent the alternative cost of winner's asset/effort necessary for the project to succeed.

where $\underline{z} \geq X$. Conditional on z_i , bidder i cashflow is distributed according to an atomless distribution $h(Z_i|z_i)$.⁵

We assume that $h(Z_i|z_i)$ has full support $[0, \infty)$ and satisfies the strict Monotone Likelihood Ratio Property (SMLRP).⁶ That is, for $z > z'$, the likelihood ratio $h(Z|z)/h(Z|z')$ increases in Z (a higher estimate implies a stronger distribution of cashflow realizations). Normalize $E[Z_i|z_i] = z_i$, so that a bidder type is his expected revenue if he wins.

If the seller runs a standard cash auction, this model is equivalent to the textbook symmetric independent private values model in which bidder valuations are $v_i = z_i - X$.

Throughout the analysis, a (feasible) bid is an offer of a contingent payment to the seller as a function of the realized cashflow, $S(Z_i)$. Canonical examples of such bids are royalty contracts (or equity in a finance application), debt and call option (or royalty rate combined with an advance) and hence we refer to such contingent payments as *contracts* or *securities* or *security bids* (so I also use terms *security-bid auction* or *contract-bid auction*).

We restrict attention to bids that satisfy that $S(Z)$ and $Z - S(Z)$ are increasing and $S(Z) \geq 0$ (so the seller cannot subsidize the bidders).⁷ Some of the analysis also assumes that bidders do not have cash and have limited liability so that $S(Z) \leq Z$.

Define $ES(z) \equiv E[S(Z)|z]$. Our assumptions imply that $ES(z)$ is continuously increasing (SMLRP implies first-order stochastic dominance) and has a slope between 0 and 1 (with the slope being 0 only for $S(Z) = 0$ and slope being 1 only for $S(Z) = Z$).

3 Formal Auctions

A formal auction is described by an *ordered set of contracts/securities* and an *auction format*. The set of allowed contracts \mathbf{S} is indexed by $s \in [s_0, s_1]$. We abuse notation and write $S(s, Z)$ as the ex-post payment of contract with index s if the realized revenue is Z . In the model, the difference between formal and informal auctions is that in informal auctions bidders choose directly contracts, $S(Z)$, from a large feasible set, while in formal auctions bidders choose

⁵The realizations of Z_i can be correlated across bidders if they depend on a common ex-post shock that no bidder has private information about. Independent private values means that the estimates z_i are independent and z_{-i} is not informative about Z_i .

⁶Additionally, $h(Z|z)$ is twice differentiable in both arguments and functions $Zh(Z|z)$, $|zh_z(Z|z)|$, $z|h_{zz}(Z|z)|$ are integrable on the domain of Z , so the expected values and derivatives we discuss are well-defined. An example of such a distribution is $Z_i = \theta z_i$ where θ is a log-normally distributed random variable with mean 1.

⁷These are standard assumptions in security design literature and are motivated by limited liability and moral hazard considerations. See for example Nachman and Noe (1994) and DeMarzo and Duffie (1999). As discussed below, monotonicity of $Z - S(Z)$ is used to establish existence of a monotone equilibrium.

only an index. The index chosen by a bidder identifies its contract as $S(s, Z)$.

Denote

$$ES(s, z) = E[S(s, Z_i) | z_i = z].$$

as the expected payment of the contract s given bidder type z .

For the set of contracts to be ordered, we require that the expected payout is increasing in s for every z : a bid with a higher index promises a higher payout keeping the type of a bidder fixed. We also require that the allowable contracts (e.g. royalty contracts) can be ranked in a continuous way (condition 1 below) and the space of contracts is rich enough so that a monotone equilibrium can exist (condition 2 below). Formally:

Definition 1 *The function $S(s, Z)$ for $s \in [s_0, s_1]$ defines an ordered set of contracts/securities if:*

1. For all z_i , $\frac{\partial}{\partial s} ES(s, z_i) > 0$ (and the derivative exists).
2. $ES(s_0, \underline{z}) \leq \underline{z} - X$ and $ES(s_1, \bar{z}) \geq \bar{z} - X$.

Examples of ordered sets of contracts/securities include:

- **Cash:** $S(b, Z) = b$, so that $ES(b, z) = b$.
- **Royalty/Equity:** The seller receives some fraction $\alpha \in [0, 1]$ of the revenue: $S(\alpha, Z) = \alpha Z$. In this case $ES(\alpha, z) = \alpha z$.
- **Fixed royalty plus cash bids:** $S(b, Z) = \alpha Z + b$, where α is a fixed royalty rate. In this case $ES(b, z) = \alpha z + b$.
- **Fixed contract plus cash bids:** $S(b, Z) = b + S(Z)$ for some fixed $S(Z)$ the same for all bidders. For example, in a procurement auction, a contract may stipulate a cost-over-run sharing rule for costs over the estimated \$10 million capped at a total of \$15 million. In the auction bidders would compete down a fixed payment for this contract.
- **Debt:** The seller gets the first $d \geq 0$ dollars of the revenue, the rest goes to the buyer: $S(d, Z) = \min\{Z, d\}$.
- **Allowance plus royalty contract:** The seller gets the maximum of a fixed allowance $d \geq 0$ and a fraction $\alpha \in [0, 1]$ of the revenue: $S(s, Z) = \max\{\alpha Z, d\}$. This class of contracts can be turned into an ordered set by either fixing α and letting bidders bid on d or vice versa (one can also pick a one-dimensional family in which both α and d increase with the index s).

- **Royalty contract with a cost deduction:** The seller receives a fraction $\alpha \in [0, 1]$ of the revenues after the buyer is allowed to deduct $d \geq 0$ from them, to allow for costs: $S(s, Z) = \alpha \max\{Z - d, 0\}$. As in the previous example, there are many ways to choose how the index relates to the parameters α and d .
- **Call option:** This is an extreme case of the previous contracts: the seller receives 100% of the revenue over an allowance k for the bidder. In finance language, the seller receives a call option on the revenue with strike price k : $S(-k, z) = \max\{Z - k, 0\}$. Higher bids correspond to lower strike prices/lower allowance for the buyer. This is also equivalent to the bidder retaining a debt claim.

Given an ordered set of contracts, it is straightforward to generalize to our setting standard auction formats: a first-price and a second-price auction. For example, in a second-price auction the bidder with the highest submitted index s_i wins and pays according to the contract offered by the second-highest bidder.

A (pure) strategy of a bidder is a mapping from his type to a chosen index. We abuse notation and denote the strategy as $s(z)$.

The following Lemma 2 in DKS describes equilibrium bidding in second-price auctions:

Lemma 1 (Lemma 2 in DKS) *A formal second-price auction has a unique equilibrium in weakly undominated strategies. The equilibrium bidding strategy $s(z)$ is the unique solution to*

$$ES(s(z), z) = z - X \tag{1}$$

The equilibrium strategy $s(z)$ is increasing.

The proof that this is a dominant strategy is the same as in cash auctions: the equilibrium bid is such that if a bidder wins and has to pay his bid, he breaks even. To see that there is a unique and increasing solution to (1) note that the derivative of the RHS with respect to z is 1 and since we assumed that $Z - S(s, Z)$ is increasing in Z , the derivative of the LHS with respect to z is less than 1. Finally, since we assumed that $ES(s, z)$ is increasing in s , as z goes up, s needs to increase for (1) to continue to hold.⁸ This reasoning illustrates the role played by our assumption that the set of contracts is ordered. If it was not, i.e. if $ES(s, z)$ was decreasing in s for some z , an increasing equilibrium may fail to exist.

Before we present general results, consider the following motivating example.

⁸Equilibrium characterization in first-price auctions is more complicated and to prove existence of a monotone equilibrium additional assumptions are needed. For details see DKS. In what follows, anytime we discuss a first-price auction we assume that the fundamentals are such that a monotone equilibrium exists.

3.1 Hansen (1985) Example.

The first paper to study auctions with contingent payments is Hansen (1985). That paper describes the following two possibilities: the seller of the project can run a second-price cash auction or a second-price royalty/equity auction. In the latter case, all bidders make royalty offers, specifying what fraction α_i of the revenue they offer to the seller in case they win. The highest bidder wins and pays according to the second-highest offer.

In case of the cash auction, it is a weakly dominant strategy to bid true value, $b_i(z_i) = z_i - X$. In equilibrium, revenue is equal to the second-highest valuation:

$$REV_{Cash} = z_i^{(2)} - X$$

where $z_i^{(k)}$ is the k 'th-highest estimate.

By Lemma 1, in the royalty/equity auction it is a dominant strategy for bidder i to bid $\alpha(z_i)$ that solves:

$$z_i - X - \alpha(z_i) z_i = 0,$$

The equilibrium strategy is hence:

$$\alpha(z_i) = \frac{z_i - X}{z_i}. \tag{2}$$

Since $\alpha(z_i)$ is increasing, the bidder with the highest estimate wins the auction. Both cash and equity auctions are efficient.

The revenue in the equity auction is⁹

$$REV_{Equity} = \alpha(z_i^{(2)}) z_i^{(1)} = \frac{z_i^{(1)}}{z_i^{(2)}} (z_i^{(2)} - X)$$

Notice that the ratio $z_i^{(1)}/z_i^{(2)} > 1$, almost surely. Moreover, the last term in the expression is equal to the revenue in the cash auction. Therefore, the equity auction yields strictly higher expected revenue than the cash auction if and only if $z_i^{(1)} > z_i^{(2)}$.

⁹This is expected revenue conditional on the highest and second-highest types. The actual revenue depends on the realization of Z_i of the winner. Since the payout of the royalty contract is linear in the winner's revenue, the expected auction revenue depends only on the types and not on the full distribution of $h(Z|z)$. This is not true for non-linear contracts.

3.2 Cash vs. any other Contract

It may be at first puzzling why the revenue equivalence theorem (RET) does not apply in the Hansen (1985) model. After all, we have a symmetric independent private values setup, the equilibria with both contracts are efficient (the allocations are the same), and the lowest type gets zero payoff. These are seemingly all the conditions for RET.

The answer is that the proof of RET applies to situations in which the seller can condition transfers only on bids/reports of bidders. However, auctions with contingent payments by definition allow to condition payments on an additional variable, Z_i . That variable is correlated with the private type of a bidder, allowing the seller to reduce bidders' information rents.

The characterization of equilibrium strategy in Lemma 1 allows us to further explain and generalize the result in Hansen (1985). In a cash auction, the revenue is $z^{(2)} - X$. In an auction with contingent payments the revenue can be written as:

$$\begin{aligned} ES(s(z^{(2)}), z^{(1)}) &= ES(s(z^{(2)}), z^{(2)}) + ES(s(z^{(2)}), z^{(1)}) - ES(s(z^{(2)}), z^{(2)}) \\ &= z^{(2)} - X + \Delta \end{aligned} \tag{3}$$

where $s(z^{(2)})$ is the contract index chosen by the second-highest bidder. The second equality follows from the equilibrium characterization: the second-highest bidder offers a contract that would cost him his full profit, just as in a cash auction. The expected seller revenue is equal to the profit of the second-highest type plus an additional term Δ . That term is strictly positive for any contract such that $ES(s, z)$ is strictly increasing in z , which is the case as long as $S(s, Z)$ increases over a positive measure of Z (recall we have assumed that $h(Z|z)$ has full support). The implication is that *any auction with contingent payments*, not only the royalty/equity auction, yields higher expected revenue than a pure cash auction.

Expression (3) also allows for an immediate comparison of auctions with bids that consist of a fixed royalty rate and a variable cash component, commonly used in mineral rights auctions. In an auction with a royalty rate α , the extra term is $\Delta = \alpha(z^{(1)} - z^{(2)})$, and a higher α implies higher auction revenues. These observations about royalty auctions (generalized to a common values environment) have been made in Riley (1988).

3.3 Mechanism Design Argument

While the direct proof of why second-price auctions with contingent payments raise higher revenue than cash auctions is simple and transparent, it is not immediate how to extend it

to other auction formats (for example, the first-price auction).

Consider a general efficient direct revelation mechanism in which each bidder reports \tilde{z}_i and based on the reports the seller allocates the good to the bidder with the highest report. For simplicity, consider mechanisms in which only the winner pays. Let the probability of winning be $P(\tilde{z}_i)$. Since we assumed independent types, $P(\tilde{z}_i)$ depends only on the reported type; since we are discussing efficient mechanisms, $P(\tilde{z}_i) = F^{N-1}(\tilde{z}_i)$. Let the expected payment conditional on winning be $T(\tilde{z}_i, z_i)$. In cash auctions $T(\tilde{z}_i, z_i)$ depends only on the reported type. In contrast, in any auction with contingent payments, $T(\tilde{z}_i, z_i)$ strictly increases in z_i .

The equilibrium expected payoff of a type z_i is:

$$U(z_i) = \max_{\tilde{z}_i} P(\tilde{z}_i) [z_i - X - T(\tilde{z}_i, z_i)]$$

Using the envelope theorem and that in equilibrium each bidder reports $\tilde{z}_i = z_i$, we get:

$$U'(z_i) = P(z_i) [1 - T_2(z_i, z_i)], \quad (4)$$

where T_2 is the derivative with respect to the second argument, the true type.

In cash auctions $T_2 = 0$ and we get the famous result that the bidder's expected surplus depends only on the probability of winning and hence any auction format with any corresponding efficient equilibrium yields the same expected bidder surplus (unless losers are subsidized). From this we get the RET: the same allocations imply the same total surpluses, so if bidder surpluses are the same, expected auction revenues have to be the same as well.

The same reasoning can be used to rank security-bid auctions and cash auctions: for any formal auction with security-bids (first-price or second-price or any other format) $T_2 > 0$ for all types $z_i > \underline{z}$. If the game has an efficient equilibrium then (4) implies that the surplus of type $z_i > \underline{z}$ is strictly lower in the security-bid auction and the expected revenue of the seller is higher than in a cash auction.

This argument immediately extends to any auctions that combine a fixed contract increasing in Z with cash bids. Let auctions A and B use a fixed contract plus variable cash design and assume that the contract in auction A has a point-wise higher slope than the contract in auction B (for example, a A has a higher royalty rate). That is, $S_A(b, z) = b + S_A(Z)$ and $S_B(b, z) = b + S_B(Z)$ for some fixed S_A and S_B that satisfy our assumptions above and such that $S_A(Z) - S_B(Z)$ is increasing. Restrict attention to efficient equilibria (assume they exist). Then the expected surplus of bidders is smaller in auction A because our assumption

that $h(Z|z)$ satisfies SMLRP implies that for every $b, b' : \frac{\partial}{\partial z} ES_A(b, z) > \frac{\partial}{\partial z} ES_B(b', z)$ and hence $T_2(z_i, z_i)$ is strictly higher in auction A .¹⁰ Since the surplus is the same in the two auctions, seller revenues are higher in auction A .

3.4 Ranking Formal Auctions

So far we have compared auctions with cash to auctions with contracts. How about ranking different contracts beyond the special example in the end of the previous section? For example, does a royalty auction or debt auction yield higher expected revenue? Looking at (4) suggests that it may be in general difficult to answer this question: while we can rank $T_2(\tilde{z}_i, z_i)$ between a cash auction and any security-bid auction, it may not be possible for general contracts/securities. For example, in case of royalty/equity vs. debt bids, the slopes of $S(Z)$ are ranked differently for different levels of Z : debt has a higher slope than equity for low realizations of Z but the opposite ranking is true for high realizations of Z . Moreover, comparing $T_2(\tilde{z}_i, z_i)$ for different contracts/securities requires comparing $ES_2(s, z)$ and not just the slopes of the underlying contracts, $S_2(s, Z)$ (and we need to take into account that s is an equilibrium choice).

Nevertheless, DKS have shown that many standard sets of contracts/securities can be ranked under the assumption that $h(Z|z)$ satisfies SMLRP (although, it is a partial order). It turns out that what matters is the relative slope of the securities *at the point they cross*. Formally:¹¹

Definition 2 *An ordered set of contracts/securities \mathbf{S}_A is steeper than an ordered set \mathbf{S}_B if, for all indices s_A and s_B from the two sets, $ES_A(s_A, z^*) = ES_B(s_B, z^*)$ implies that $ES_2^A(s_A, z^*) > ES_2^B(s_B, z^*)$. If that is true we say that " $S_A(s_A, z)$ strictly crosses $S_B(s_B, z)$ from below."*

This definition is written in terms of slopes of $ES(s, z)$ for a fixed s , but we want to compare contracts in terms of the slopes of $S(s, Z)$ since these are the primitives of a formal auction. Lemma 5 in DKS establishes a sufficient condition:

Lemma 2 (Lemma 5 in DKS) *If $h(Z|z)$ satisfies SMLRP then a sufficient condition for $S_A(s_A, z)$ to strictly cross $S_B(s_B, z)$ from below is that there exists Z^* such that $S_A(s_A, Z) \leq S_B(s_B, Z)$ for $Z < Z^*$ and $S_A(s_A, Z) \geq S_B(s_B, Z)$ for $Z > Z^*$.*

¹⁰SMLRP implies first-order stochastic dominance.

¹¹I apologize for another abuse of notation. I write the expected cost of a security from set A as $ES_A(s, z)$ but the shorthand for a partial derivative with respect to the second argument is $ES_2^A(s, z)$.

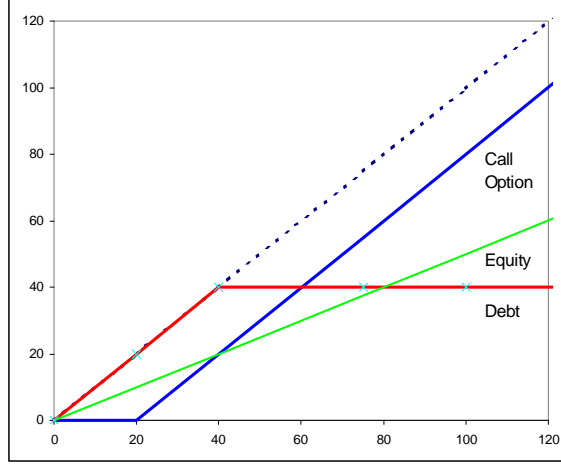


Figure 1: Comparing $S(s, Z)$ for debt, equity and call options.

This lemma implies that equity is steeper than debt and a call option is steeper than either one of them, as seen in Figure 1.

The general ranking result is:

Proposition 1 (Proposition 1 in DKS) *Suppose the ordered set of contracts/securities \mathbf{S}_A is steeper than \mathbf{S}_B . Then for either a first-price or a second-price auction, for any realization of types (almost surely), the seller's revenues are higher using \mathbf{S}_A than using \mathbf{S}_B .*

As a corollary of this proposition and the previous lemma, debt auctions yield lower revenue than royalty/equity auctions and both are dominated by auctions with call options.

The proof of this proposition for second-price auction uses the characterization of equilibrium strategies, (3). Recall that the auction with contingent payments raises revenues that are higher than in a cash auction by:

$$\Delta = ES(s(z^{(2)}), z^{(1)}) - ES(s(z^{(2)}), z^{(2)})$$

The equilibrium condition (1), implies that in equilibrium:

$$ES_A(s_A(z^{(2)}), z^{(2)}) = ES_B(s_B(z^{(2)}), z^{(2)}).$$

By definition of steepness $\Delta_A > \Delta_B$ almost surely (i.e. anytime $z^{(1)} > z^{(2)}$) because ES_A is increasing faster than ES_B as we increase the type from $z^{(2)}$ to $z^{(1)}$.

The proof for first-price auctions uses the mechanism design approach and the proof technique from Milgrom and Weber (1982) known as "the linkage principle." While we cannot

rank $T_2(z, z)$ for the two sets for every z , it turns out that it is sufficient to rank them at any point where the levels of payments, $T(z, z)$ are equal.

Formally, in a first-price auction winner pays his bid so $T(z, z) = ES(s(z), z)$. Therefore, $T_2(z, z) = ES_2(s(z), z)$. Restrict attention to efficient equilibria in both auctions. Using the envelope formula (4), we get that the ranking of $U'_A(z^*)$ and $U'_B(z^*)$ is opposite to the ranking of $T_2(z, z)$ in the two mechanisms.

Fix any $z^* > \underline{z}$, such that $U_A(z^*) = U_B(z^*)$. That implies $T^A(z^*, z^*) = T^B(z^*, z^*)$. In turn, the assumption that set \mathbf{S}_A is steeper than \mathbf{S}_B implies then $ES_2^A(s_A(z^*), z^*) > ES_2^B(s_B(z^*), z^*)$ at this type. That implies the ranking of $T_2^A(z^*, z^*) > T_2^B(z^*, z^*)$ and so $U'_A(z^*) < U'_B(z^*)$. Therefore, if $U_A(z^*) = U_B(z^*)$, then for all $z > z^*$ it must be that $U_A(z) < U_B(z)$. The final step of reasoning is a similar argument about the ranking of derivatives of $U(z)/P(z)$ at the lowest type (in the limit $z \rightarrow \underline{z}$ this ratio is zero for both sets since the lowest type bids up to his value in a first-price auction), to conclude that $U_A(z) < U_B(z)$ for all $z > \underline{z}$. As usual, since the total expected surplus is the same in the two auction formats, the ranking of expected seller revenues is the opposite to the ranking of bidder surpluses.

Analogous reasoning can be used to rank auction formats for a given set of contracts. The key observation is that in a first-price auction the winner pays according to the contract he submits. In the second-price auction, from the point of view of the winner, the payment is uncertain since it depends on the realization of $z^{(2)}$. If a convex combination of two contracts from the ordered set is still in the set, as is the case of equity bids or cash plus fixed royalty bids, then this uncertainty does not matter and the expected bidder surplus and revenue are the same in first-price and second-price auctions (and in any other pricing rule as long as the equilibrium is efficient). However, in case of debt contracts, a convex combination of two debt contracts with a different face value is not another debt contract. That convex combination is steeper than any debt contract. The opposite is true for call options: a convex combination of two call options with a different strike price is not a call option and call options are steeper than this convex combination. This ranking of steepness yields a ranking of expected revenues.¹²

Figure 2, borrowed from DKS, summarizes via a numerical example the ranking of different formal auctions. In this example there are 2 bidders, z'_i s are distributed uniformly over $[120, 210]$ and $X = 100$. $Z_i = z_i\theta$ where θ is distributed log-normally with mean 1 and standard deviation of $\ln(\theta)$ equal to $\frac{1}{2}$. The expected revenue from a cash auction is

¹²The ranking of derivatives at the lowest type is more subtle when we compare auction formats than when we compare sets of contracts. The economic intuition remains the same, see DKS for details.

Expected Seller Revenues		
Contract/Security	First-Price Auction	Second-Price Auction
Cash	50.00	50.00
Debt	50.05	50.14
Royalty/Equity	58.65	58.65
Call Option	74.53	74.49

Figure 2: Comparing Revenues in Formal Auctions

$50 = E \min \{z_1, z_2\} - X$. By the RET is the same for the two formats. The upper bound on the expected revenue is full surplus extraction $80 = E \max \{z_1, z_2\} - X$. It is worth noticing that the first-order differences are between security/contract designs and not auction formats. This is not a coincidence: auction formats deliver different revenues only because randomization over contracts changes their slope, so changing the slope directly can do just as well and even better.

Remark 1 (Relationship to scoring auctions.) *Formal auctions are closely related to scoring auctions, where bidders choose contracts from a set that is not ordered and the auction rules specify a scoring rule that maps bids into a one-dimensional score used to determine a winner. One can decompose the bidding in a scoring auction into two steps: picking a score (which solely affects the probability of winning) and choosing the cheapest contract that achieves that score. The result of the second step optimization creates endogenously the relevant set of contracts in our model of formal auctions. If this endogenous set is ordered, our analysis can be applied to it. However, for some scoring rules, it may be cheaper for a lower type to achieve a given score than it is for a higher type, breaking the monotonicity condition. In that case a monotone equilibrium may fail to exist.*

Additional (small) difficulty in analyzing scoring auctions is how to define a second-price auction since the highest and second-highest bids may not be point-wise ranked. In some situations the natural approach is to let the winner match the score of the second-highest bidder with the type of contract used by the winner. For example, in some online ad auctions bidders choose from a menu of pay-per-impression, pay-per-click and pay-per-action bids. These bids are then translated into a common score using click-rate or action-rate estimates and the winning bidder pays according to his chosen type of contract an amount to match the second-highest score.

3.5 Multi-unit example

The intuition presented so far can be extended to multi-unit auctions. Consider the following example. There is a unit mass of firms, each interested in buying a single business license. The gross value of those licenses is distributed over a range $[0, 1]$ according to some distribution $F(v)$. The government is going to auction off licenses to a fraction ρ of the firms via a uniform-price auction in which the highest losing bid determines the price for all winners. In addition to raising money directly from the auction, the government is going to raise tax revenue from the observed profits of all winners, with a tax rate t .¹³

How does the tax rate t affect bidding and overall revenue? In a uniform-price auction, when each bidder is interested in a single license, it is optimal to bid one's ex-post value, so bidder with type v will bid $(1 - t)v$. Equilibrium price is set by the bidder with type v^* such that $(1 - F(v^*)) = \rho$ (recall that in this example we have a continuum of firms, but the arguments can be generalized to a finite number of firms and objects). Changes in t do not change the marginal winner but they affect his willingness to pay.

The price per license is $(1 - t)v^*$ so the auction revenue is decreasing in the tax rate. But that is offset by higher tax revenues ex-post. Which effect is larger?

It is easy to see graphically that as the tax rate goes up, the overall revenues increase. In Figure 3 the black line presents gross valuations for a low tax rate and the red line for a higher tax rate. The yellow rectangle shows the reduction in auction revenues if the tax rate is increased while the blue trapezoid shows the corresponding increase in tax revenue from the licenses. The area of the trapezoid is strictly larger because the red and black lines get further apart as we move to the left.¹⁴

The economic intuition is easy to explain and applies not only to the large-market limit. A lower tax rate increases auction revenues by the value of the tax savings to the marginal type, while it reduces post-auction tax revenue proportionally to the average value of a winner. Only if the average value and the marginal value were the same, there would be no effect on revenues. This reasoning implies a clear connection between the single-unit and multi-unit models: bids depend on the marginal bidder and revenues accrue from the (average) winner. Since this is true in both setups, the theory presented above for single-unit auctions applies

¹³A real-life example of such auctions is sales of gambling machine permits. First, net revenues are monitored, taxed at rates often higher than regular profits and the tax is a non-trivial source of revenue. Second, often out of concerns to limit gambling, the government limits the number of licenses it allocates (increasing revenues would be another reason to limit the number of licenses).

¹⁴The base of the trapezoid at the supply, $1 - F(v^*)$ is the same as the shorter side of the rectangle. The longer side of the rectangle is the same as the height of the trapezoid. The areas are then ranked because the other base of the trapezoid is larger than the shorter side of the rectangle.

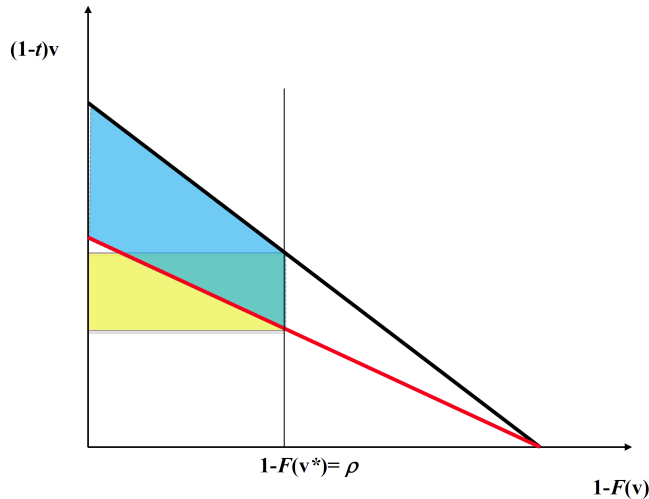


Figure 3: Multiunit Example: Higher Taxes, Higher Overall Revenue

as well to multi-unit auctions, at least as long as every bidder demands one unit.

Multi-unit models are also relevant for auctions of patent license contracts. There, unlike our simple license example, the value of a contract is a function of the number of contracts awarded and the terms of the contracts, because the winners compete in the consumer market post-auction (and hence a reduced-form model of such auctions would include negative externalities between bidders). For recent papers that study such markets see for example Sen, and Tauman (2007) or Giebe and Wolfstetter (2008). The relationship between auctions with contingent payments and auctions with externalities is beyond the scope of this survey.

Given this general result one may wonder why any government would reduce the future tax rate on revenues in an attempt to increase auction revenues. We discuss many such reasons in Section 5 (they include surplus-distorting moral hazard). In addition to those reasons, which apply generally to security-bid auctions, if the government has a higher discount rate than the bidders (for example, because of a re-election risk), it may find it optimal to sacrifice total revenue to obtain part of it sooner.

4 Informal Auctions

Many of the real-life examples discussed in the Introduction are not formal auctions. Often the auctioneer either does not announce any preferences over the contracts/securities he would like to receive or even if he does, he lacks the commitment power to reject offers that are not consistent with these preferences. For example, consider an author of a book

selecting a publishing house. The author may state that he would like a contract with at least a 10% royalty rate, but any publishing house that offers him a contract could ignore that request. If the offered advance is attractive enough, it may be hard for the author to reject it even if the royalty rate is less than 10%.¹⁵

A way to model such markets (for details see Section III in DKS) is to consider a game in which the bidders simultaneously submit contracts $S_i(Z)$, the seller then forms beliefs about the value of each contract and selects the bidder with the best bid. The winner pays according to his offer (it is not clear how to define a "second-price" auction in this case since the winning bidder and the seller may disagree which contract is the second-highest and the bidder may even consider his winning bid to be cheaper than some of the competing bids).

That game is difficult to analyze since it has elements of a signaling game: the seller needs to form beliefs about z_i from the chosen costly action, i.e. the bid S_i . The freedom to choose off-equilibrium path beliefs (i.e. when a bidder offers a contract that no type uses in equilibrium) leads to a multiplicity of equilibria.

Valuation of only one type of bids does not depend on beliefs: cash. That is, even if the seller believes that a bidder making a given cash bid has a very low type, the value of the bid is not affected.

That leads to the first result: if bidders are not budget constrained (so can make cash bids), then in the unique symmetric equilibrium bidders use only cash bids.¹⁶ The intuition is that for a bidder i with type z_i to outbid any bidder with a lower type, $z_j < z_i$ it is cheaper to do it by submitting cash bid than any contingent bid. It is so because it costs i more than it costs j to offer the same contract, while the cost of a cash offer is the same to all types.

The reasoning that steeper contracts are cheaper for lower types can be generalized to auctions without cash if off-path beliefs are restricted to satisfy a standard refinement $D1$ from Cho and Kreps (1987). This refinement implies that if a bidder deviates to a contract that is more expensive to types below him than their equilibrium-path contracts (and not more expensive to him or higher types), then the seller belief about the bidder type cannot deteriorate. With this refinement if bidders do not have access to cash (so bids have to satisfy $S(Z) \leq Z$), the unique symmetric equilibrium is for all bidders to use debt offers. As with cash, it is the cheapest way to out-bid lower types (Proposition 5 in DKS).

¹⁵The author's agent can try to use his reputation to argue that he cannot recommend his client a contract with an unusually low royalty rate. Reputation can provide some commitment power in private markets even if the sellers lack legal commitment.

¹⁶All statements in this section are "almost surely" since for every equilibrium we describe there exists a continuum of essentially equivalent equilibria in which the lowest type uses an arbitrary contract with expected cost $ES(\underline{z}) = \underline{z} - X$.

Summing up, if the choice of contracts/securities is left to bidders then in equilibrium they will prefer to use cash or, if they are cash-constrained, as flat contract as possible. Combining it with the results discussed in the previous section, it implies that being able to commit to any formal auction (as long as it results in an efficient outcome) would raise seller revenues.

5 Why Steeper is not Always Better for the Seller and why do Bidders Sometimes Prefer Contingent Payments?

The analysis so far points in the direction that in informal contests we should observe bidders using cash and/or debt bids but the auctioneers should always try to commit to steeper contracts. As Crémer (1987) pointed out, this steepness argument can be taken to the extreme: for example, in the Hansen (1985) model if the seller subsidized most of the up-front cost X , he would extract arbitrarily close to the full surplus. To see this, note that the bidding function (2) converges to 100% as $X \rightarrow 0$.

In practice, both in formal and informal auctions, we observe a rich variety of contracts and forms of payments used. One possible explanation is that in reality rarely one of the sides has all the bargaining power to decide on the forms of payment. Another can be efficient risk sharing - our model assumed risk neutrality while in practice some of the players (e.g. actors, authors, entrepreneurs) may be risk averse and contract design is chosen to allocate risk efficiently. Depending on a particular market, yet other factors may be more important. This section discusses papers that study models deviating from the DKS benchmark discussed so far, and identify tradeoffs in the use of contingent payments for the auctioneer and for the bidders.

5.1 Adverse Selection when X is not Constant.

Che and Kim (2010) point out that in many situations X is higher for bidders with higher z . For example, a publishing house that sees a larger opportunity for a book may plan a larger marketing campaign. Let $X(z)$ be the cost to start the project if the estimate is z and assume that $X'(z) \in (0, 1)$ so that while it is more costly to start more promising projects, the expected gross profit from a project increases in z .

In this case the equilibrium condition for the second-price royalty/equity auction changes

to:

$$\alpha(z) = \frac{z - X(z)}{z},$$

and the equilibrium condition (1) for an arbitrary formal auction generalizes to:

$$ES(s(z), z) = z - X(z) \tag{5}$$

First, we can see what can go wrong with the royalty/equity auction: the strategy $\alpha(z)$ is decreasing if $X(z)/z$ is increasing and that can be consistent with $z - X(z)$ increasing. If that happens, the winner of the auction will be the lowest type instead of the highest type! If so, the revenue will be lower than in a cash auction: in a cash auction the revenue is equal to the expected profit of the second highest bidder. In the equity auction, if $\alpha(z)$ is decreasing, revenue is smaller than the expected profit of the lowest bidder. Moreover, the steeper is the contract/security the higher is the discount the lowest type receives. Therefore, once we are in the decreasing-strategy regime, steeper contracts are worse for revenue (and for the same reason, the ranking of first-price and second-price auctions we discussed above is reversed).

When are the contracts too steep? The derivative of the RHS of (5) is $1 - X'(z)$. For the equilibrium strategy $s(z)$ to be increasing we need that at s such that (5) holds, the derivative $ES_2(s, z) < 1 - X'(z)$. In the DKS model $X' = 0$ so a sufficient condition is that $S'(Z) \leq 1$. With X increasing, stronger assumptions are needed to avoid a negative selection on X . In general, whether the equilibrium is efficient depends on the distributions of z and Z . However, a sufficient condition is to restrict bids to contracts that satisfy $S'(Z) \leq \min_z (1 - X'(z))$. For example, if $\min_z (1 - X'(z)) = 0.6$ then a second-price auction with cash bids plus a fixed royalty of less than 60% is guaranteed to have an efficient equilibrium.

Negative selection concerns, similar in spirit to the one in Che and Kim (2010), arise also when X and z are two-dimensional private information of the bidders (independent across bidders): using contingent payments reduces the information rent of the winner but can also reduce overall efficiency of the auction.

For example, suppose z_i and X_i are distributed in a way that X_i is uniform $[0, 1]$ and the expected gross profit, $z_i - X_i$, is also uniform on $[0, 1]$ (and independent of X_i). If we compare cash and equity auctions with two bidders, the first effect dominates and the equity-bid auction generates higher expected revenue. However, if the number of bidders gets large, the second effect dominates and the cash auction is better for the seller. In the limit, as the number of players grows, the cash auction revenue converges to 1, while the equity auction revenue is strictly less than 1, even in the limit (for the equity auction to be inefficient even

in the limit it is important that the domain of X_i starts at 0: the intuition is that even if there are many bidders, the bidder willing to offer the highest royalty rate is not necessarily the one with the highest profit, $z_i - X_i$, but it can be instead a bidder with a really small X_i who has little to lose if he wins by offering a very high royalty rate).

5.2 Moral Hazard

An obvious drawback of using steep contracts is that it can create moral hazard problems: when the winner of the auction is not a residual claimant of Z_i , it may not take profit-maximizing actions. There are many ways of incorporating moral hazard considerations to this framework. For example, Kogan and Morgan (2010) study a model where type and effort enter multiplicatively (Z depends on $z_i(1 + \delta e_i)$), while Jun and Wolfstetter (2012) consider an additive model ($Z = z_i + e_i$). Whether cash or equity auctions are more profitable in this setup depends on the number of bidders and on how important moral hazard is for creation of value. The intuition is that using contingent payments creates a tradeoff: on the one hand, it allows extracting a larger fraction of the surplus from the winner. On the other hand, it creates a distortion in effort that reduces total surplus. When effort is not important or very important, the efficiency loss from the effort distortion in an equity auction is small because either the distortion does not matter or because the winning agent cares enough about choosing the right amount of effort that even in the equity auction the winner chooses almost the efficient effort. In these cases, surplus extraction is more important and equity auctions dominate, while cash auctions tend to dominate for intermediate levels of effort importance. Similarly, when there are enough many bidders, a cash auction extracts already close to full surplus and hence effort distortions are of first-order importance.

As pointed out by McAfee and McMillan (1986), even with moral hazard, it is possible to always find auctions with contingent payments that dominate pure cash auctions. The intuition is that introducing a small royalty component (or cost sharing in case of procurement auctions), creates only a second-order loss in total efficiency (due to distorted incentives), but it creates a first-order gain of the extraction of surplus from the winner, for reasons discussed above.

Moral hazard issues connect the literature on auctions with contingent payments to the literature on optimal mechanisms for selling incentive contracts. Classic papers in this literature include Laffont and Tirole (1987) and McAfee and McMillan (1987). These papers study the design of the optimal contracts rather than comparing particular mechanisms (see

Jun and Wolfstetter (2012) for an application of this approach and for comparisons between equity and cash auctions to the optimal mechanism).

Another robust result in this literature is that since the seller does not extract full value from the winner but only the marginal revenue/virtual value, an optimal contract induces the winner to take actions to maximize marginal revenue rather than his gross value. Therefore cash auctions do not maximize revenue. Moreover, since for the highest type marginal revenue is equal to value, the optimal contract has the familiar "no distortion at the top" feature. The literature on optimal contracting in procurement (including procurement contests and selling off a regulated monopoly as in Riordan and Sappington 1987 and related literature) is large and reviewing it is beyond the scope of this paper.

Board (2007a) is a recent paper where the contract design is applied to auctions for oil lease contracts. The model assumes that bidders have private information about costs to start exploration but not about the amount of oil in the field, prices of oil, or costs to continue oil extraction. In the optimal mechanism firms compete with up-front payments and additionally, once the winner develops the oil field (and he chooses when to do so based on the price of oil and his costs), he makes an additional payment. As firms are assumed not to have private information about the amount of oil in the ground, using royalties is suboptimal. However, since the decision to start exploration depends on costs, it is optimal to have an additional payment at that point. For more on auctions (formal and informal) of real options see Cong (2012).

DKS discuss another form of moral hazard: if the investment of X is non-verifiable, then trying to extract additional surplus by subsidizing some of the up-front investment would necessarily backfire for the seller. The intuition is that in this case even the lowest type could guarantee itself a payoff equal to that subsidy by submitting a very high bid and then, instead of making the investment, just cashing out the subsidy. Finally, DKS point out that in case of informal auctions certain moral hazard concerns can actually *increase* the seller revenue. The intuition is as follows. Suppose that the winning bidder observes Z_i privately and can divert some of the revenue to private use at a cost of γ per dollar of revenue. Then, in equilibrium he has to restrict offers to contracts with a slope no higher than $(1 - \gamma)$ since otherwise the seller would expect cash diversion to happen ex-post. It does not matter for the equilibrium if bidders have access to cash, but if they do not, that excludes debt bids since if Z is smaller than the face value of the debt, the bidder has nothing to lose by diverting cash. Hence, the equilibrium bids need to be *steeper than debt*.¹⁷

¹⁷This may be a bit confusing: why does an upper bound on the slope of contracts force bidders to use steeper contracts? The answer is that our definition of steepness refers to the relative slopes of securities at

All this discussion is solely about winner's moral hazard. In some markets moral hazard on the side of the auctioneer may be as important. For example, consider an author auctioning the rights to publish her book. After the sale, the author may still need to finish the book and go on the road to promote it. A contract that splits the revenue between the author and the winning publishing house makes sure that both sides have incentives to contribute to the success of the publication. That can account partially why even in informal auctions bidders use contingent payments.

5.3 Competition between Auctioneers

Gorbenko and Malenko (2011) provide a different answer for why sellers may not always choose very steep contracts/securities: competition to attract bidders. The tradeoff they analyze is that a formal auction with steeper contracts is good at extracting additional surplus from a given set of bidders, but if the bidders have to choose between auctioneers, they would prefer an auction with less steep contracts since it leaves more surplus to the winner. They also relate their model to previous literature on competition between sellers in reserve prices. In the model competing sellers choose reserve prices and the set of contracts in the auction. When competition is weak, the sellers use both reserve prices and the steepest contracts possible. As competition increases, reserve prices drop to zero first and only then securities get less steep. The intuition is that while both reserve prices and contingent payments are helpful in reducing information rents of the winners, reserve prices additionally reduce total surplus while, as we discussed above, auctions with contingent payments are efficient under appropriate conditions. In the large-market limit (many buyers and sellers), the sellers in equilibrium announce cash auctions with no reserve prices. This limiting result follows the same reasoning as the results in McAfee (1993), Levin and Smith (1994) and Peters and Severinov (1997).

5.4 Seller's Selection and Common Values

In many applications, including mergers and acquisitions, common value concerns are likely of first order importance. Those concerns are especially acute if the seller has private information about the value of the company/asset being auctioned. If so, bidders in an informal auction face the risk that if they bid less-steep security than their competitors, they will suffer adverse selection from the seller.

the point they cross and debt contract crosses any other contract from above.

The following example illustrates the problem. There are two bidders and they can bid with any combination of cash and equity for an asset. The asset has the same gross (expected) value to both bidders, z . The seller knows z but bidders are uninformed. They have the same prior belief about z that is non-degenerate. The winner needs to spend a fixed cost X to use the asset.

In this game the seller's selection problem is so strong that in the unique equilibrium both bidders use only equity. The intuition is as follows: Bertrand competition between buyers drives equilibrium profits down to zero. Suppose that in equilibrium bidder 1 makes an offer that is at least partially in cash. Then bidder 2 could offer a bid that has less cash and more equity, such that for the average seller has the same value as bidder 1 offer. Despite being equally attractive on average, these two offers will be selected by different types of the seller. In particular, better-than average seller types strictly prefer offer 2 and worse-than average sellers strictly prefer offer 1 because the high types value more the additional equity offered by bidder 2 (from their perspective the equity is undervalued). As a result, bidder 2 payoff, conditional on being selected, is strictly higher than bidder 1. That violates that in equilibrium they both make zero profit.

Fishman (1989) makes related observations in a model where a bidder chooses between a cash and debt bid and trades-off the advantage of inducing target management to make an efficient accept/reject decision (steeper bid makes them act on their information that is relevant to the bidder) and the risk of attracting competition (in his model bidding with cash reduces competition by signaling a high valuation for the target, as in entry predation models.)

Moreover, contingent bids interact with the winner's curse. For example, in a first-price auction with royalties, a bidder is less worried that he over-estimates the value of the oil field since if he does, his payment is automatically partially reduced. If bidders bid in royalty rates (as opposed to bidding with cash with a fixed royalty rate), a bidder in a second-price auction knows that if he over-estimates the value of the asset, the second-highest bid will be smaller and hence he will get an additional payment reduction because of the lower rate he will have to pay.

5.5 Other Considerations

We have assumed so far that keeping the type of the winner fixed (and post-auction actions fixed in case of moral hazard), the overall surplus generated by the project is independent of the contract. Board (2007b) changes this assumption pointing out that bankruptcy costs

are often non-negligible, creating new tradeoffs between division of surplus and surplus creation.¹⁸ Debt auctions connect the literature on contingent payments to auctions with budget constraints - see for example Che and Gale (1998) and Zheng (2001).

Formal auctions require auctioneer's commitment power not to accept offers outside the chosen set of contracts. In many markets such commitment is hard to achieve and the theory presented so far would suggest that bidders without budget constraints would bid with cash. Povel and Singh (2010) illustrate how a seller without a commitment power can nevertheless extract extra surplus by enticing bidders to use security-bids voluntarily. The idea is to offer a subsidy to a certain security that makes it more attractive than cash to bidders. The real-life example they study is "stapled finance," which is a commitment by the investment bank advising the seller in an M&A transaction to offer the winner a non-recourse loan at below-market rate (backed by the asset being sold). While the bank offering the loan makes a negative profit on it, the increased competition among bidders increases the seller's profit by more than the subsidy since the subsidy is competed away (and hence the target firm can pay the bank to make that offer).

Vladimirov (2012) analyzes cash-only auctions with budget-constrained bidders who need to raise financing from third parties. Financing is secured with the proceeds of the auctioned project. He points out that even though the auction is in cash, the reasoning we described applies to the financing problem. For example, if a financier has a monopoly power over a bidder, financing will be structured via the steepest security: intuitively, this is what a seller would like to do in a formal auction, but instead of the seller gaining additional profits from the use of contingent payments, it is the financier that does (if the financing market is competitive, we are back at informal auctions though and financing is with debt).

So far we have assumed that only the winning bidder invests X to start the project. Alhirakiy and Ganuza (2012) argue that in many procurement situations (for example, road construction) the public sponsor also needs to commit some resources. Suppose the sponsor internalizes the consumer surplus from the project and would find it efficient to make the investment if and only if Z turns out to be high enough. If bidders have private information about costs and about Z , offering them some fraction of the toll revenues can help the seller learn information about Z and decide whether to proceed with the project or not. Should the seller run a simple cash auction, offering to pay the winning bidder a fixed amount to build the road, it would not be able to infer from the bids anything about Z . That would result in possibly suboptimal decisions. For example, building roads that construction companies

¹⁸For other papers on bidding with debt see for example Waehrer (1995) and Rhodes-Kropf and Viswanathan (2000) and (2005).

expect to be underutilized, so-called "white elephants," investment projects with negative social surplus (a concern also expressed in Engel, Fischer and Galetovic 1997 and 2003, and made worse if the winner holds-up the procurer and re-negotiates the contract ex-post).

Finally, the analysis presented in this paper is static in the sense that the timing of the auction and the set of potential bidders is taken as exogenous (with the exception of the discussion of competition between auctioneers). Gorbenko and Malenko (2012) study the interaction between takeover timing, means of payment (cash versus stock), and premiums in a dynamic model of takeovers with partially cash-constrained bidders. They show that since winning stock bids leave fewer gains to the bidder (for reasons discussed above), higher-synergy bidders, who start takeover contests earlier in time when they are smaller, end up acquiring targets in cash. Lower-synergy bidders start contests for larger targets, and as a result are cash-constrained and have to pay using stock. Because of this self-selection, average takeover premium in stock deals can be below that in cash deals. Moreover, in initiated contests, less constrained bidders have, on average, lower synergies and lose more frequently.

6 Conclusions

Auctions with contingent payments are common in practice, from procurement contracts to labor contracts to merger and acquisition contests. Due to a broad spectrum of applications, it is of interest beyond mechanism design literature: the literature surveyed in this paper spans finance, industrial organization and contract theory. As in other applications of auction theory, there is no one solution that fits all. The specifics of the market determine which economic forces are of first-order importance and hence what auction designs would be good for the auctioneer and what design would be used by the bidders absent a seller's commitment.

The benchmark model of DKS lacks a lot of the important features present in the papers discussed in Section 5. Hence, the interpretation of the results is not a guide to find revenue-maximizing mechanisms. Instead, the model is designed to capture an important economic intuition about security-bid auctions that identifies one of the reasons the type of securities used affects the outcomes of the auction.

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