Assignment 3 (Choice Under Uncertainty and Auctions)

1. Prove the equivalence of first order stochastic dominance to one lottery above another definition when the set of outcomes is finite.

2. Consider three monetary outcomes - 1 dollar, 2 dollars, 3 dollars. One can represent the set of all lotteries using a two dimensional simplex (the equilateral triangle where each corner point represents a degenerate lottery).

   - For a given lottery L over these outcomes, show the regions in the simplex where any lottery first order stochastically dominates the lottery L.
   - Verify that this region is the same region where $F(x) \leq G(x)$ for all x, where F is the distribution corresponding to L' lotteries that dominate L and G is the distribution corresponding to L.
   - Repeat this exercise for second order stochastic dominance and its equivalent definitions.

3. Consider a second-price auction with a **reserve price**. The role of reserve price is as follows. If a reserve price of r is used, then only bids of bidders above r are considered. If there are no such bidders, then the object is not sold. Else, the highest bidder wins the object and pays an amount equal to the maximum of the reserve price and the second highest bid.

   Let G be the distribution of the highest value among $n-1$ bidders. Suppose the values are drawn from $[0, w]$ for all the bidders.

   - Show that it is a weakly dominant strategy to bid your value in this auction.
   - Show that the expected payment of a bidder with value x is
   $$rG(r) + \int_r^x yg(y)dy.$$
   - Use this to show that the expected payment of a bidder (this is computed before he realizes his value) is given by
   $$rG(r)(1 - F(r)) + \int_r^w (1 - F(y))yg(y)dy.$$
   - Show that the optimal value of reserve price solves $r = \frac{1-F(r)}{f(r)}$.
   - If values are uniformly distributed in $[0, 1]$, what is the optimal reserve price in this auction?