Auctions with Contingent Payments

By ROBERT G. HANSEN*

There now exists a host of results concerning the revenue performance of various auction methods. This note delves deeper into auction markets by examining the effects on sellers' revenue of certain noncash means of payment. The basic result—that bidding mediums which include some contingent pricing feature generally yield the seller more revenue than do cash bids—is intriguing by itself and also points out the limitations of received theory.

Since what follows builds on the independent-preferences framework, it is useful to first note the assumptions of that model and the major results pertaining to it. To model an auction in the independent-preferences tradition, one assumes that bidders have reservation values, V_i, that are known only privately and that can be depicted as being drawn independently from some distribution $F(V)$. The most important result for this model is the revenue equivalence theorem: as given in John Riley and William Samuelson (1981), any auction involving risk-neutral bidders for which the following four conditions hold:

(a) a buyer can make any bid above some minimum "reserve" price,

(b) the buyer making the highest bid is awarded the object,

(c) the auction rules are anonymous, and

(d) there is a common equilibrium bidding strategy in which each buyer makes a bid b_i, which is a strictly increasing function of his reservation value V_i, yields an expected revenue of

$$(1)\quad E(\mathit{revenue}) = \int_{V^*}^{V} (VF'(V)+F(V)-1)F(V)^{n-1}\,dV,$$

where n = number of bidders and V^* = reservation value below which it is unprofitable to submit a bid.[1]

The limitations of this result can be pointed out by considering two common noncash bidding methods, stock bidding in the market for corporate control, and profit-share bidding in oil lease auctions. Under independent-preferences assumptions (and the additional condition that some variable correlated with the V_i becomes observable *ex post*), both of these auction methods yield expected revenue strictly greater than that given by (1).[2] The examples are quite simple yet it should be clear that they imply a general result on the dominance of contingent-payment bidding mediums.

I. Stock Bidding

Consider an auction involving one "target" firm and n potential "acquiring" firms. Each acquiring firm has value of itself equal to X (X will later be allowed to vary); the target has value to acquiring firm A_i of V_i. To meet independent-preferences assumptions, assume that the V_i can be represented as independent draws from the distribution $F(V)$, that V_i is known only to A_i, and that X is publicly known.

Suppose then that the target firm is sold by an open (progressive), cash-bid auction with zero reserve price. From standard auction theory, the high bid b^* for this auction will be

$$(2)\qquad b^* = V_2,$$

where V_2 equals the second highest of the V_i.

*Amos Tuck School of Business Administration, Dartmouth College, Hanover NH 03755. I thank Richard Bower, Dennis Logue, John Riley, and two referees for helpful comments. Financial support was received from the Tuck Associates.

[1] See Riley and Samuelson (pp. 382–83).

[2] The findings here do not technically contradict Riley and Samuelson's proposition because their (implicit) assumption is that V_i is unobservable even *ex post*. Also, for the auction rules considered here, property (c) concerning anonymity does not hold: the same bid can entail different payments for different bidders.

Furthermore, the auction will be won by the firm that values the target highest, and the seller's expected revenue equals the expectation of the second-highest value.

Suppose instead that the target requires open bidding of stock offers, where a stock offer is defined as the percentage of the merged entity that the target will own after trade. The highest-percentage bid naturally decides the winner.

Then it is clear that a dominant bidding strategy is to bid up to

$$p(V_i) = (V_i/(X+V_i)). \tag{3}$$

The firm with the highest V_i still wins this auction, and the high bid p^* will be

$$p^* = p(V_2) = (V_2/(X+V_2)),$$

where V_2 is again the second-highest value. Interestingly, the value of p^* to the target is given as

$$\textit{value to target} = \frac{V_2}{X+V_2}(X+V_1) > V_2$$
$$= \textit{high cash bid},$$

where V_1 is the highest value. For every possible set of V_i, the open stock auction therefore yields more revenue than the open cash auction (the revenue for which equals revenue from other common auction rules, for example, cash-sealed high bid).[3]

Of course, the assumption that all acquiring firms have the same value X is not very realistic. However, we can allow acquiring firm i to have value X_i—as long as this is known publicly[4]—and a stock auction will still yield more revenue than a cash auction (which is unaffected by acquiring-firm values). The auction procedure now becomes more complex, but, at the same time, more interesting.

For this new model, the bidding variable for an open auction is still p_i, the percentage of ownership that firm i offers. Now, however, the target cannot simply choose the highest p-offer, for firms with low X_i can make high p-offers. Suppose that the target announces that p-offers will be evaluated according to a function Z, where

$$Z(p_i) = (p_i X_i/(1-p_i)). \tag{4}$$

The firm offering the highest Z will win and pay its associated p-offer.

Faced with this evaluation rule, acquiring firms have a dominant strategy of bidding up to

$$p_i^* = (V_i/(X_i+V_i)). \tag{5}$$

There would be no reason to stop bidding at a lower p and forego possible gains; likewise, there would be no reason to bid more than p_i^* since upon winning that would entail paying more than value.

Notice that (5) implies each firm offers a Z up to

$$Z_i^* = (p_i^* X_i/(1-p_i^*)) = V_i \tag{6}$$

This is, of course, why the target's rule (4) is a sensible one. Furthermore, it then follows that the firm with the highest V_i (call this firm W) will win the auction and will have to offer a p_w such that

$$(p_w X_w/(1-p_w)) = V_2 \tag{7}$$

since the second highest of the Z_i will equal V_2. Equation (7) implies

$$p_w = (V_2/(X_w+V_2)),$$

so that the value of the winning p-offer to the target is given by

$$\textit{value of } p_w \textit{ to target} = p_w(X_w+V_1) > V_2.$$

The stock auction still dominates a cash auction.

II. Profit-Share Bidding in Oil Lease Auctions

Here, I must resort to some specific assumptions so that equilibrium does not degenerate to one where everybody bids a

[3] Notice that V^* does not differ between the cash and stock auctions; in both cases $V^*=0$. Also, expected revenue is still equal across open and sealed-bid auctions using stock bids.

[4] My 1984 paper discusses cases where X_i is only known privately.

profit share of 100 percent.[5] Specifically, assume that the lease up for sale either has no oil, or it has oil worth V. All of n potential bidders agree there is a $(1-p)$ chance of no oil and a p chance of "striking" V.[6] Unfortunately for the bidders, the cost to firm i is C_i to find out the state of nature, and sellers share only in profits. To meet the independent preferences assumption, assume that the C_i can be represented as independent draws from a distribution $F(C)$. Firm i then calculates expected value of the lease as $pV-C_i$.

In an open cash auction with zero reserve price, the high bid b^* will be

$$b^* = pV - C_2, \tag{8}$$

where C_2 is the second lowest of the C_i. The auction will, as usual, be won by the firm having the highest value (lowest C_i); expected revenue for the seller is the expected value of b^*.

With open profit-share bidding where the highest share wins, firm i will bid up to[7]

$$q(C_i) = ((pV - C_i)/(pV - pC_i)). \tag{9}$$

The high bid q^* will be

$$q^* = ((pV - C_2)/(pV - pC_2)),$$

and the firm with the lowest C_i will win. Expected value of q^* to the seller is

$$\textit{value of } q^* = ((pV - C_2)/(pV - pC_2))(p)(V - C_1) > pV - C_2 = b^*$$

As for stock offers in the merger market, profit-share bidding increases expected revenue.[8]

III. Conclusions

A general principle can be drawn from the results for these two examples. The source of the gain from using contingent-payment bidding methods is that these methods allow the seller to capture some portion of the difference between the two highest reservation values, $V_1 - V_2$. Cash auctions, of course, yield expected revenue equal only to V_2. With contingent payments, the "package" offered by the winning bidder (for instance, the profit share offered) is such that, if applied to the second-highest bidder, would yield that bidder a zero profit upon winning and would yield the seller revenue of V_2. The winning package will not, however, be applied to the second highest but to the first highest; it is this twist that allows the seller to capture some of $V_1 - V_2$.

It should also be pointed out that independent preferences do not capture all the aspects of any auction completely; however, to the extent that values do differ across bidders (for example, even in the generalized independent-preferences/mineral rights model of Paul Milgrom and Robert Weber, 1982), the effects of contingent bidding rules should continue to hold.

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[5] Douglas Reece (1979) and Walter Mead et al. (1984) have previously considered profit-share bidding and made similar assumptions (except for the informational assumptions; see fn. 6). Mead et al. cannot conclude that profit-share bidding unambiguously increases revenue; Reece finds an unambiguous increase but does not give an explanation such as that given here.

[6] Notice I am not employing the usual "mineral rights" assumptions of independent estimates on oil potential.

[7] The firm loses C_i without a strike and gets $(1-q)(V-C_i)$ with a strike.

[8] It should be noted that all contingent-pricing rules will involve incentive problems that could reduce the seller's expected revenue. For this reason, royalty-rate bidding in oil lease auctions has not been presented as another example.

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