Diffusing Coordination Risk*

Deepal Basak and Zhen Zhou†

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Abstract

This paper designs an optimal mechanism to correct coordination failure. We consider a borrower who wants her creditors to coordinate on their decisions to roll over their debts. Creditors are learning the liquidity of the borrower and making their decision based on noisy private signals. The global game literature uniquely identifies the risk of debt run where the coordination risk is concentrated at one point in time. We analyse what happens when the borrower diffuses the coordination risk over time. The borrower approaches the creditors sequentially - only a proportion of creditors at a time and advancing further only when she has recovered what has already been withdrawn. The public information of survival works as a coordination device and helps in mitigating the coordination risk. We show that (1) truncated information is essential for diffusion to help reducing the chance of default, (2) if the borrower can diffuse the term structure enough then she can achieve the first best as the unique equilibrium outcome, (3) when creditors differ in terms of their willingness to roll over, a cautious or max-min borrower should approach the more reluctant creditors first.

1 Introduction

Coordination failure leads to economic turmoils or recessions. Pessimestic investors worry about the non-participation of other potential investors and decide to walk away from the new investment opportunity. Is there a way to lower the coordination risk? To what extent, the coordination risk can be lowered? Is it possible to have a mechanism to completely avoid the coordination failure? - These are very pertinent questions. This paper seeks to study these issues by focusing on the coordination problem among a mass of creditors who have to decide whether to roll over their debt or not. Even if the project is profitable, fear of panic-based runs caused by other creditors may lead to costly pre-mature liquidation. In this paper, we ask how a country or a firm can design its debt structure to minimize the chance of such coordination failure.

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†Department of Economics, New York University, 19 West 4th Street, New York, NY 10012. Email: db1839@nyu.edu, zhen.zhou@nyu.edu

We focus on the effect of debt structure on the belief of creditors and the coordination risk among them. Diffusing debt structure enables the borrower to approach the creditors sequentially in groups. For instance, suppose the project's return will be realized in one year, instead of letting creditors make their rollover decisions at the end of the sixth month, the borrower can design the debt contract which asks 10% share of creditors to make their decisions at the end of each month from the first month to the tenth month. Diffusing debt structure is feasible and commonly used. Choi et al. (2014) show that corporate bond issuers diversify debt rollovers across maturity dates and there is substantial variation in the granularity of debt across firms and across time. Hedge fund managers also set redemption gate, which limits the amount of withdrawals from the fund during a redemption period. [1]

Diffusing the debt structure does not necessarily reduce the coordination risk because the success of the project depends on the *aggregate* withdrawals and the outside liquidity of the borrower. The creditors facing less coordination risk today will have dynamic concerns for the future coordination failure. We show that if there are only private information transmissions, the *diffused coordination risk* when the debt structure is dispersed is exactly the same as the *concentrated coordination risk,* when all creditors make their decisions at one time. However, the *diffused coordination risk* is lower than the *concentrated coordination risk*, if the borrower approaches the next group of creditors *only* if she successfully sustained all the previous withdrawals. Our main result states that, if the borrower can make the debt structure dispersed enough, she can achieve the first best as the unique equilibrium outcome. In that equilibrium, the profitable project with any positive outside liquidity will have zero probability of pre-mature liquidation.

How can diffusing coordination completely avoid coordination risk and achieve the first best outcome? Creditors make their rollover decisions based on their belief of borrower's outside liquidity. If creditors understand that the borrower sustained all the previous withdrawals, this public truncated information will make incumbent creditors more positive towards the borrower's liquidity in sustaining current and late withdrawal. Creditor also understands that other creditors with the same truncated information are less likely to withdraw. Thus, with the public truncated information, *diffused coordination risk* is lower than the *concentrated coordination risk*. Consider the last group of creditors who are playing a static game. If the mass of creditors in the last period is small enough, we will show that the truncated public information overcomes all the coordination risk. The unique rationalizable action for them is to ignore their private information and roll over. The second last group of creditors can rationally forecast the strategy of the last group and so on. By backward induction, we show that diffusing the debt structure enough unravels the coordination risk from the end and thus enable the borrower to completely avoid the coordination failure.

The borrower can be thought of as an information designer (as defined in Kamenica and Gentzkow (2011) or Bergemann and Morris (2013a)) but with limited means to manipulate the creditors beliefs. By diffusing the debt structure enough, the borrower manipulates the creditors' beliefs in a way that her favorite outcome is the unique outcome. Suppose the borrower cannot diffuse the term structure enough. Suppose the groups are predefined and the borrower cannot make any finer groups. The only thing she can do is to choose the order in which she will approach

[1] The example of hedge fund is slightly different from our model. The group of creditors (or investors) who make their decisions during a certain redemption time is endogenuous.

the groups. Suppose creditors are heterogenous in terms of their willingness to rollover. Which group the borrower should approach first? What is the best order to approach these heterogenous creditors? We show that the *max-min*, or *cautious*, borrower should always approach the group of creditors who are most reluctant to roll over first. The borrower should rank the groups of creditors by the reluctance of rolling over and approach creditors from the most reluctant to the least reluctant in that order.

Our main result is not specific to the debt run model but have more general applicability in games with coordination risk. For example, think about a new technological innovation, e.g. online chat application, which can be successfully implemented only if it can attract a significant number of users. Adoption of this technology is costly to any potential user but it will be beneficial if successfully implemented. How could the innovator successfully market this new idea and avoid the inefficient coordination failure? Applying our result, the innovator should approach a small group of potential adopters first. Only if enough users decide to adopt this new technology such that the innovation is still implementable, the innovator will approach the next group. The same procedure continues to the last group of potential users. As long as the project is socially beneficial and implementation of it does not require 100% users to join, the innovation will be successfully implemented for sure if the innovator can divide all the potential users into small groups and approach them sequentially.

Financing the long-term investment by short-term borrowing, or the maturity mismatch problem, is in the center of current financial crisis (Brunnermeier, 2009). The coordination failure among creditors impairs the stability of financial system by inducing fire sale and drying up the market liquidity. This paper provides a feasible way to minimize this coordination risk among creditors. Our comparative statics results are in line with the recent studies (Gorton and Ordoñez, 2014) of the causes of financial crisis. In economic booms, the return rate of debt is high and creditors tend to ignore their private information of the firm's liquidity. In order to achieve the first best outcome without coordination failure, the borrower does not need to diffuse the debt structure very much.[2] The high interest rate will make creditors more likely to roll over and their decision will be based more on the public truncated information. However, creditors start to acquire information about borrower's liquidity and the expected rate of return is lower before recessions. At that time, the desired debt structure to achieve the first best outcome is much more dispersed and that is why there will be ample coordination risk and panic-based runs given the debt structure design more suitable for good times.

1.1 Related Literature

We begin with a coordination problem faced by a mass of creditors. When all creditors take their decision simultaneously, the game typically has multiple equilibria. Carlsson and Van Damme (1993) consider the refinement idea of relaxing the common knowledge of payoffs and obtain a unique equilibrium prediction. Morris and Shin (1998, 2003, 2007) (henceforth MS) developed the idea further. This strand of literature is commonly referred to as Global Games. Creditors privately gather information regarding the fundamental strength of the borrower's project and this leads to

[2]Suppose diffusing debt structure has some minor costs, we will discuss this in more detail in Section 6.

a unique equilibrium which plays out in threshold strategy. Creditors rollover if and only if they get a good enough signal. The risk of coordination is concentrated at one point in time. This is the basic problem we will start with.

When the borrower diffuses this coordination risk over time, the early creditors have concerns regarding actions of future creditors. There have been several works which focus on specific features of this dynamic concern. The paper which comes closest to ours is Dasgupta (2007). The fundamental of the project is chosen ex-ante and remains fixed (unlike Chassang (2010)). Agents gather relevant information privately before they make their decisions. Dasgupta et al. (2012) and Mathevet and Steiner (2013) also talk about a similar private learning enviornment. In addition to private information creditors publicly learn that the borrower has survived all early withdrawals. We see similar public information in Angeletos et al. (2007) (henceforth AHP). However, we have two major difference with AHP . First, unlike AHP, in our model, creditors do not get to choose the timing of their action. Creditors move at an exogenously specified point in time and only decide if they want to withdraw or roll over. Second, in AHP agents only care about whether enough agents will coordinate today, but in our model creditors not only care about what their fellow creditors will do today but also what debt holders will do in future .

Our reasearch question is very different from the works we have mentioned above. We want to design a debt structure so that the borrower can minimize the chance of default. In terms of the reasearch question, our work has close relation with the work of Bergemann and Morris (2013b) (henceforth BM) and Kamenica and Gentzkow (2011). The borrower can be thought of as an information designer but with limited means to manipulate the creditors beliefs.. Like BM we ask the question: can the borrower achieve her favorite outcome as the unique equilibrium outcome? We showed the answer is yes and we prove it by construction. Sakovics and Steiner (2012) also asked the same question: how to solve coordination failure. They designed the optimal subsidy or deposit insurance when agents are heterogeneous in terms of their willingness to invest or roll over. They showed that the borrower should subsidize the more reluctant agents first. We show that a cautious borrower should approach the more reluctant agents first.

It is a common practice for firms to spread creditors' rollover decisions over time to reduce the liquidity risk of having to roll over large quantities of debt at the same time (He and Xiong, 2012). In this paper, we justify the diffused debt structure without liquidity shocks to the borrower. Diffused debt structure is similar to the asynchronous debt structure in finance literature (Leland and Toft, 1996; He and Konstantin, 2014). Stationary debt structure requires the borrower to roll over a fixed fraction of their outstanding debt at every instant of time. Instead of taking the stationary debt structure as given, this paper rationalizes it from minimizing the coordination risk between creditors.

Unlike Diamond-Dybivg debt run model, creditors in our model have no preference shocks and the allocation of asset is absent. We focus only on the information aspect of the coordination problem. Hence, we consider a dynamic coordination problem as in the global game literature[3]. Green and Lin (2003) designed a direct mechanism to achieve the first best (no bank run and ex-ante efficient allocation) under sequential serivice constraint in Diamond-Dybivg model. The basic idea of their mechanism is to make payoff depend on the position and reported type of creditors. The opti-

[3]See Goldstein and Pauzner (2005) for a global game refinement of the Diamond-Dybivg model

mal mechanism in our paper is the design of debt structure, which gives same payoff to each creditor.

This paper is also related to the work of Gale (1995). In Gale (1995) a finite number of players facing dynamic coordination problem with endogenous delays. Efficiency can be achieved when agnets move sequentially beause of complete information. But in our model creditors have incomplete information. They collect information privately. The only publicly availabe information is that the borrower has not failed yet.

1.2 Outline

The paper is organized as follows: In section 2 we will describe the benchmark where all the risk of coordination is concentrated at one point in time. Readers familiar with the global game literature can skip this section. In section 3 we will consider a diffused term structure while creditors privately gather relevant information. We will show that diffusing the concentration does not change the coordination risk. In section 4 we will consider the case when the borrower asks the later debt holders to roll over only if she has already recovered the fund that the early creditors have withdrawn. So, when the later creditors are asked to roll over their debt, they know that the borrower has survived the early withdrawal. The public information of survival causes multiplicity of equilibria. We will show that even in the worst equilibrium diffused debt structure has less chance of default than concentrated debt structure. This section also contains the main result of the paper: if the borrower can diffuse the term structure enough, the project succeeds whenever the borrower can withstand any positive withdrawal (however small). In section 6 we will consider heterogeneous debt holders who differs in terms of their willingness to roll over. We will show that a cautious or max-min borrower should approach the more reluctant debt holders first. In section 7 we discuss some extensions and section 8 concludes. All proofs that are omitted in the main paper are in the appendix.

2 Concentrated Coordination Risk

There is a borrower with a positive net present value investment opportunity and a continuum [0,1] of creditors. The borrower and creditors are all risk neutral. The project needs an initial investment of 1 dollar. The borrower finaces her investment project by issuing collateralized debt. The collateralized debt contract specifies that each creditor will lend 1 dollar to the borrower and get a collateral, which can be a share of the underlying investment project. Before the project matures, each collaterallized creditor will have a chance (at the same time) to decide roll over or withdraw their money. The success of the project only depends on the proportion of creditors who withdraw their money w and the state of the economy θ. Specifically, if the proportion of creditors who withdraw is (weakly) less than the outside funding source θ, the project is successful. Creditors will earn interest rates on their lendings. Otherwise, the project fails. The project will have to be early liquidated and the borrower would have to default. Creditors cannot get their face value back but receive a share of the liquidation value.

The state of the world θ, in this paper, is the measure of outside funding the borrower can manage to access. It is natural to assume that θ is positive. However, being more general here, we assume that there is no lower bound for θ, i.e. $\theta \in (-\infty, +\infty)$.[4] One can interpret θ as some liquid assets owned by the borrower. In that sense, the liquidity of the borrower can be below zero when the value of assets is negative, e.g. the value of derivatives in financial market can be negative if the underlying entity is not performing well. If $\theta < 0$ (lower dominance region) then the borrower cannot sustain any withdrawal but has to sell this investment project for existing obiligations. Even if nobody withdraws the project fails. If $\theta > 1$ (upper dominance region) then even when everybody withdraws the project survives.

If θ is comonly known, then there are multiple equilibria: all creditors rolling over is an equilibrium and all creditors withdrawing is also an equilibrium. Consider two cases, the first project has a outside funding $\theta = 0.95$, which can sustain the withdrawal of 95% of its creditors, and the second project $\theta = 0.05$, which can only sustain 5% withdrawal of its creditors. It is natural to expect a $\theta = 0.95$ project is much more likely to succeed than a $\theta = 0.05$ project. Following Carlsson and Van Damme (1993) we will relax the common knowledge assumption. Suppose nature picks the state of the world θ from the commonly known prior $N(\theta_0, \sigma_0^2)$. Creditor i gets independent private signals about θ.[5]

$$s_i = \theta + \epsilon_i \text{ where } \epsilon_i \sim N(0, \sigma^2)$$

Let $\tau_0 = \frac{1}{\sigma_0^2}$ and $\tau = \frac{1}{\sigma^2}$ be the precision of the prior and the signal respectively. Before the project matures, creditors decide between two actions: 1 (roll over) and 0 (withdraw). Creditor's payoff u is a function of her withdraw decision, the aggregate withdrawal w and the strength of the fundamental θ :

$$u(1, w, \theta) = \begin{cases} 1 + r & \text{if } w \leq \theta \\ q & \text{if } w > \theta \end{cases}$$

$$u(0, w, \theta) = \begin{cases} 1 & \text{if } w \leq \theta \\ c & \text{if } w > \theta \end{cases}$$

If the total withdrawal is less than the outside funding, the project succeeds and each creditor will get paid $1 + r$. $r \in (0, 1)$ is the interest rate. If the project failed, or the aggregate withdrawal is more than the available funding, each creditor will get the liquidation value of the investment project. We only consider the case where $q < c \leq 1$, which means rolling over is a risky action for creditors. Without loss of generality, we assume that the liquidation value of the project $q = 0$ and the collateral value in liquidation $c = 1$. It worth mentioning that the sequential service constraint in Diamond-Dybivg model is absent in our model. The payoff for withdrawing under liquidation is assumed to be fixed, which doesn't depend on the aggregate withdrawal and whether withdrawal happen before liquidation or not. These assumptions are not essential to any result in the paper but rather for the convenience of computation. [6]

When creditors get higher signals they beleieve the borrower can withstand more withdrawal. Creditors with higher signal will be more optimisitic towards the other creditors' signals as well. Thus, they believe the project is more likely to succeed. Naturally, we will look into the monotone

[4]This assumption is not essential to understand the main result of this paper. However, this helps us to avoid multiplicity of equilibria in the basic setting. We will have a discussion of the prior $\theta \geq 0$ in section 6.

[5]Although there are some technical concern regarding the existence of continuum of independent random variables, Judd (1985) shows this assumption is still appropriate to work with.

[6]Goldstein and Pauzner (2005) show that the strategic complementarities will be lower but the result will be similar. Since our focus is not in this subject, we stick to the simple payment structure.

equilibrium where creditors roll over if and only if their signal is higher than some threshold $s^\star$. Given this, the higher the θ is, the lower proportion of creditors will withdraw. Thus, we get a threshold value of fundamental, $\theta^\star$, such that the project succeeds if and only if θ is greater than $\theta^\star$. $(\theta^\star, s^\star)$ will represent the monotone equilibrium.

Assumption 1 *Private signals are precise enough as compared to the prior :* $\tau_0 < \sqrt{2\pi\tau}$

Given this assumption, it can be shown (See MS) that there is a unique monotone equilibrium. FollowingMilgrom and Roberts (1990) (henceforth MR), We will argue that this is not just the unique monotone equilibrium but indeed the unique equilibrium in general.

Proposition 1 *Given Assumption 1, there is a unique equilibrium where agents roll over if they get a signal $s > s^*$, otherwise they withdraw. When $\tau \to \infty$ the project succeeds iff $\theta > \frac{1}{1+r}$.*

This is a well known result but we will reproduce the proof in the appendix for the shake of completeness.

Proof. See appendix. ■

For the rest of the paper we will consider $\frac{\tau_0}{\tau} \to 0$, which means either $\tau_0 \to 0$ and/or $\tau \to \infty$. So prior plays no role. For simplicity of algebra we will assume $\tau_0 \to 0$, or equivalently taking improper prior over θ, $\theta \sim U(\mathbb{R})$. Since our concern is only conditional probabilities, $U(\mathbb{R})$ is an appropriate assumption. To think about the unconditional probabilities, take the normal prior approximation with variance tending to infinity. We can also think of $U[a, b]$, where $a \to -\infty$ and $b \to \infty$. Therefore, minimizing the ex-ante probability of default is equivalent to minimizing $\theta^\star$. The unique equilibrium in this case is $(\theta^*,\ s^*)$ where, $\theta^* = \frac{1}{1+r}$ and $s^* = \frac{1}{1+r} + \frac{1}{\sqrt{\tau}}\Phi^{-1}(\frac{1}{1+r})$. If we restrict the prior to be $\theta \geq 0$, the lower dominance region is absent by iterated elimination. So, $\theta^\star = 0$ can never be ruled out. Observe that, when $\theta \geq 0$ is commonly known, creditors roll over for any signal is indeed an equilibrium resulting $\theta^\star = 0$, which means the project always succeeds. We will discuss this in more detial in Section 6.

3 Diffusing Coordination Risk: Private Information Transmission

3.1 Bifurcated Term Structure

Suppose the investment project ends at T and creditors make their rollover decision at t $(t < T)$ In the previous section, all debt holders are taking their rollover decision at the same time, where the strategic uncertainty is concentrated at time t. In this section, we want to investigate whether the borrower can decrease the risk of default by diffusing concentration of risk, i.e., by allowing creditors to make their rollover decisions at different points in time.

We will consider the following term structure: $\alpha_1 = \alpha(\alpha \in (0, 1))$ proportion of debt holders take their rollover decision at some other time, say at $t' < t$. And the rest of $\alpha_2 = 1 - \alpha$ proportion of creditors take their decision at t. To keep the notation simple, suppose $t' = 1,\ t = 2,\ T = 3$. Creditors still have the same contract: $(1 + r)$ if the project succeeds and 0 if fails. If they withdraw

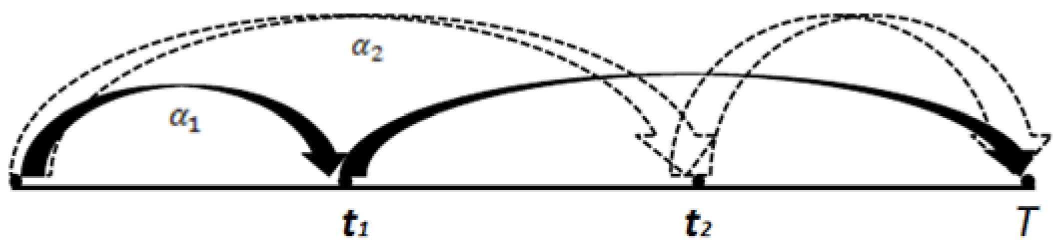

Figure 1: Bifurcated Term Structure: α_1 proportion of creditors take their decision at t_1, α_2 proportion of creditors take their action at t_2

they still get 1. The debt structure $\{\alpha_1, \alpha_2\}$ is common knowledge. Consistent with the benchmark model, we assmue that both the debt contracts are identical except the time of rollover decision

As in the basic model, the fundamental $\theta \sim U(\mathbb{R})$. At $t = 1$, creditors will receive independent private signals $s_{1i} = \theta + \epsilon_{1i}$, $\epsilon_{1i} \sim N(0, \sigma^2)$. Let $w_1(\theta)$ denote the proportion of creditors withdrawing at $t = 1$. Let $\theta_2 = f_1(\theta) = \theta - \alpha_1 w_1(\theta)$ denote residual outside funding. So the borrower can effectively withstand θ_2 withdrawal at $t = 2$. The creditors at $t = 2$ are *instanteneous information gatherers* i.e. they gather some information regarding the fundamental strength of the borrower just before they make their decision. Gathering information over time may be costly and the fundamental strength of the borrower that the late creditors are interested in is θ_2 (how much withdraw the borrower can still withstand) rather than θ. [7] As before we will assume that agents make their decision based on their private information rather than prior i.e. the private information is very precise compared to the prior information. For simplicity we will assume that creditors in period 2 share the common uninformative prior $\theta_2 \sim U(\mathbb{R})$ and will receive independent private signals about how much withdrawal the borrower can still withstand, i.e. $s_{2i} = \theta_2 + \epsilon_{2i}$, $\epsilon_{2i} \sim N(0, \sigma^2)$.

At $t = 2$, the problem creditors are facing is exactly the same to the static benchmark case with

[7]However, if the creditors also choose the timing of their decision then it is not a natural assumption. The creditors will rather gather information over time as in Dasgupta(2007). We will extend our model to T many periods in later section, where T can be very large. It seems that creditors moving very late will be neither willing to gather information about all previous withdrawal nor it is feasible to do so. So we assume that agents just gather information reagarding how much withdrawal the borrower can still witstand. Our results will go through even if we assume information structure as in Dasgupta: agents in period 2 acquire private information $s_{1i} = \theta + \epsilon_{1i}$ and $s_{2i} = \Phi^{-1}(w_1) + \epsilon_{2i}$.

the measure of creditors is α_2. Hence, there is a unique equilibrium (θ_2^*, s_2^*), where

$$\theta_2^* = \frac{\alpha_2}{1+r},\ s_2^* = \frac{\alpha_2}{1+r} + \frac{1}{\sqrt{\tau}}\Phi^{-1}(\frac{1}{1+r})$$

At $t = 1$, creditors are not only concerned about the withdrawal today but also the potential withdrawal tomorrow. Only if the project can withstand the aggregate withdrawal of both periods they can make profit. Given the equilibrium map f_1 :$\mathbb{R} \to \mathbb{R}$, creditors at $t = 1$ form their belief about $\theta_2 = f_1(\theta)$ after receiving their private signals. Creditors at $t = 1$ roll over only if

$$P(f_1(\theta) > \theta_2^*|s_1) \geq \frac{1}{1+r}$$

Let $(\theta_1^\star,\ s_1^\star)$ be the equilibrium, then threshold signal $s_1^\star$ should be such that the creditor who gets signal $s_1^\star$ is indifferent between rolling over and withdrawing.

$$P(f_1(\theta) > \theta_2^*|s_1^*) = \frac{1}{(1+r)}$$

The threshold fundamental $\theta_1^\star$ should be such that when $\theta = \theta_1^\star$, the aggregate withdrawal of both periods should be equal to the fundamental itself.

$$\alpha P(s_1 \leq s_1^\star|\theta_1^\star) + \theta_2^\star = \theta_1^\star \Leftrightarrow f_1(\theta_1^\star) = \theta_2^\star$$

Solving this we get $\theta^* = \frac{1}{1+r}$ and $s_1^* = \frac{1}{1+r} + \frac{1}{\sqrt{\tau}}\Phi^{-1}(\frac{1}{1+r})$. Hence the ex-ante chance of default is $P(\theta < \frac{1}{1+r})$, which is exactly the same as in the benchmark model. This result is similar to proposition 1.3 in Dasgupta (2007).

Proposition 2 *When there is only private information, the risk of default $P(D)$ remains the same for any $\alpha \in [0, 1]$*

Proof.
See Appendix ■

Had there been only finitely many agents then individual action in period 1 would have affected the signal agents get in period 2. Continuum of agents assumption shuts down this force. Only the aggregate action affects the signal and not individual action.

3.2 General Term Structure

In this subsection, we extend the above result to the general case when the borrower approaches the creditors sequentially in T many rounds. Assume that for any $t \in \{1, 2, 3, \ldots, T\}$, the creditors at t share the uninformative prior

$$\theta_t \sim U(\mathbb{R}).$$

They receive independent private signals regarding how much withdrawal the borrower can withstand currently

$$s_{ti} = \theta_t + \epsilon_{ti},\ \epsilon_{ti} \sim N(0, \sigma^2).$$

Let w_t :$\mathbb{R} \to \mathbb{R}$ denote the equilibrium proportion of withdrawal at t given θ_t and $f_t : \mathbb{R} \to \mathbb{R}$ is defined as follows:

$$f_t(\theta_t) := \theta_t - \alpha_t w_t(\theta_t) = \theta_{t+1}$$

Proposition 3 *For any $\alpha \equiv (\alpha_1, \alpha_2, \ldots \alpha_T)$ such that $\alpha_t \in [0,1]$ for all $t = 1, 2, \ldots T$ and $\sum_{t=1}^{T} \alpha_t = 1$, the chance of default is same.*

Proof. Following the same arguments as in proposition 2 we can show that for all $t = 1, 2, \ldots T$

$$\theta_t^* = \frac{\alpha_t}{1+r} + \theta_{t+1}^* \text{ with } \theta_{T+1}^* = 0$$

Therefore for any α, $\theta_1^* = \frac{1}{1+r}$. ■

Since this proposition is true for any T, however diffused the term structure is designed, the probability of failure always stays the same. Therefore, given the fact that later creditors cannot receive any public information regarding how early creditors behave, diffusing term structure cannot change the ex-ante probability of default. Observe that, if all creditors receive information only about θ (rather than period t creditors receiving information about θ_t), then the diffused game is exactly the same as the static benchmark game. Here creditors do get some partial information about how early creditors behave, but we still have the equivalence with the benchmark case. As mentioned in the previous subsubsection, a continuum of agents is necessary to this equivalence.

4 Diffusing Coordination Risk: Truncated Public Information

4.1 Bifurcated Term Structure

Consider the two period case. The borrower approaches the early creditors first and ask them to roll over their debt. Based on their private signals, some of the creditors roll over and some withdraw. The borrower now goes to the outside funding source to recover what has been withdrawn. If she can manage to refinance those withdrawals, then she approaches the late creditors. The late creditors get the public information that the borrower has already sustained the withdrawal in the first period, along with that they get their private information about how much the borrower can still withstand. Based on both the private signal and the public information, the late creditors then decide whether to roll over or withdraw. The project succeeds if the residual outside funding can withstand the withdrawal in second period. If the outside funding source is insufficient to sustain the withdrawal of early creditors, then the borrower has no reason to approach the late creditors because the project has to be liquidated independent of the later creditors' decisions. In the earlier model, the borrower was waiting until all the withdrawal have taken place before she approach the outside source of funding.

Thus, when the borrower approaches the late creditors, the late creditors know that the borrower has already recovered what has been withdrawn by early creditors. In other words, they learn that $\theta_2 = \theta - \alpha_1 w_1 \geq 0$. This is what we refer to as the truncated public information. The currency attack model of AHP contains this type of information as well, where the agents see the central bank has survived past currency attacks. There are two substantial differences between this model and AHP. First, creditors in this model move at a pre-specified time while in AHP agents decide when to attack the currency. Second, in this model, the early creditors are not only concerned about what their fellow creditors will do today but also what creditors will do later, while in AHP agents attacking the currency are only worried about what other agents will do today.

Consider a threshold equilibrium $(\theta_1^\star, \theta_2^\star, s_1^\star, s_2^\star)$. Given any $\theta_2^\star$, the corresponding threshold signal $s_2^\star$ has to be such that when $\theta_2 = \theta_2^\star$, the aggregate withdrawal in period 2 is equal to $\theta_2^\star$. This gives us $s_2^\star = \theta_2^\star + \frac{1}{\sqrt{\tau}}\Phi^{-1}(\frac{\theta_2^\star}{\alpha_2})$. The belief of the threshold creditor that the project will succeed is

$$P(\theta_2 > \theta_2^\star | s_2^\star, \theta_2 \geq 0) = \frac{\frac{\theta_2^*}{\alpha_2}}{\Phi(\sqrt{\tau}\theta_2^* + \Phi^{-1}(\frac{\theta_2^*}{\alpha_2}))}.$$

The definition of $(\theta_1^\star, s_1^\star)$ is the same as in the benchmark model. The following proposition describes the equilibrium formally.

Proposition 4 *There exists monotone equilibria* (s_t^*) *such that debt holders roll over in period* t *iff* $s_t > s_t^*$, $t = 1, 2$. *Consequently, the project succeeds iff* $\theta > \theta_1^*$, *where*

$$\theta_1^* = \frac{\alpha_1}{1+r} + \theta_2^* \text{ where}$$

$$\frac{\frac{\theta_2^*}{\alpha_2}}{\Phi(\sqrt{\tau}\theta_2^* + \Phi^{-1}(\frac{\theta_2^*}{\alpha_2}))} = \frac{1}{1+r} \text{ or } \theta_2^* = 0$$

Proof. This is a special case of proposition 6. ■

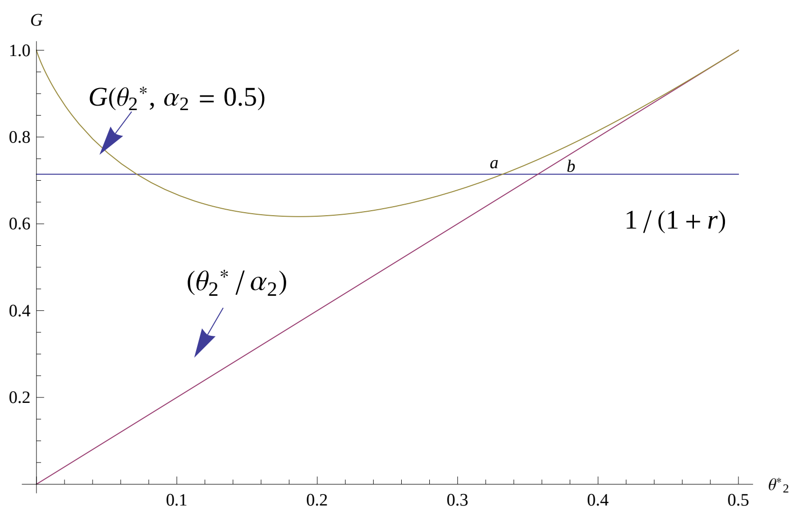

Figure 2: Effect of Truncated Information ($\tau = 10, r = 0.4, \alpha_2 = 0.5$)

As can be seen in Figure 2, when there is no truncated information, the threshold creditor believes that the project will succeed with probability $\frac{\theta_2^\star}{\alpha_2}$, which is an increasing function of $\theta_2^\star$. This gives the unique solution (point b in Figure 1) of $\frac{\theta_2^\star}{\alpha_2} = \frac{1}{1+r}$. However, when there is truncated information, the belief of the threshold creditor is not monotonic (G in Figure 2), which gives us multiple solutions for $\theta_2^\star$. As in AHP, we have multiple monotone equilibria. Also, there may exist non-monotonic equilibria. Oberseve that $\theta_2^* = 0$ is always an equilibrium. To see this suppose agents always roll over irrespective of whatever signal they get in period 2. If time 2 creditor believes that, then she would have rolled over only if she believes that $\theta_2 \geq 0$ with probability greater than $\frac{1}{1+r}$.

If it is already publicly known that $\theta_2 \geq 0$, then we can not eliminate any never best responses. So, we can never rule out the strategy that creditors always roll over irrespective of their signal as a never best response. Therefore, conditional on reaching period 2, there is no chance of default if $\theta_2^\star = 0$. Consequently, given the bifurcated term structure (α_1, α_2), the equilibrium with least chance of default is the monotonic equilibrium with fundamental thresholds $(\theta_1^\star = \frac{\alpha_1}{1+r},\ \theta_2^\star = 0)$. The following proposition shows the effects of truncated information on coordination risk.

Proposition 5 *The truncated information helps to reduce the coordination risk in any possible equilibrium.*

Proof. In equilibrium, this threshold creditor is indifferent between rolling over and withdrawing. Thus, $P(\theta_2 > \theta_2^\star | s_2^\star, \theta_2 \geq 0) = \frac{1}{1+r}$. The belief of the threshold agent is

$$P(\theta_2 > \theta_2^\star | s_2^\star, \theta_2 \geq 0) = \frac{\frac{\theta_2^*}{\alpha_2}}{\Phi(\sqrt{\tau}\theta_2^* + \Phi^{-1}(\frac{\theta_2^*}{\alpha_2}))} \geq \frac{\theta_2^*}{\alpha_2}.$$

The last inequality follows from $\Phi(.) \leq 1$. So in any possible monotone equilibrium, $\theta_2^\star < \frac{\alpha_2}{1+r}$ and $\theta_1^\star = \theta_2^\star + \frac{\alpha_2}{1+r} < \frac{1}{1+r}$. While in the case without truncated information, $\theta_1^\star = \frac{1}{1+r}$.
■

Although multiple equilibria may arise with truncated information, the result in proposition 5 is robust to any equilibrium being selected. In period 2, given any private signal s_2, the creditor assigns weakly higher probability to success when she gets the truncated information, $\theta_2 \geq 0$. Therefore, the equilibrium $\theta_2^\star$ is always below the threshold when there is no truncated information. Compared to the case without truncated information, when there is threshold signal, for any signal a creditor gets in period 1, she assigns weakly higher probability to success. This is because there is some fundamental strength for which the borrower would not have survived in period 2 had there been no public truncated information. Thus even if all debt holders in period 1 behave the same way as in no public information transmission case, any debt holder in period 1 will find withdrawing less attractive than in the no publuc information transmission case. This makes any debt holders less likely to withdraw in period 1. Because of strategic complementarity this will make other debt holders less likely to withdraw and so on. Thus we expect the threshold of fundamental in period 1, $\theta_1^\star$, to be lower than the case without truncated information. To be more precise, given $\theta_2^\star$, $\theta_1^\star$ is such that aggregate withdrawal in period 1 exactly leaves $\theta_2^\star$ for period 2, i.e. $\theta_1^\star = \frac{\alpha_1}{1+r} + \theta_2^\star$. Since $\theta_2^\star$ is smaller, the threshold of period 1 fundamental $\theta_1^\star$ is smaller and hence the chance of default will go down.

4.2 General Term Structure

Suppose the borrower is able to separate the creditors into T groups and approach them sequentially. At any time t $(1 \leq t < T)$, the borrower will ask mass α_t creditors to roll over their debt. If the borrower can recover what has been withdrawn then the borrower asks the next group of mass α_{t+1} creditor to roll over their debt and keeps doing so until period T, where she exhausts the whole set of debt holders, i.e. $\sum_{t=1}^{T} \alpha_t = 1$. The end period creditors is exactly the same as in the bifurcated case. Given the equilibrium $\theta_T^\star$, at $T-1$, the creditors believe that the project succeeds if $\theta_{T-1} - \theta_T^\star \geq \alpha_{T-1} w_{T-1}$. If $\theta_{T-1}^\star$ is the equilibrium threshold, then there is a threshold for private signal, $s_T^\star$, such that $s_{T-1}^\star = \theta_{T-1}^\star - \theta_T^\star + \frac{1}{\sqrt{\tau}}\Phi^{-1}(\frac{\theta_{T-1}^\star - \theta_T^\star}{\alpha_T})$. The belief of the threshold creditor that the project will succeed is

$$P(\theta_{T-1} > \theta_{T-1}^\star | s_{T-1}^\star, \theta_{T-1} \geq 0) = \frac{\frac{\theta_{T-1}^* - \theta_T^\star}{\alpha_{T-1}}}{\Phi(\sqrt{\tau}\theta_{T-1}^* + \Phi^{-1}(\frac{\theta_{T-1}^* - \theta_T^\star}{\alpha_{T-1}}))} \geq \frac{\theta_{T-1}^* - \theta_T^\star}{\alpha_{T-1}}$$

This solves for $\theta_{T-1}^\star$ and go backwards we can solve for the sequence of $\{\theta_t^\star\}_{t=1}^T$. In the equilibrium, the creditor with the threshold private signal must believe the probability of success is $\frac{1}{1+r}$. Similar to the argument we made in 2 period model, the resulting $\theta_1^\star$ will be smaller than the case without truncated information.

Proposition 6 *There exists monotone equilibria* (s_t^*) *such that debt holders roll over in period* t *iff* $s_t > s_t^*$, $t = 1, 2 \dots T$. *Consequently, the project succeeds iff* $\theta > \theta_1^*$, *where*

$$\theta_1^* = \frac{\alpha_1}{1+r} + \theta_2^* \textit{ where}$$

$$\textit{for any } t = 2, \dots T, \quad \frac{\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t}}{\Phi(\sqrt{\tau}\theta_t^* + \Phi^{-1}(\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t}))} = \frac{1}{1+r} \textit{ or } \theta_t^* = 0 \textit{ and } \theta_{T+1}^* = 0 \tag{1}$$

Proof. See appendix. ■

The above recursive relation does not have unique solution similar to the bifurcated term structure case. $\theta_t^\star = 0$ for all $t = 2, 3, \dots, T$ is always a solution. Therefore, there is an equilibrium where conditional on reaching period 2, there is no chance of default. If creditors in period 1 believes that this will happen from period 2 onwards, then the equilibrium thresholds are $(\theta_1^\star = \frac{\alpha_1}{1+r},\ \theta_2^\star = 0 \dots, \theta_T^\star = 0)$. This monotone equilibrium has the least chance of default among all possible equilibria.

4.3 Optimal Term Structure

We have already seen that when the borrower approaches the creditors sequentially and there is truncated information, the probability of default is always lower (for any equilibrium) than the case where there is no truncated information. Thus diffusion of coordination risk helps in reducing the probability of default if there is truncated information. We would like to know how the borrower can design a term structure $(T, (\alpha))$ where $T \in \mathbb{N}$ and $(\alpha) \equiv (\alpha_1, \alpha_2, \dots \alpha_T) \in \Delta^{T-1}$,[8] such that the probability of default is minimized. Let $P(T, (\alpha))$ be the probability of default given term structure $(T, (\alpha))$. The borrwer chooses $(T, (\alpha))$ to minimize $P(T, (\alpha))$. If each policy induces a unique equilibrium then the objective function is straight forward. However, as we have seen there can be multiple equilibria when there is truncated information. So we first need to define the objective function when there are multiple equilibria corresponding to any policy $(T, (\alpha))$.

Let us first rank all the equilibria in order of probability of default[9]. Following MR we can say the best and worst equilibrium are in monotone strategy. We have already shown that given

[8]where $\Delta^{T-1} := \{(\alpha_1, \alpha_2, \dots, \alpha_T) \in \mathbb{R}^T |\ \alpha_t \geq 0\ \forall t = 1, 2 \dots T,\ \sum_{t=1}^T \alpha_t = 1\}$ is the standard $T-1$ simplex

[9]We donot need a complete order, we only need a lower bound and an upper bound on $P(T, (\alpha))$ for any $(T, (\alpha))$.

any term structure $(T,(\alpha))$, the *best* equilibrium is the monotone equilibrium with threshold fundamental $(\theta_1^{\star l}=\frac{\alpha_1}{1+r},\ \theta_2^{\star l}=0\ldots,\theta_T^{\star l}=0)$. In this equilibrium, creditors always roll over from $t=2$ onwards irrespective of their private signals. The *worst* equilibrium is the monotone equilibrium corresponding to the maximum solution to equation 1. Let $\{\theta_t^{*h}(T,(\alpha))\}_{t=2}^T$ be the maximum solution to 1. Then the *worst* equilibrium is a monotone equilibrium with threshold fundamental $(\theta_1^{\star h}=\frac{\alpha_1}{1+r}+\theta_2^{\star h},\ \theta_2^{\star h}\ldots,\theta_T^{\star h})$. Let us define $\underline{P}(T,(\alpha)):=P(\theta<\theta_1^{*l}(T,(\alpha)))$ be the probability of default corresponding to the *best* equlibrium and $\bar{P}(T,(\alpha)):=P(\theta<\theta_1^{*h}(T,(\alpha)))$ for the *worst* equlibrium. So, given any term structure $(T,(\alpha))$,

$$\bar{P}(T,(\alpha))\geq P(T,(\alpha))\geq \underline{P}(T,(\alpha))$$

P is the prior belief of the borrower. It is possible that the borrower has better information than the creditors and so P may be different from the prior beleif of the creditors. If $(T,(\alpha))$ is designed after the borrower has acquired this information then in equilibrium the creditors would have learned more about the fundamental after the borrower announces the policy. We are assuming that either the borrower announces $(T,(\alpha))$ before she acquires more information or she shares the same prior as the creditors. Thus the creditors do not learn anything from the chosen term structure. Since we have assumed uninformative prior, the unconditional probabilty is not well defined. But we can think of the normal approximation where the variance goes to infinity and use the basic property that the cdf is increasing. So, if there is a threshold value of fundamental such that the project succeeds whenever θ is beyond this threshold, the borrower would like to reduce the threshold.

Given the potential multiplicity of equilibria, we can say that for any term structure $(T,(\alpha))$, there is $\theta_1^{*l}(T,(\alpha))$ and $\theta_1^{*h}(T,(\alpha))$ such that : (1) If $\theta\geq\theta_1^{*h}(T,(\alpha))$ the project succeeds irrespective of whatever equilibrium is played. (2) If $\theta<\theta_1^{*l}(T,(\alpha))$ the project fails irrespective of whatever equilibrium is played and (3) if $\theta\in[\theta_1^{*l}(T,(\alpha)),\theta_1^{*h}(T,(\alpha)))$, there is an equilibrium such that the project may fail.

Let us define

$$\underline{\theta}_1^*:=\inf_{(T,(\alpha))}\theta_1^{*l}(T,(\alpha))\text{ and }\bar{\theta}_1^*:=\inf_{(T,(\alpha))}\theta_1^{*h}(T,(\alpha)).$$

We know $\theta_1^{*l}(T,(\alpha))=\frac{\alpha_1}{1+r}$ for all $(T,(\alpha))$. Therefore, $\underline{\theta}_1^*=0$. One way to define the borrower's objective will be to mimimize $\theta_1^{*h}(T,(\alpha))$. We will call such a borrower a *Cautious* borrower or a *max-min* borrower. The borrower wants to minimize the possibility of default anticipatng the worst can happen (see Gilboa and Schmeidler (1989)). Of course this is not the only reasonable objective of the borrower when there are multiple equilibria. For example, we can think of a borrower who is *optimistic* i.e. always anticipates the best possible equilibrium will be played. The following theorem represents the main result of this paper. The validity of this theorem is not limited to the specific assumption regarding the borrower's objective. Theorem 1 says that when the borrower can diffuse the term structure enough, she can place small enough α_t creditors in every round and she can make sure that the project succeeds for all $\theta>0$.

Theorem 1 *When the borrower approaches the creditors sequentially and there is truncated information, $\bar{\theta}_1^*=\underline{\theta}_1^*=0$. Also, given (r,τ) there exists $T^*<\infty$, such that for any $\eta>0$ (however small), the borrower can design a term structure $(T,(\alpha))$ such that the project succeeds for all $\theta\geq\eta$, if the borrower can diffuse the term structure enough i.e. $T\geq T^*$.*

Proof. See appendix. ■

It follows from definition of $\bar{\theta}_1^*$ that for any $\eta > 0$ the borrower can design a term structure $(T, (\alpha))$ such that the project will always succeed for any $\theta \geq \eta$. The theorem claims more than that. It says for any $\eta > 0$ (however small), there is a uniform bound on how much the borrower needs to diffuse the term structure to make sure the project succeeds whenever $\theta \geq \eta$. In practice, the borrower can diffuse the concentration of coordination risk at almost no cost and thus almost costlessly make sure the project succeeds for any $\theta > 0$. [10]

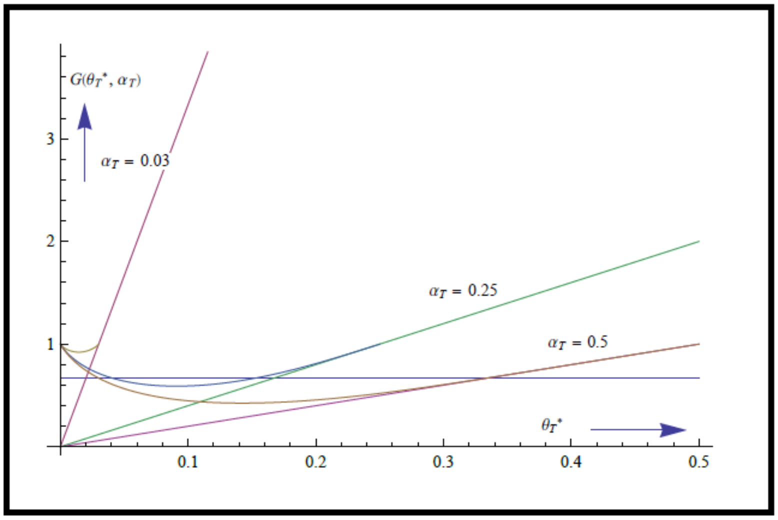

Figure 3: Effect of α_T on θ_T^*

The proof is constructive. When the borrower separates the crediors in several groups but there is no truncated information, the last group of creditors problem is just a scaled down version of the static benchmark problem. We can think of the problem with effective fundamental per capita $\frac{\theta_T}{\alpha_T}$. Thus in equilibrium $\frac{\theta_T^*}{\alpha_T} = \frac{1}{1+r}$. Now suppose there is truncated information $\theta_T \geq 0$. Consider two cases: (1) the mass of creditors moving at time is α_T, (2) the mass of creditors moving at time is α_T', $(\alpha_T > \alpha_T')$. We will see how the two groups are differently affected by the threshold signal. If θ_T^* be the threshold value of fundamental then the threshold agent believes the probability of success is higher for case 2 than for case 1 i.e. when smaller mass of agents are moving in period T, the threshold agent is more optimistic about success (as shown in figure 2).

We will show that there is a critical mass α^* such that if $\alpha_T < \alpha^*$ the truncated information completely overcomes the coordination risk i.e. the threshold agent believes the probability of suc-

[10]It does not seem likely that the borrower will face some strict constraint $\bar{T} < T^*$ such that she can diffuse the term structure only upto $\bar{T}$. If such is the case we can still design the optomal term stucture but the result will depend on the precise definition of the borrower's objective. We have solved for the optimal term structure for a cautious borrower with $\bar{T} = 2$ but the result is more mechanical than intuitive and so we omit this from the paper.

cess is higher than $\frac{1}{1+r}$. Therefore when the borrower approaches the last group, the creditors do not withdraw irrespective of their private signals. Thus if the borrower designs the term structure such that $\alpha_T < \alpha^*$ then conditional on reaching the last group of creditor there is no chance of default, i.e. $\theta_T^* = 0$.

Now consider the last but one group. In absense of any truncated signal and given any θ_T^*, their problem is similar to the static problem with effective fundamental per capita $\frac{\theta_{T-1}-\theta_T^*}{\alpha_{T-1}}$. Thus when there is no truncated information $\frac{\theta_{T-1}^*-\theta_T^*}{\alpha_{T-1}} = \frac{1}{1+r}$. So if the borrower has designed a term structure such that $\alpha_T < \alpha^*$, then the $T-1$ group of creditors know $\theta_T^* = 0$, i.e. there is no chance of default at T, if the borrower can withstand the withdrawal in period $T-1$. Thus their problem is exactly the same as the group T creditors. Therefore if the borrrower chooses $\alpha_{T-1} < \alpha^*$, then $\theta_{T-1}^* = 0$ is the only equilibrium.

Continuing this way until group 1, if the borrower design a term structure that places $\alpha_t < \alpha^*$ for all $t = 2, 3 \dots T$, then $\theta_2^* = 0$ i.e. group 1 creditors know that if the borrower can withstand withdrawal at time 1, then there is no chance of default. So they are playing a static game. Let $T^* := \frac{1}{\alpha^*}+1$. Consider the term stucture $(T^*, (\alpha))$ such that $\alpha_1 = \epsilon$ and $\alpha_t < \alpha^*$ for all $t = 2, 3 \dots, T$. Then, there is unique equilibrium with fundamental threshold $\left(\theta_1^* = \frac{\epsilon}{1+r},\ \theta_2^\star = 0, \dots,\ \theta_T^\star = 0\right)$. Now take $\epsilon \to 0$. So the projects succeeds for all $\theta > 0$.

5 Heterogeneity

In the previous section, we consider a borrower who has full flexibility in separating creditors into different groups. We have seen that when the borrower can diffuse the term structure enough, she can make sure the project succeeds for any positive fundamental. In this section, we will consider the case when the groups are exdogenously defined and the borrower can only choose the sequence of approaching different groups. For simplicity, we assume the groups are of equal mass and have access to equally informative signals. The only heterogeneity among groups is in term of the willingness to roll over. A possible explanation could be that one group is more financially constrained than the other group and thus values the return more when the project succeeds. Or one group has much lower discount rate of future cash flow, and thus they will value the project return more than the other group.

Suppose there are only two groups with equal mass. Group i values the return $(1 + r_i)$ if the project succeeds, $i = 1, 2$. Let us assume that $r_1 < r_2$. Group i creditors roll over if they believe that the project will succeed with probability (weakly) higher than $\frac{1}{(1+r_i)}$. Thus, group 1 is more reluctant to roll over than group 2, i.e. $\frac{1}{(1+r_1)} > \frac{1}{(1+r_2)}$. The information structure is as before (with truncated information). From our analysis before, multiple equilibria may arise because of the truncated information structure. So we need an objective criteria for the borrower. The borrower is assumed to be *cautious* or *max-min* economic agent, i.e. she wants to minimize the chance of default anticipating the *worst* equilibrium can be played. Let us denote $\mathcal{T}_2$ as the set of all possible permutation of $(1, 2)$, i.e. $\mathcal{T}_2 := \{(t(1), t(2)) | t : \{1, 2\} \to \{1, 2\} \text{ is a permutation}\}$. The borrower's problem is to choose $(t(1), t(2)) \in \mathcal{T}_2$ to minimize the highest possible threshold $\theta_1^{\star h}$. In other words, the borrower needs to decide whether she should approach the more reluctant group first or the less reluctant group first. The following proposition answers this question.

Proposition 7 *Suppose there are two groups of equally informed debt holders with equal mass and* $r_1 < r_2$. *A* cautious *borrower will (weakly) prefer the permutation* $(1,2)$ *over* $(2,1)$.

Proof. See Appendix. ■

This proposition says that the *cautious* borrower should approach the less reluctant creditors later. If the borrower approaches the less reluctant creditors later, then conditional upon reaching the group moving later, there is higher chance of success, i.e. $\theta_2^{\star h}(1,2) < \theta_2^{\star h}(2,1)$, where $\theta_2^{\star h}(t(1),t(2))$ is the highest fundamental threshold in period 2 when the borrower chooses the permutation $(t(1),t(2))$. Therefore, the group moving earlier would have higher effective fundamental per capita $\frac{\theta-\theta_2^{\star h}}{1/2}$, if less reluctant creditors were approached later. But, the more reluctant group of creditors face higher strategic uncertainty from their fellow creditors. In the proof, we showed that the first effect dominates when there is truncated information. If there was no truncated information then the two contradictory forces offset each other. So, $\theta_1^* = \frac{1/2}{1+r_1} + \frac{1/2}{1+r_2}$ irrespective of which group is approached first. When there is truncated information θ_2^* falls and consequently θ_1^* is lower. Comparing with the no truncated information case then the magnitude of the effect of any policy can be evaluated as the reduction in θ_2^*. As figure 3 shows this effect is higher if less reluctat group is approached later.

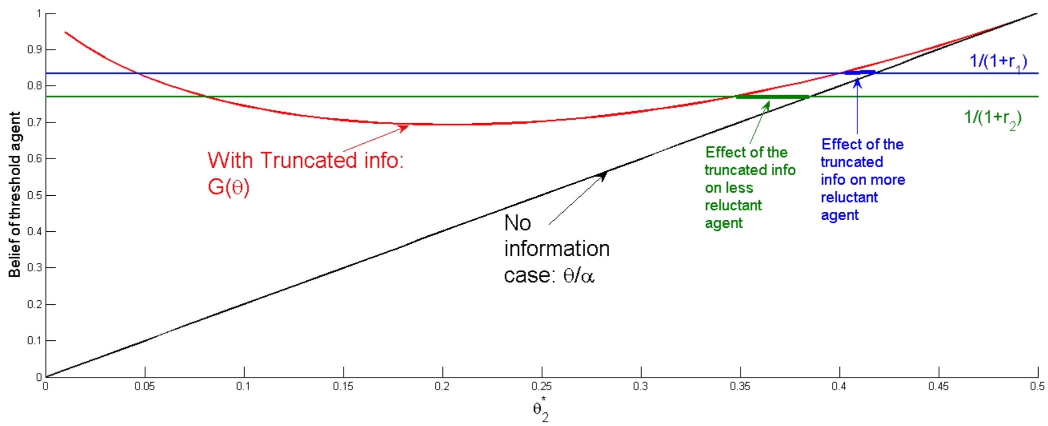

Figure 4: Effect of permutation (1,2) and (2,1)

Extending the argument we can show that if there are more than 2 groups then $(\ldots k, k', \ldots)$ permutation is better than $(\ldots k', k, \ldots)$ if group k is more reluctant than group k'. Therefore we can extend this two groups intuition to the general case.

Proposition 8 *Suppose there are* n *equally informed groups of mass* $\frac{1}{n}$ *each. Suppose* $r_1 < r_2 < \ldots < r_n$. *Then a cautious borrower should optimally pick the permutation* $(1,2,\ldots n)$.

Proof. See appendix. ■

6 Discussion

6.1 Information Design

Think of the borrower as the information designer who wants her creditors to coordinate on rolling over their debts. An information designer can commit to the information process that the agents have acess to. But she cannot influence the realisation of the signal. Unlike the grand information designer as in Kamenica and Gentzkow (2011), the borrower cannot implement all Bayes-plausible distribution of posterior belief of the creditors. The borrower can influence the belief of the creditors only by designing a debt structure. In this sense the borrower has only limited power of manipulation.

The borrower's favourite equilibrium is when there is no default i.e. all creditors roll over as long as the borrower can withstand non-negative withdrawal. We will refer to this as Borrower's favorite Bayes Correlated Equilibrium [11] (BFBCE). So, the best outcome the borrower can hope to achieve is that the project succeeds for all $\theta \geq 0$. We know that if creditors know θ then this is one possible outcome but so is project fails for all $\theta < 1$. Now suppose creditors do not know the fundamental strength i.e. share uninformative prior. Further assume creditors privately gather some independent noisy information about the fundamental strength of the borrower. How should the information designer design the information structure? *Can the information designer design the information structure so that the resulting outcome is always her favorite outcome?*

Suppose, the borrower does not intervene at all. Then the standard global game argument tells us there is a unique Bayes Nash Equilibrium (BNE). The project succeeds if the fundamental strength is beyond a threshold level, otherwise not. The threshold is not zero as the borrower wants. Now think of what way the information designer can possibly influence the outcome? Suppose the borrower can commit to publicly disclosure of a truncated information of the following form: whenever the borrower can withstand non-negative withdrawal it sends a good signal, otherwise a bad signal. Although the borrower's favorite equilibrium is a BNE, multiple equilibria arise. But the borrower does not have unlimited manipulation power. So let us qualify our earlier question: *Can the borrower deign an informtion structure with her limited manipulation power such that her favorite BCE is the unique BNE?*

The answer is yes and we have proven it by construction. The design of this debt structure is based on the idea of diffusing coordination risk coupled with truncated public information. Note that diffusion of coordination risk alone does not reduce coordination risk. As we have seen in section 3. However, if the bowrrower approaches the late creditor only when she can recover the early withdraw then there is a natural truncated information that late creditors get. This public information that things are good changes the creditors' beliefs. Consequently the chance of default is less even for the worst possible equilibrium. So the next question we ask:

Is there a limit to how much the borrower can improve by designing such diffused term structure?

[11]See Bergmann and Morris (2013) for formal definition of Bayes Correlated Equilibrium (creditors are *obedient*) and Bayes Nash Equilibrium (creditors are *obedient* and *belief invariant*)

We showed that borrower can achieve her favorite equilibrium as the unique BNE if she can diffuse the term structure enough. The basic argument is that the effect of public news of survival can completely overcome the coordination risk if the risk comes from a small enough mass of creditors. Then the coordination risk unravels from back. Thus, whenever the borrower has full freedom to make the groups and diffuse the term structure she can achieve the her favorite equilibrium as the unique BNE.

6.2 No lower Dominance Region

As we have mentioned in static benchmark case, it might be reasonable to assume that the outside funding source is known to be non-negative, i.e. $\theta \geq 0$. In this case, we lose the lower dominance region, i.e. the project cannot fail when all creditors roll over. The truncated information raises the threshold creditor's belief about the probability of success. With this modification in the benchmark model, multiple equilibria arise but the probabilty of failure will be lower no matter which equilibrium is selected. The best equilibrium threshold is $\theta_1^{\star l} = 0$ and worst equilibrium threshold is $\theta_1^{\star h}$, which is the maximum solution to equation 1 with $T = 1, \ \alpha_T = 1$.

Consider the diffused term structure $(T, (\alpha))$ with truncated information such that $\alpha_t < \alpha^\star$for all $t = 1, \ldots, T$. If the first period creditors know $\theta \geq 0$, then for such term structure, $\theta_1^\star = 0$. Compare this with $\theta_1^\star = \frac{\alpha_1}{1+r}$ in case when the information of $\theta \geq 0$ was not available. The borrower can achieve the best case scenario with zero probability of default with out restricting α_1 to be tending to zero. Therefore, it is easier to construct the term structure for the validity of Theorem 1.

Consider the concentrated coordination risk model and supoose $\theta \geq 0$ information was available. There are multiple BNE including Borrower's favorite BCE. If the borrower can control the precision of private signals then she can acheieve BFBCE as the unique BNE by choosing a small enough precision so that the good news overcome the coordination risk. But the borrower is not likely to have such unlimited manipulation power. The diffusion of term structure is serving the same purpose. Although the borrower can not control the precision of private signal, she can control the mass of agents taking their decision at any point in time. There is a critical mass such that the good news effect overcomes the coordination risk. Using this result we design a debt structure such the coordination risk unrave from the end.

6.3 Other Applications

In this paper, we provide a way to lower (or completely avoid) the coordination risk in games with strategic complementarities. Although we present the model and our result in the model of debt runs, our main result is robust to any coordination game.

Consider a new technological innovation, e.g. online chat application, which can be successfully implemented only if it can attract a signicant number of users. The fundamental strength θ is one minus this significant number, i.e. the maximum share of potential users this innovation can lose to succeed. For example, the new technology requires 40% of Apple product users to join to make a success. $\theta = 60\%$ means that the project will fail only if more than 60% of the target consumers reject to adopt this techology. For any individual potential user, adoption of this technology is costly but it will be benecial if it is successfully implemented. This is a standard coordination problem. Coordination failure will be socially costly. How can the innovator successfully market this new idea and aviod the ineffcient coordination failure?

According to our main result, the innovator is able to design a mechanism to completely avoid the coordination risk. The innovator should approach a small group of potential adopters first. Only if the share of rejecting target consumers is smaller than θ, which makes the innovation still implementable, the innovator will approach the next group. The same procedure continues to the last group of potential users. For example, the new technology can be successfully adopted if the share of rejection is less than 60%, or $\theta = 60\%$. After presenting the new idea and talking to 80% of target customers, if 50% of target group (40% of total mass) reject to pay to adopt the technology, the developer will proceed to approach the next group of potential users. If there are more than 60% of potential users have already rejected the adoption, the innovator will stop. As long as the project is socially benecial and the fundamental $\theta > 0$, this mechanism will make the innovation successful for sure if the innovator can divide potential users of this innovation into small enough groups and approach them sequentially.

6.4 General Equilibrium Concern

In Theorem 1, we assume that the interest rate paid to creditors is fixed at r, which does not depend on the probability of default in equilibrium. It is worthwhile to consider the incentive of creditors in a general equilibrium framework. In order to make the diffused debt contract feasible, creditors would ask a higher interest rate if the coordination failure is more likely to happen. Consider the debt contract $(T, (\alpha))$, where $T \in \mathbb{N}$ and $\alpha \equiv (\alpha_1, \alpha_2, ..., \alpha_T) \in \Delta^{T-1}$. As defined in Section 4, the probability of default corresponding to the worst equilibrium is $\bar{P}(T, (\alpha)) := P(\theta < \theta_1^{*h}(T, (\alpha)))$. The participation constraint for any creditors is

$$\left[1 - \bar{P}(T, (\alpha))\right] \times [1 + r(T, (\alpha))] \geq 1 + R$$

in which R is the creditors outside option and $r(T, (\alpha))$ is the interest rate paid to creditor given the project succeeds. In equilibrium, borrower will choose the optimal debt contract to maximize the expected profit by taking the creditors' participation constraint into consideration. Here, we assume that the borrower cannot set the interest rate high or low to subsidize all creditors or a group of creditors. The interest rate is uniform to each creditor. Borrower will set the interest rate at $r(T, (\alpha)) = \frac{1+R}{1-\bar{P}(T,(\alpha))} - 1$ to make creditors breakeven. Given this interest rate, creditors act exactly the same as in the partial equilbrium model discussed before.

The main result of the paper stays the same when endogenizing the interest rate. The first best outcome is still implementable in the general equilibrium model. Borrower's objective is still to minimize the probability of coordination failure as the interent rate $r(T, (\alpha))$ is increasing with the coordination risk. When the $\alpha_t < \alpha^\star$ for $t = 2, 3, ...T$, and $\alpha_1 \to 0$, the unique equilibrium gives the first best outcome $\bar{P}(T, (\alpha)) = \underline{P}(T, (\alpha)) = P(T, (\alpha)) \to 0$ and the interest rate would be the outside option of creditors R. Deviating from this debt structure will increase the probability of default and thus increase the cost of borrowing, so the best debt structure is exactly the same.

6.5 Financial Crisis

We have shown that the borrower can almost costlessly design a term structure such that whenever she can withstand non negative withdrawal, creditors always rollover. There may be some negligible costs of diffusion. For instance, there might be some issuance cost for each type of debt. One can interpret the cost of diffusing debt structure as the illiquidity discount, since more debt issues with smaller sizes will have a less liquid secondary market than few debt issues with larger sizes (Choi et al., 2014). It does not seem like something of the borrower's first order proirity.[12] We can see that $\frac{\partial T^*}{\partial \tau} > 0$ i.e. if the precision of private signal is higher then the borrower needs to diffuse more to achieve her favorite equilibrium as the unique Bayesian Nash Equilibrium. This is because it is more difficult to overcome the coordition risk with the good news effect. Thus if the borrower has any control over the precision of private signal it is in her interest to keep it more noisy. Gorton and Ordoñez (2014) argues that creditors becomes more skeptical during bad times, i.e. they rely less on public good news and try to gather more precise information. Thus it may happen that due to some external shocks creditors starts gathering more precise private information and consequently the old diffused debt structure is no longer good enough to achieve the first best. As a result crisis may occur. Choi et al. (2014) found empirical evidence that right before the financial crisis financial institutions adopted more diffussed term structures.

7 Conclusion

8 Appendix

Proof of Proposition 1 Rolling over is a risky decision which may return $(1+r)$, in case the project succeeds and 0, in case it fails. On the other hand withdrawal is a safe action which returns 1. Therefore, debt holders roll over if she believes $P(\text{success}) > \frac{1}{(1+r)}$. Agent updates her belief of θ conditional on her private signal s is

$$\theta | s \sim N(\frac{\tau_0}{\tau_0+\tau}\theta_0 + \frac{\tau}{\tau_0+\tau}s, \frac{1}{\tau_0+\tau})$$

Let s_- denote the signal of other creditors (We are dropping the suffix i from now on in the proof). The updated belief of s_-given private signal s is

$$s_- | s \sim N(\frac{\tau_0}{\tau_0+\tau}\theta_0 + \frac{\tau}{\tau_0+\tau}s, \frac{1}{\tau_0+\tau} + \frac{1}{\tau})$$

[12]Suppose the borrower has lexicographic preference where she wants to minimize the chance of default first and the given the chance of default wants to minimize the cost of diffusion. If cost of diffusion is equally important for the borrower as reducing the chance of default then she needs to diffuse it so much that the marginal benefit is equal to the marginal cost. However, since there are multiple equilibria, there is no unquestionable way of defining the borrower's objective. The optimum is based on what the borrower believes which equilibrium will be selected and also there is no way to justify why equilibrium selection has to be invariant to policy choice.

Step 1: Iterated Elimination of Never Best Responses(nbr):

Let us suppose creditors always roll over irrespective of her signal. If a creditor believes that other creditors always roll over then she will roll over only if she $\frac{1}{1+r}$ believes that $\theta > 0$ (p believe of event E means $P(E) \geq p$) i.e. s is such that $p(\theta > 0|s) > \frac{1}{1+r}$. Solving this we get $s > s^{1(1)}$. $s^{1(n)}$ is the threshold after n^{th} round of elimination of nbr for over (1 stands for the action of rolling over). $s^{0(n)}$ is the threshold after n^{th} round of elimination of nbr for withdrawal (0 stands for the action of withdrawal). If creditors know that others will never roll over unless $s > s^{1(1)}$ then they will roll over only if $p(p(s_- > s^{1(1)}|\theta) > 1 - \theta|s) > \frac{1}{1+r} \Rightarrow s > s^{1(2)}$ and so on. $s^{1(n)}$ can be solved using the following recursive relation with $s^{1(0)} = -\infty$

$$s^{1(n+1)} = \frac{\tau_0 + \tau}{\tau}\sqrt{\frac{1}{\tau_0 + \tau}}\Phi^{-1}(\frac{1}{1+r}) - \frac{\tau_0}{\tau}\theta_0 + \frac{\tau_0 + \tau}{\tau}s^{-1}(s^{1(n)})$$

where

$$s(\theta) = \theta + \frac{1}{\sqrt{\tau}}\Phi^{-1}(\theta) \tag{2}$$

is an increasing function.

This gives us $s^{1(n)} \to s^{1(\infty)}$ (The limit exists because this is a bounded increasing squence) i.e. by iterated elimination of never best responses we can say that the agent will never roll over if $s \leq s^{1(\infty)}$.

Similarly, starting from agents always withdraw and following iterated elimination of never best responses we can say agents will never withdraw if $s \geq s^{0(\infty)} \geq s^{1(\infty)}$

If $s^{0(\infty)} = s^{1(\infty)}$, then we have unique rationalizable strategy: agents roll over if $s > s^{0(\infty)} = s^{1(\infty)}$ and withdraw otherwise. Hence there is unique equilibrium. But if $s^{0(\infty)} > s^{1(\infty)}$, then both the following monotone strategy constitute equilibrium: (1) agents roll over if $s > s^{0(\infty)}$ and withdraw otherwise and (2) agents roll over if $s > s^{1(\infty)}$ and withdraw otherwise. This result follows from Milgrom and Roberts (1990).

Therefore, if there is unique equilibrium in monotone strategy then it is indeed the unique equilibrium.

Step 2: Equilibrium in Monotone strategy:

Let $s^\star$ be a monotone equilibrium. Let us define $\theta^\star$ as the value of fundamental where aggregate withdrawal is exactly equals what the borrower can withstand:

$$p(s \leq s^\star|\theta^\star) = \theta^\star \tag{3}$$

This gives $\theta^* = s^{-1}(s^*)$, where $s(\theta)$ is as defined in equation 2. Therefore when $\theta > \theta^\star$ the project succeeds otherwise it fails. The creditor must be indifferent between rolling over and withdrawing when she gets the threshold signal s^*. Hence,

$$p(\theta > \theta^\star|s^\star) = \frac{1}{(1+r)} \tag{4}$$

This gives us

$$\Phi(\frac{\frac{\tau_0}{\tau_0+\tau}\theta_0 + \frac{\tau}{\tau_0+\tau}s^\star - s^{-1}(s^\star)}{\sqrt{\frac{1}{\tau_0+\tau}}}) = \frac{1}{1+r}$$

Given assumption 1, LHS is increasing in s^* and thus there is unique solution.

Finally, taking $\tau \to \infty$, we have

$$\lim_{\tau\to\infty} \Phi(\frac{\frac{1}{\sqrt{\tau}}\Phi^{-1}(\theta^\star) - \frac{\tau_0}{\tau_0+\tau}(\theta_0 + s^\star)}{\sqrt{\frac{1}{\tau_0+\tau}}}) = \frac{1}{1+r}$$

Hence,

$$\theta^\star = \frac{1}{1+r}$$

□

Proof of Proposition 2 Consider $t = 2$, in equilibrium the creditor who gets the threshold signal s_2^* must be indifferent between rolling over and withdrawing. Hence,

$$P(\theta_2 > \theta_2^* | s_2^*) = \frac{1}{(1+r)}$$

This gives us

$$s_2^* = \theta_2^* + \frac{1}{\sqrt{\tau}}\Phi^{-1}(\frac{1}{1+r})$$

The threshold fundamental must be such that when $\theta_2 = \theta_2^*$, the aggregate withdrawal in period 2 is same as θ_2^*, i.e.

$$\alpha_2 p(s_2 \le s_2^* | \theta_2^*) = \theta_2^*$$

Hence,

$$\theta_2^* = \frac{\alpha_2}{1+r}, \quad s_2^* = \frac{\alpha_2}{1+r} + \frac{1}{\sqrt{\tau}}\Phi^{-1}(\frac{1}{1+r})$$

Now consider $t = 1$, the creditor who gets the threshold signal is indifferent between withdrawing and rolling over. So,

$$P(f_1(\theta) > \theta_2^* | s_1^*) = \frac{1}{(1+r)}$$

where

$$f_1(\theta) := \theta - \alpha_1 \Phi(\sqrt{\tau}(s_1^* - \theta))$$

Given monotonic equilibrium, $f_1(\theta)$ is increasing in θ and hence invertible. Therefore the indifference condition gives us

$$s_1^* = f_1^{-1}(\theta_2^*) + \frac{1}{\sqrt{\tau}}\Phi^{-1}(\frac{1}{1+r})$$

The threshold fundamental must be such that when $\theta = \theta_1^*$, the agrregate withdrawal in two periods equals θ_1^*. Hence,

$$f_1(\theta_1^*) = \theta_2^*$$

Hence,

$$s_1^* = \theta_1^* + \frac{1}{\sqrt{\tau}}\Phi^{-1}(\frac{1}{1+r})$$

Now replacing s_1^* in the definition of $f_1(\theta_1^*)$ we get

$$\alpha_1(\frac{1}{1+r}) + \theta_2^* = \theta_1^* \tag{5}$$

Hence,

$$\theta_1^* = \frac{1}{1+r},\ s_1^* = \frac{1}{1+r} + \frac{1}{\sqrt{\tau}}\Phi^{-1}(\frac{1}{1+r})$$

Thus we see that there is a unique monotone equilibrium. Extending the MR argument [13] we can then say that it is indeed the unique uquilibrium in general. So, probability of default given any α is $P(D|\alpha) = P(\theta \leq \theta_1^*)$ is independent of α. □

Proof of Proposition 6 Let $(\theta_t^*, s_t^*)_{t=1}^T$ be the equilibrium threshold. At $t = 1$ following the same steps as proposition 2 we get equation 5.

$$\theta_1^* = (\frac{\alpha_1}{1+r}) + \theta_2^* \tag{6}$$

For any $t \geq 2$ let us define the net pay off from rolling over when an agent gets a signal s_t and it is commonly known that $\theta_t \geq 0$ as follows:

$$u(s_t, \theta_t^*) = \begin{cases} (1+r)\frac{\Phi(\sqrt{\tau}(s_t - \theta_t^*))}{\Phi(\sqrt{\tau}s_t)} - 1 & \text{if } \theta_t^* > 0 \\ r & \text{if } \theta_t^* = 0 \end{cases} \tag{7}$$

since

$$P(success|s_t, \theta_t \geq 0) = \frac{P(\theta_t \geq \theta_t^*|s_t)}{P(\theta_t \geq 0|s_t)} = \frac{\Phi(\sqrt{\tau}(s_t - \theta_t^*))}{\Phi(\sqrt{\tau}s_t)}$$

Let $s_t^*(\theta_t^*)$ be the threshold signal such that if debt holders roll over iff $s_t \geq s_t^*(\theta^*; \tau, r, \alpha)$ then $\theta_t \geq \alpha_t w(\theta_t) + \theta_{t+1}^*$(or $f_t(\theta) < \theta_{t+1}^*$) iff $\theta_t \geq \theta_t^*$. Therefore,

$$s_t^*(\theta_t^*) = \theta_t^* + \frac{1}{\sqrt{\tau}}\Phi^{-1}(\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t}) \tag{8}$$

Finally define the net payoff from rolling over for the marginal agent who gets the signal $s_t^*(\theta^*; \tau, r, \alpha)$ and it is publicly known that $\theta_t \geq 0$ as follows:

$$U(s_t, \theta_t^*) = \begin{cases} \lim_{s_t \to -\infty} u(s_t, \theta_t^*) & \text{if } \theta_t^* = 0 \\ u(s_t^*(\theta^*), \theta_t^*) & \text{if } \theta_t^* \in (0,1) \\ \lim_{s_t \to \infty} u(s_t, \theta_t^*) & \text{if } \theta_t^* = 1 \end{cases} \tag{9}$$

In equilibrium the marginal agent is indifferent. So $U(s_t, \theta^*) = 0$. Also, $\theta_t^* = 0$ is always a solution but $\theta_t^* = 1$ is never a solution. This gives us the recursive relation 1. □

The following lemma is a technical lemma. It gives us some properties of the belief of the threshold creditor when there is truncated information.

[13]It is a very straight forward extension where we just need to treat the creditors in period 1 and 2 diffrently. So we ommit the details.

Lemma 1 *Define*

$$G(x,\alpha) := \frac{\frac{x}{\alpha}}{\Phi(\sqrt{\tau}x + \Phi^{-1}(\frac{x}{\alpha}))},\ \alpha \in [0,1],\ x \in [0,\alpha] \tag{10}$$

Then,

1. *$G(\theta^*,\alpha)$ is differentiable*
2. *given α, $G(0,\alpha) = 1 = G(\alpha,\alpha)$*
3. *$G(x,\alpha)$ first decreases and then increases with x.*
4. *$G_x(x,\alpha) \le \frac{1}{\alpha}$ forall $x \in [0,\alpha]$*

Proof. lemma 1.1 is obvious and lemma 1.2 follows from use of L'Hospital rule. Since the denominator is convex until some x^* and then concave beyond it, we have lemma 1.3. One can check that $G'(x,\alpha)$ is maximum when $x \to \alpha$, where $\lim_{x\to\alpha} G_x(x,\alpha) = \frac{1}{\alpha}$ (using L'Hospital rule). ■

Proof of Theorem 1 Let us define

$$G(\theta^*,\alpha) := \frac{\frac{\theta^*}{\alpha}}{\Phi(\sqrt{\tau}\theta^* + \Phi^{-1}(\frac{\theta^*}{\alpha}))},\ \alpha \in [0,1],\ \theta^* \in [0,\alpha] \tag{11}$$

$$x(\alpha) := \arg\min_{\theta^*} G(\theta^*,\alpha) \tag{12}$$

$$y(\alpha) := G(x(\alpha),\alpha) \tag{13}$$

Given lemma 1.3, first order condition (FOC) is sufficient to identify $x(\alpha)$.

Lemma 2 *There exists $\alpha^* > 0$ such that*

$$y(\alpha) > \frac{1}{1+r} \textit{if } \alpha < \alpha^*$$

$$y(\alpha) \le \frac{1}{1+r} \textit{if } \alpha \ge \alpha^*$$

Proof.

By envelope theorem

$$\frac{d}{d\alpha}y(\alpha) = \frac{\partial}{\partial\alpha}G(\theta^*,\alpha)|_{\theta^*=x(\alpha)} \tag{14}$$

Now,

$$\frac{\partial}{\partial\alpha}G(\theta^*,\alpha)|_{\theta^*=x(\alpha)} = -\frac{x(\alpha)}{\alpha^2\Phi(.)^2}[\Phi(.) - \frac{x(\alpha)}{\alpha}\frac{\phi(.)}{\phi(\Phi^{-1}(\frac{x(\alpha)}{\alpha}))}] \tag{15}$$

where $(.) = \sqrt{\tau}x(\alpha) + \Phi^{-1}(\frac{x(\alpha)}{\alpha})$. From the FOC $\frac{\partial}{\partial\theta^*}G(\theta^*,\alpha)|_{\theta^*=x(\alpha)} = 0$ we have

$$\frac{x(\alpha)\phi(.)}{\Phi(.)}[\sqrt{\tau} + \frac{1}{\alpha\phi(\Phi^{-1}(\frac{x(\alpha)}{\alpha}))}] = 1 \tag{16}$$

Therefore,

$$[\Phi(.) - \frac{x(\alpha)}{\alpha}\frac{\phi(.)}{\phi(\Phi^{-1}(\frac{x(\alpha)}{\alpha}))}] = \sqrt{\tau}x(\alpha)\phi(.) \tag{17}$$

Replacing this in equation 15 we have $\frac{d}{d\alpha}y(\alpha) < 0$.

By theorem of maximum we know $y(\alpha)$ is continuous. Hence, $y(0) = \lim_{\alpha\to 0} y(\alpha) = 1 > \frac{1}{1+r}$. If $y(1) < \frac{1}{1+r}$ then $\exists\alpha^* > 0$ if $y(0) > \frac{1}{1+r}$ and if $y(1) > \frac{1}{1+r}$ then $\alpha^* = 1$,

■

Consider period T, from proposition 6 we know in equilibrium either $G(\theta_T^*, \alpha_T) = \frac{1}{1+r}$ or $\theta_T^* = 0$. Therefore using lemma 2 if $\alpha_T < \alpha^*$ then $\min_{\theta_T^*} G(\theta_T^*, \alpha_T) > \frac{1}{1+r}$. Hence $\theta_T^* = 0$ is the unique equilibrium. Also note that if $\alpha_T \geq \alpha^*$ then there is always equilibrium with $\theta_T^* > 0$. Therefore if less than α^* agents move in period T, then the borrower will not fail if it reaches period T.

Suppose $\alpha_T < \alpha^*$, then debt holders in period $T-1$ knows that the borrower will survive the next period if it survives the current period. So they are essentially facing static coordination risk while it is publicly known that $\theta_{T-1} > 0$. Therefore using lemma 2 if $\alpha_{T-1} < \alpha^*$ then $\theta_{T-1}^* = 0$ is the unique equilibrium. Proceeding the same way if $\alpha_t < \alpha^*$ for all $t \geq 2$ then $\theta_2^* = 0$.

Let $T^* = \frac{1}{\alpha^*} + 1 < \infty$. If $T > T^*$ then the borrower can design a term structure such that it assigns ϵ (however small) proportion of debt holders in period 1 and rest $(1-\epsilon)$ proportion of debt holders in the rest $T-1$ periods such that $\alpha_t < \alpha^*$. Then $\theta_1^* = \frac{\epsilon}{1+r}$. Take $\epsilon \to 0$ and then $\theta_1^* \to 0$.Therefore, $\bar{\theta}_1^\star = 0$. □

Proof of Proposition 7 The borrower can either pick $(1,2)$ i.e. approach group 1 first and then group 2 or the other way i.e. $(2,1)$. If $y(1/2) > \frac{1}{1+r_g}$,($y(\alpha)$ is as defined in equation 13) for some group $g = 1,2$, then the borrower would approach him at the end. If both groups are like that then the order does not matter. So, we will consider the case when $\frac{1}{1+r_g} \geq y(1/2)$ for all $g = 1,2$. Suppose there is no information transmission. Then irrespective of whether the borrower picks $(1,2)$ or $(2,1)$, the project succeeds whenever $\theta > \theta_1^*(0)$, where

$$\theta_1^*(0) := \frac{1}{2}(\frac{1}{1+r_1} + \frac{1}{1+r_2})$$

If the borrower picks $(1,2)$ then the project succeed whenever $\theta > \theta_1^*(1,2)$, where

$$\theta_1^*(1,2) = \frac{1}{2}(\frac{1}{1+r_1}) + \theta_2^*(1,2) \text{ where } G(\theta_2^*) = \frac{1}{1+r_2}$$

where $G(.)$ is the belief of the threshold agent as defined in equation 11. Since the mass of groups are constant we are dropping it from the argument. If the borrower picks $(2,1)$ then the project succeed whenever $\theta > \theta_1^*(2,1)$, where

$$\theta_1^*(2,1) = \frac{1}{2}(\frac{1}{1+r_2}) + \theta_2^*(2,1) \text{ where } G(\theta_2^*) = \frac{1}{1+r_1}$$

Therefore

$$\theta_1^*(1,2) - \theta_1^*(2,1) = \{\theta_1^*(0) - \theta_1^*(2,1)\} - \{\theta_1^*(0) - \theta_1^*(1,2)\}$$

$$= \{\frac{1}{2}(\frac{1}{1+r_1}) - \theta_2^*(2,1)\} - \{\frac{1}{2}(\frac{1}{1+r_2}) - \theta_2^*(1,2)\}$$

$$= \{\frac{1}{2}(\frac{1}{1+r_1}) - G^{-1}(\frac{1}{1+r_1})\} - \{\frac{1}{2}(\frac{1}{1+r_2}) - G^{-1}(\frac{1}{1+r_2})\}$$

Since the borrower is cautious we are looking at the maximum solution for θ_2^* and from the proof of theorem 1 we know $G(.)$ is increasing and hence invertible and the inverse function is increasing. Let us define $F(x) := \frac{1}{2}x - G^{-1}(x)$. We have dropped the argument α from the notation since $\alpha = \frac{1}{2}$ is fixed. We can say $\theta_1^*(1,2) - \theta_1^*(2,1) < (>)0$ if $F(.)$ is decreasing (increasing) function (since $\frac{1}{1+r_1} > \frac{1}{1+r_2}$). Now, $F'(x) = \frac{1}{2} - \frac{1}{G'(G^{-1}(x))}$. Therefore, $F(x)$ is decreasing (increasing) if $0 < G_x(.) < (>)2$. Using lemma 1.4 we can say $F(.)$ is a decreasing function. Hence, $\theta_1^*(1,2) < \theta_1^*(2,1)$. Therefore the borrower should approach the more reluctant group first. □

Proof of Proposition 8 If there exists some group g for which r_g is so high that $y(1/n) > \frac{1}{1+r_g}$,($y(\alpha)$ is as defined in equation 13) then the borrower would approach him at the end. So, let us focus only on the case where $\frac{1}{1+r_g} \geq y(1/n)$ for all $g \in \{1, 2 \ldots n\}$. Let us define

$$G(x,y,\alpha) := \frac{\frac{x}{\alpha}}{\Phi(\sqrt{\tau}x + \Phi^{-1}(\frac{x}{\alpha}) + \sqrt{\tau}y)}, \;\alpha \in [0,1],\; x \in [0,\alpha], y \in [0,\alpha] \tag{18}$$

The only difference from the earlier definintion of G as in equation 10 is that we have one more argument in $G(.)$. Let us define $G^{-1}(q|y,\alpha)$ such that $G(G^{-1}(q|y,\alpha), y, \alpha) = q$. There are muliple roots and we consider the maximum root (since we will consider only the worst equilibrium). It can be checked that $G_x(G^{-1}(q|y,\alpha), y, \alpha) > 0$.

Lemma 3 *Given $G(x,y,\alpha)$ as defined above,*

1. $G(x,0,\alpha) \leq G(x,y,\alpha) \leq x/\alpha$
2. $\frac{\partial}{\partial y}G(x,y,\alpha) < 0$
3. $\frac{\partial}{\partial y}G_x(G^{-1}(q|y,\alpha), y, \alpha) > 0$

Proof. 3.1 and 3.2 are obvious.G is decreasing in y. $\forall\, y' > y$, $G^{-1}(q|y',\alpha) > G^{-1}(q|y,\alpha)$ (since $G_x(.) > 0$). Now both $G(x,y,\alpha)$ and $G(x,y',\alpha)$ are increasing and smoothly pastes with x/α. So, $G_x(G^{-1}(q|y',\alpha), y', \alpha) > G_x(G^{-1}(q|y,\alpha), y, \alpha)$. ■

The equilibrium thresholds are such that

$$G(\theta_t^* - \theta_{t+1}^*, \theta_{t+1}^*, 1/n) = \frac{1}{1+r_{(t)}} \; for\; t = 2, \ldots T$$

If thete is truncated information in period 1 as well then the above equation should be true for $t = 1$ as well. Otherwise,

$$\frac{\theta_t^* - \theta_{t+1}^*}{1/n} = \frac{1}{1+r_{(1)}}$$

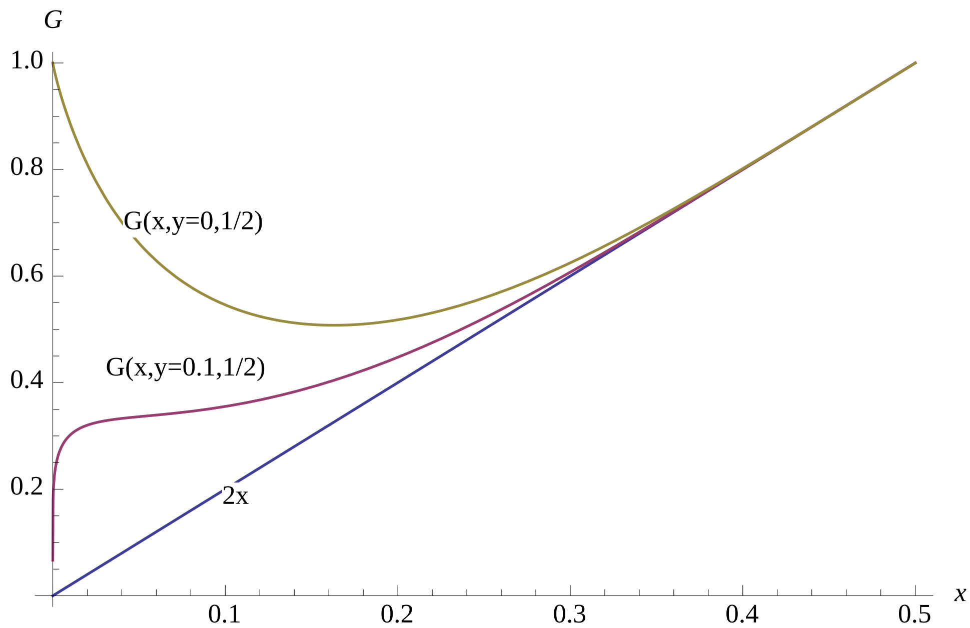

Figure 5: Effect of y on $G(x, y, \alpha)$

where $r_{(t)}$ is the payoff of group moving at period t. For the rest of proof, for simplicity, we drop the argument α from $G(.)$ because it is fixed at $1/n$. Let $\theta_t^*(p)$ be the threshold at period t given permutation p.

Consider two permutations (kk') and $(k'k)$, where the only difference between them is that- in kk', group k moves at $(t-1)$ and k' moves at t while in $k'k$, group k' moves at $(t-1)$ and k moves at t. Since all the groups moving after t are exactly the same, we can write $\theta_{t+1}^*(kk') = \theta_{t+1}^*(k'k) = \theta_{t+1}^*$. The threshold at time t depends on the group moving at time t and the threshold at time $t-1$ depends on the order of k and k'. Let $r_k < r_{k'}$. Given lemma 3.2 we can show that

$$\text{sgn}(\theta_1^*(kk') - \theta_1^*(k'k)) = \text{sgn}(\theta_{t-1}^*(kk') - \theta_{t-1}^*(k'k))$$

$$\theta_{t-1}^*(kk') - \theta_{t-1}^*(k'k) = \{G^{-1}(\frac{1}{1+r_k}|\theta_t^*(kk')) - G^{-1}(\frac{1}{1+r_k}|\theta_{t+1}^*)\}$$
$$-\{G^{-1}(\frac{1}{1+r_{k'}}|\theta_t^*(k'k)) - G^{-1}(\frac{1}{1+r_{k'}}|\theta_{t+1}^*)\}$$

Define $F(q, y) := G^{-1}(q|y) - G^{-1}(q|z)$, where z is fixed and $y \geq z$. If $\theta_t \geq \theta_t^*$, then the project survives all period after t. This implies $\theta_t \geq \theta_{t+1} \geq \theta_{t+1}^*$. So, by definition $\theta_t^* \geq \theta_{t+1}^*$. Therefore,

$$\text{sgn}(\theta_{t-1}^*(kk') - \theta_{t-1}^*(k'k)) < (>) 0 \text{ iff } \frac{dF}{dq} < (> 0)$$

Now $\frac{dF}{dq} = F_q + F_y \frac{dy}{dq}$. Since, $G(G^{-1}(q|y), y) = q$, $F_y = -\frac{G_y}{G_x} > 0$ (Lemma 3.2). Since $\frac{1}{1+r_k} > \frac{1}{1+r_{k'}}$, we have $\theta_t^*(kk') < \theta_t^*(k'k)$. So, $\frac{dy}{dq} < 0$.

$$F_q = \frac{1}{G_x(G^{-1}(q|y), y)} - \frac{1}{G_x(G^{-1}(q|z), z)}$$

Lemma 3.3 implies $G_x(G^{-1}(q|z), z) < G_x(G^{-1}(q|y), y)$. So, $F_q < 0$. Therefore, $\frac{dF}{dq} < 0$. Hence, $\theta_1^*(kk') < \theta_1^*(k'k)$. Using this argument repeatitively, we can say the optimal permutation is $(1, 2 \ldots n)$, when $r_1 < r_2 < \ldots < r_n$. $\square$

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