Assignment 4 (Auctions and Moral Hazard)

1. Consider a second-price auction with a **reserve price**. The role of reserve price is as follows. If a reserve price of r is used, then only bids of bidders above r are considered. If there are no such bidders, then the object is not sold. Else, the highest bidder wins the object and pays an amount equal to the maximum of the reserve price and the second highest bid.

   Let G be the distribution of the highest value among $n-1$ bidders. Suppose the values are drawn from $[0, w]$ for all the bidders.

   - Show that it is a weakly dominant strategy to bid your value in this auction.
   - Show that the expected payment of a bidder with value x is
   $$rG(r) + \int_r^x yg(y)dy.$$
   - Use this to show that the expected payment of a bidder (this is computed before he realizes his value) is given by
   $$rG(r)(1 - F(r)) + \int_r^w (1 - F(y))yg(y)dy.$$
   - Show that the optimal value of reserve price solves $r = \frac{1-F(r)}{f(r)}$.
   - If values are uniformly distributed in $[0, 1]$, what is the optimal reserve price in this auction?

2. Consider the following bilateral trading model. A buyer and a seller want to trade an indivisible object. The seller can produce the object at a cost c which is his private information and is drawn uniformly from $[0, 1]$. The buyer has a value v for the object which is his private information and is drawn uniformly from $[0, 1]$. The trading happens as follows. The buyer proposes a price p_b and the seller proposes a price p_s simultaneously. If $p_b < p_s$, no trade happens and both the agents get zero net utility. If $p_b > p_s$, then trade happens at price $p = \frac{p_b+p_s}{2}$ and the net utilities of the buyer and the seller are $v - p$ and $p - c$ respectively.

   - Show that $p_b = \frac{2}{3}v + \frac{1}{12}$ and $p_s = \frac{2}{3}c + \frac{1}{4}$ is a linear Bayesian equilibrium of this bilateral trading game.
   - Argue how this equilibrium is inefficient.
   - Assume linear strategies of the form $\beta(v) = \alpha_b v + \gamma_b$ and $\beta(c) = \alpha_s c + \gamma_s$. Is this Bayesian equilibrium unique in the class of linear strategies.

3. Consider the following hidden action principal agent model (moral hazard) with three possible actions $E = \{e_1, e_2, e_3\}$. There are two possible profit levels $\pi_H = 10$ and $\pi_L = 0$. The probabilities of π_H conditional on the effort levels are

$$f(\pi_H|e_1) = \frac{2}{3}, f(\pi_H|e_2) = \frac{1}{2}, f(\pi_H|e_3) = \frac{1}{3}.$$

The agent's effort cost function $g(\cdot)$ is given by

$$g(e_1) = \frac{5}{3}, g(e_2) = \frac{8}{5}, g(e_3) = \frac{4}{3}.$$

Finally, let the value of wage to the agent is given by $v(w) = \sqrt{w}$, and the reservation utility of the agent is $\bar{u} = 0$.

- What is the optimal contract when effort is observable?
- Show that if effort is not observable then e_2 is not implementable.
- What is the minimum expected wage for implementing e_1 and e_3 when effort is not observable?
- Determine the optimal contract when effort is not observable.