

# Aspirations and Inequality

ISI Annual Conference, 2014

Garance Genicot and Debraj Ray

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“[UPA rule] is a period during which growth accelerated, Indians started saving and investing more, the economy opened up, foreign investment came rushing in, poverty declined sharply and building of infrastructure gathered pace . . . [But] growth can also unleash powerful aspirations as well as frustrations, and political parties who can tap into these emotions reap the benefits.” Ghatak-Ghosh-Kotwal, “Growth in the time of UPA: Myths and Reality,” *Economic and Political Weekly*, April 19, 2014.

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“[T]hose made to wait unconscionably long for ‘trickle-down’ — people with dramatically raised but mostly unfulfillable aspirations — have become vulnerable to demagogues promising national regeneration. It is this tiger of unfocused fury . . . that Modi has sought to ride from Gujarat to New Delhi.” Mishra, “Narendra Modi and the new face of India,” *The Guardian*, May 16, 2014.

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“Economists, mired as they generally are in a context-less description of human preferences, are nowhere close to a theory of socially defined aspirations and for the double-edged way in which they might influence individual behavior — either constructively, via a profitable chain of investment and reward, or destructively, via frustration and violent conflict.” Ray, *Journal of Economic Perspectives*, 2010

# Aspirations and Society

- Two-way interaction:
  - Aspirations → inspiration or frustration → investment  
(and investment shapes growth and distribution)
  - Society → aspirations  
(aspirations are shaped by the lives of others around us)

# Literature

- **Internal constraints** on growth and stagnation:
  - Hirschman's tunnel
  - The capacity to aspire: Appadurai 2004, Ray 1998, 2006
  - Poverty and cognitive function: Mani, Mullainathan, Shafir and Zhao 2013
  - Psychological poverty traps Banerjee-Mullainathan 2010, Bernheim-Ray-Yeltekin 1999, 2013

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  - Poverty and cognitive function: Mani, Mullainathan, Shafir and Zhao 2013
  - Psychological poverty traps Banerjee-Mullainathan 2010, Bernheim-Ray-Yeltekin 1999, 2013
- To be contrasted with **external constraints**:
  - Nonconvexities: Azariadis-Drazen 1990, Dasgupta-Ray 1986
  - imperfect credit markets: Banerjee-Newman 1993, Galor-Zeira 1993

## The Setting

- **Society:** single-parent single-child strings (**dynasties**)
- Lifetime income or wealth  $y$ ;  $y = c + k$ .
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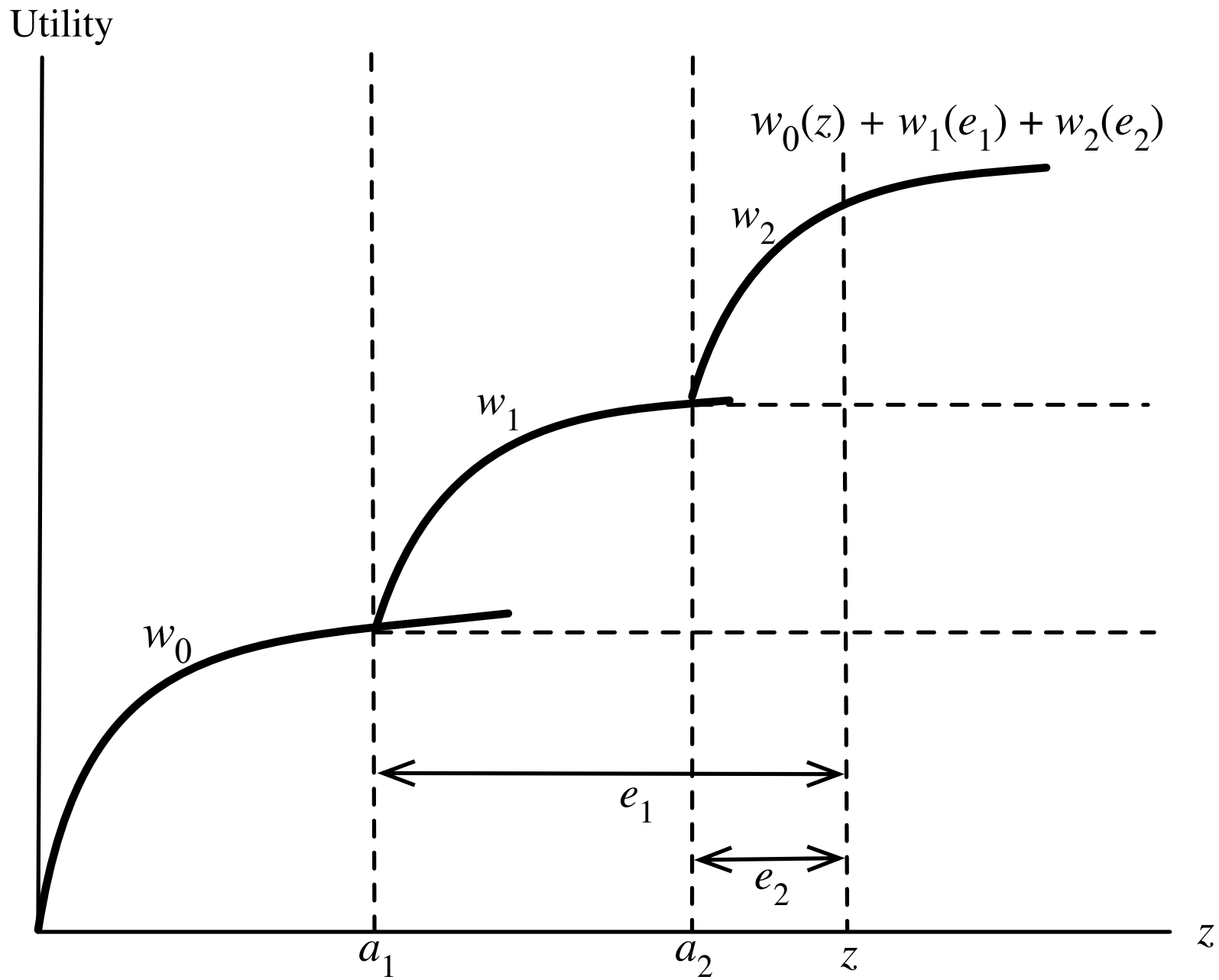
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- $w_0$  **intrinsic** utility;  $w_i$  **milestone** utility.

- Increasing, smooth, concave; unbounded steepness at zero.



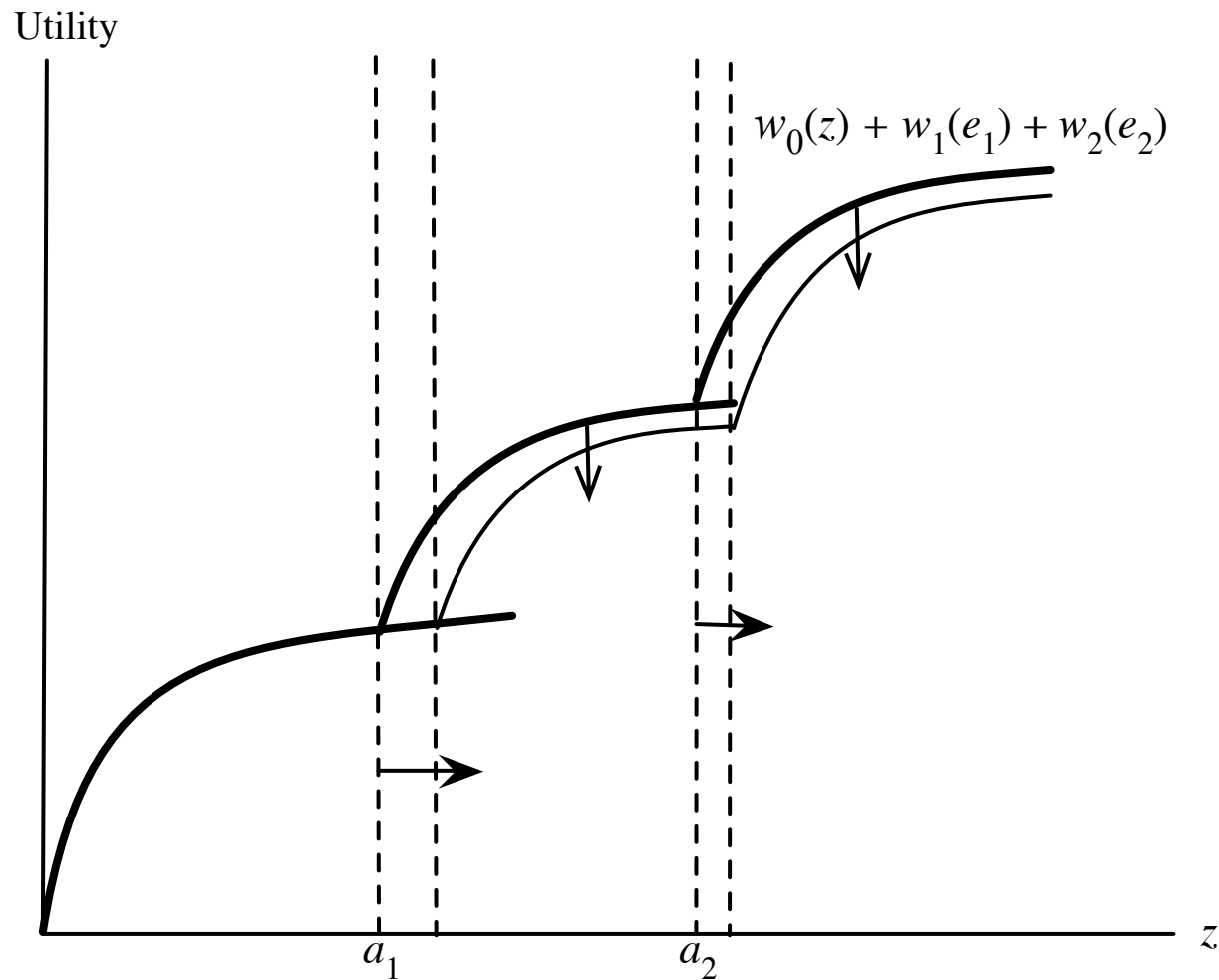
- $\mathbf{a} = (a_1, a_2)$

- Notes:

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- The entire map  $\mathbf{a}$  is always “present,” though different components are relevant at different income levels.
- Higher aspirations always bad for happiness in the short-run:



## The Formation of Aspirations

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- $\Psi$  is continuous (in weak convergence topology on distributions)
- $\min F \leq a(1) \leq \dots \leq a(n) \leq \max F$ .
- $\Psi$  nondecreasing (in FOSD on distributions)
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- $n = 1$  a leading special case: **single-step aspirations**.
- $n > 1$ : different milestones become salient with higher wealth.



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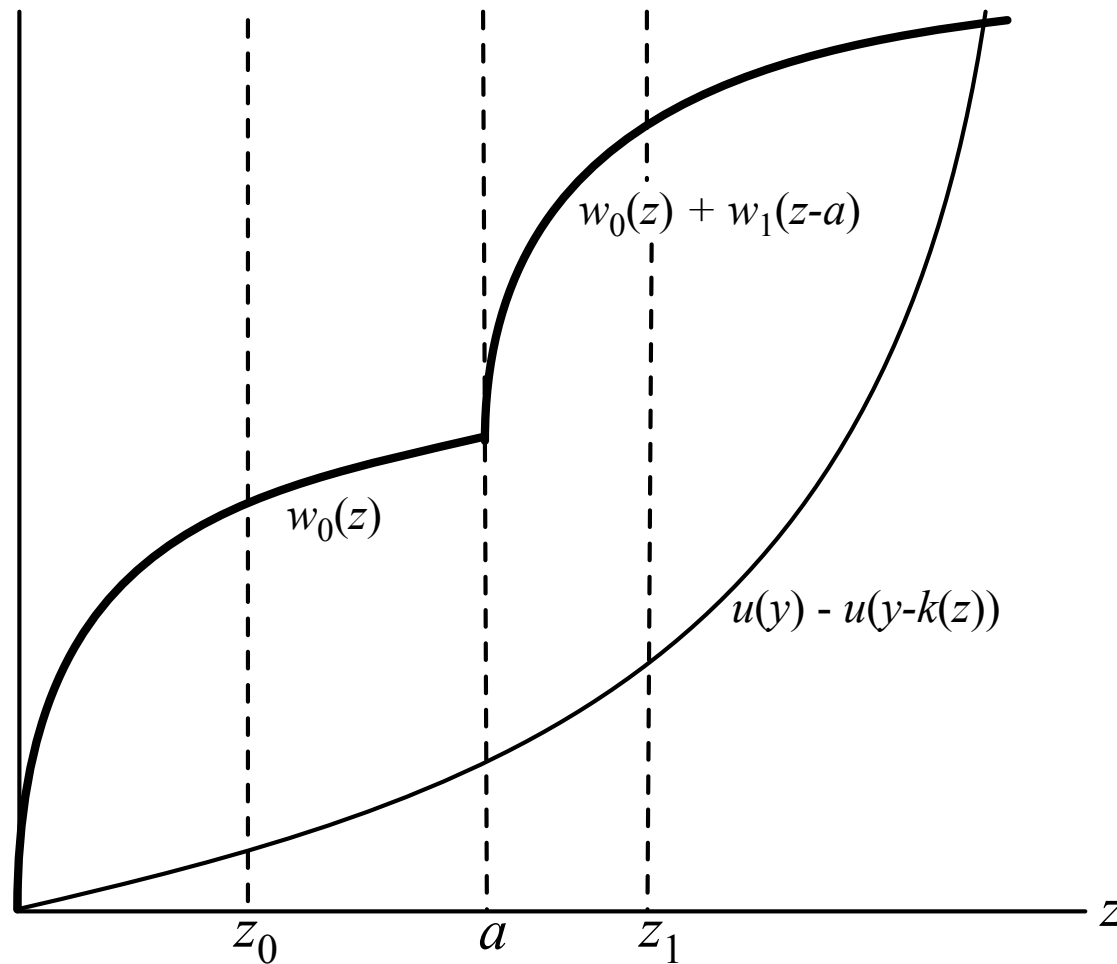
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- Starting from  $F_0$ , recursively generates a sequence  $\{F_t\}$ :
- an equilibrium.

# From Aspirations To Investment

## From Aspirations To Investment



- At most one “local” solution on either side of  $a$ .
- Compare, and pick the one with the higher payoff.

- Easily extended to  $n$ -step aspirations.
- At most  $n + 1$  possible choices for  $z$ , given by  $z_j \in (a(j), \infty)$  for  $j = 0, \dots, n$ :

$$\sum_{i=0}^j w'_i(z_j - a(i)) = u'(y - k(z_j)) / f'(k(z_j)).$$

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- **When is a milestone satisfied, and when is it frustrated?**

useful partial equilibrium exercise, aspirations “exogenous.”

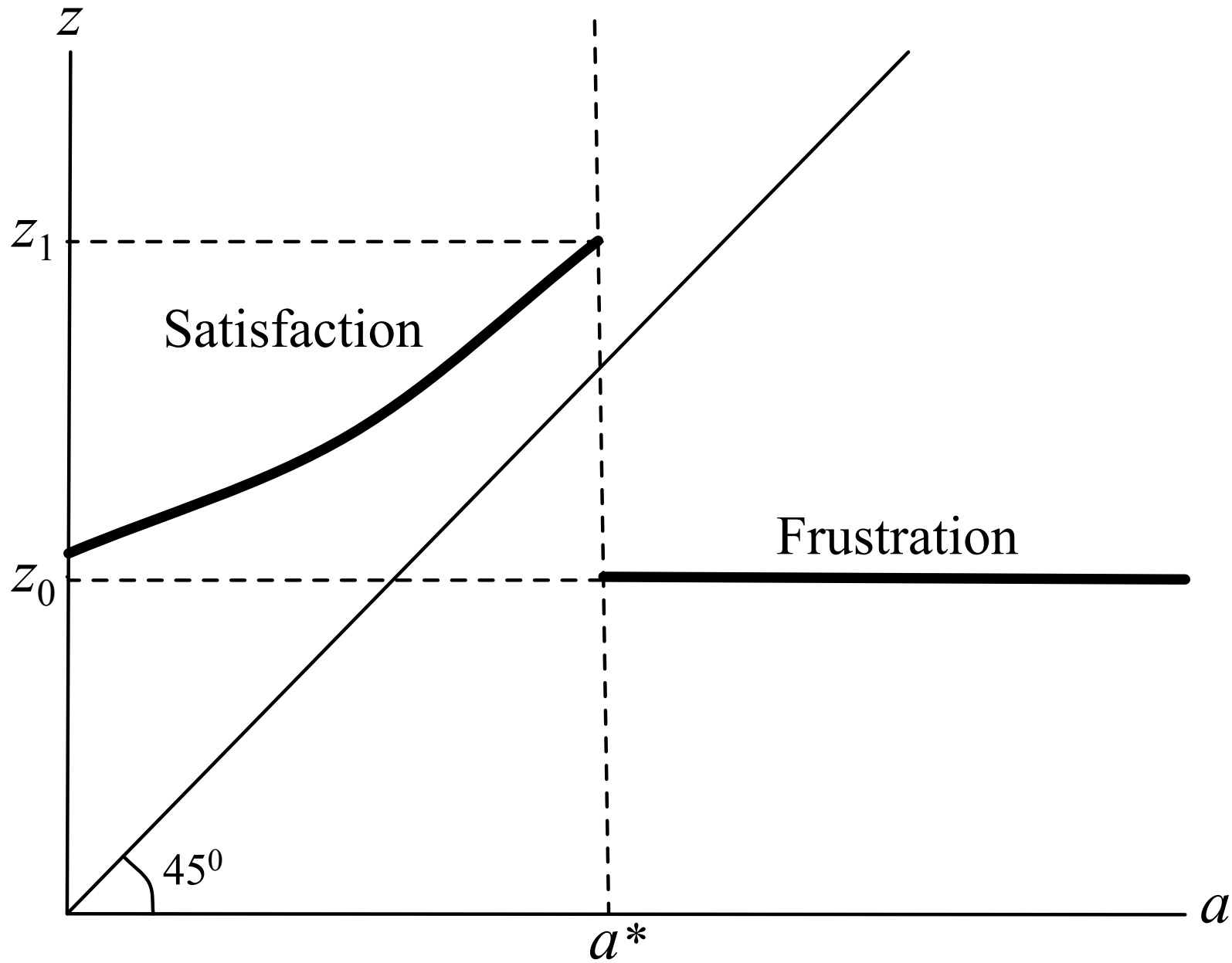


## Exogenously Changing Aspirations

- rise of television, advertising or the internet
- change in the income distribution

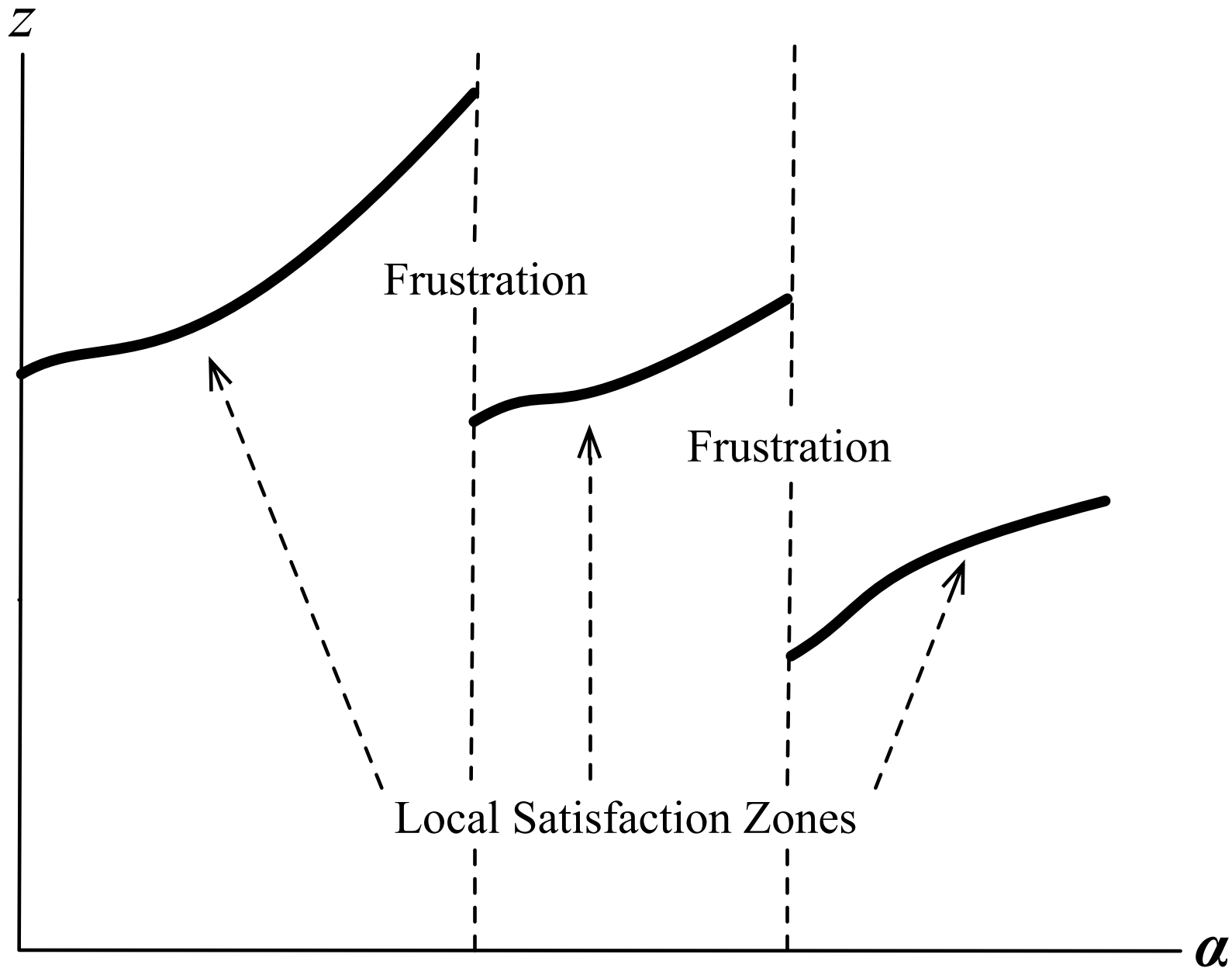
# Exogenously Changing Aspirations

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- change in the income distribution
- Begin with the single-step case:
  - **Proposition 1.**
    - Fix current wealth. Under single-step aspirations, there is a unique threshold value of the milestone below which aspirations are satisfied, and above which they are frustrated.
    - As long as aspirations are satisfied, chosen wealth grows with the milestone. Once aspirations are frustrated, chosen wealth becomes insensitive to the milestone.



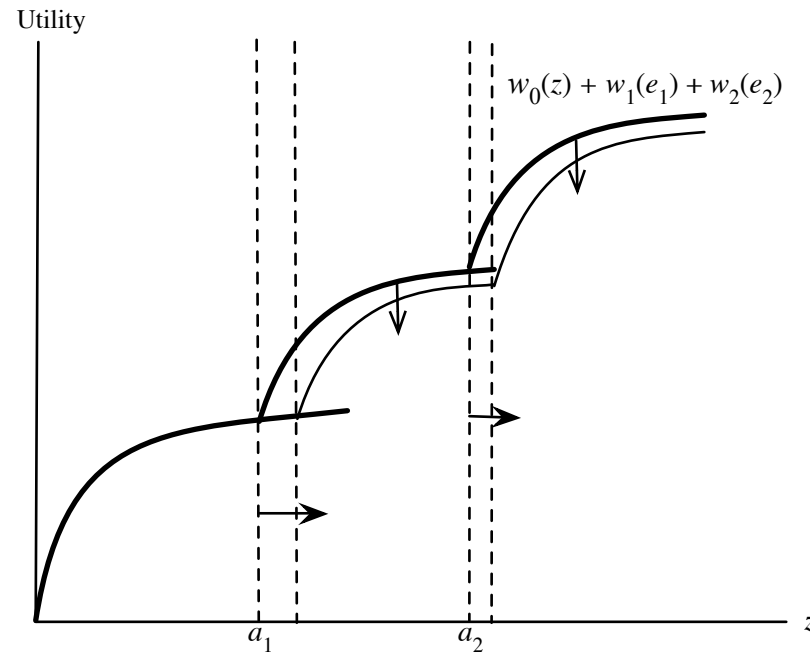
- Note the discontinuous jump-down.

■ Extension to multi-step aspirations



## On Frustration

- Recall that aspirational growth always lowers direct utility:

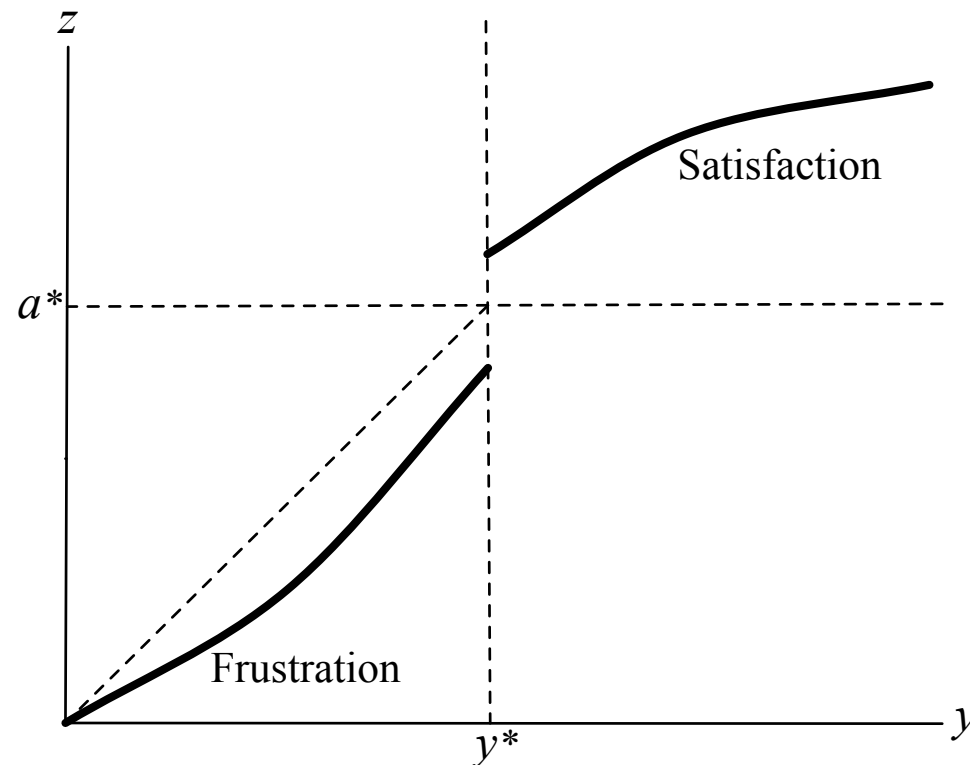


- If aspirations are frustrated, no inspirational role either:
- “The French found their position all the more intolerable as it became better.” de Tocqueville, 1856
- Lowered aspirations of low income students reduces school dropout  
Kearney-Levine, 2014, for the US; Goux-Gurgand-Maurin, 2014, for France

## From Wealth To Investment (More partial equilibrium)

### ■ Proposition 2.

- For each milestone  $a^*$ , there is a unique wealth threshold  $y^*$  above which  $a^*$  is satisfied.
- Optimally chosen wealth for the next generation is nondecreasing in current wealth.



## From Wealth To Investment, contd.

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- Introducing the **canonical linear model**:
  - **Linear production**:  $f(k) = \rho k$ .
  - **Constant-elasticity utility**:

$$u(c) = c^{1-\sigma}, w_0(z) = \delta z^{1-\sigma}, \text{ and } w_i(e_i) = \delta \pi_i e_i^{1-\sigma}$$



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- Dividing through by  $y$ .

■ Single-step aspirations:

- Let  $r \equiv y/a$ . Choose growth  $g \equiv z/y$  to max

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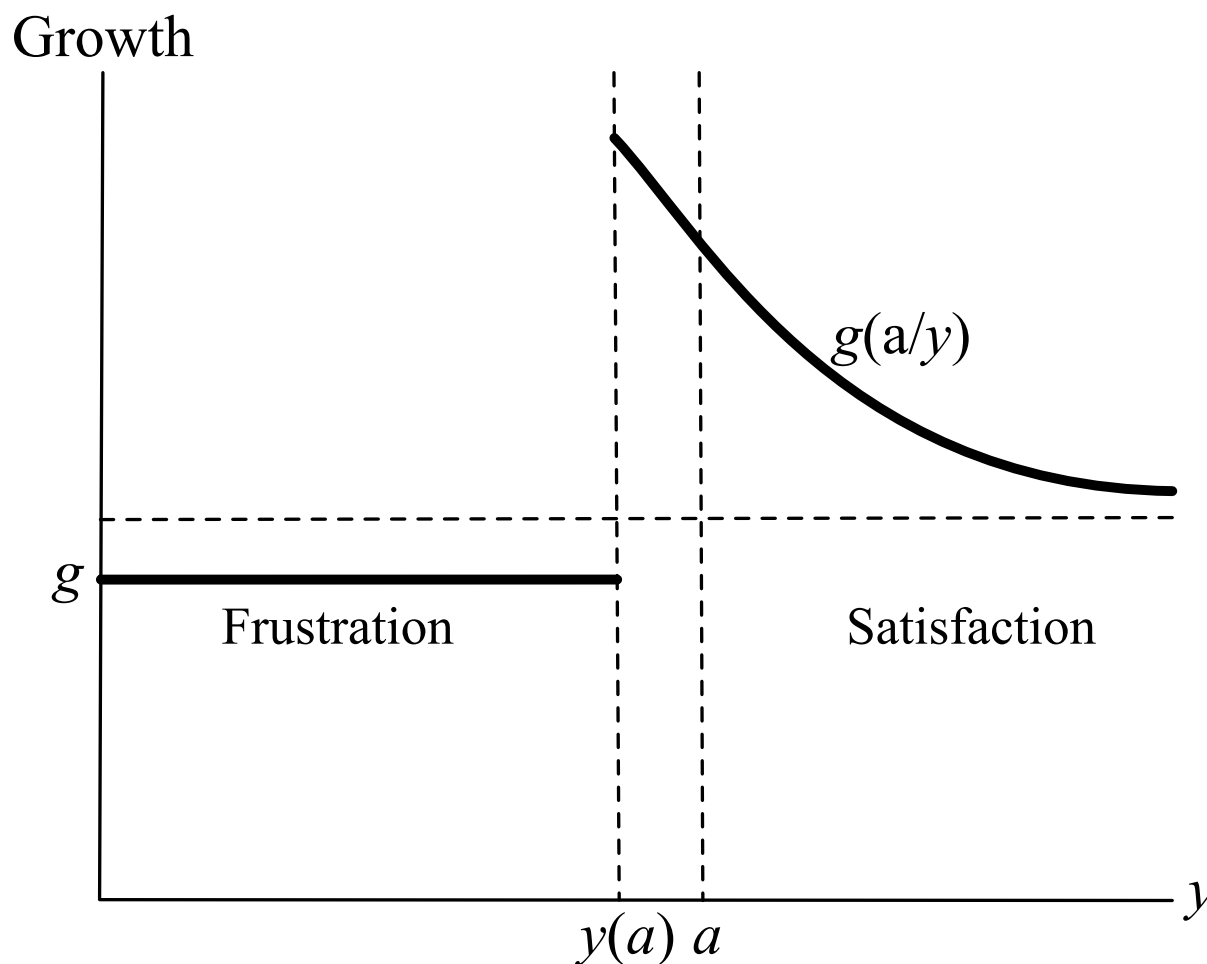
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- Satisfied aspiration; solution  $g(r)$  depends on  $r$ :

$$\left(1 - \frac{g(r)}{\rho}\right)^{-\sigma} = \delta \rho \left[ g(r)^{-\sigma} + \pi_1 \left( g(r) - \frac{1}{r} \right)^{-\sigma} \right].$$

- **Proposition 3.** Fix single milestone  $a$  in canonical linear model.
- Then there is a unique  $y(a)$  such that for  $y < y(a)$ , wealth grows at rate  $\underline{g}$ , and for all  $y > y(a)$ , wealth grows at rate  $g(y/a)$ .
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■ The multi-step case.

■ FOC for crossing  $m$  milestones:

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- “Globally frustrated” individuals have constant growth  $\underline{g}$ .
- For two individuals with same (nonempty) satisfied aspirations,  $g \downarrow$  as  $y \uparrow$ .
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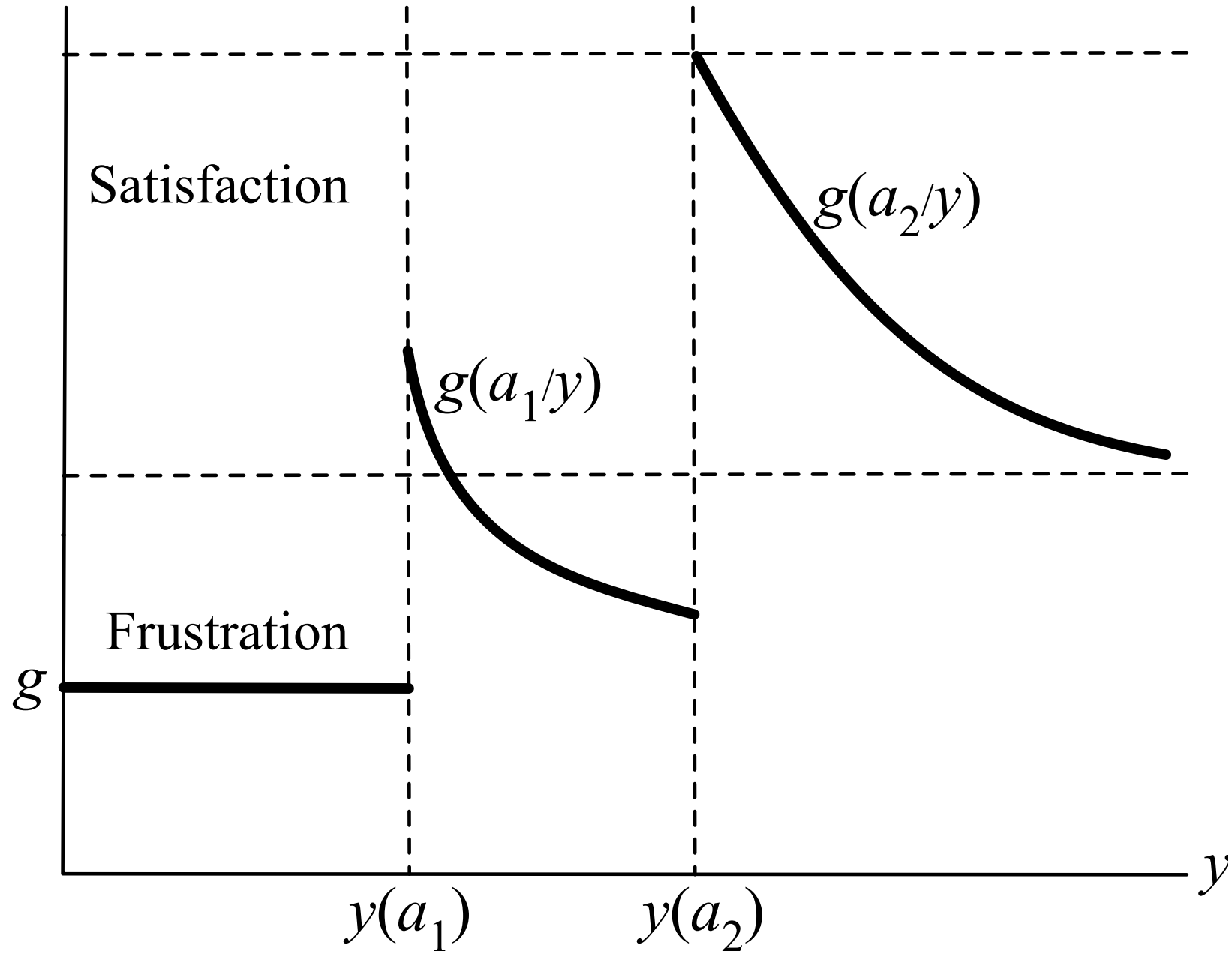
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- Succession of wealth thresholds at which  $g$  jumps up.
- In summary, a complex growth incidence curve with rising and falling segments.
- Overall tendency:  $g$  rises with wealth, because each decline is bounded below by a rate that exceeds the lower bound of the previous segment.

Growth



General Equilibrium:

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- Questions:
  - Persistent or growing inequality, or convergence?
  - Connections between initial distribution and subsequent growth.

## Steady States

- Distribution  $F^*$  such that  $\{F^*, F^*, F^*, \dots\}$  equilibrium from  $F^*$ .
- Natural setting: incomes in compact support, as in Solow model:  
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- Proposition 4.
  - Under Assumption C, there exists a steady state distribution.
  - No steady state can involve perfect equality of wealth.
- Proof:
  - Perfect equality implies concentration of  $y$  and  $a$  at same point.
  - Contradiction: everyone wants to move away from  $y = a$ .
  - Related: Matsuyama, Freeman, Mookherjee-Ray, Ray-Robson

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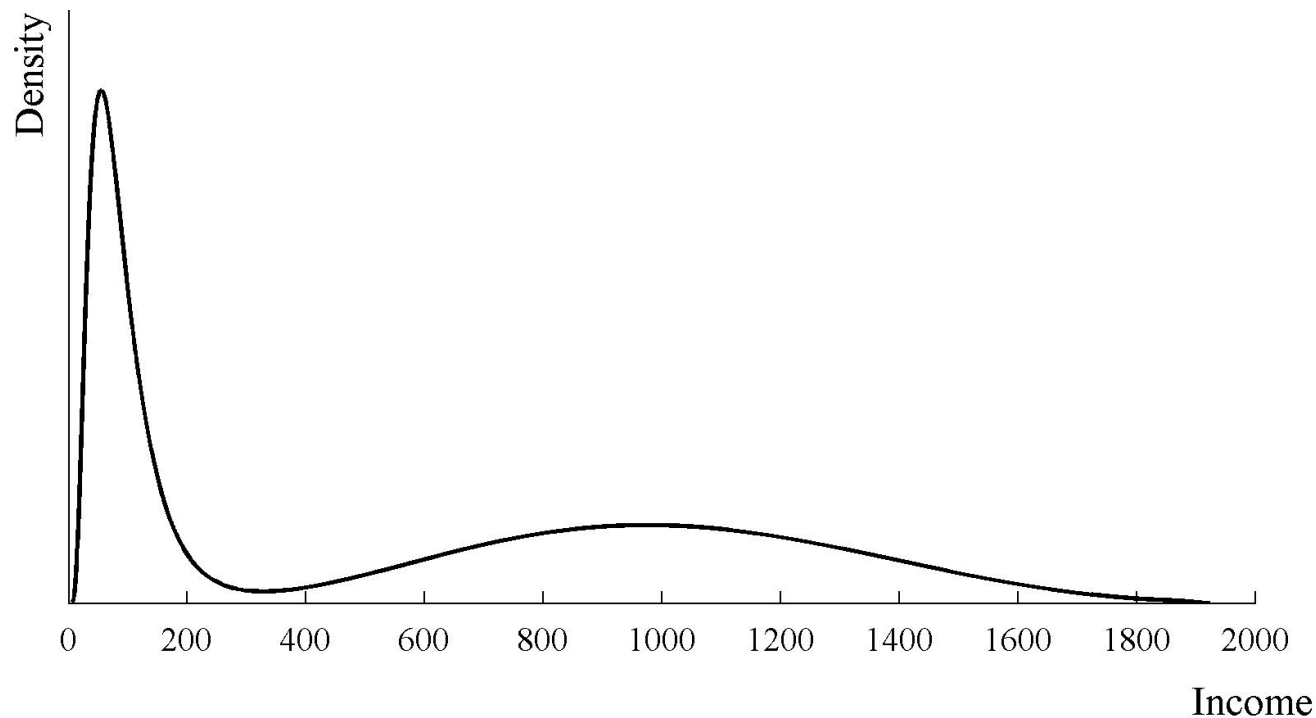
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- **Proposition 5.** Assume D and  $n$ -step aspirations.

- Steady state consists of at least 2 and at most  $n + 1$  mass points.
- In particular, the single-step case exhibits bimodal steady states.

## Remarks on Clustering

- Of course, convergence to **degenerate** poles is an artifact (akin to single steady-state income in Solow model.)
- With stochastic shocks (e.g., Brock-Mirman 1972): smoothly dispersed but multimodal distribution.



Constant-elasticity utility:  $\sigma = 0.8$ ,  $\delta = 0.8$  and  $\pi_1 = 1$ ;  $f(k, \theta) = \theta(A/\beta)k^\beta$ , where  $\beta = 0.8$ ,  $A = 4$  and  $\theta$  lognormal with mean 1.  $a = \text{mean } y$ .

## ■ Multimodality in the literature:

- [US] Pittau-Zelli 2004, Sala-i-Martin 2006, Zhu 2005
- [world] convergence clubs (Durlauf and Johnson 1995, Quah 1993, 1996, Durlauf-Quah, 1999).
- Quah uses the term “twin peaks.”
- Bimodality also a feature of polarized distributions (Esteban-Ray 1994, Wolfson 1994).

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# Aspirations, Inequality and Endogenous Growth

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  - constant-elasticity utility, linear production.
  - Begin with **single-step aspirations**.
- An uninteresting case:
  - $F_0$  is such that **everyone** is frustrated at date 0.
  - By linear homogeneity of  $\Psi$ , must have perpetual decay.
  - We don't consider this case.

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**II. Persistent Divergence.**  $F_t$  “separates” into two components defined by threshold  $y^* \in \text{int Range } F_0$ :

- If  $y < y^*$ , income grows forever after at  $\underline{g}$ .
- If  $y > y^*$ , income has asymptotic growth  $\bar{g} > \underline{g}$ , with  $\bar{g} - 1 > 0$ , and  $y_t/\bar{g}^t$  has the same limit independent of  $y_0$ , as long as  $y_0$  exceeds  $y^*$ .
- $\underline{g} < \bar{g} < g^*$ : equality exhibits faster growth.
- In Case II, **relative inequality never settles, it perpetually widens.**

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- **Note:** Both cases contrast strongly with the Solow setting.
  - Equality not possible, stable inequality possible.

- Multi-Step Aspirations



## ■ Multi-Step Aspirations

■ **Proposition 7.** Assume that everyone is not frustrated at date 0. Then one of the following must hold:

1. Some individuals are frustrated with respect to every milestone at date 0. Then there is persistently widening relative inequality over time, with  $\sup y_t / \inf y_t \rightarrow \infty$  as  $t \rightarrow \infty$ .

2. Every individual is satisfied with respect to some milestone at date 0. Then full convergence to perfect equality, stable relative inequality, and unbounded relative inequality are all possible outcomes.

## ■ Multi-Step Aspirations

■ **Proposition 7.** Assume that everyone is not frustrated at date 0. Then one of the following must hold:

1. Some individuals are frustrated with respect to every milestone at date 0. Then there is persistently widening relative inequality over time, with  $\sup y_t / \inf y_t \rightarrow \infty$  as  $t \rightarrow \infty$ .

2. Every individual is satisfied with respect to some milestone at date 0. Then full convergence to perfect equality, stable relative inequality, and unbounded relative inequality are all possible outcomes.

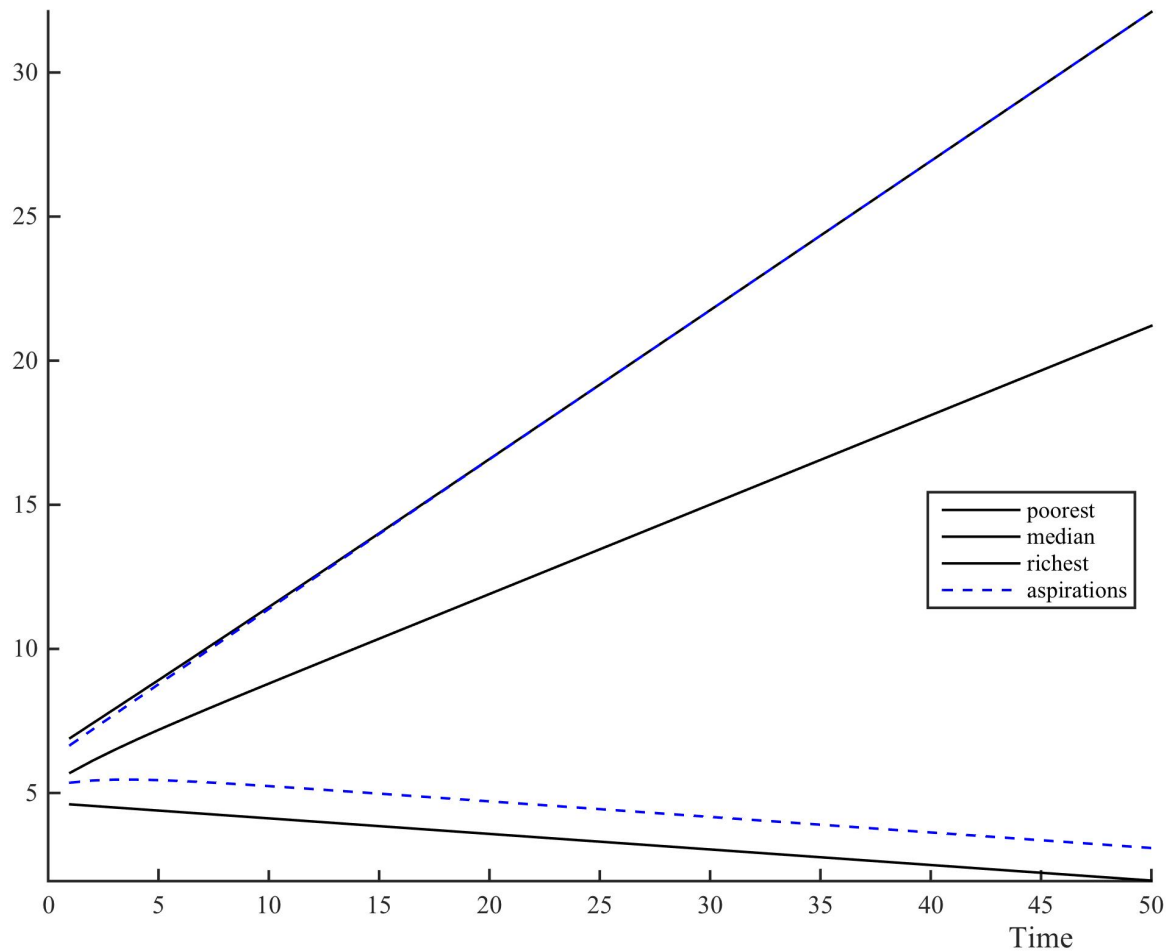
■ **Note:**

- These results are preliminary.
- Don't yet have a full description of asymptotic behavior.

## A Two-Step Example

- Both aspirations of the form  $a = \sum w_i y_i$ .
- $a(1)$  uses weights  $w_i = y_i^{-1} / \sum y_j^{-1}$
- $a(2)$  uses weights  $w_i = y_i / \sum y_j$ .
- $F_0$  given by three-point distribution.
- Consider four cases.

- Case A. Poorest group globally frustrated at date 0.



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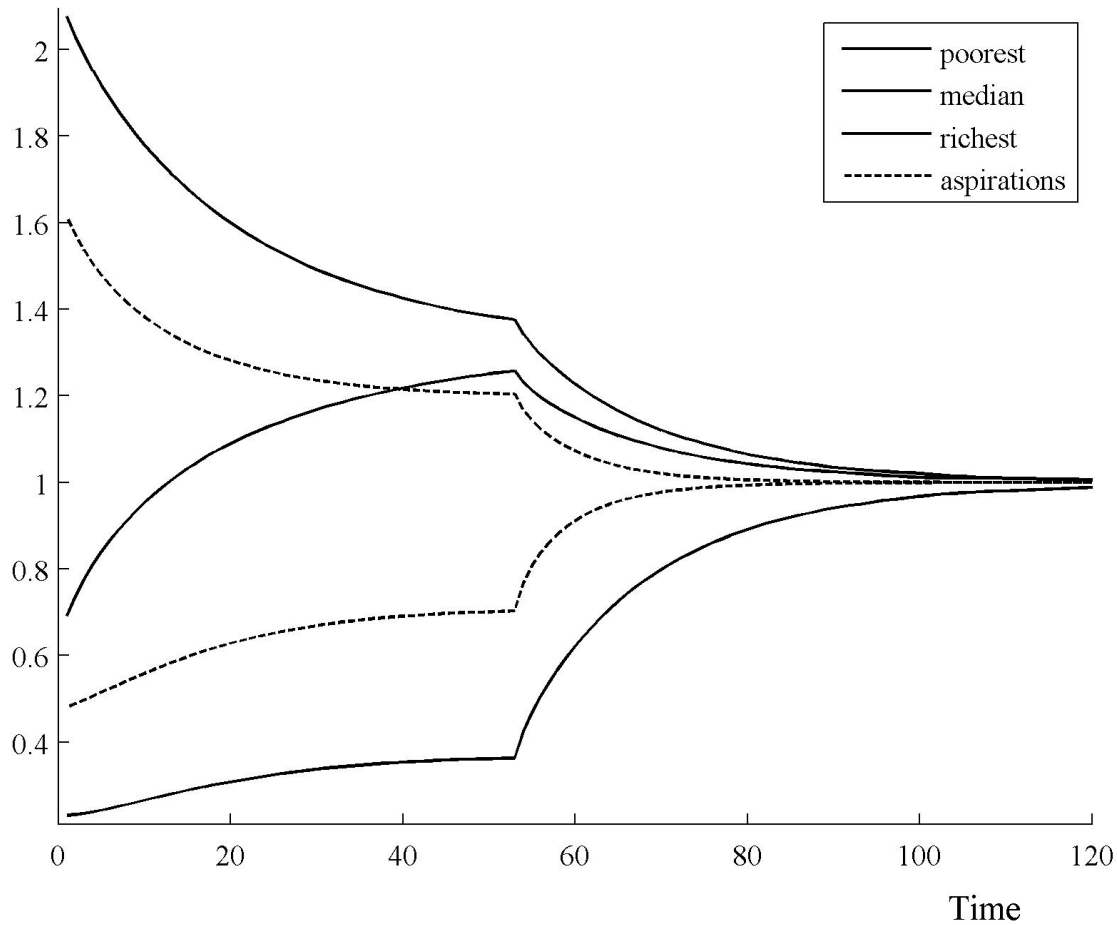
### Asymptotic Growth Rates

Poorest	Median	Richest
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-0.05	0.36	0.68
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■ Case B. Full convergence.

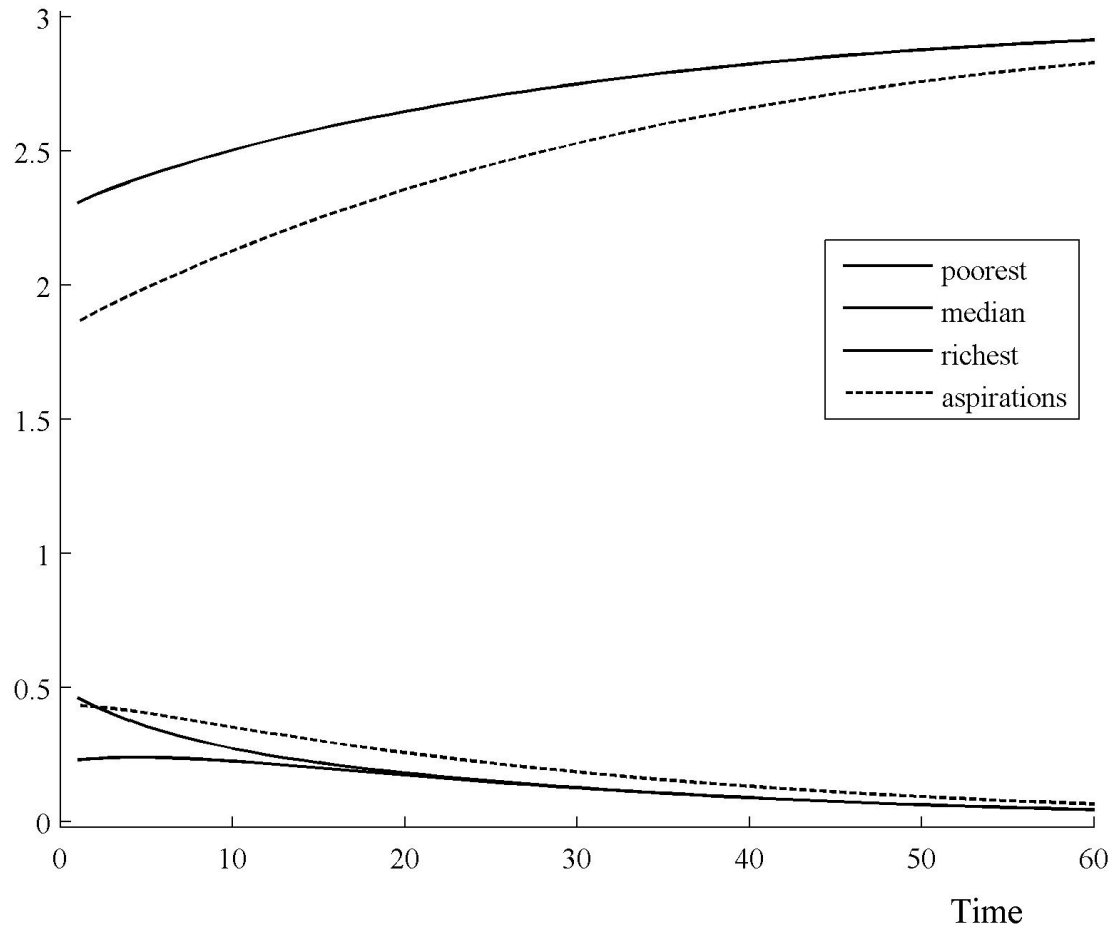


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Asymptotic Growth Rates		
Poorest	Median	Richest
1.82	1.82	1.82

---

- Case C. Poorest catch up to the median, richest grow faster.

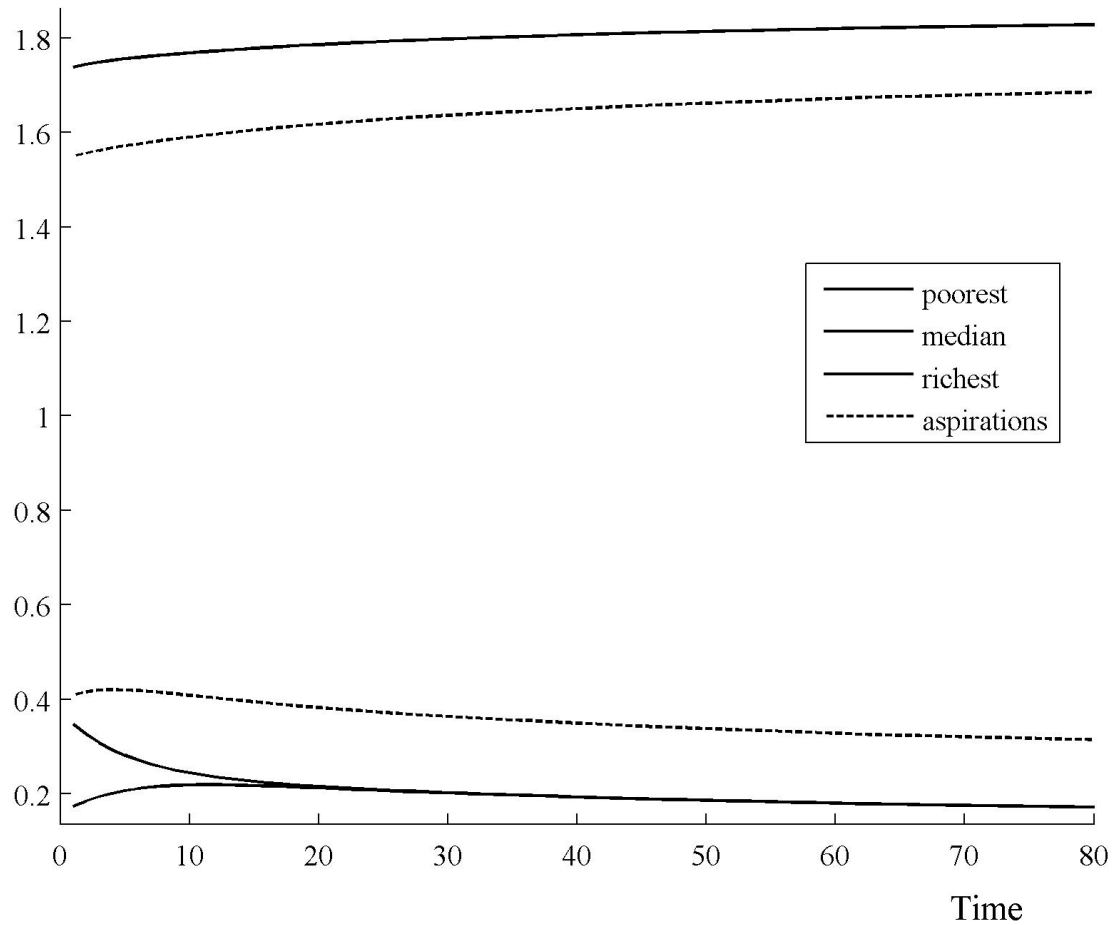



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Asymptotic Growth Rates		
Poorest	Median	Richest
1.66	1.66	1.76

---

■ Case D. Stable asymptotic inequality.



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Asymptotic Growth Rates

Poorest | Median | Richest

---

1.76 | 1.76 | 1.76

---

# Extensions

## ■ 1. Collective Action and Violence

### ■ Choose actions:

- $k$  (investment) and  $t$  (fraction time spent in violence).
- budget constraint:  $y = k + ty + c$
- $t$  reduces well-being of others, or permits looting and exclusion
- $z = f(k)$  as before.



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- $z = f(k)$  as before.

- maximize  $u(c) + w_0(z) + \sum_{i=1}^n w_i(e_i)$ .

#### ■ Proposition 8.

- As aspirations fail, investment  $\downarrow$  and violence  $\uparrow$ .

- As own wealth grows, investment  $\uparrow$  and violence  $\downarrow$ .

## ■ 2. Connected versus Polarized Societies

How far apart are the steps in multistep aspirations?

Connected versus polarized societies.

If milestones are polarized, the incentives to accumulate from lower step are minimal.

If there are lots of milestones, can approximate the concave case.

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## ■ 3. Calibrating Growth Incidence Curves

(Work in progress)

## Summary

- We build a theory of **aspirations formation**.
- Emphasizes the **social** foundations of individual aspirations
- Relates those aspirations to investment and growth.
- Such behavior can be aggregated, thus closing the model.

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- We build a theory of **aspirations formation**.
- Emphasizes the **social** foundations of individual aspirations
- Relates those aspirations to investment and growth.
- Such behavior can be aggregated, thus closing the model.
- Central feature: aspirations can incentivize and frustrate.
- Aspirations above incomes can encourage high investment.
- But aspirations that are too high will discourage investment.
- Rising aspirations have instrumental value — but up to a point.

- Steady state distributions **must exhibit inequality**.
- They are concentrated on a small number of local attractors.
- With single-step aspirations, steady states are **bipolar**.
- More generally, number of modes related to number of aspirational steps.

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- Single-step aspirations: either convergence to equal division, or perennially widening inequality.
- With multi-step aspirations there is a finer range of predictions.
- The model is tractable and may be useful in other contexts.