

Poverty Trap with Convex Production Function: The role of Public and Private Capital

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ABSTRACT. The objective of the paper is to explain why poverty traps may emerge with an entirely convex aggregate production function. It does so by delineating the role private capital plays in the accumulation of public capital and vice-versa. Governments accumulate public capital stock from the tax revenues and entrepreneurs accumulate private capital stock from private savings. The output of the economy is a function of stock of both types of capital and the two types of capital are complementary. Both the private sector and government leverage their respective income streams to borrow from the capital markets. If returns to total capital (public and private) is decreasing, the economy converges to a unique per-capita output level and exhibits the standard convergence properties of the Solow growth model. Conversely, if the total returns to capital is increasing, there is a poverty trap region or a threshold below which the economy is in a low steady state.

The paper shows that the private sector's ability to leverage their income stream to borrower interacts with the public sector's ability to leverage their tax revenues to borrow and determines either the steady-state per-capita output or the size of the poverty trap, depending on the returns to total capital. Increasing government's and private sector's leverage factors increases the per-capita output or decreases the poverty trap depending on the returns to total capital. Thus, the policy makers have to put equal emphasis on processes that facilitate the augmentation of public and private capital. It also explains why foreign aid is ineffective in springing an economy from a poverty trap if it only augments public capital stock, leaving private capital stock as it is.

Keywords: Economic Growth, Poverty Trap, Financial Market, Public Capital, Infrastructure

JEL Classification: O11, O16, O41, O43

1. INTRODUCTION

Lets start with the story of a town called Hathin in India to motivate the paper. Hathin is typical nondescript small town like so many others in the developing world. Even though it is merely, 75.1 kilometres away from New Delhi, the glistening capital of India, it could be a world away with its pot-holed roads, lack of basic infrastructure, absent civil servants, impecunious inhabitants and lack of any perceptible economic activity.

In spite of its physical proximity to a lucrative market like Delhi, there is hardly any entrepreneurial activity in Hathin. Among other things, the lack of basic infrastructure and pot-holed roads add significantly to the cost of any production process and ensure that there is very little entrepreneurial activity in Hathin (Aniket, 2006). So, why doesn't the government simply invest upfront in roads and infrastructure to encourage economic activity. The first obvious answer is that that there is no entrepreneurial activity in Hathin and the government gets no tax revenues from Hathin. A rational resource constrained government may thus feel that there is no point in investing in Hathin's infrastructure.

So, there is no infrastructure because there is no private entrepreneurial activity and there is no private entrepreneurial activity because there is no infrastructure.¹ This feedback loop creates a poverty trap. It also sounds like a typical coordination failure, which can be easily solved if the government is able to invest upfront in infrastructure and wait for the entrepreneurial activity to build up. If the economic activity flourishes, it would allow the government to recover its upfront investment in infrastructure. If it is that easy, why does Hathin, like large parts of the development world, still have such woefully inadequate infrastructure.

There is another added layer of complication beyond the coordination failure. Both the government and potential entrepreneurs in Hathin are credit constrained. Both the government and the private entrepreneurs need to borrow from the financial markets in order to invest in public capital and private capital respectively. The financial markets, if it exists, may allow the government and entrepreneurs to leverage their current income by a certain factor. Consequently, this lack of access to capital which results from imperfect capital markets makes it more difficult to solve the coordination problem. It also have increases the size of the poverty trap dramatically.

In what follows, we build a simple of model to illustrate how imperfect capital markets create this poverty trap. Even though the example above of Hathin may seem to leave a linger doubt that it is indivisibilities in the investment process that leads to the poverty trap, in what follows, we have a convex production process with no indivisibilities in either public

¹Gollin and Rogerson (2010), Fan and Rosegrant (2008), Teravaninthorn and Raballand (2008) and Gollin and Rogerson (2010) are papers that present empirical evidence from Africa that road infrastructure significantly restricts economic growth.

capital or private capital. In spite of a convex production process, we obtain a poverty trap that increases in size with the imperfection² in the credit market in an unexpected way.

This paper takes its primary motivation from Lucas (1990), which has a self-explanatory title, “Why Doesn’t Capital Flow from Rich to Poor Countries?” The rich countries have higher capital stock and presumably a lower marginal productivity than poor countries, which should lead to capital flows from the rich to the poor countries. Actually, the global capital flows are dominated either by flows between developed countries or between the developed and the emerging countries. (Aguiar and Gopinath, 2007; Neumeyer and Perri, 2005) In comparison, capital flows to the poor developing countries are nothing more than a trickle.

Lucas (1990) proposed two main candidates, differences in the complementary inputs, like human capital and total factor productivity, and frictions in the global capital markets. The paper did not satisfactorily answer the question it set for itself. If it is indeed the differences in the complementary inputs that lead to differences in per-capita income, why can’t free flow of capital eliminate the differences in the complementary inputs across the world? For instance, if an entrepreneur starts a production process in a poor country, the human capital of the local labour would get upgraded over time through the process of learning-by-doing. In another influential paper, Lucas (1990) concludes that a very large part of the growth in fast growing economies like the East Asian economies can be attributed to accumulation of human capital through the process of learning by doing, which, in turn has been stimulated by trade.

If indeed there are large differences in the complementary inputs across the world, arguing in a similar vein, it is not clear why free flow of capital along with international trade and factor mobility cannot eliminate the differences in factor productivity across countries in the world. If it is the case that capital market frictions stem the flow of capital, then the determinants of these frictions should be pinned down precisely.

This paper attempts to pin down how the lack of public capital, i.e., fiscal capacity and provision of public goods, in conjunction with frictions in the credit market can potentially trap a country in poverty. Our focus in this paper is going to remain on the potential impact credit market frictions could have on economic activity and tax revenues.

We assume that the government is benign and conscientiously invests all its tax revenues into public good.³ Using this assumption, we will try to pin down the full impact of the credit market frictions in terms of how the *government’s limited ability to borrow interacts with the entrepreneur’s limited ability to borrow*. What emerges out of this interaction is a poverty

²That is, the size of the poverty trap increases as the government and private entrepreneur’s ability to leverage their current revenues decrease.

³Acemoglu (2005) models the impact of corrupt rulers that divert tax revenues to benefit themselves and finds that neither very weak or very strong states are good for the economy.

trap. The government's ability to leverage its tax revenues interact with the entrepreneur's ability to leverage their incomes to create the poverty trap.

This may explain why the flow of capital extremely poor countries is almost negligible. What is holding back is not just the government's inability to borrow or the entrepreneur's inability to borrow on its own but actually the interaction of these two factors. The model also predicts that once a country crosses a particular output threshold, there are large volume of capital flows to the country as both the government and the private sector starts borrowing simultaneously as their respective leverage factors interact and in turn increase their respective borrowing capacities. Indeed, we have seen a dramatic increase in capital flows to the BRIC countries at the end of the last century or the Latin American countries in the 1970s.

Walter Wriston, the Chairman of Citibank from 1967 to 1984, argued in that there was no cause for concern about lending to the Latin American countries in the 1970s because "countries don't go bankrupt". Of course, he was proved wrong but he may have been arguing that lending to the sovereign countries is creates lead to private economic activity and thus improve the fiscal position of the country. What he may not have appreciated is that both the government and private entrepreneurs need access of credit in order for a country to realise its full potential.

Lack of fiscal capacity of state is both the symptom and cause of lack of development. Richer countries tend to raise more taxes, provide better public goods and have more economic activity than poor countries. (Baunsgaard and Keen, 2010; Besley and Persson, 2009) There is an endogenous relationship between provision of public goods or fiscal capacity of the state and the extent of private economic activity in the economy.⁴

Unravelling this endogenous relationship between public goods provision and private economic activity is critical to getting to grips with the problems faced by the poorest countries in the world. Lack of public goods provisions is blamed on inefficiency and corruption of the governments in the poor countries (Acemoglu, 2006, 2008). In this paper, we want to abstract from the political economy questions and focus on the resource constraints that conscientious governments in the poor country face when they aspire to accumulate public capital stock. The objective of the paper is to find conditions under which the poor countries are not able to accumulate public and private capital in spite of access to global capital markets.

Quah (1997) was the first one to emphasise the stratification in world income distribution and a possible shift towards a bimodal distribution. Even if there is some conditional

⁴Besley and Persson (2008) suggests that external conflicts serve as an exogenous impulse that increases the tax revenue, allowing a country to increase its investment in its fiscal capacity. They find that civil wars lead to smaller investments in fiscal capacity, whereas prospects of external war generally lead to larger investments.

convergence across the world, it is painfully slow. In contrast there is a lot of evidence of convergence within the OECD countries (Acemoglu, 2009). This suggests the economic environment faced by the poor countries are distinct from the rest of the world and there are some specific factors that sustain this divergence. The poorest of the poor countries thus stagnate while the rest of the world continues to grow. Our wider focus is on the set of conditions that trap a country or a region in a vicious cycle to poverty, independent of the inefficiencies and corruption of the government. The paper thus aims to explore why even conscientious governments in the poor countries may get caught in a poverty trap, in spite of some access to global capital markets.

Lucas (1990) looks at private capital flows but the same question can be rephrased in term of why capital does not flow to the governments of poor countries for accumulation of public goods. Trefler (1993) and Caselli and Feyrer (2007) find very little difference in returns to capital at the margin across across the countries. Yet, at the same time, there is considerable variation in the stock of public capital capital across the world. Why can't the conscientious governments of poor countries borrow and invest in public capital stock?

The fact that capital flows are dominated either between the developed countries or from the developed to emerging countries suggests that the returns to capital maybe increasing in capital stock. Only countries that cross a threshold in terms of capital stock can attract the international flow of capital and the flow of capital is dominated by the countries that have the highest capital stock.

In our model, each country has two types of capital stock, public and private capital stock. The two types of capital stock are complementary with respect to each other. The process that determines the accumulation of one type of capital stock is influenced by the level of the other type of capital stock. The accumulation of private capital stock moves hand in hand with public capital stock.

The paper models poverty trap with an aggregate production function that is convex in both public and private capital. It examines the role capital markets can play in reducing the extent of the poverty trap, allowing poor countries at the margin to escape. The poverty trap in our model emerge only when the return to capital, widely defined as including both public and private capital stock, is increasing in capital.

The supply of capital from the global financial markets is homogenous. The capital is then differentiated by the end users. The end users in the model are the government and the private entrepreneurs, who use the capital for accumulating public capital⁵ and private capital stock respectively. For poverty traps to exist, the returns at the margin need to be

⁵We make a strong assumption that the government is conscientious and invests all its resources into public goods and does not divert any resource.

increasing in the total capital stock, that is the sum of private and public capital. At the margin, the returns to both, public capital and private capital, are decreasing on their own.

If returns to the total capital stock is increasing there is a *low stable steady-state* and a *high unstable steady-state*, in terms of per-capita income. The low stable steady state has a catchment area which we pin down. An economy that gets caught in the *deadly waters*, that is the catchment area surrounding the low steady-state, cannot escape and would converge to a low per-capita income. If the economy starts away from the catchment area, it grows perpetually. Conversely, if the returns to total capital stock is diminishing, there a unique *stable steady-state* and the economy converges to this steady state.

Papers like Galor and Zeira (1993) Aghion and Bolton (1997) and Matsuyama (2000) have previously modelled poverty traps as a result of credit frictions in presence of non-convexities technology. In this paper, the technology is convex. We attempt to explain the bi-modal distribution obtained by Quah (1997), while keeping the model as close as possible to the Solow model.

In Section 3 we model the economy in a non-co-operative environment under autarky. The government imposes a tax on entrepreneurs and funds the public capital investment in the economy through those tax revenues. In section 4 we model the economy in a non-co-operative environment with access to an imperfect global markets. Like Section 3, the public capital is financed through tax revenues. There is an enforcement problem associated with debt contracts, which implies that the government and entrepreneurs are constrained in the amount they borrow.

We find that there is a poverty trap or a catchment area of the low steady state in both Section 3 and Section 4. In autarky, the size of the poverty trap is decreasing in the saving rate and has a non-monotonic relationship with the tax rate. When the economy has access to imperfect capital market, as in the earlier specification, the size of the poverty trap is decreasing in the saving rate and has a non-monotonic relationship with the tax rate. The size of the poverty trap is also decreasing in both the government and the private entrepreneurs ability to borrow and increasing in global interest rate. This suggests that imperfections in the capital market may worsen the situation of the poor countries by increasing the size of the poverty trap but these frictions are not the fundamental cause of the poverty trap. The fundamental cause of the poverty trap remain the increasing returns to the total (public and private) capital stock. Having said that, easing the borrowing restrictions for one category of end-users has a impact on both types of capital stock. Thus, policy makers have to think of easing borrowing conditions for the private sector and government simultaneously. It also explains why foreign aid is not usually effective if it just augments the public capital stock, leaving the private capital as it is. The economy is likely to remain in the poverty trap

and the effect of the increase in public capital stock is going to disappear as the economy converges back to the initial equilibrium.

2. THE ENVIRONMENT

The economy has a constant population that is normalised to 1. The tax rate is t and the agents save s proportion of their net-tax income and consume the rest. There is no exogenous or endogenous technological progress in the economy. The output of the economy y is a function of the installed public capital \bar{k} and private capital k in the economy.

$$y = \bar{k}^\beta \cdot k^\alpha \quad (1)$$

where $\beta \in (0, 1)$ and $\alpha \in (0, 1)$. We assume that private capital k is exclusively installed by the entrepreneurs and public capital \bar{k} is exclusively installed by the government in the economy.

We normalise the production function so that $\bar{k} = 0$, $k = 0$ and $y = 0$ represents the point where production takes place without any capital and just using natural resources, i.e., the circumstances as they exist in the poorest countries in the world where there is almost no capital used in production process.

The production function may or may not have diminishing returns to capital depending on the value of $\alpha + \beta$. We assume that both \bar{k} and k depreciate at an identical rate δ .

3. AUTARKY

In this section, we analyse the evolution of public capital \bar{k} and private capital k in autarky where the entrepreneurs and government have no access to global capital markets. We have a dichotomy where the government only invests to accumulate public capital and entrepreneurs only invest to accumulate private capital. The government and the private sector make their respective decisions independent of each other.

The government pays for public capital by imposing a proportional tax t on the entrepreneurs and the entrepreneurs are able to keep $(1 - t)$ for themselves. The per-capita saving of entrepreneurs is given by $s_p = s(1 - t)y$ which goes towards augmenting the private capital stock k after the depreciated private capital stock is replaced. The government is able to garner resources to the tune of $s_g = ty$ to augment public capital stock \bar{k} after the depreciated public capital stock has been replaced. For simplicity, we assume that both \bar{k} and k depreciates at the same rate δ .

The capital accumulation conditions for k and \bar{k} are given by $\Delta k_t = s_{pt} - \delta k_t = s(1 - t)y_t - \delta k_t$ and $\Delta \bar{k}_t = s_{gt} - \delta \bar{k}_t = ty_t - \delta \bar{k}_t$ respectively, where $\Delta k_t = (k_{t+1} - k_t)$ and $\Delta \bar{k}_t = (\bar{k}_{t+1} - \bar{k}_t)$. §

This gives use the following system of difference equations.

$$k_{t+1} = s(1-t)\bar{k}_t^\beta k_t^\alpha + (1-\delta)k_t \quad (2)$$

$$\bar{k}_{t+1} = t\bar{k}_t^\beta k_t^\alpha + (1-\delta)\bar{k}_t \quad (3)$$

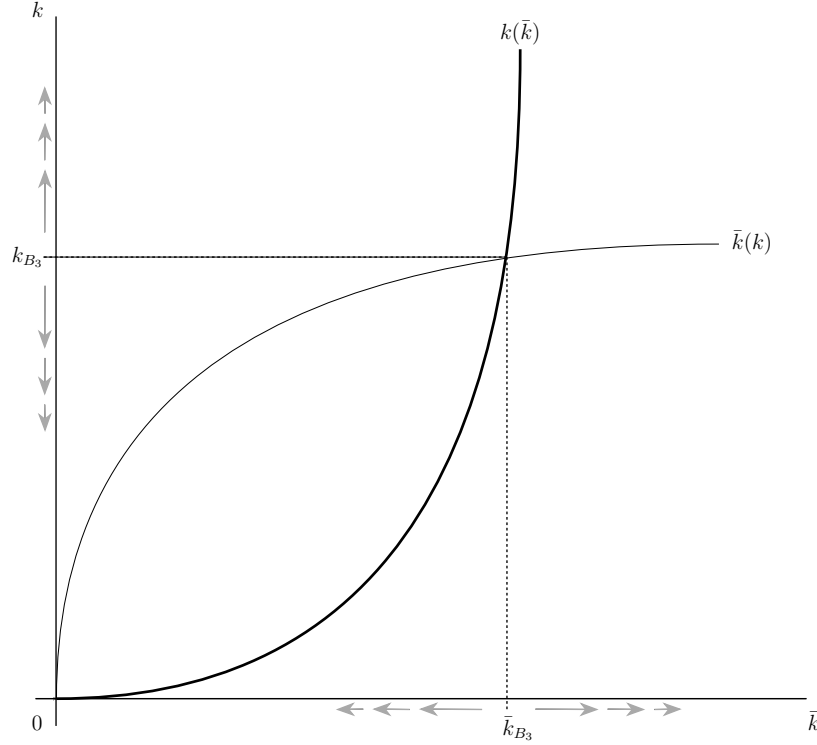


FIGURE 1. Autarky: $k(\bar{k})$ and $\bar{k}(k)$ locus curves if $\alpha + \beta > 1$.

Setting $\Delta k_t = 0$ in (2) gives us the locus $k(\bar{k})$ in (4) along which private capital does not change.

$$k(\bar{k}) = \left(\frac{s(1-t)}{\delta} \right)^{\frac{1}{1-\alpha}} \bar{k}^{\frac{\beta}{1-\alpha}} \quad (4)$$

Setting $\Delta \bar{k}_t = 0$ in (3) gives us the $\bar{k}(k)$ locus in (5) along which public capital does not change.

$$\bar{k}(k) = \left(\frac{t}{\delta} \right)^{\frac{1}{1-\beta}} k^{\frac{\alpha}{1-\beta}} \quad (5)$$

The shape of the locus $k(\bar{k})$ and $\bar{k}(k)$ in (4) and (5) depends on $(\alpha + \beta - 1)$. If $\alpha + \beta > 1$, then $\frac{d^2 k}{d\bar{k}^2} \Big|_{k(\bar{k})} > 0$ and $\frac{d^2 \bar{k}}{dk^2} \Big|_{\bar{k}(k)} > 0$ and the resulting loci $k(\bar{k})$ and $\bar{k}(k)$ are depicted in Figure

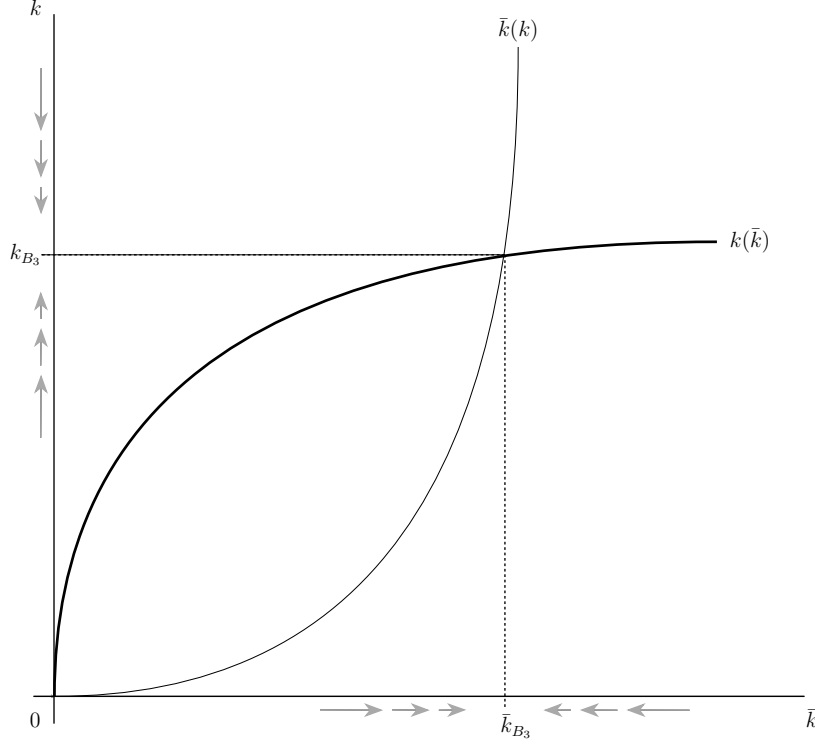


FIGURE 2. Autarky: $k(\bar{k})$ and $\bar{k}(k)$ locus curves where $\alpha + \beta < 1$.

1.⁶Conversely, if $\alpha + \beta < 1$, then $\frac{d^2 k}{d\bar{k}^2} \Big|_{k(\bar{k})} < 0$ and $\frac{d^2 \bar{k}}{dk^2} \Big|_{\bar{k}(k)} < 0$ and the resulting loci $k(\bar{k})$ and $\bar{k}(k)$ are depicted in Figure 2.⁷

Lets define $k_A = \bar{k}_A = 0$. k_{B_3} and \bar{k}_{B_3} are defined as follows.

$$\begin{aligned} k_{B_3} &= \left[s^{1-\beta} \cdot \frac{(1-t)^{1-\beta} t^\beta}{\delta} \right]^{\frac{1}{1-(\alpha+\beta)}} \\ \bar{k}_{B_3} &= \left[s^\alpha \cdot \frac{(1-t)^\alpha t^{1-\alpha}}{\delta} \right]^{\frac{1}{1-(\alpha+\beta)}} \end{aligned} \quad (6)$$

It follows from the analysis above that economy has an unique stable steady-state if $\alpha + \beta < 1$ and one stable and one unstable steady-state if $\alpha + \beta > 1$. We summarise with Proposition 1.

⁶If $\alpha + \beta > 1$, in Figure 1, $\Delta k_t > 0$ is on the right of $k(\bar{k})$ locus, $\Delta k_t < 0$ is on the left of $k(\bar{k})$ locus, $\Delta \bar{k}_t > 0$ above the $\bar{k}(k)$ locus and $\Delta \bar{k}_t < 0$ below the $\bar{k}(k)$ locus.

⁷If $\alpha + \beta < 1$, in Figure 2, $\Delta k_t > 0$ below the $k(\bar{k})$ locus, $\Delta k_t < 0$ above the $k(\bar{k})$ locus, $\Delta \bar{k}_t > 0$ on the left of $\bar{k}(k)$ locus and $\Delta \bar{k}_t < 0$ is on the right of $\bar{k}(k)$ locus.

Proposition 1. *In autarky with public capital funded by a proportional tax, the economy would have following steady-state(s).*

- i. If $\alpha + \beta < 1$, the economy has a unique stable steady-state at (k_{B_3}, \bar{k}_{B_3}) .*
- ii. If $\alpha + \beta > 1$ the economy has a stable steady-state at $(k_A, \bar{k}_A) = (0, 0)$ and a unstable steady-state at (k_{B_3}, \bar{k}_{B_3}) .*
- iii. If $\alpha + \beta = 1$, then there is stable steady-state at $(k_A, \bar{k}_A) = (0, 0)$ if $\frac{t^{1-\alpha}}{(1-t)^{1-\beta}} < \delta^{\alpha-\beta} \left(\frac{1}{s}\right)^{1-\beta}$ and unstable steady-state at $(k, \bar{k}) = (0, 0)$ if $\frac{t^{1-\alpha}}{(1-t)^{1-\beta}} > \delta^{\alpha-\beta} \left(\frac{1}{s}\right)^{1-\beta}$.
If $\frac{t^{1-\alpha}}{(1-t)^{1-\beta}} = \delta^{\alpha-\beta} \left(\frac{1}{s}\right)^{1-\beta}$, then there is a continuum of steady-states at $k = \bar{k}$, where $k = [0, \infty)$.*

Proposition 1 basically says that the economy behaves like the traditional Solow growth model and converges to the steady state at (k_{B_3}, \bar{k}_{B_3}) if $\alpha + \beta < 1$. Conversely, if $\alpha + \beta > 1$, the economy exhibits a poverty trap. It converges to $k_A = \bar{k}_A = 0$ if either $k < k_{B_3}$ and $\bar{k} < \bar{k}_{B_3}$. Corollary 3.1 summarises the growth and convergence properties of this economy.

Corollary 3.1. *In autarky with public capital funded by a proportional tax,*

- i. If $\alpha + \beta < 1$, the economy will converge to a stable steady-state (k_{B_3}, \bar{k}_{B_3}) where*

$$y_{B_3} = y(\bar{k}_{B_3}, k_{B_3}) = \left(\frac{s^\alpha}{\delta^{\alpha+\beta}} \cdot (1-t)^{\alpha t^\beta} \right)^{\frac{1}{1-(\alpha+\beta)}} \quad (7)$$

- ii. if $\alpha + \beta > 1$,*
 - (a) k and \bar{k} will have perpetual negative growth till the economy converges to the stable steady-state $(k_A, \bar{k}_A) = (0, 0)$ if $k \in [0, k_{B_3})$ and $\bar{k} \in [0, \bar{k}_{B_3})$*
 - (b) there will be no growth in k and \bar{k} if it starts from $k = k_{B_3}$ and $\bar{k} = \bar{k}_{B_3}$*
 - (c) k and \bar{k} will perpetual positive growth if it starts from $k \in (k_{B_3}, \infty)$ and $\bar{k} \in (\bar{k}_{B_3}, \infty)$*

If $\alpha + \beta < 1$, the economy converges to the steady state per-capita output level y_{B_3} . In this case, y_{B_3} is increasing in s and $(1-t)^{\alpha t^\beta}$. Just like the traditional Solow growth model, the output per-capita is increasing in the savings rate.

If $\alpha + \beta > 1$, the economy diverges from y_{B_3} . That is the economy explodes if it is above the y_{B_3} threshold and converges to $(0, 0)$ if it is below the y_{B_3} threshold. In this case, y_{B_3} is decreasing in s and $(1-t)^{\alpha t^\beta}$. y_{B_3} also determines the size of the poverty trap, that is the size of the poverty trap decreases as saving rate increases.

It is important to interpret s correctly. $s = \frac{s_p}{(1-t)y}$ is the proportion of economy's net tax income that is invested into private capital. Thus, s is not affected by savings under the pillow. Thus, if for a poor economy, s increases due the financial sector maturing, then according to (7), the size of the poverty trap would decrease. We summarise with Corollary 3.2.

Corollary 3.2. *In autarky,*

- i. if $\alpha + \beta < 1$, the steady-state output of the economy is increasing in s*
- ii. if $\alpha + \beta > 1$, the poverty trap for the economy is decreasing s .*

4. CAPITAL MARKETS WITH ENFORCEMENT PROBLEM

The credit market has an *enforcement problem*, where the lenders find it difficult to enforce contracts and extract the full repayment from the borrower. If the borrower chooses to default, then lender can only wrench μ proportion of the output from the borrower. We assume that the borrower can choose to default but cannot choose not to pay their taxes.

If an entrepreneur borrows b at interest rate r and the lender can only wrench μ_p proportion of her output, she will only repay if $rb \leq \mu_p(1-t)y$. b_p , the maximum amount that the borrower can borrow from the capital markets, is given by

$$b_p = \frac{\mu_p(1-t)y}{r} \quad (8)$$

Similarly, if the government borrows b at interest rate r and the lender can only wrench μ_g proportion of the government's tax revenues, it will only repay if $rb \leq \mu_g ty$. b^g , the maximum amount that the borrower can borrow from the capital markets, is given by

$$b_g = \frac{\mu_g ty}{r} \quad (9)$$

If private entrepreneur's borrow b_p or less and government borrows b_g or less, it is in their interest to repay back and there would be no defaults on loans in equilibrium. μ_g and μ_p and can be considered leverage factors. Holding the level of level of output, interest rate and tax rate constant, if we increase μ_p and μ_g , it increases the amount that the government and the private entrepreneurs can borrow.

We continue with the assumption that there is a clear dichotomy where only entrepreneurs invest in private capital and government invests only in public capital. The entrepreneur's savings are given by $s_p = s(1-t)y$ and government's available resources for public capital investment are $s_g = ty$. Both public capital and private capital stock depreciate at the same rate δ .

Public and private accumulation of capital are determined by the equations $\Delta k_t = \left(s + \frac{\mu_p}{r}\right)(1-t)y_t - \delta k_t$ and $\Delta \bar{k}_t = \left(1 + \frac{\mu_g}{r}\right)ty_t - \delta \bar{k}_t$. This gives use the following system of difference equations.

$$k_{t+1} = \left(s + \frac{\mu_p}{r}\right)(1-t)\bar{k}_t^\beta k_t^\alpha + (1-\delta)k_t \quad (10)$$

$$\bar{k}_{t+1} = \left(1 + \frac{\mu_g}{r}\right)t\bar{k}_t^\beta k_t^\alpha + (1-\delta)\bar{k}_t \quad (11)$$

Imposing the condition that $\Delta k = \Delta \bar{k} = 0$ on (10) and (11) gives us the loci $k(\bar{k})$ and $\bar{k}(k)$.

$$k(\bar{k}) = \left[\left(s + \frac{\mu_p}{r} \right) \frac{(1-t)}{\delta} \right]^{\frac{1}{1-\alpha}} \bar{k}^{\frac{\beta}{1-\alpha}} \quad (12)$$

$$\bar{k}(k) = \left[\left(1 + \frac{\mu_g}{r} \right) \cdot \frac{t}{\delta} \right]^{\frac{1}{1-\beta}} k^{\frac{\alpha}{1-\beta}} \quad (13)$$

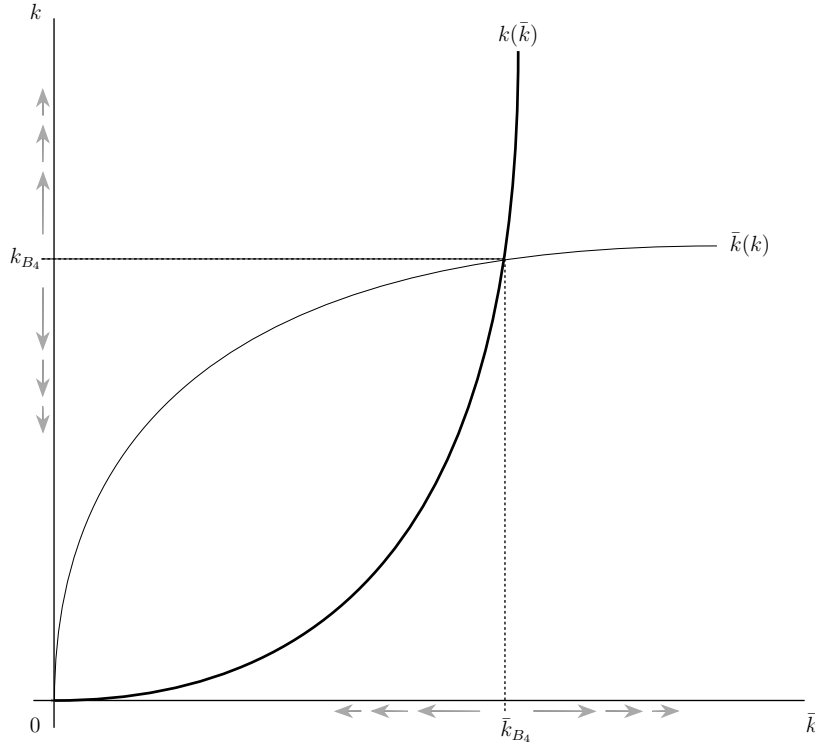


FIGURE 3. Capital Markets with Enforcement Problem: $k(\bar{k})$ and $\bar{k}(k)$ reaction curves when $\alpha + \beta > 1$

The shape of the locus $k(\bar{k})$ and $\bar{k}(k)$ in (12) and (13) depends on $(\alpha + \beta - 1)$. If $\alpha + \beta > 1$, then $\frac{d^2 k}{d\bar{k}^2} \Big|_{k(\bar{k})} > 0$ and $\frac{d^2 \bar{k}}{dk^2} \Big|_{\bar{k}(k)} > 0$ and the resulting loci $k(\bar{k})$ and $\bar{k}(k)$ are depicted in Figure 3. Conversely, if $\alpha + \beta < 1$, then $\frac{d^2 k}{d\bar{k}^2} \Big|_{k(\bar{k})} < 0$ and $\frac{d^2 \bar{k}}{dk^2} \Big|_{\bar{k}(k)} < 0$ and the resulting loci $k(\bar{k})$ and $\bar{k}(k)$ are depicted in Figure 4. For a given \bar{k} , k is higher in (12) than in (4) if $\mu_p > 0$.⁸ Similarly, for a given k , \bar{k} is higher in (13) than in (5) if $\frac{\mu_g}{r} > 0$ or $\mu_g > 0$. Access to global

⁸Assuming r is less than infinity.

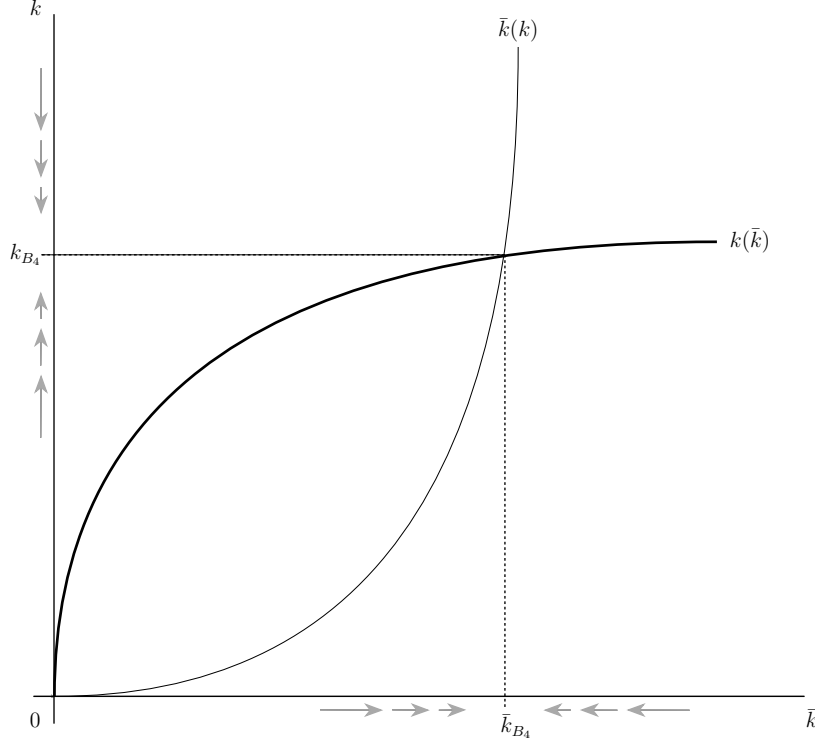


FIGURE 4. Capital Markets with Enforcement Problem: $k(\bar{k})$ and $\bar{k}(k)$ reaction curves when $\alpha + \beta < 1$

capital markets stimulate both the investment of public and private capital if $\mu_p > 0$ and $\mu_g > 0$.

k_{b_4} and \bar{k}_{b_4} are defined as follows.

$$\begin{aligned}
 k_{B_4} &= \left[\left(s + \frac{\mu_p}{r} \right)^{1-\beta} \left(1 + \frac{\mu_g}{r} \right)^\beta \frac{(1-t)^{1-\beta} t^\beta}{\delta} \right]^{\frac{1}{1-(\alpha+\beta)}} \\
 \bar{k}_{B_4} &= \left[\left(s + \frac{\mu_p}{r} \right)^\alpha \left(1 + \frac{\mu_g}{r} \right)^{1-\alpha} \frac{(1-t)^\alpha t^{1-\alpha}}{\delta} \right]^{\frac{1}{1-(\alpha+\beta)}}
 \end{aligned} \tag{14}$$

Proposition 2 describes the steady-state properties of the economy.

Proposition 2. *With an enforcement problem in the global capital markets and public capital funded by a proportional tax,*

- i. *if $\alpha + \beta < 1$, the economy has a unique stable steady-state at (k_{B_4}, \bar{k}_{B_4}) .*
- ii. *if $\alpha + \beta > 1$, the economy has a stable steady-state at $(k_A, \bar{k}_A) = (0, 0)$ and a unstable steady-state at (k_{B_4}, \bar{k}_{B_4}) .*

iii. if $\alpha + \beta = 1$, then there is stable steady-state at $(k_A, \bar{k}_A) = (0, 0)$ if $t < \frac{sr + \mu_p}{(1+s)r + 2\mu_g}$ and unstable steady-state at $(k, \bar{k}) = (0, 0)$ if $t > \frac{sr + \mu_p}{(1+s)r + 2\mu_g}$. If $t = \frac{sr + \mu_p}{(1+s)r + 2\mu_g}$, then there is a continuum of steady-states at $k = \bar{k}$, where $k = [0, \infty)$.

If there is no access to credit, then the whole problem reduces to the autarky analysis in Section 3. To see this, if we set $\mu_p = \mu_g = 0$, loci (12) and (13) reduces to loci (4) and (5) in Section 3.⁹ Corollary 4.1 describes the convergence properties of the economy.

Corollary 4.1. *With an enforcement problem in the global capital markets and public capital funded by a proportional tax,*

i. if $\alpha + \beta < 1$, the economy will converge to a stable steady-state (k_{B_4}, \bar{k}_{B_4}) where

$$y_{B_4} = y(\bar{k}_{B_4}, k_{B_4}) = \left[\left(s + \frac{\mu_p}{r} \right)^\alpha \left(1 + \frac{\mu_g}{r} \right)^\beta \cdot \frac{(1-t)^{\alpha t \beta}}{\delta^{\alpha + \beta}} \right]^{\frac{1}{1 - (\alpha + \beta)}} \quad (15)$$

ii. if $\alpha + \beta > 1$,

(a) k and \bar{k} will have perpetual negative growth till the economy converges to the stable steady-state $(k_A, \bar{k}_A) = (0, 0)$ if $k \in [0, k_{B_4})$ and $\bar{k} \in [0, \bar{k}_{B_4})$

(b) there will be no growth in k and \bar{k} if it starts from $k = k_{B_4}$ and $\bar{k} = \bar{k}_{B_4}$

(c) k and \bar{k} will have perpetual positive growth if it starts from $k \in (k_{B_4}, \infty)$ and $\bar{k} \in (\bar{k}_{B_4}, \infty)$

If $\alpha + \beta < 1$, the economy converges to the steady state per-capita output level y_{B_4} . In this case, y_{B_4} is increasing in s , μ_p , μ_g and $(1-t)^{\alpha t \beta}$. If $\alpha + \beta > 1$, the economy behaves just like the traditional Solow growth model.

If $\alpha + \beta > 1$, the economy diverges from y_{B_4} . That is the economy explodes if it is above the y_{B_4} threshold and converges to $(0, 0)$ if it is below the y_{B_4} threshold. In this case, y_{B_4} is decreasing in s , μ_p , μ_g , and $(1-t)^{\alpha t \beta}$. y_{B_4} also determines the size of the poverty trap, that is the size of the poverty trap decreases as saving rate increases.

What is interesting is that y_{B_4} is a function of $\left(s + \frac{\mu_p}{r} \right)^\alpha \left(1 + \frac{\mu_g}{r} \right)^\beta$. Thus, μ_g and μ_p complement each other's impact on y_{B_4} , the output of the economy if $\alpha + \beta < 1$ and the size of poverty trap if $\alpha + \beta > 1$.

Using Equation (15), we show in Section A that $\frac{\partial y_{B_4}}{\partial \mu_p} > 0$ if $\alpha + \beta < 1$ and $\frac{\partial y_{B_4}}{\partial \mu_p} < 0$ if $\alpha + \beta > 1$. We also show that $\frac{\partial^2 y_{B_4}}{\partial \mu_p \partial \mu_g} > 0$ holds irrespective of the value of $\alpha + \beta$. This shows that steady state output is increasing in μ_g and μ_p and the poverty trap is decreasing in μ_g and μ_p . Further, the marginal impact each leverage factor has on y_{B_4} increases if the other leverage factor increases. Thus, an increase in μ_p makes the effect the μ_g has on y_{B_4} more effective and vice-versa.

⁹From Equation (14) we can see that $t = \beta$ maximises k_{B_4} if $\alpha + \beta < 1$ and minimises it if $\alpha + \beta > 1$. Similarly, $t = 1 - \alpha$ maximises k_{B_4} if $\alpha + \beta < 1$ and minimised it if $\alpha + \beta > 1$.

Corollary 4.2. *When the global capital markets are imperfect,*

- i. if $\alpha + \beta < 1$, the steady-state output of the economy is increasing in μ_p , μ_g , s and decreasing in r .*
- ii. if $\alpha + \beta > 1$, the economy's poverty trap area is decreasing in μ_p , μ_g , s and increasing in r .*

5. IMPACT OF ACCESS TO INTERNATIONAL CAPITAL MARKETS

In this section we evaluate the effect having access to imperfect international capital markets has on an economy. In autarky, the supply of capital is inelastic and determined by the previous period's output. With access to imperfect capital markets, the supply of capital is stock is entirely elastic at r . The government and the entrepreneurs can leverage their output and tax revenues by the leverage factors μ_g and μ_p , which augments the supply of capital.

By comparing y_{B_4} with y_{B_3} , we can examine the impact moving from a situations where the economy is in autarky to a situation where it has access to imperfect global capital markets has on the economy.¹⁰

$$\left[\frac{y_{B_4}}{y_{B_3}} \right] = \left[\frac{\left(s + \frac{\mu_p}{r} \right)^\alpha \left(1 + \frac{\mu_g}{r} \right)^\beta}{s^\alpha} \right]^{\frac{1}{1-(\alpha+\beta)}} \begin{cases} > 1, & \text{if } \alpha + \beta < 1 \\ < 1, & \text{if } \alpha + \beta > 1 \end{cases}$$

For an economy in autarky, access to perfect capital market always increases the per-capita output if $\alpha + \beta < 1$. Conversely, if $\alpha + \beta > 1$, it decreases the size of the poverty trap. This is because $\left(1 + \frac{\mu_p}{sr} \right)^\alpha \left(1 + \frac{\mu_g}{r} \right)^\beta > 1$ holds if $\mu_p > 0$, $\mu_g > 0$, $s > 0$ and $\infty > r > 0$ are greater than zero. We can summarise with Lemma 5.1.

Lemma 5.1. *Starting from autarky, gaining access to imperfect capital markets always*

- i. increases the output levels of the an economy if $\alpha + \beta < 1$ and*
- ii. decreases the size of the economy's poverty trap if $\alpha + \beta > 1$*
if $\mu_p > 0$, $\mu_g > 0$, $s > 0$ and $\infty > r > 0$ are greater than zero.

6. CONCLUSION

The paper attempts to understand why the poorest countries in the world are not able to catch up with the richest countries in the world. We explore the role capital markets could potentially play in facilitating this catch up process.

We modelled the output as a function of public and private capital stock. The government's role in the model is to impose a tax on private entrepreneurs and use the tax revenues to install the public capital stock. The government of the poor countries are able to access the

¹⁰Assuming that the tax rate is identical in both situations.

capital markets to borrow and invest in public capital. We make a strong assumption here and assume that the government is always efficient and conscientious, that is, it invests all the tax revenues into the installing public capital.

We were able to show that if returns to total capital stock (public and private) in the economy was increasing, at low level of per capita income, the economy would be caught in a poverty trap. The size of the poverty trap in autarky is decreasing in the saving rate of the economy.

We also analysed the situation where the global capital markets were imperfect due to an enforcement problem and borrowers could leverage their income by a certain leverage factor. In this case, the size of the poverty trap depends on the both the government and entrepreneur's leverage factor. Further, the two leverage factors are complementary. This implies that constraints on government's ability to borrow in the capital markets constrains the entrepreneur's ability to borrow. Similarly, constraints on the entrepreneur's ability to borrow in the capital markets constrains the government's ability to borrow.

Conversely, if the returns to total capital stock in the economy were decreasing, there would be no poverty trap. The economy would converge to the unique steady-state just like it does in the traditional Solow Model. In this case, adding public capital to the economy's production function makes no substantial difference in the converges properties of the economy. We find that the steady state per-capita output level is increasing in the saving rate in autarky and decreasing in global interest rate if the global capital markets are perfect. With imperfect access to capital markets, the steady-state output level is increasing in the two leverage factors.

This paper attempts to explain why middle income economies like Brazil, China and India seem on track to converge with the rest of the OECD countries, where as other countries like sub-saharan Africa do not seem to be converging at all. [Burgess and Besley \(2003\)](#) show that poverty alleviation in China and India has far exceeded expectation of the Millennium Development Goal and Sub-Saharan Africa is lagging behind. Similarly, [Collier \(2007\)](#) bemoans the fact that growth in Indian and China has moved the focus away from the Africa, where things have not changed.

The model in this paper suggests that if returns to the total capital stock (public and private) is increasing, there is a clearly defined poverty trap. The economy thus needs a big push to emerge from the poverty trap in form of foreign aid. In the absence of aid, has access to imperfect capital markets decreases the size of the poverty trap. The size of the poverty trap in this case is determined by the interaction of government and private leverage factors. Policy makers in the past have chosen to either target public or private capital. Our model suggests that the most effective way to push an economy out of the poverty trap is to simultaneous give both the government and private sector greater access of capital.

access to capital global markets only helps the economy has once it has crossed a public and private capital threshold. If it does not cross the threshold, it remains forever caught in a poverty trap.

APPENDIX A. SIGNS FOR $\frac{\partial y_{B_4}}{\partial \mu_p}$ AND $\frac{\partial^2 y_{B_4}}{\partial \mu_p \partial \mu_g}$

Equation (15) can be written as

$$y_{B_4} = (\psi K)^{\frac{1}{1-\alpha-\beta}} \quad (16)$$

where $K = \frac{(1-t)^{\alpha} t^{\beta}}{\delta^{\alpha+\beta}}$, $\psi = \left(s + \frac{\mu_p}{r}\right)^{\alpha} \left(1 + \frac{\mu_g}{r}\right)^{\beta}$ and $\ln(\psi) = \alpha \ln\left(s + \frac{\mu_p}{r}\right) + \beta \ln\left(1 + \frac{\mu_g}{r}\right)$. We can take logs and write equation (16) as $\ln(y_{B_4}) = \frac{\ln(\psi) + \ln(K)}{1-(\alpha+\beta)}$. This gives us

$$\begin{aligned} \frac{\partial \ln(\psi)}{\partial r} &= \left(\frac{-1}{r^2}\right) \left[\frac{\alpha \mu_p}{s + \frac{\mu_p}{r}} + \frac{\beta \mu_g}{s + \frac{\mu_g}{r}}\right] < 0 \\ \frac{\partial \ln(\psi)}{\partial s} &= \frac{\alpha}{\left(s + \frac{\mu_p}{r}\right)} > 0 \\ \frac{\partial \ln(\psi)}{\partial \mu_p} &= \frac{\alpha}{\left(s + \frac{\mu_p}{r}\right)} \left(\frac{1}{r}\right) > 0 \\ \frac{\partial^2 \ln(\psi)}{\partial \mu_p \partial \mu_g} &= 0 \end{aligned} \quad (17)$$

Using (15) and (17) we can show that

$$\begin{aligned} \frac{\partial y_{B_4}}{\partial r} &= \frac{y}{1 - (\alpha + \beta)} \left(\frac{\partial \ln(\psi)}{\partial r}\right) \begin{cases} < 0, & \text{if } \alpha + \beta < 1 \\ > 0, & \text{if } \alpha + \beta > 1 \end{cases} \\ \frac{\partial y_{B_4}}{\partial s} &= \frac{y}{1 - (\alpha + \beta)} \left(\frac{\partial \ln(\psi)}{\partial s}\right) \begin{cases} > 0, & \text{if } \alpha + \beta < 1 \\ < 0, & \text{if } \alpha + \beta > 1 \end{cases} \\ \frac{\partial y_{B_4}}{\partial \mu_p} &= \frac{y}{1 - (\alpha + \beta)} \left(\frac{\partial \ln(\psi)}{\partial \mu_p}\right) \begin{cases} > 0, & \text{if } \alpha + \beta < 1 \\ < 0, & \text{if } \alpha + \beta > 1 \end{cases} \\ \frac{\partial^2 y_{B_4}}{\partial \mu_p \partial \mu_g} &= \frac{y}{1 - (\alpha + \beta)} \left[\frac{\partial^2 \ln(\psi)}{\partial \mu_p \partial \mu_g} + \frac{1}{1 - (\alpha + \beta)} \left(\frac{\partial \psi}{\partial \mu_p}\right)^2\right] \\ &= y \left[\frac{1}{1 - (\alpha + \beta)} \left(\frac{\partial \psi}{\partial \mu_p}\right)\right]^2 > 0 \end{aligned}$$

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