1 Assignment 1 (General Equilibrium)

1. Consider a pure exchange economy with two consumers 1 and 2 and two goods x and y. Consumer 1's initial endowment is $(1, 0)$, i.e., 1 unit of x and 0 of y. Consumer 2's initial endowment is $(0, 1)$, i.e., 0 unit of x and 1 unit of y. Each consumer i's utility function is $u_i(x_i, y_i) = x_i y_i$, where (x_i, y_i) represents i's consumption bundle of x and y.

   (a) Draw an Edgeworth box and show all features of this economy.

   (b) Identify the points in the Edgeworth box that are Pareto optimal. Is the initial endowment Pareto optimal?

   (c) Find a Walrasian equilibrium of this economy and indicate it in the Edgeworth box.

   (d) For any pair of prices $(p(x), p(y))$, each consumer demands a certain amount of x and a certain amount of y (beyond their initial endowment) by maximizing their preferences. This is called the excess demand of each consumer (in term of x units). Multiplying these excess demand by respective prices give the excess demand amount in money units. Adding these excess demand (in money units), we get aggregate excess demand. Show that the aggregate excess demand for good x and good y must be zero.

2. Consider an Edgeoworth box economy with two goods x and y and two consumers. The initial endowment of agent 1 is $(2, 2)$ and utility function is $u_1(x_1, y_1) = x_1 y_1$. The initial endowment of agent 2 is $(3, 3)$ and utility function is $u_2(x_2, y_2) = x_2 y_2^2$. Consider the price $p(x) = p(y) = 1$ with a consumption bundle of $(4, 1)$ for agent 1 and $(1, 4)$ for agent 2.

   (a) Is the suggested allocation an improvement over initial endowment?

   (b) Show the budget line for this price on the Edgeworth box and the optimal consumption bundles of each consumer.

   (c) Is $p(x) = p(y) = 1$ with a consumption bundle of $(4, 1)$ for agent 1 and $(1, 4)$ for agent 2 a Walrasian equilibrium?

   (d) Solve for the Walrasian equilibrium of this problem.

3. Consider an Edgeworth box economy with two goods x and y and two consumers. The initial endowment of agent 1 is $(4, 4)$ and utility function is $u_1(x_1, y_1) = x_1 y_1^3$. The initial endowment of agent 2 is $(0, 0)$ and utility function is $u_2(x_2, y_2) = x_2 y_2$.

(a) Is the initial endowment Pareto optimal?

(b) Suppose the planner adds 4 utils to agent 2's budget and takes away 4 utils from agent 1's budget. Draw the new budget line in the Edgeworth box. Solve for the price equilibrium with transfers.

4. Can a boundary point of an Edgeworth economy be a Pareto optimal point? Can a boundary point of an Edgeworth economy be a Walrasian equilibrium? Can a boundary point of an Edgeworth economy be a price equilibrium with transfer?