Assignment 5 (Not to be handed over)

1. Consider the marriage market model (with n men and n women) where every man m ranks women as:

$$w_1 \succ_m w_2 \succ_m \ldots \succ_m w_n$$

and every woman w ranks men as:

$$m_1 \succ_w m_2 \succ_w \ldots \succ_w m_n.$$

What are the outcomes of the men-proposing and the women-proposing versions of the deferred acceptance algorithm at this profile?

Solution. The unique efficient matching in this profile is man m_i is matched to woman w_i for all $i \in \{1, \ldots, n\}$. Hence, both the versions of the deferred acceptance algorithm must terminate at this matching.

2. Consider the house allocation model with three agents $N = \{1, 2, 3\}$ and three objects $M = \{a, b, c\}$. Let f be a mechanism defined as follows. At any preference profile $\succ \equiv (\succ_1, \ldots, \succ_n)$, if $\succ_2 (1) = a$, then agent 1 gets the best element in $\{b, c\}$ according to his preference ordering $\succ_1$, agent 2 gets a, and agent 3 gets the remaining object (i.e., a serial dictatorship with the highest priority to agent 2, followed by agent 1, and finally to agent 3). In all other cases, agent 1 gets the best object in M, agent 2 gets the best remaining object according to $\succ_2$, and agent 3 gets the remaining object (i.e., a serial dictatorship with the highest priority to agent 1, followed by agent 2, and finally to agent 3).

Is f strategy-proof? Is f non-bossy, (a mechanism is bossy if an agent change the outcome at a profile without changing the object assigned to him; it is non-bossy if it is not bossy)?

Answer: f is strategy-proof. Agents 1 and 3 cannot change the priority. So, they have no incentive to manipulate. Agent 2 can change the priority. But he will not manipulate if he gets the top priority. When agent 2 gets the second priority, he can change the priority by saying that his top is a, and in this case he gets a. But a is not his top according to his true preference. So he gets an object which is at least his second preferred object. But that he could have got even if he did not change the priority. So, he does not gain by manipulation.

f is also non-bossy. Note that if the serial dictatorship with a given priority is non-bossy. So, if an agent does not change his own allocation in f, it does not change the priority in f. So, by the same reasoning, it is non-bossy - other agents will continue to choose the best from same set of available objects to them.

3. Consider a two-sided matching model with men and women. Let $\succ$ be a profile of preference orderings as shown in Table 1.

$\succ_{m_1}$	$\succ_{m_2}$	$\succ_{m_3}$	$\succ_{w_1}$	$\succ_{w_2}$	$\succ_{w_3}$
w_1	w_2	w_2	m_2	m_1	m_1
w_3	w_1	w_1	m_1	m_2	m_2
w_2	w_3	w_3	m_3	m_3	m_3

Table 1: Preference orderings of men and women

Suppose μ is the outcome of the women-proposing deferred acceptance algorithm for the preference profile $\succ$. Let μ' be the outcome of the fixed-priority TTC mechanism where the priorities of men are fixed according to their preference orderings in $\succ$ in Table 1, and then each woman points to the woman with her favorite man in every stage of the TTC.

(a) Verify that $\mu \neq \mu'$.

(b) Verify that μ' is not stable by identifying a blocking pair.

(c) Verify that μ' women-dominates μ.