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Auctions with Contingent Payments: Comment

By WILLIAM SAMUELSON*

In a recent paper in this *Review* (1985), Robert Hansen demonstrates that an auction allowing for contingent payments yields a greater expected revenue to the seller than an auction with fixed price bids. This "competitive" benefit is a potential explanation for why contingent pricing schemes are common in actual practice, where examples range from corporate acquisitions via exchange of securities, to revenue sharing in oil lease auctions, and incentive contracts in defense procurement.[1] This comment extends Hansen's analysis by noting several limitations of such contingent price schemes and emphasizing significant problems of adverse selection (an inefficient firm may win the auction) accompanying their use. It is shown that there is a basic tradeoff between the competitive benefits and the selection costs of increasing the extent of contingent payment in the auctioned contract.

In Hansen's model, a single seller faces a number of competing buyers who bid for the right to obtain an item. Let v_i denote the value of the item to firm i, and, following Hansen, suppose that these values are independently and identically distributed according to the probability distribution $F(v)$. If bidding is limited to fixed prices, then any of the standard auctions, sealed high bid or English, yields an expected revenue to the seller equal to the expectation of the second-highest value among the bidders, $E[R]=E[v_2]$. Now consider instead an auction based on profit share bids: the bidder allowing the seller to retain the greatest percentage profit share wins the auction. As Hansen shows, this contingent pricing scheme increases the seller's expected revenue, $E[R']>E[R]$.

Somewhat hidden in Hansen's demonstration is the origin of the contingent payment's advantage. As Jacques Crémer points out in his comment (1987), the source is easily seen by considering an auction where the seller predetermines the profit share that the winning firm will acquire. Call the firm's share s; the seller retains share $1-s$. Given this sharing rate, firms submit "bonus" bids, and the highest bidding firm wins the auction. For convenience, attention is restricted to a sealed second-bid auction—a member of a wide class of optimal (i.e., revenue-maximizing) auctions. In this auction, the high bidder pays the seller a price equal to the second-highest bid. Define the reservation price of firm i as the value of its retained profit share, $r_i=sv_i$. It is well-known that each firm's dominant strategy is to bid its reservation price, $b_i=sv_i$. Henceforth, this auction will be referred to as an English auction since it is strategically equivalent to the common auction of that name. Let the bids be ordered $b_1>b_2\ldots>b_n$, and let $v_{(i)}$ denote the ith highest value among the n firms.

In equilibrium, the seller receives expected revenue

$$(1)\quad E[R]=E[b_2]+(1-s)E[v_1] = sE[v_{(2)}]+(1-s)E[v_{(1)}] = E[v_{(1)}]-sE[v_{(1)}-v_{(2)}].$$

It follows immediately that it is optimal for the seller to set $s=\varepsilon$, where ε is in the positive neighborhood of zero.[2] The seller

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[1] This general point has been highlighted recently by Paul Milgrom (1985). M. S. Robinson (1984) presents an in-depth discussion of the potential benefits and limitations of contingent contracts in oil lease auctions.

[2] Strictly speaking, the seller's problem has no optimum due to the discontinuity in the revenue function at $s=0$. Nonetheless, the seller can achieve as near to the first-best level of revenue as desired by setting $s=\varepsilon$.

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obtains a near 100 percent share of the highest value among the competing bidders —that is, it obtains the first-best level of expected revenue, $E[R]=v_{(1)}$.

This finding is easily understood in terms of a fundamental result for resource allocation problems under asymmetric information —that an economic agent earns rents reflecting the degree to which it holds (payoff relevant) *private* information. Here, the winning firm's bidding profit (or rent) is $\pi_i = r_1 - r_2 = s(v_1 - v_2)$. Making the firm's payment depend on v_i, assuming this is observable *ex post*, reduces the variation in private information (i.e., reservation prices) across bidders, and, therefore, reduces this rent. In the limit where $s = \varepsilon$, this bidding profit is driven to zero. As Paul Milgrom (1985) notes, the use of contingent pricing is one example of the "linkage" principle—linking price to a variable which is affiliated with bidders' private information reduces the winning bidder's rent. Dollar for dollar, this raises the seller's expected revenue since (1) can be rewritten

$$(1') \qquad E[R] = E[v_1] - E[\pi_1].$$

In its simplest version, the lessons of Hansen's model for real-world applications are straightforward. In a corporate acquisition the target would prefer to retain a near 100 percent profit share while transferring management control to the most efficient acquirer. Similarly, the federal government would "sell" mineral and oil leases by awarding management contracts. Finally, in a competitive procurement, the buyer would award a cost-plus contract to the low-cost winning bidder.

I. Limitations

In principle, auctioning an *epsilon* share to the highest bidder achieves a first-best outcome, provided: (*i*) the value v_1 is observable *ex post*, (*ii*) this value is unaffected by *ex post* actions of the firm, and (*iii*) the auction procedure indeed selects the high value firm. In a more realistic model of the contracting setting, some or all of these provisos are likely to be violated, causing a significant reduction in the performance of (extreme) contingent price schemes. The present analysis first points out the welfare costs due to the possibility of *moral hazard* (items *i* and *ii*) before taking up my main topic, the costs stemming from *adverse selection*.

Under fixed price bidding, the seller's payoff is determined at the conclusion of the auction. By contrast, under contingent pricing, the seller enters into a contractual relationship with the winning firm. Thus, the seller and winning bidder face the standard set of principal-agent problems that (by now) are very well known. The revenue of the seller (the principal) will be reduced relative to the first-best optimum to the extent that 1) monitoring v_i (perfectly or imperfectly) is costly, and/or 2) the winning firm (the agent) is prone to *moral hazard*, that is, has the opportunity or the incentive to take actions *ex post* that reduce the value of v_1, so reducing the seller's revenue. Because the value v_1 typically incorporates revenues and costs that are more or less difficult for the principal to monitor, one type of moral hazard occurs when the firm misrepresents elements of v_1, such as exaggerating actual costs, and profits thereby. A second type of moral hazard stems from the firm's incentive to take inefficient actions. As is well known, under contingent price contracts, the agent firm will generally have insufficient incentives to undertake an efficient level of "effort." Thus, it may exhibit managerial slack or pay insufficient attention to cost control, passing on "unnecessary" costs to the seller thereby reaping (internal) benefits.

As is well known, the effects of imperfect monitoring and moral hazard are mitigated by increasing the agent's profit share—that is, reducing the degree of price contingency in the contract.[3] (Moral hazard problems are eliminated altogether by holding the agent to a fixed-price contract.) Thus, in the first instance, there is a tradeoff between the competitive benefit and the moral hazard

[3]The most well-known analyses are in noncompetitive contexts where the tradeoff is between risk-sharing and moral hazard effects. A recent representative treatment is Martin Weitzman (1980).

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cost of increasing the degree of contingent pricing in the auctioned contract.[4]

II. Adverse Selection

I now make my main point: that increasing the degree of price contingency in the auction leads to adverse selection, that is, it reduces the likelihood that an efficient (high value) firm will be selected in the first place.[5] Intuitively, this is clear from the knife-edge nature of the *epsilon* profit share contract. Since firm reservation prices $r_i = \varepsilon v_i$ are nearly identical (and nearly zero), the bids, though they follow the same ordering as values, must be nearly identical also. But this means that any added factor (even pure white noise) can destroy the identity of the bid and value orderings and, therefore, lead to inefficient selection in the auction.

Inefficient selection is also a potential problem under Hansen's proposal for profit-share bidding. This is explicit in his model of stock bidding for a target firm. Here, potential acquirers differ with respect to their own market valuations, x_i, as well as their potential acquisition values v_i for the target. Hansen demonstrates the strong result that the target can use profit-share bids to identify correctly the high-value acquirer ($v_{(1)}$), *provided* it knows and can separate out the effects of the x_i. In the absence of such knowledge, however, profit share bidding will be inefficient—that is, the high-value acquirer need not be selected for the transaction.

To model the effects of adverse selection, let us suppose that in place of $r_i = sv_i$, firm reservation prices are given by

$$r_i = sv_i + z_i, \tag{2}$$

where z_i is an additive factor. Assume that each z_i is independent of v_j and z_k, for $j = 1, \ldots, n$ and $k \neq i$, and that each z_i comes from probability distribution $G(z)$ having a positive variance. In the English auction, each firm bids $b_i = r_i$ as before and the firm with the highest reservation price wins the auction. However, this firm need not hold the highest value since r_i depends on z_i as well as v_i. Indeed, as the firm's profit share shrinks, the bid ordering reflects less and less the underlying value ordering. In the limit $s = \varepsilon$, the bids b_i are ordered solely by the additive factors z_i and so are independent of values.

Returning to (1′), it is easy to trace the effect on seller revenue of changes in the firm's cost-sharing rate. A reduction in s has two countervailing effects: As before, it has the competitive benefit of reducing the winning firm's expected bidding profit (the second bracketed term) and so raises seller revenue. At the same time, lowering s leads to less efficient selection of high value firms, thus lowering revenue via the first bracketed term.

A good illustration of the inherent tradeoff between selection and competitive effects is furnished by the case of a competitive procurement. Here, numerous potential suppliers vie to win a production or service contract offered by a single buyer. Suppose that firm i's total cost of fulfilling the contract is the sum of a production cost component c_i and a required profit amount z_i. Firms differ with respect to c_i and z_i, and only the firm itself knows its characteristics *ex ante*. (I abstract from the possibility of moral hazard by assuming that a firm cannot influence its characteristics.) After contract award and completion, the buyer is able to observe the winning firm's cost c_1, but the firm's profit requirement z_1 remains unobservable. In response to this last fact, the buyer's procurement contract is written to be contingent on c_1 alone (and not on total cost $c_1 + z_1$). Under a linear contract, the buyer's payment to the winning firm is

$$P = b + (1 - s)c_1. \tag{3}$$

Here, the term b is the firm's fixed-profit fee. In turn, s is the share of observable production cost that the firm bears; the buyer's reimbursement is the amount $(1 - s)c_1$. At

[4] For recent treatments (including formal models) of this tradeoff, see R. Preston McAfee and John McMillan (1984), R. Engelbrecht-Wiggans (1985), and Jean-Jacques Laffont and Jean Tirole (1985).

[5] John McCall (1970) presents a simple example of adverse selection in contracting.

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$s=1$, the contract is fixed price, $P=b$. At $s=0$, it is cost-plus, $P=b+c_1$. The buyer is said to institute a linear "incentive" contract for $s \in (0,1)$.

The auction procedure is as follows. 1) The buyer sets s. 2) Firms submit profit fee bids b_i. 3) The firm submitting the lowest profit fee bid b_1 is chosen to undertake the contract at terms b_2 (the second-lowest bid) and s. Note that firm i's reservation price (its break-even fee) is $r_i = sc_i + z_i$, and firms bid according to $b_i = r_i$. The buyer's expected payment is

$$(4)\qquad E[P] = E[b_2] + (1-s)E[c_1] = E[c_1+z_1] + E[\pi_1],$$

after using the fact $\pi_1 = b_2 - r_1 = b_2 - sc_1 - z_1$. The buyer's payment must cover the winning firm's production cost and profit requirement plus its bidding profit. As noted earlier, there is a basic tradeoff between the "selection" and "competitive" effects of varying s. As s falls, the first term increases (since selection of the low total-cost firm becomes less and less likely) but the second term falls: $\partial E[c_1+z_1]/\partial s < 0$ for $s \in [0,1)$ and $\partial E[\pi_1]/\partial s > 0$ for $s \in (0,1]$. One can also show that the former and latter effects diminish to zero as the contract becomes respectively fixed price or cost-plus—that is, $\partial E[c_1+z_1]/\partial s = 0$ at $s=1$ and $\partial E[\pi_1]/\partial s = 0$ at $s=0$. Consequently, the optimal sharing rate s^* lies in the open interval $(0,1)$—that is, neither a cost-plus nor a fixed-price contract is optimal.

III. Example

Suppose each c_i is drawn independently from normal distribution $N(\mu_c, \sigma_c)$ and each z_i is drawn from $N(\mu_z, \sigma_z)$. Then, the reservation prices $r_i = sc_i + z_i$ across firms are also independently and normally distributed. To compute the buyer's expected payment, first write $E[c_1+z_1]$ as $E_{r_1}[E[c_1+z_1|r_1]]$. Since r_i and c_i are bivariate normal, as are r_i and z_i, the conditional expectation is *linear* in r_1 and can be easily expressed in terms of the data of the problem. Furthermore, the expectation of the jth lowest (i.e., jth order statistic) of n independent draws from a normal distribution can be written $E[r_j] = \mu - k_j\sigma$, where k_j is a statistical constant that depends on n. Thus, it is straightforward for one to compute $E[r_1]$ and $E[\pi_1] = E[r_2] - E[r_1]$. After carrying out these computations, one obtains

$$(5)\qquad E[P] = \left[\mu_c + \mu_z - k_1\sigma_r - k_1 s(1-s)\sigma_c^2/\sigma_r\right] + \left[(k_1-k_2)\sigma_r\right],$$

where $\sigma_r = (s^2\sigma_c^2 + \sigma_z^2)^{1/2}$ is the standard deviation of r_i. It is easy to check that $\partial E[P]/\partial s$ is negative at $s=0$ and positive at $s=1$ so that the optimal sharing rate occurs in the interior of the interval $(0,1)$.

Table 1 lists the expected cost consequences under fixed-price, cost-plus, and optimal contracts. In all cases, the distribution means are normalized to zero, $\mu_c = \mu_z = 0$. Thus, $E[P]$ and $E[c_1+z_1]$ should be interpreted as the buyer's reduction in cost (relative to a zero mean) under a given cost-sharing rate. In addition, the standard deviation of z_i is normalized to one. Two types of conditions are varied: (i) the number of competing firms ($n = 3$, 6, or 10) and (ii) the standard deviation of production cost across firms ($\sigma_c = 5$ or 10). Since a firm's profit requirement is a small fraction of its total cost, one would expect that the z_i would vary less across firms than would the c_i. Thus, we examined the cases $\sigma_c/\sigma_z = 5$ or 10. One can make the following observations from Table 1:

1) As noted earlier, $E[c_1+z_1]$ and $E[\pi_1]$ are respectively increasing and decreasing with s, other things equal. For $n > 3$, the magnitude of the former effect is (considerably) larger than the latter. Thus, in the majority of cases, the potential benefit from superior selection dwarfs the potential gain from rent reduction. Accordingly, the fixed-price contract dominates the cost-plus (i.e., *epsilon*) contract.

2) Other things (including s) equal, $E[P]$ decreases as n and/or σ_c increase—that is, the buyer is better off selecting from a larger number of firms or when cost variations across firms increase.

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TABLE 1—EXPECTED PROCUREMENT COSTS

Number of Firms		$E[P]$	=	$E[c_1+z_1]$	+	$E[\pi_1]$
$n=3$						
$\sigma_c=5$	$s=.00$	0		$-.85$		$+.85$
	$s^*=.27$	-2.49		-3.92		$+1.43$
	$s=1.00$	0		-4.33		$+4.33$
$\sigma_c=10$	$s=.00$	0		$-.85$		$+.85$
	$s^*=.18$	-6.09		-7.84		$+1.75$
	$s=1.00$	0		-8.54		$+8.54$
$n=6$						
$\sigma_c=5$	$s=.00$	$-.64$		-1.27		$+.63$
	$s^*=.34$	-4.87		-6.12		$+1.25$
	$s=1.00$	-3.26		-6.47		$+3.21$
$\sigma_c=10$	$s=.00$	$-.64$		-1.27		$+.63$
	$s^*=.24$	-10.57		-12.21		$+1.64$
	$s=1.00$	-6.43		-12.76		$+6.33$
$n=10$						
$\sigma_c=5$	$s=.00$	-1.00		-1.54		$+.54$
	$s^*=.38$	-6.37		-7.53		$+1.16$
	$s=1.00$	-5.10		-7.85		$+2.75$
$\sigma_c=10$	$s=.00$	-1.00		-1.54		$+.54$
	$s^*=.27$	-13.42		-14.98		$+1.56$
	$s=1.00$	-10.05		-15.48		$+5.43$

Note: $\sigma_z=1, \mu_z=\mu_c=0$. Thus, $E[P]$ and $E[c_1+z_1]$ should be interpreted as the buyer's reduction in cost (relative to a zero mean) under a given cost-sharing rate.

3) Most important, an appropriately designed incentive contract is far superior to either the fixed-price or cost-plus extremes. Optimal sharing rates fall in the 20–40 percent range. Thus, superior performance is achieved by making the firm share a relatively small portion of costs. Such contracts perform well in terms of selection (compare $E[c_1+z_1]$ at s^* and $s=1$). At the same time, optimal sharing rates are low enough to limit firm bidding profits (compare $E[\pi_1]$ at s^* and $s=0$).

4) As one would expect, the optimal sharing rate rises when the number of competing firms and/or the ratio σ_z/σ_c increases. Either effect increases the marginal gain from better firm selection, implying a higher sharing rate s. (Of course, $s=\varepsilon$ is optimal as σ_z/σ_c approaches zero.)

Before leaving the example, it is worth noting the effects of allowing correlation between c_i and z_i. The most likely case is that c_i and z_i are negatively correlated reflecting the fact that a low-cost producer for a particular procurement may very well enjoy cost advantages in its other alternatives and, therefore, demand a high profit rate. Given negative correlation, contracts with low firm-sharing rates perform poorly—they tend to select firms with low profit requirements but high total costs. Furthermore, with negative correlation, the profit-reducing effect of low sharing rates is lessened or even reversed. As can be checked analytically, the optimal sharing rate increases with the degree of negative correlation between c_i and z_i other things equal. An extreme illustration (correlation of -1) is provided in the case that $z_i=C-(1-\varepsilon)c_i$, where ε is in the positive neighborhood of zero. Here, the variation in reservation prices *declines* as s increases. Thus, firm profits are reduced by *raising* the sharing rate (the reverse of the usual effect). Note also that a firm's total cost declines as c_i falls (z_i rises). As one can easily confirm, this leads to an extreme case of adverse selection: for any sharing rate $s<1-\varepsilon$, the *high* total cost firm from the

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competing population is selected. Nonetheless, by setting a fixed-price contract, $s=1$, the buyer secures a first-best outcome (perfect selection and a zero bidding profit for the winning firm).

IV. Conclusion

The discussion and analysis has demonstrated that the competitive benefits of auctions having contingent payments (clearly identified by Hansen and more recently by others) must be weighed against the accompanying costs stemming from moral hazard and (especially) adverse selection. Appreciation of this tradeoff offers a strong justification for the use of optimal incentive contracts in actual practice.[6]

[6] The buyer can improve upon an optimal incentive contract by allowing each competing firm to submit a bid consisting of a cost-sharing rate and fixed fee, and rewarding high sharing rates and low fees in selecting the contractor. Such a procedure allows firms to better "signal" its private characteristics c_i and z_i. (In particular, a firm's submission of a high sharing rate signals a low production cost c_i.) This same point also applies to Hansen's stock bidding model in which a pair of private characteristics must be identified. The optimal structure of such signaling contracts is discussed in my 1986 article and is a promising field of future research.

REFERENCES

Crémer, J., "Auctions with Contingent Payments: Comment," *American Economic Review*, September 1987, *77*, 746.

Engelbrecht-Wiggans, R., "Optimal Incentive Contracting," University of Illinois, Working Paper No. 1200, 1985.

Hansen, R., "Auctions with Contingent Payments," *American Economic Review*, September 1985, *75*, 862–65.

Laffont, J.-J. and Tirole, J., "Auctioning Incentive Contracts," Working Paper No. 403, Department of Economics, MIT, 1985.

McAfee, R. P. and McMillan, J., "Bidding for Contracts: A Principal Agent Analysis," Working Paper, University of Western Ontario, 1984.

McCall, J. J., "The Simple Economics of Incentive Contracting," *American Economic Review*, December 1970, *60*, 837–46.

Milgrom, P. R., "Auction Theory," Lecture for the Fifth World Congress of the Econometric Society, mimeo., 1985.

Robinson, M. S., "Oil Lease Auctions: Reconciling Economic Theory with Practice," Discussion Paper No. 292, UCLA, 1984.

Samuelson, W., "Bidding for Contracts," *Management Science*, December 1986, *32*, 1533–50.

Weitzman, M. L., "Efficient Incentive Contracts," *Quarterly Journal of Economics*, August 1980, *94*, 719–40.

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