Market Capitalisation, Growth and Inflation in a New-Keynesian framework

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Abstract

Yearly data from 1988 to 2012 for four emerging economies suggest a positive and significant correlation coefficient between market capitalisation and growth. It is shown that a New-Keynesian theoretical framework with endogenous growth and imperfect inflation indexation,can replicate the empirical findings for a low value of the price stickiness parameter. Further, along the balanced growth path inflation is shown to have an effect on the long run welfare through the channels of balanced growth and price dispersion. Also if inflation is below a threshold level, a rise in long run inflation increases both growth and market capialisation ratio.

1 Introduction

Throughout the world perceptions about the stock market are often extreme. A section of the media treats booms or busts in the stock market as major indicators of growth or recession of the economy. This is done in spite of the fact that stock market participation of the average citizen in any country is still not very significant. On the other hand, there are sceptics who would like to view the stock market as a world-wide casino where agents participate to speculate and gamble. This latter group would like to deny any positive role the stock market could play in the development of the general economy. The truth probably lies somewhere in between. While it is certainly true that a large part of the activities of the stock market is speculative in nature, historically the stock market has indeed played an important role in mobilizing funds for growth in countries which are now considered developed. On the other hand, growth itself has played a role in the development of stock markets in countries all over the world. It is therefore necessary to understand and analyse the mutual relationship between the stock market and the process of economic growth. The basic purpose of the present research is to contribute to this understanding.

There is a vast theoretical literature relating GDP or its growth to the levels of stock market activities of a country. It is possible to divide this theoretical literature into two broad groups. References to journals and articles that belong to these two groups are mentioned in details in section 2 of this chapter where I discuss the theoretical review of literature. The first and relatively older group identifies stock markets as one of the most important determinants of growth. In this strand of literature, stock markets are viewed as major financial intermediaries that channel savings into investments thereby facilitating capital formation and production. It is argued that a more developed stock market, supports a higher level of economic activity, per capita income and growth by mitigating liquidity risks (Levine(1991), Jappelli and Pagano (1994), Bencivenga, Smith and Starr (1995)), investment risks through portfolio diversification (Saint-Paul (1992), Devereux and Smith (1994), King and Levine (1993) and helping the agents acquire information with lumpy cost (Boyd and Prescott (1986), Greenwood and Jovanovic (1990).

The second group of literature looks at the stock market-growth relationship from the opposite side. It envisages the stream of per capita income or its growth as the determinant of stock market activity. In this second strand of literature, stocks are viewed as one of the major instruments which attract savings. With intertemporal utility maximizing agents, an increase in current income is likely to affect savings and therefore the stock market. However, the direction of the effect of an increase in income on stock market activities is not clear from the theory. There are two separate effects. One is the normal intertemporal substitution effect which should induce the agent to save more in the present period by substituting present consumption by future consumption in the act of smoothing out consumption if future income is uncertain. But there is also an 'information' effect. If higher income now signals still higher income in the future, there is little requirement for intertemporal substitution. In this case, an increase in current income may actually reduce current savings reducing current demand for stocks and stock prices. If it is the other way round, that is, if current high income signals lower income in future, then due to intertemporal substitution, demand for stocks should go up now leading to an increase in stock prices. This strand of thought was pioneered by Lucas (1978) and further developed by Abel (1988) and Cochrane (2001).

The present research departs from these two strands of literature in one important way. The existing literature treats either stock market development as exogenous and looks at its effect on growth, or takes income or its growth as exogenous and analyses its effect on the stock market. In contrast, the present research develops a DSGE model where growth and the stock market development (represented by market capitalization to income ratio in the model) are *simultaneously* determined. This allows me to look at the correlation between these two variables and the extent to which one can predict the other, without going into the issue of causality between them. In particular, I look at the effects of various exogenous shocks on growth and market capitalisation ratio in the short run as well as in the long run.

My primary interest in this paper is in emerging economies. In particular, I look into the yearly data on market capitalisation as a ratio of GDP and growth from 1988 to 2012[1] for four emerging economies viz. India, Brazil, Russia and South Africa. Market capitalisation is measured by the price of a share times number of outstanding shares. The values of the correlation coefficients beween market capialisation and growth for these countries are positive and significant

[1] Data Source: World Development Indicators

at 5% level of significance and are reported in Table 1.

Table 1: Correlation coefficient between market capialisation and growth

Countries	Correlation Coefficient
India	0.62
Brazil	0.57
Russia	0.75
South Africa	0.55

One of the main purposes of this paper is to explain these instantaneous short run correlations, which turn out to be positive and significant, within the framework of a DSGE model. In a companion research (Sarkar(2015)) I have shown that in an otherwise frictionless model with either perfect competition or monopolistic competition, it is not possible to obtain such correlations. To obtain positive and significant correlations, it is necessary to have some kind of a friction in the model. In the companion research referred above, I have also shown that it is possible to obtain the empirically observed positive correlations in a DSGE model with perfect competition and credit constraint. In the present paper I show that positive and significant correlations can be obtained in a DSGE model with monopolistic competition, sticky prices and imperfect inflation indexation. More precisely, using a New-Keynesian theoretical framework with endogenous growth and imperfect inflation indexation, I investigate the relationship between market capitalisation and growth and find that the model can replicate the empirical findings reported above for a low value of the price stickiness parameter.

The other purpose of the present research is to look at the long run balanced growth path. I show that along the balanced growth path the market capitalisation ratio is related to the rate of growth of the economy via the long run value of the Tobin's q among other parameters and, upto a certain threshold level of inflation, a rise in inflation will cause both market capitalisation and growth to increase.

There are other interesting implications of the long run equilibrium. I establish that in this particular theoretical framework, inflation will have an effect on the long run growth and welfare; this result crucially hinging on the fact that inflation is imperfectly indexed by the sticky price firms. In general, higher long run inflation will have conflicting effects on steady state welfare through two channels: (i) growth channel (ii) price dispersion channel. Regarding (i), due to partial inflation indexation, firms adhering to last period's price level (called *fix price firms*) will gain a higher market share than the firms who reset prices every period (called *flex price firms*). If the proportion of the *fix price firms* is sufficiently large, it raises the rental price of capital up to a threshold by boosting the demand for capital which translates into higher capital formation and higher growth. Beyond this threshold, the higher markup charged by the price setting firms imposes an implicit tax on the rental income. Regarding (ii), due to partial inflation indexation a higher long run inflation by raising the steady state markup translates into greater price dispersion among sticky and flexible price firms. This elevated price dispersion lowers steady state welfare of citizens. A hump-shaped relationship between inflation and steady state welfare thus emerges.

The present research may also be viewed as a contribution to the currently existing New Keynesian DSGE models by introducing a stock market and by deriving the dynamics of stock prices along with that of other macro variables. It is also related to the very recent literature on financial deepening.

In what follows, I develop the basic model in section 2. In section 3, some balnced growth implications of inflation are derived. Section 4 contains some quantitative analysis. I calibrate the model and show that correlations obtained from the calibrated model are indeed very close to those obtained empirically and reported in Table 1 above. It also analyses the impulse response of the endogenous variables to various exogenous shocks. Section 5 concludes the paper.

2 The Model

2.1 Basic components

In this section I discuss the main features of the theoretical framework. There are three main players: Households, Intermediate good producing firms (I firms) and Final good producing firms (F firms).

2.1.1 Firms

There are two types of firms: intermediate good producing firms (I firms) and final good producing firms (F firms). I firms produce different varieties of intermediate goods i.e. each firm produces an intermediate good that is different from that of the other firms. The I firms thus have some monopoly power. There is a large number of I firms in a continuous interval of 0 to 1. The number of I firms (and hence the number of varieties) is assumed to be fixed. Since there is no free entry or exit, I firms make positive profits. These profits are distributed as dividends to households who are owners of the I firms. I firms produce output with capital only which they sell to the F firms. The amount earned by the I firms as revenue is spent in paying rent and dividends to the households.

As in Calvo (1983), all I firms are *exante* identical. Each period a firm receives a random "price change" signal with a probability $(1-\theta)$. Since there are a large number of I firms within the economy, by the law of large numbers, at a given time period, the economy will consist of θ fraction of I firms who will not set a new price, but stick to their previous period price and $(1-\theta)$ fraction of I firms who will not stick to their previous period price, but set an optimum price based on the maximisation of their discounted stream of future dividends. In the spirit of Yun (1996), if the I firm does not receive a price signal, its price is increased at the steady state rate of inflation (Π) subject to an inflation indexation parameterized by $\gamma \in (0,1)$. Lower γ means less indexation. The partial inflation indexation formulation is borrowed from Smets and Wouters (2003).

F firms, on the other hand, are perfectly competitive. They assemble the intermediate goods to produce the final output. The F firms being price takers sell the final goods at a fixed price to the households.The amount paid by the F firms to the I firms is in terms of the final good and it is further transferred from the I firms to the households in the form of dividend income and rental income. However, in the theoretical framework, I will work with a nominal price for the final good and a nominal price for the intermediate good.

2.1.2 Households

Just like the I firms, there is also a continuum of a large number of identical households. The representative household accumulates physical capital each period and supplies this to the I firms. In return they get paid a fixed rental income. Households also own the I firms and by virtue of this hold shares in the I firms. As a result of this dividend income becomes their other source of income apart from rental income. Households spend their income on consumption and also on investment in capital (physical investment) and shares of the I firms (financial investment). Consumption and intended investment in shares and physical capital are obtained through intertemporal maximisation.

2.1.3 Markets

I firms exchange intermediate goods for final goods with F firms. The final good that they get is distributed to households as dividends and rent. Marginal Cost on part of the I firms involves rental cost only. There is no market for labour. In equilibrium households' addition to the number of shares is zero and total value of output must be equal to the sum of representative household's rental income and dividend income.

2.2 F firm's problem

I define:

x_{it} = amt of ith intermediate good used to produce the final good.

y_t = amt of final good.

P_{it}= nominal price of the ith intermediate good.

P_t= nominal price of the final good.

All the intermediate goods get bundled by the F firm in order to produce the time t final good y_t. The final good producion technology is

$$y_t = \left(\int_0^1 x_{it}^{\rho} di \right)^{\frac{1}{\rho}}$$

This type of producion technology is called a constant elasticity of substitution (CES) bundler where $\sigma = \frac{1}{1-\rho}$ is the elasticity of substitution between inputs and $0 < \rho < 1$.

This means the production technology can be written as

$$y_t = \left(\int_0^1 x_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

A profit maximising F firm's objective function becomes

$$\begin{aligned} Max &: P_t y_t - \int_0^1 P_{it} x_{it} di \\ st &: y_t = \left(\int_0^1 x_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

The above optimisation problem gives rise to the general form of the demand function of the ith intermediate good which is given by

$$x_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\sigma} y_t \tag{2}$$

and the general price aggregation equation given by

$$P_t = \left(\int_0^1 P_{it}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \tag{3}$$

σ is also the elasticity of demand for each input. Derivation of the demand function of the ith intermediate good is discussed in the appendix.

2.3 I firm's problem

There is a continuum of intermediate goods firms in the economy in the unit interval. Each variety (i) of such goods is produced wih a linear technology as follows:

$$x_{it} = \epsilon_t k_{it} \tag{4}$$

where k_{it} is the capital used in the production of the ith variety of the intermediate good and each firm faces the same Total Factor Productivity (TFP) shock ϵ_t. The linear technology (AK type as in Rebelo,1991) is the vehicle of endogenous growth. Each variety is produced by a firm with a patent right which disallows the entry of new firms to replicate this variety.

As mentioned earlier, all I firms are *exante* identical. However, after receiving a random "price change" signal with a probability $(1-\theta)$, they can be bunched into two distinct categories: (a) firms which do not choose a new optimised price and stick to their inflation indexed past period price and (b) firms which reset a new optimal price. I call the first category *fix price firms* and the second category *flex price firms*. Since there are a large number of I firms, in each time period, θ fraction of the total firms is fix price firms and the remaining $(1-\theta)$ fracion consists of flex price firms.

The profit maximisation of the F firm yields the conditional input demand function represented by equation (2) which is

$$x_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\sigma} y_t$$

where $P_{it} = \Pi^{\gamma} P_{it-1}$ if $i \in (0, \theta)$ and $P_{jt} = P_t^*$ otherwise. P_t is the general price level at date t as mentioned earlier. The ith variety fix price firm sticks to its previous period's price (P_{it-1}) adjusted by the long run gross inflation Π, γ being the parameter of inflation indexation, where $\gamma \in (0,1)$. $\gamma = 1$ and $\gamma = 0$ imply full indexation and no indexation respectively. A fraction value of γ indicates paritial inflation indexation.

Following Grohe-Schmitt and Uribe (2011), I define a price dispersion term:

$$s_t = \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\sigma} di \tag{5}$$

Using equation (2) and categorising two different sets of prices for fix price firms and flex price firms, I can rewrite equation (5) as

$$s_t = \theta \Pi^{-\gamma\sigma} \Pi_t^{\sigma} s_{t-1} + (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{-\sigma} \tag{6}$$

Derivation of equation (6) from equation (5) is shown in the appendix.

Aggregate demand for fix price firms is given by

$$\begin{aligned} x_{1t} &= \int_0^\theta \left(\frac{\Pi^\gamma P_{it-1}}{P_t} \right)^{-\sigma} y_t di \\ &= \theta \Pi_t^\sigma \Pi^{-\gamma\sigma} s_{t-1} y_t \end{aligned} \tag{7}$$

where

$$\Pi_t = \frac{P_t}{P_{t-1}}$$

denotes inflation at time period t.

Aggregate demand for flex price firms

$$\begin{aligned} x_{2t} &= \int_\theta^1 \left(\frac{P_t^*}{P_t} \right)^{-\sigma} y_t di \\ &= (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{-\sigma} y_t \end{aligned} \tag{8}$$

Relative demand of fix price firms w.r.t. flex price firms

$$\frac{x_{1t}}{x_{2t}} = \left[\Pi^{-\gamma} \Pi_t \left(\frac{P_t^*}{P_t} \right) \right]^{\sigma} s_{t-1} \left[\frac{\theta}{1-\theta} \right] \tag{9}$$

Now I come to the price setting problem for the ith flex price firm.

Let the ith I firm set its price at P_t^* in time period t, subject to the fact that this price will stay intact k periods after this i.e. the time period $(t+k)$ with probability θ^k.

Maximising with respect to P_t^*, the expected sum of inflation adjusted discounted stream of profits for this I firm, I can obtain an expression for P_t^*. The firm's profit maximisation problem can be written more formally as:

$$Max \sum_{k=0}^{\infty} \theta^k M_{t,t+k}(\Pi^{\gamma k} P_t^* x_{t+k|t} - TC_{t+k|t}(x_{t+k|t}))$$

subject to the demand functions,

$$x_{t+k|t} = (\frac{\Pi^{\gamma k} P_t^*}{P_{t+k}})^{-\sigma} y_{t+k} \tag{10}$$

where

$$M_{t,t+k} = \beta^k \left(\frac{P_t}{P_{t+k}}\right) \left(\frac{u'(c_{t+k})}{u'(c_t)}\right) \tag{11}$$

is the firm's nominal stochastic discount factor and $TC_{t+k|t}$ is the price setter's date t forecast of the nominal total cost at time $t+k$.

The law of motion of the general price level which follows from the price aggregation in equation (3) is given by:

$$P_t = \left[\theta(\Pi^{\gamma} P_{t-1})^{1-\sigma} + (1-\theta) P_t^{*1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{12}$$

The optimal price (P_t^*) is nonstationary and thus it is normalized by the general price level P_t. We get:

$$\frac{P_t^*}{P_t} = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{E_t\sum_{k=0}^{\infty}(\theta\Pi^{-\sigma\gamma})^k M_{t,t+k}\Pi_{t,t+k}^{\sigma+1} mc_{t,t+k}(\frac{y_{t+k}}{y_t})}{E_t\sum_{k=0}^{\infty}(\theta\Pi^{\gamma(1-\sigma)})^k M_{t,t+k}\Pi_{t,t+k}^{\sigma}(\frac{y_{t+k}}{y_t})}\right) \tag{13}$$

where $mc_{t,t+k}$ is the k period ahead forecast of the real marginal cost.[2] Given the linear production function (4),

$$mc_{t,t+k} = r_{t,t+k}/\epsilon_{t+k} \tag{14}$$

where $r_{t,t+k}$ is the k period ahead forecast of the real rental price of capital.

Equation (13) can be written as

$$\frac{P_t^*}{P_t} = w_t^{-1}\left(\frac{\sigma}{\sigma-1}\right) mc_t + (1-w_t^{-1})\Pi^{-\gamma}\left(\Pi_{t,t+1}\left(\frac{P_{t+1}^*}{P_{t+1}}\right)\right) \tag{15}$$

where

$$w_t = E_t\sum_{k=0}^{\infty}(\theta\Pi^{\gamma(1-\sigma)})^k M_{t,t+k}\Pi_{t,t+k}^{\sigma}\left(\frac{y_{t+k}}{y_t}\right) \tag{16}$$

Detailed derivation of equations (13) and (15) is shown in the appendix.

At time t, capital demanded by fix price firms = k_{1t} and capital demanded by flex price firms =k_{2t}.

[2] The real marginal cost (mc_t) is the same for all firms facing the same technology.

The production function of the fix price firms:

$$x_{1t} = \epsilon_t k_{1t}$$

and production function of the flex price firms:

$$x_{2t} = \epsilon_t k_{2t}$$

Also from equation (9) the relative demand of goods produced by fix price firms w.r.t. those produced by flex price firms is given by

$$\frac{x_{1t}}{x_{2t}} = \left[\Pi^{-\gamma}\Pi_t\left(\frac{P_t^*}{P_t}\right)\right]^{\sigma} s_{t-1}\left[\frac{\theta}{1-\theta}\right]$$

Therefore, for equilibrium in the intermediate goods market,

$$\frac{k_{1t}}{k_{2t}} = \left[\Pi^{-\gamma}\Pi_t\left(\frac{P_t^*}{P_t}\right)\right]^{\sigma} s_{t-1}\left[\frac{\theta}{1-\theta}\right] = \psi_t \tag{17}$$

Equilibrium of intermediate goods market determines the optimal demand for k_{1t} and k_{2t} coming from the two types of I firms. This demand for capital coming from the I firms will be a derived demand and will be generated through the demand for intermediate goods coming from the F firms. As θ fraction of firms demand k_{1t} units of capital and $(1-\theta)$ fraction of firms demand k_{2t} units of capital, I have

$$k_t = \theta k_{1t} + (1-\theta)k_{2t} \tag{18}$$

From equation (17) and equation (18) it follows that

$$k_{1t} = \left(\frac{\psi_t}{\theta\psi_t + 1 - \theta}\right) k_t \tag{19}$$

and

$$k_{2t} = \left(\frac{1}{\theta\psi_t + 1 - \theta}\right) k_t \tag{20}$$

Once the price signal is received at time period t, the price charged by each of the fix price firms will be $\Pi^{\gamma} P_{it-1}$, which is its previous period's inflation indexed average price. Each of the flex price firms, on the other hand, will be charging the newly set optimal price P_t^*. However, the total price charged by all the fix price firms taken together will be $\theta(\Pi^{\gamma} P_{t-1})^{1-\sigma}$ and the total price charged by all the flex price firms taken together will be $(1-\theta)P_t^{*1-\sigma}$. This is reflected in the law of motion of the general price level P_t represented by equation (12). Let D_{1t} represent the total nominal dividend of the fix price firms if the measure of fix price firms were unity and D_{2t} represent the total nominal dividend of the flex price firms if the measure of fix price firms were unity. d_{1t} and d_{2t} represent the total real dividends of the fix price firms and the flex price firms respectively. Then for fix price firms

$$D_{1t} = \Pi^{\gamma} P_{t-1} \epsilon_t k_{1t} - R_t k_{1t}$$

which implies

$$d_{1t} = \left(\left(\frac{\Pi^{\gamma}}{\pi_{t-1,t}} \right) \epsilon_t - r_t \right) \left(\frac{\psi_t}{\theta \psi_t + 1 - \theta} \right) k_t \tag{21}$$

and for flex price firms

$$D_{2t} = P_t^* \epsilon_t k_{2t} - R_t k_{2t}$$

which implies

$$d_{2t} = \left(\left(\frac{P_t^*}{P_t} \right) \epsilon_t - r_t \right) \left(\frac{1}{\theta \psi_t + 1 - \theta} \right) k_t \tag{22}$$

2.4 Representative household's problem

Just like the I firms, there is a continuum of identical households within an unit interval. The representative household owns the physical capital and rents it to intermediate goods firms. Households also have ownership claims to all these firms. At date t, the household receives its proceeds from rental income, dividends from the ownership of firms and interest income from holding of a risk-free bond. The household uses its income at date t by consuming final consumption goods, investing in physical capital and buying new stocks and bonds. There is no aggregate risk in this environment.

The representative home-consumer has the following expected utility function over an infinite horizon.

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t - \gamma_c C_{t-1}) \tag{23}$$

where E_0 denotes the conditional expectation at date t, β the subjective discount factor with $0 < \beta < 1$. Due to aggregate habit formation (external), the consumer receives utility from the current consumpion, c_t after adjusting for the previous period's aggregate level of consumption, c_{t-1}. Utility function is logarithmic and is given by

$$u(.) = \ln(c_t - \gamma_c c_{t-1}) \tag{24}$$

The representative household's capital accumulation facing the investment technology is given by the following equation

$$k_{t+1} = (1 - \delta_t)k_t + \left[1 - s\left(\frac{\chi_t}{\chi_{t-1}}\right)\right] \chi_t \xi_t \tag{25}$$

where χ_t denotes investment undertaken by the household, k_{t+1} the amount of accumulated capital, δ_t the rate of depriciation of the physical capital stock and $s(.)$ captures the investment adjustment costs as in Christiano et al. (2005). I make the standard assumption that in the long run $s(.) = s'(.) = 0$ and $s''(.) > 0$ implying that the adjustment cost disappears in the long run. There is also an investment specific technology shock (IST) represented by ξ_t.

The following budget constraint summarises the choice set facing the representative home consumer:

$$P_t c_t + P_t \chi_t + \int_0^1 P_{it}^z (z_{it+1} - z_{it}) di + B_{t+1} = \int_0^1 D_{it} z_{it} di + R_t k_t + B_t(1 + i_t) \quad (26)$$

The right hand side of this constraint represents the total income of the household, which consists of the nominal dividend income given by $\int_0^1 D_{it} z_{it} di$, nominal rental income given by $R_t k_t$ and nominal bond income given by $B_t(1+i_t)$ where as before R_t represents the nominal rental rate, k_t represents the amount of capital supplied by the representative household to the I firms and B_t represents the number of risk free bonds the household consumed at time period $(t-1)$ where each bond yielded a nominal return of $(1+i_t)$, i_t being the nominal interest rate. The usual solvency condition, $\lim_{T \longrightarrow \infty} E_t B_{t+T} \geq 0$ holds for all t.

The left hand side of the constraint represents the household's nominal consumption given by $P_t c_t$, nominal physical investment (accumulation of physical capital) given by $P_t \chi_t$, nominal asset investment given by $\int_0^1 P_{it}^z (z_{it+1} - z_{it}) di$ and nominal investment on risk free bonds given by B_{t+1}. P_{it}^z represents the nominal price of an asset, $z_{it+1} - z_{it}$ represents the household's net addition to its stock of the ith I firm's asset i.e. number of additional stocks bought by the household at time period t.

Denoting the derivaive of the utility function with respect to c_t by u_{c_t}, the relevant first order conditions of the household can be written as

$$c_t : \beta^t u_{c_t} - \lambda_t P_t = 0 \quad (27)$$

$$k_{t+1}:\lambda_{t+1}R_{t+1}-\mu_t+\mu_{t+1}(1-\delta_{t+1})=0 \tag{28}$$

$$i_t:-\lambda_t P_t+\mu_t\left[\{1-s\left(.\right)\}\xi_t-\xi_t\left(\frac{i_t}{i_{t-1}}\right)s'\left(.\right)\right]+\mu_{t+1}\left[\left(\frac{i_{t+1}}{i_t}\right)^2\xi_{t+1}s'(.)\right]=0 \tag{29}$$

$$z_{it+1}:-\lambda_t P^z_{it}+\lambda_{t+1}\left(D_{it+1}+P^z_{it+1}\right)=0 \tag{30}$$

$$B_{t+1}:-\lambda_t+\left(1+i_{t+1}\right)\lambda_{t+1}=0 \tag{31}$$

where λ_t and μ_t are the Lagrangian multipliers associated with the nominal flow budget constraint (26) and the capital accumulation technology (25) respectively.

The Tobin's q (the opportunity cost of investment in terms of foregoing consumption) is defined as:

$$q_t=\frac{\mu_t}{\lambda_t P_t} \tag{32}$$

Using this definition of q the Euler Equation (29) can be rewritten as:

$$q_t\left[1-s\left(.\right)\right]\xi_t-\xi_t q_t\left(\frac{\chi k_t}{\chi k_{t-1}}\right)\left(kg_{t-1}\right)s^{'}\left(.\right)+E_t q_{t+1}m_{t+1}\xi_{t+1}kg_t^2\left(\frac{\chi k_{t+1}}{\chi k_t}\right)^2 s^{'}\left(.\right)=1 \tag{33}$$

where χk_t represents the investment to capital ratio and m_{t+1} is the household's real stochastic discount factor and is expressed as: $m_{t+1} = \beta \frac{u_{c_{t+1}}}{u_{c_t}} = \frac{\lambda_{t+1} P_{t+1}}{\lambda_t P_t}$ (which follows from equation (27)). Using the logarithmic nature of the utility function given by equation (24), the household's stochastic discount factor can be written as

$$m_{t+1} = \beta \left(\frac{c_t - \gamma_c c_{t-1}}{c_{t+1} - \gamma_c c_t} \right) \tag{34}$$

Also from equation (11) and using the logarithmic nature of the production function from equation (24) the firm's nominal discount factor can be written as

$$M_{t+1} = \beta \left(\frac{c_t - \gamma_c c_{t-1}}{c_{t+1} - \gamma_c c_t} \right) \left(\frac{P_t}{P_{t+1}} \right) \tag{35}$$

Thus I have

$$M_{t+1} = \frac{m_{t+1}}{\Pi_{t+1}} \tag{36}$$

where $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ is a measure of inflaion at time period t.

The equation (28) can be written as:

$$q_t = E_t m_{t+1} r_{t+1} + E_t q_{t+1} m_{t+1} (1 - \delta_{t+1}) \tag{37}$$

where r_{t+1} denotes the real rental rate.

Equation (30) can be written as

$$q_t = E_t m_{t+1} \left[dk_{t+1} + q_{t+1} kg_{t+1} \right] \tag{38}$$

where $dk_t = \frac{d_t}{k_t}$ represents average dividend to capital ratio and is given by

$$dk_t = \theta dk_{1t} + (1 - \theta) dk_{2t} \tag{39}$$

with $dk_{1t} = \frac{d_{1t}}{k_t}$ and $dk_{2t} = \frac{d_{2t}}{k_t}$ being the average dividend to capital ratio for the fix price firms and the flex price firms respectively.

Equation (31) boils down to

$$(1 + i_{t+1}) \frac{m_{t+1}}{\Pi_{t+1}} = 1 \tag{40}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ denotes inflation at time period t.

2.5 Equilibrium

In equilibrium, the household's net holding of bonds is nil. So

$$c_t + i_t = y_t \tag{41}$$

where

$$\begin{aligned} y_t &= \Omega_t \epsilon_t k_t \qquad (42) \\ &= \text{sum of dividend income and rental income.} \end{aligned}$$

and

$$\Omega_t = \left[\theta\left(\frac{\psi_t}{1-\theta+\theta\psi_t}\right)^{\frac{\sigma-1}{\sigma}} + (1-\theta)\left(\frac{1}{1-\theta+\theta\psi_t}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{43}$$

Derivation of (43) is shown in the appendix.

I define

$$a_t = \frac{\psi_t}{1-\theta+\theta\psi_t} \tag{44}$$

and

$$b_t = \frac{1}{1-\theta+\theta\psi_t} \tag{45}$$

2.6 Forcing Process

There are four endogenous variables, namely Total Factor Productivity (TFP) shock given by ϵ_t, Investment Specific Technology (IST) shock given by ξ_t, Monitary Policy (MP) shock given by i_t and Capital Quality (CQ) shock given by δ_t. Each of these shocks follow an AR(1) process.

TFP shock:

$$\epsilon_t - \epsilon = \rho_\epsilon(\epsilon_{t-1} - \epsilon) + \zeta_t^\epsilon \tag{46}$$

The steady state value of ϵ_t is ϵ. ζ_t^ϵ is the disturbance term.

IST shock:

$$\xi_t - \xi = \rho_\xi(\xi_{t-1} - \xi) + \zeta_t^\chi \tag{47}$$

The steady state value of ξ_t is ξ. ζ_t^χ represents the disturbance term.

MP shock:

$$i_t - i = \rho_m(i_{t-1} - i) + (1 - \rho_m)(\phi_\Pi(\Pi_t - \Pi) + \phi_y(yg_t - yg)) + \zeta_t^i \tag{48}$$

The inerest rate sequence follows a standard Taylor rule in the short run and is specified by equation (48). The monetary authority responds by raising interest rate if it anicipates a higher inflation rate or experiences a higher output growth gap. i is the steady state interest rate and yg is the steady state growth. ζ_t^i denotes the disturbance term.

CQ shock:

$$\delta_t - \delta = \rho_\delta(\delta_{t-1} - \delta) + \zeta_t^\delta \tag{49}$$

A capital quality shock is represented by the depriciation δ_t of capital. A positive capital quality shock means higher depriciation whereas a negative capital quality shock implies lower depriciation of capital. Capital depriciation is measured as a difference from its steady state value δ. ζ_t^δ denotes the disturbance term.

2.7 Balanced Growth variables

In this theoretical framework, I will deal with variables which are all stationary in the long run. Most of these variables are either expressed in growth or are normalised by capital in order to make them staionary. The long run balanced growth (BG) values of all these variables can be solved in terms of the deep parameters γ, θ, σ, β, δ, Π, ϵ, s'', γ_c, q, ξ, ρ_ϵ, ρ_ξ, ρ_δ, ϕ_Π and ϕ_y. I am going to discuss more about the deep parameters in the section on quantitative analysis.The relevant variables that I am going to work with are:

1. ck_t = consumption to capial ratio

At BG

$$ck = yk - \chi k \tag{50}$$

where ck is the BG value of consumption to capial ratio, yk the BG value of ouput to capital ratio and χk the BG value of investment to capital ratio.

2. χk_t = investment to capial ratio

At BG

$$\chi k = \frac{1}{\xi}\left(kg + \delta - 1\right) \tag{51}$$

where kg represents the BG value of the capital growth rate.

3. yk_t = output to capial ratio

At BG

$$yk = \Omega\epsilon \tag{52}$$

where Ω represents the BG value of Ω_t represented by equation (43) and ϵ is the BG value of the TFP component.

4. r_t = real rental rate

At BG

$$r = \left(\frac{P^*}{P}\right)\left(\frac{1-(1-w^{-1})\Pi^{1-\gamma}}{w^{-1}}\right)\left(\frac{\sigma-1}{\sigma}\right)\epsilon \tag{53}$$

where r represents the BG value of the rental rate of capital, w represents the BG value of the variable w_t represented by equation (16) and $\left(\frac{P^*}{P}\right)$ represents the BG value of the optimal price normalised by the general price level and given by equation (13).

5. ϵ_t = TFP shock

At BG this becomes ϵ.

6. Π_t = inflation rate defined as $\frac{P_t}{P_{t-1}}$.

At BG this becomes Π.

7. $a_t = \frac{\psi_t}{1-\theta+\theta\psi_t}$ from equation (44)

At BG

$$a = \frac{\psi}{1-\theta+\theta\psi} \tag{54}$$

where ψ is the BG value of the ratio of capital demanded by fix price firms to that demanded by flex price firms.

8. $b_t = \frac{1}{1-\theta+\theta\psi_t}$ from equation (45)

At BG

$$b = \frac{1}{1-\theta+\theta\psi} \tag{55}$$

9. $\Omega_t = \left[\theta\left(\frac{\psi_t}{1-\theta+\theta\psi_t}\right)^{\frac{\sigma-1}{\sigma}} + (1-\theta)\left(\frac{1}{1-\theta+\theta\psi_t}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$ and is represented by equation (43)

At BG

$$\Omega = \left[\theta\left(\frac{\psi}{1-\theta+\theta\psi}\right)^{\frac{\sigma-1}{\sigma}} + (1-\theta)\left(\frac{1}{1-\theta+\theta\psi}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{56}$$

10. $\frac{P_t^*}{P_t}$ = optimal price (P_t^*) normalized by the general price level P_t and is

given by $\frac{P_t^*}{P_t} = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{\sum\limits_{k=0}^{\infty}(\theta\Pi^{-\sigma\gamma})^k M_{t,t+k}\Pi_{t,t+k}^{\sigma+1} mc_{t,t+k}(\frac{y_{t+k}}{y_t})}{\sum\limits_{k=0}^{\infty}(\theta\Pi^{\gamma(1-\sigma)})^k M_{t,t+k}\Pi_{t,t+k}^{\sigma}(\frac{y_{t+k}}{y_t})}\right)$ in equation (13).

Also from the price aggregation equation in (12) I have $P_t = \left[\theta(\Pi^{\gamma}P_{t-1})^{1-\sigma} + (1-\theta)P_t^{*1-\sigma}\right]^{\frac{1}{1-\sigma}}$.

The BG expression for $\frac{P^*}{P}$ according to the price aggregation in equation (12) is

$$\frac{P^*}{P} = \left(\frac{1-\theta\Pi^{(1-\gamma)(\sigma-1)}}{1-\theta}\right)^{\frac{1}{1-\sigma}} \tag{57}$$

The BG expression for $\frac{P^*}{P}$ according to equation (13) is

$$\frac{P^*}{P} = \left(\frac{1-\theta\beta\Pi^{(1-\gamma)(\sigma-1)}}{1-\theta\beta\Pi^{(1-\gamma)\sigma}}\right)\left(\frac{\sigma}{\sigma-1}\right)mc \tag{58}$$

where mc denotes the BG value of marginal cost.

11. $w_t = E_t\sum\limits_{k=0}^{\infty}(\theta\Pi^{\gamma(1-\sigma)})^k M_{t,t+k}\Pi_{t,t+k}^{\sigma}\left(\frac{y_{t+k}}{y_t}\right)$

At BG

$$w = \frac{1}{1-\theta\beta\Pi^{(1-\gamma)(\sigma-1)}} \tag{59}$$

12. mc_t = real marginal cost

At BG

$$mc = \frac{r}{\epsilon} \tag{60}$$

13. s_t = price dispersion term given by $\int\limits_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\sigma} di$

At BG

$$s = \frac{\left(1-\theta\Pi^{(1-\gamma)(\sigma-1)}\right)^{\frac{\sigma}{\sigma-1}}}{\left(1-\theta\Pi^{\sigma(1-\gamma)}\right)(1-\theta)^{\frac{1}{\sigma-1}}} \tag{61}$$

14. ψ_t = relative capial share given by $\left[\Pi^{-\gamma}\Pi_t\left(\frac{P_t^*}{P_t}\right)\right]^{\sigma} s_{t-1}\left[\frac{\theta}{1-\theta}\right]$

At BG

$$\psi = \frac{\theta\Pi^{\sigma(1-\gamma)}}{1-\theta\Pi^{\sigma(1-\gamma)}} \tag{62}$$

15. m_{t+1} = household's discount factor given by $\beta\frac{u_{c_{t+1}}}{u_{c_t}}$

At BG

$$m = \frac{\beta}{kg} \tag{63}$$

16. M_{t+1} = firm's discount factor given by $\beta^k\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)$

At BG

$$M = \frac{m}{\Pi} \tag{64}$$

17. yg_t = output growth rate given by

$$yg_t = \frac{y_t}{y_{t-1}} = \frac{\Omega_t\epsilon_t k_t}{\Omega_{t-1}\epsilon_{t-1}k_{t-1}} \tag{65}$$

At BG

$$yg = \beta(1+r-\delta) \tag{66}$$

18. $yg\_\exp_t$ = expected growth rate at time period $t+1$ and is given by

$$yg\_\exp_t = \frac{y_{t+1}}{y_t} \tag{67}$$

At BG

$$yg\_\exp_t = yg \tag{68}$$

19. i_t = nominal interest rate on the riskless bond

At BG

$$i = \frac{\Pi}{m} - 1 \tag{69}$$

20. kg_t = growth rate of capial given by $\frac{k_{t+1}}{k_t}$

At BG

$$kg = yg = G(\text{say}) \tag{70}$$

21. dk_{1t} = total dividend to capital ratio for fix price firms given by $\left(\left(\frac{\Pi^\gamma}{\pi_{t-1,t}}\right)\epsilon_t - r_t\right)\left(\frac{\psi_t}{\theta\psi_t+1-\theta}\right)$

At BG

$$dk_1 = \left(\Pi^{\gamma-1}\epsilon - r\right)\left(\frac{\psi}{\theta\psi + 1 - \theta}\right) \tag{71}$$

22. dk_{2t} = dividend to capital ratio for fix price firms given by $\left(\left(\frac{P_t^*}{P_t}\right)\epsilon_t - r_t\right)\left(\frac{1}{\theta\psi_t+1-\theta}\right)$

At BG

$$dk_2 = \left(\left(\frac{P^*}{P}\right)\epsilon - r\right)\left(\frac{1}{\theta\psi + 1 - \theta}\right) \tag{72}$$

23. $dk_t = \theta dk_{1t} + (1-\theta)dk_{2t}$

At BG

$$dk = \theta dk_1 + (1-\theta)dk_2 \tag{73}$$

24. q_t = Tobin's q

At BG this becomes q

25. mk_t = market capitalisation ratio and is defined by

$$\frac{p_t}{y_t} = (q_t.kg_t)/\Omega_t \epsilon_t \tag{74}$$

where

$$q_t = \frac{p_t}{k_{t+1}} \tag{75}$$

At BG

$$mk = (q.kg)/\Omega\epsilon \tag{76}$$

26. ξ_t = IST component

At BG this becomes ξ

27. δ_t = depriciation rate of capial.

At BG this becomes δ.

2.8 Solution strategy

I am chiefly interesed to explore the short run dynamics between market capialisaion ratio mk_t and ouput growth yg_t. For this I loglinearise the non-linear opimal conditions and the resource constraints around the BG values of the respective variables which have been solved in terms of the deep parameters. A hat ($\hat{}$) over a variable represents proportional change from its balanced growth path value. The loglinearised system of equations are as follows:

1. Equilibrium Budget Constraint represented by equation (41)

$$ck\ \widehat{ck_t} + \chi k\ \widehat{\chi k_t} = yk\ \widehat{yk_t}$$

2. Investment equation with Investment Adjustment Cost represented by equation (25)

$$\chi k\ \xi\left(\ \widehat{\chi k_t} + \widehat{\xi_t}\right) = kg\ \widehat{kg_t} + \delta\ \widehat{\delta_t}$$

3.Production function represented by equation (42)

$$\widehat{yk_t} = \ \widehat{\Omega_t} + \widehat{\epsilon_t}$$

4. Equation (43)

$$\widehat{\Omega_t} = \left(\frac{\psi\theta(1-\theta)\left(\frac{1}{\theta\psi+1-\theta}\right)^{-\frac{1}{\sigma}}\left(\psi^{-\frac{1}{\sigma}}-1\right)}{\theta a^{\frac{\sigma-1}{\sigma}} + (1-\theta)b^{\frac{\sigma-1}{\sigma}}}\right)\widehat{\psi_t}$$

5. Price Dispersion recursion represented by equation (6)

$$\widehat{s_t} = \theta\Pi^{\sigma(1-\gamma)}\ \widehat{s_{t-1}} + \theta\Pi^{\sigma(1-\gamma)}\sigma\ \widehat{\Pi_t} - \sigma\left(1-\theta\Pi^{\sigma(1-\gamma)}\right)\widehat{\frac{P_t^*}{P_t}}$$

6. Equation (44)

$$\widehat{a_t} = \left(\frac{1-\theta}{\theta\psi+1-\theta}\right)\widehat{\psi_t}$$

7. Equation (45)

$$\widehat{b_t} = \left(\frac{-\theta\psi}{\theta\psi+1-\theta}\right)\widehat{\psi_t}$$

8.capital allocation ratio of fix price firms to flex price firms given by ψ_t in equation (17)

$$\widehat{\psi}_t = \sigma \left(\widehat{\Pi}_t + \widehat{\frac{P_t^*}{P_t}} \right) + \widehat{s_{t-1}}$$

9. Price aggregation eqn given by equation (12)

$$\widehat{\frac{P_t^*}{P_t}} = \left(\frac{\theta \Pi^{(1-\sigma)(\gamma-1)}}{1 - \theta \Pi^{(1-\sigma)(\gamma-1)}} \right) \widehat{\Pi}_t$$

10. Price optimisation eqn given by equation (15)

$$\begin{aligned} \widehat{\frac{P_t^*}{P_t}} \left(\frac{P^*}{P} \right) \quad &= \quad \left[\left(\frac{\sigma}{\sigma - 1} \right) \frac{mc}{w} + \frac{\Pi^{1-\gamma}}{w} \left(\frac{P^*}{P} \right) \right] \widehat{w_t} + \\ &\left(\frac{\sigma}{\sigma - 1} \right) \frac{mc}{w} \widehat{mc_t} + \left[\left(1 - w^{-1}\right) \Pi^{1-\gamma} \left(\frac{P^*}{P} \right) \right] \widehat{\frac{P_{t+1}^*}{P_{t+1}}} + \left[\left(\frac{P^*}{P} \right) \left(1 - w^{-1}\right) \Pi^{1-\gamma} \right] \widehat{\Pi_{t+1}} \end{aligned}$$

11. Rental equation represented by equation (14)

$$\widehat{r_t} = \widehat{mc_t} + \widehat{\epsilon_t}$$

12.Recursion for w represented by equation (16)

$$\widehat{w_t} = \theta \Pi^{\nu(1-\sigma)} M \Pi^{\sigma} yg \left[\widehat{yg_{t+1}} + \widehat{w_{t+1}} + \widehat{\Pi_{t+1}} + \widehat{M_{t+1}} \right]$$

13. Firm's discount factor given by eqn (35)

$$\widehat{M_{t+1}} = \left[\frac{kg + \gamma_c}{kg - \gamma_c} \right] \widehat{ck_t} + \left[\frac{\gamma_c}{kg - \gamma_c} \right] \left[\widehat{kg_{t-1}} - \widehat{ck_{t-1}} \right] - \left[\frac{kg}{kg - \gamma_c} \right] \left[\widehat{kg_t} + \widehat{ck_{t+1}} \right] - \widehat{\Pi_{t+1}}$$

14. Dividend to capital for fix price firms which follows from equation (21)

$$\left[\Pi^{\gamma-1}\epsilon - r\right]\widehat{dk_{1t}} = \epsilon\Pi^{\gamma-1}\left[\widehat{\epsilon_t} - \widehat{\Pi_t}\right] - \widehat{r_t}\; r + \left[\Pi^{\gamma-1}\epsilon - r\right]\widehat{a_t}$$

15. Dividend to capital for flex price firms which follows from equation (22)

$$\left[\left(\frac{P^*}{P}\right)\epsilon - r\right]\widehat{dk_{2t}} = \left(\frac{P^*}{P}\right)\epsilon\left[\widehat{\epsilon_t} + \widehat{\frac{P_t^*}{P_t}}\right] - \widehat{r_t}\; r + \left[\left(\frac{P^*}{P}\right)\epsilon - r\right]\widehat{b_t}$$

16. Average Dividend to capital represened by equation (39)

$$dk\,\widehat{dk_t} = \theta dk_1\,\widehat{dk_{1t}} + (1-\theta)\,\widehat{dk_{2t}}$$

Euler eqn w.r.t. i_t represented by equation (33)

$$\widehat{q_t} = \widehat{m_{t+1}} + \frac{r\,\widehat{r_{t+1}} + (1-\delta)q\,\widehat{q_{t+1}}}{r + (1-\delta)q} - \delta q\,\widehat{\delta_{t+1}}$$

Euler eqn w.r.t. k_{t+1} represented by equation (37)

$$\widehat{q_t} = s''kg^2\chi\left[(1+mkg)\widehat{\chi k} - \widehat{\chi k_{t-1}} + \widehat{kg_{t-1}} - mkg\left(\widehat{\chi k_{t+1}} + \widehat{kg_t}\right)\right]$$

17.Using the no arbitrage condition, the two Euler Equations can be combined as

$$\widehat{m_{t+1}} + \frac{r\,\widehat{r_{t+1}} + (1-\delta)q\,\widehat{q_{t+1}}}{r + (1-\delta)q} - \delta q\,\widehat{\delta_{t+1}} = s''kg^2\xi\left[(1+mkg)\widehat{\chi k} - \widehat{\chi k_{t-1}} + \widehat{kg_{t-1}} - mkg\left(\widehat{\chi k_{t+1}} + \widehat{kg_t}\right)\right]$$

18. Asset Euler Equation given by equation (38)

$$\widehat{q_t} = \widehat{m_{t+1}} + \left[\frac{dk}{dk + qkg}\right] \widehat{dk_{t+1}} + \left[\frac{qkg}{dk + qkg}\right] \left[\widehat{q_{t+1}} + \widehat{kg_{t+1}}\right]$$

19. Bond Euler Equation given by equation (40)

$$\left[\frac{i}{1+i}\right] \widehat{i_t} + \widehat{m_{t+1}} = \widehat{\Pi_{t+1}}$$

20. Household discount facor given by equation (36)

$$\widehat{m_{t+1}} = \widehat{M_{t+1}} + \widehat{\Pi_{t+1}}$$

21. Market capitalisation given by equation (74)

$$\widehat{mk_t} = \widehat{q_t} + \widehat{kg_t} - \widehat{\Omega_t} - \widehat{\epsilon_t}$$

22. Growth given by equation (65)

$$\widehat{yg_t} = \widehat{\Omega_t} - \widehat{\Omega_{t-1}} + \widehat{kg_{t-1}} + \widehat{\epsilon_t} - \widehat{\epsilon_{t-1}}$$

23. Expected growth given by equation (67)

$$\widehat{yg\_\mathrm{exp}_t} = \widehat{yg_{t+1}}$$

24. Monitary policy shock represented by equation (48)

$$\widehat{i_t} = \rho_m(\widehat{i_{t-1}}) + (1-\rho_m)(\phi_\Pi \widehat{\Pi_t} + \phi_y \widehat{yg_t}) + \widehat{\zeta_t^i}$$

25. TFP shock represented by equation (46)

$$\widehat{\epsilon_t} = \rho_\epsilon \ \widehat{\epsilon_{t-1}} + \widehat{\zeta_t^\epsilon}$$

26. IST shock represented by equation (47)

$$\widehat{\xi_t} = \rho_\xi \ \widehat{\xi_{t-1}} + \widehat{\zeta_t^\xi}$$

27. CQ shock represented by equation (49)

$$\widehat{\delta_t} = \rho_\delta \ \widehat{\delta_{t-1}} + \widehat{\zeta_t^\delta}$$

In the above system of equations I have 27 equations and 27 unknowns which indicates that the model is solvable. I propose to solve the model in Dynare to obtain a rational expectation equilibrium solution.

3 Balanced Growth Implications of Inflation

Using the household's Euler equations (33) and(37) and assuming a logarithmic utility function, one obtains the following expression for the balanced growth rate, G:

$$G = \beta(r + 1 - \delta)$$

Using (13) one gets the following steady state expression for the optimal price along the balanced growth path [3]

$$\frac{P_t^*}{P_t} = \mu_n(\frac{\sigma}{\sigma - 1})mc \tag{77}$$

where

$$\mu_n = \left[\frac{1 - \theta\beta\Pi^{(1-\gamma)(\sigma-1)}}{1 - \theta\beta\Pi^{(1-\gamma)\sigma}}\right] \tag{78}$$

The steady state markup over the real marginal cost has a useful decomposition property. The standard flexible price markup, $\sigma/(\sigma - 1)$ is inversely related to the demand elasticity parameter σ. The additional markup μ_n (which we call a *nominal* markup) is primarily due to the existence of nominal rigidity. If either $\theta = 0$ or $\gamma = 1$, this nominal markup disappears ($\mu_n = 1$) and the total markup just equals the flexible price markup $\sigma/(\sigma - 1)$.

Note the useful property of the nominal markup term: $\partial\mu_n/\partial\Pi > 0$ as long as $\Pi > 1$. Next from the aggregate individual demand functions in (7) and (8) one obtains the following aggregate demand functions for fix and flex price firms along the BGP:

[3] For this steady state price ratio to exist one needs the convergence condition that $\Pi < \theta^{\frac{1}{(\gamma-1)\sigma}}$. For our calibrated values of θ, γ and σ, this upper bound is about 11.39% which is above the 2% inflation target.

$$x_{1t} = \theta \Pi^{\sigma(1-\gamma)} \bar{s} y_t \text{ and } x_{2t} = (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{-\sigma} y_t \tag{79}$$

where $\bar{s}$ is the steady state price dispersion given by,

$$\bar{s} = \frac{(1 - \theta \Pi^{(1-\gamma)(\sigma-1)})^{\sigma/(\sigma-1)}}{(1 - \theta \Pi^{\sigma(1-\gamma)})(1-\theta)^{1/(\sigma-1)}} \tag{80}$$

Along the BGP, the relative market share of sticky and flexible price firms based on (79) is given by:

$$\frac{x_{1t}}{x_{2t}} = \frac{\theta \Pi^{\sigma(1-\gamma)} \bar{s}}{(1-\theta)} \left(\frac{P_t^*}{P_t} \right)^{\sigma}$$

which upon substituting the value of $\bar{s}$ from (80) and the balanced growth value of the price aggregator from (12) simplifies to

$$\frac{x_{1t}}{x_{2t}} = \left[\frac{\theta \Pi^{\sigma(1-\gamma)}}{1 - \theta \Pi^{\sigma(1-\gamma)}} \right] \tag{81}$$

Both $\bar{s}$ and (x_{1t}/x_{2t}) are increasing in Π. This property crucially hinges upon the partial indexation of inflation ($0 < \gamma < 1$).[4]

Next using the price aggregator (12) one obtains:

$$\frac{P_t^*}{P_t} = \left[\frac{1-\theta}{1 - \theta \Pi^{(\sigma-1)(1-\gamma)}} \right]^{1/(\sigma-1)} \tag{82}$$

Given the target inflation Π, the steady state rental price (r) adjusts to

[4] If inflation is fully indexed ($\gamma = 1$), $\bar{s} = 1$ which means no price dispersion and x_{1t}/x_{2t} equals $\theta/(1-\theta)$, independent of Π.

equate (77) and (82) which yields:

$$r = \left[\frac{1-\theta}{1-\theta\Pi^{(1-\gamma)(\sigma-1)}} \right]^{1/(\sigma-1)} \mu_n^{-1} \left(\frac{\sigma-1}{\sigma} \right) \epsilon \tag{83}$$

A higher inflation has conflicting effects on the balanced growth (**??**) via the steady state rental price of capital (r). It raises r via the first square bracket term in (83) which reflects the increased demand for capital of sticky price firms gaining market share for a large θ,i.e. if the proportion of fix price firms is sufficiently large. On the other hand, it lowers r by raising the steady state markup μ_n. As a result, for plausible parameter values, it is found that a rise in inflation will lead to a rise in growth upto a certain inflation level, beyond which a rise in inflation will be associated with a fall in growth (demonstrated by figure 1 in the appendix).

Lemma 1 *The steady state welfare* (W) *function is given by:*

$$W = \frac{\ln c_0}{1-\beta} + \frac{\beta \ln G}{(1-\beta)^2} \tag{84}$$

where the initial consumption (c_0) is given by:

$$c_0 = (\Omega A + 1 - \delta) - G \tag{85}$$

with

$$\Omega = \frac{\left[\theta(x_{1t}/x_{2t})^{(\sigma-1)/\sigma} + 1 - \theta\right]^{\sigma/(\sigma-1)}}{[[\theta(x_{1t}/x_{2t}) + 1 - \theta]]} \tag{86}$$

Proof:

The Balanced Growth welfare is given by

$$\begin{aligned}
W &= \sum_{t=0}^{\infty} \beta^t \ln c_0 G^t \\
&= \ln c_0 + \beta \ln c_0 G + \beta^2 \ln c_0 G^2 + ... \\
&= \ln c_0 + \beta(\ln c_0 + \ln G) + \beta^2(\ln c_0 + \ln G^2) + ... \\
&= (\ln c_0 + \beta \ln c_0 + \beta^2 \ln c_0) + (\beta \ln G + \beta^2 \ln G^2 + ...) \\
&= \frac{\ln c_0}{1-\beta} + \beta \ln G(1 + 2\beta + 3\beta^2 + ...) \\
&= \frac{\ln c_0}{1-\beta} + \frac{\beta \ln G}{(1-\beta)^2}
\end{aligned}$$

The equilibrium budget constraint is given by

$$\begin{aligned}
c_t + k_{t+1} - (1-\delta)k_t &= y_t \\
&= \left[\theta x_{1t}^{\frac{\sigma-1}{\sigma}} + (1-\theta)x_{2t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[\theta A k_{1t}^{\frac{\sigma-1}{\sigma}} + (1-\theta)A k_{2t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}
\end{aligned}$$

The equilibrium capital allocation is given by:

$$k_{1t} = \frac{\psi_t}{\psi_t \theta + 1 - \theta} k_t$$

and

$$k_{2t} = \frac{\psi_t}{\psi_t \theta + 1 - \theta} k_t$$

where

$$\frac{k_{1t}}{k_{2t}} = \psi_t = \frac{x_{1t}}{x_{2t}}$$

Hence using this the equilibrium budget constraint becomes

$$\frac{c_0}{k_0} + G - (1-\delta) = A\Omega$$

where

$$\begin{aligned}\Omega &= \left[\theta\left(\frac{\psi}{\psi\theta+1-\theta}\right)^{\frac{\sigma-1}{\sigma}} + (1-\theta)\left(\frac{1}{\psi\theta+1-\theta}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\ &= \frac{\left[\theta(x_{1t}/x_{2t})^{(\sigma-1)/\sigma}+1-\theta\right]^{\sigma/(\sigma-1)}}{\left[\theta(x_{1t}/x_{2t})+1-\theta\right]}\end{aligned}$$

This implies, assuming $k_0 = 1$,

$$c_0 = A\Omega + 1 - \delta - G$$

From equation (84), (85) and (86), the steady state welfare depends on the long run inflation rate (Π) through two channels: (i) the long run growth (G) which depends positively on Π for large θ up to a threshold Π and (ii) the price dispersion ($\bar{s}$) via the term Ω which depends negatively on Π. The proof that $\partial\Omega/\partial\Pi < 0$ for admissible range of Π is shown below.

Lemma 2 *If* $(2\theta)^{\frac{1}{\sigma(\gamma-1)}} < \Pi < \theta^{\frac{1}{\sigma(\gamma-1)}}$, $\frac{\partial\Omega}{\partial(x_{1t}/x_{2t})} < 0$

Proof: Since in the Balanced Growth, $x_{1t}/x_{2t} = \psi$, we can rewrite (86) as:

$$\Omega = \frac{\left[\theta\psi^{(\sigma-1)/\sigma} + 1 - \theta\right]^{\sigma/(\sigma-1)}}{[\theta\psi + 1 - \theta]}$$

We have:

$$\frac{\partial \ln \Omega}{\partial \psi} = \frac{\theta\psi^{-\frac{1}{\sigma}}}{\theta\psi^{\frac{\sigma-1}{\sigma}} + 1 - \theta} - \frac{\theta}{\theta\psi + 1 - \theta}$$

Now, $\frac{\partial \ln \Omega}{\partial \psi} < 0$ iff $\frac{\psi^{-\frac{1}{\sigma}}}{\theta\psi^{\frac{\sigma-1}{\sigma}}+1-\theta} < \frac{1}{\theta\psi+1-\theta}$ $<=> \psi > 1 <=> \left[\frac{\theta\Pi^{\sigma(1-\gamma)}}{1-\theta\Pi^{\sigma(1-\gamma)}}\right] > 1$ $<=> \Pi > (2\theta)^{\frac{1}{\sigma(\gamma-1)}}$.

Therefore, for $\frac{\partial \ln \Omega}{\partial \psi} < 0$, it is necessary and sufficient that

$$(2\theta)^{\frac{1}{\sigma(\gamma-1)}} < \Pi \tag{87}$$

Next note from the convergence condition, we get the upper bound on Π as

$$\Pi < \theta^{\frac{1}{\sigma(\gamma-1)}} \tag{88}$$

Combining 87 and 88 I have

$$(2\theta)^{\frac{1}{\sigma(\gamma-1)}} < \Pi < \theta^{\frac{1}{\sigma(\gamma-1)}}. \tag{89}$$

This proves the lemma.

Proposition For $(2\theta)^{\frac{1}{\sigma(\gamma-1)}} < \Pi < \theta^{\frac{1}{\sigma(\gamma-1)}}$, $\frac{\partial\Omega}{\partial\Pi} < 0$.[5]

[5] For plausible parameter values the lower bound on gross inflation Π is 0.63, i.e. a negative

Proof: Note from eq (81) of the paper that $\partial\psi/\partial\Pi > 0$. This together with the previous lemma proves the proposition. (Figure 2 in the appendix demonstrates the price disortionary effect of inflation i.e. the effect of long run inflation on Ω.)

For plausible parameter values, the relationship between Π and welfare (W) after factoring (i) and (ii) is hump shaped (demonstrated by figure 3 in the appendix).

Thus in a New-Keynesian theoratical framework with endogenous growth and partial inflation indexation, nominal rigidity will have an impact on the steady state growth and welfare.

Also, along the balanced growth path (BGP), the expression for market capitalisation is

$$mk = (q.G)/\Omega\epsilon$$

Thus, along the balanced growth path the market capitalisation ratio i.e. mk is related to the rate of growth of the economy i.e. G via q, the long run value of the Tobin's q, the long run value of the TFP parameter ϵ and Ω. This implies that a rise in ψ i.e. relative demand of fix price firms with respect to flex price firms (which in turn depends on the long run price dispersion) leads to a fall in Ω and a subsequent rise in the market capitalisation ratio. Upto a certain threshhold value of inflation, a rise in inflation will lead to a rise in growth. Also a rise in Π will steadily diminish Ω through an effect that will be

net inflation or disinflation; this combined with an upper bound of 11.39% creates a wide and reasonable range of Π.

generated via the long run price dispersion. Hence, before the inflation threshold is reached, market capitalisation will increase on both counts due to an increase in Π. Beyond this threshold level, however, the effect of a rise in inflation on market capitalisation in the long run will be ambiguous as a rise in inflation will lead to a fall in Ω as well as a fall in G. Thus upto a certain level of inflation, a rise in long run inflation will increase both growth and market capitalisation ratio unambiguously.

4 Quantitative Analysis

As the purpose of the quantitative analysis is predominantly illustrative, I do not formally estimate the structural parameters. I have relied mostly on existing studies to estimate the structural parameters except the TFP parameter.

4.1 Baseline parameterization

I fix $\beta = 0.96$ and $\delta = 0.1$ at the conventional levels (Prescott, 1986) consisent with low frequency annual data.The demand elasticity parameter σ is fixed at 6.00 as in Kollmann (2002). The habit persisence parameter γ_c is fixed at 0.6 as in Basu and Thoenissen (2011). The adjustment cost parameter $s''(.)$ is fixed at 2.5 as in Christiano et al. (2005). The long run inflation rate is set at the popular 2% target inflation rate for major indusrial countries. For a developing country like India it is set at 4% according to the recent Patel commission report.[6] There is a considerable disagreement in the literature about the range of values for the price sickiness parameter θ. While Kollmann (2002) uses 0.75 as the baseline value, Smets and Wouers (2003) estimate a higher value of θ around 0.91. In a current working paper on Technology shocks and business cycles in India, Banerjee and Basu (2015) have calculated the value of the price sickiness index for India based on the micro level commodity-wise monthly CPI data for indusrial workers. They estimated that the probability of price change within a year is 0.22. This estimate of nominal rigidity is considerably lower than the esimates used in the literature. A similar imprecision arises from the inflation indexation parameter γ which is estimated by Smets and Wouters (2003) around 0.41 with a high standard error of 0.1. Moreover the extant baseline estimates of nominal rigidity parameters are targeted to quarterly series while our focus here is on low frequency annual data. The productivity parameter A is fixed to target the per capita annual GDP growth rate of 1.97% for the sample period 1947-2014.

Table 1 reports the baseline parameter values. These are deep parameters. The short run equations are loglinearised around the balanced growth.

Table 1: Baseline Parameterization

[6] According to the recent Patel commission report, the inflation rate is targeted to be brought down to 4% from the current 10% gradually over approximatel in three years. See https://rbi.org.in/scripts/PublicationReportDetails.aspx

γ	θ	σ	β	δ	Π	ϵ	s''	q
0.65	0.22	6	0.96	0.1	1.02	0.194	2.5	1
γ_c	ϕ_Π	ϕ_y	ρ_m	ρ_ϵ	ρ_ξ	ρ_δ	ξ	
0.6	1.5	0.01	0.9	0.9	0.9	0.9	1	

4.2 Impulse Response Analysis

Although, I am interested in investigating the effect of a TFP shock, an IST shock, a moninary policy shock and a capital quality shock on market capitalisation and growth, in this section I report the impulse response figures describing the effect of these shocks on each of the variables that have been used in the model. The impulse responses are plotted in figures 1 through 12 with the labels ck=$\widehat{ck_t}$, chik= $\widehat{\chi k_t}$, , yk=$\widehat{yk_t}$, r=$\widehat{r_t}$, z=$\widehat{\epsilon_t}$, i=$\widehat{i_t}$, delta=$\widehat{\delta_t}$, xi=$\widehat{\chi_t}$, pii=$\widehat{\Pi_t}$, omega=$\widehat{\Omega_t}$, pstar=$\widehat{\frac{P_t^*}{P_t}}$, w=$\widehat{w_t}$, mc=$\widehat{mc_t}$, psi=$\widehat{\psi_t}$, m=$\widehat{m_{t+1}}$, M=$\widehat{M_{t+1}}$, yg_exp=$\widehat{yg\_exp_t}$, kg=$\widehat{kg_t}$, dk1=$\widehat{dk_{1t}}$, dk2=$\widehat{dk_{2t}}$,dk=$\widehat{dk_t}$, q=$\widehat{q_t}$, mk=$\widehat{mk_t}$, a=$\widehat{a_t}$, b=$\widehat{b_t}$, s=$\widehat{s_t}$, yg=$\widehat{yg_t}$. It is to be noted that the correlation coefficient between market capitalisation and growth obtained from the model is the summary of the impulse response time paths between market capialisation and growth driven by the four shocks. With a low value of the price stickiness parameter θ i.e. $\theta = 0.22$, the model calculates correlation coefficient between market capitalisation and growth as 0.61. For India this value is 0.62. Hence, the correlation between market capitalisation and growth for India can be reproduced by this theoretical framework with a low value of θ.Table 4 compares the correlation between market capitalisation and growth as obtained from the data with that of few other emerging economies.

Table 2: Correlation between market capitalisation and growth

(Data and Model)

	Data	Model
India	0.62	0.61
Brazil	0.57	0.61
Russia	0.75	0.61
South Africa	0.55	0.61

Figures 4 through 6 represent the effect of a shock to TFP on the above mentioned variables. From Figure 6, it is evident that due to a TFP shock, both market capitalisation (mk) and output growth (yg) undergo a rise.

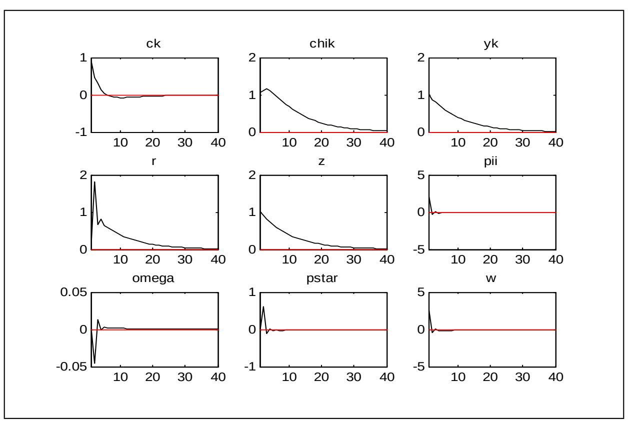


Figure 4: Effect of a TFP shock

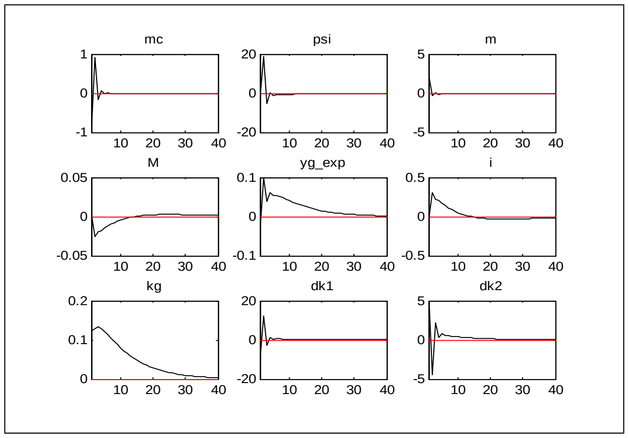


Figure 5: Effect of a TFP shock

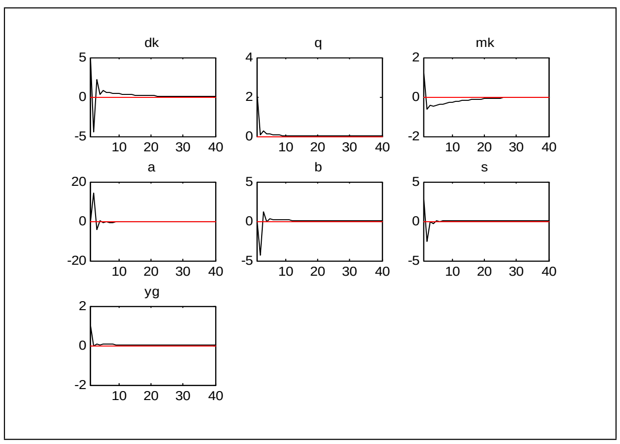


Figure 6: Effect of a TFP shock

Figures 7 through 9 represent the effect of a shock to IST on the above mentioned variables. From Figures 8 and 9, it is evident that due to an IST shock, both market capitalisation (mk) and output growth (current (yg) as well as expected (yg_exp)) undergo a rise.

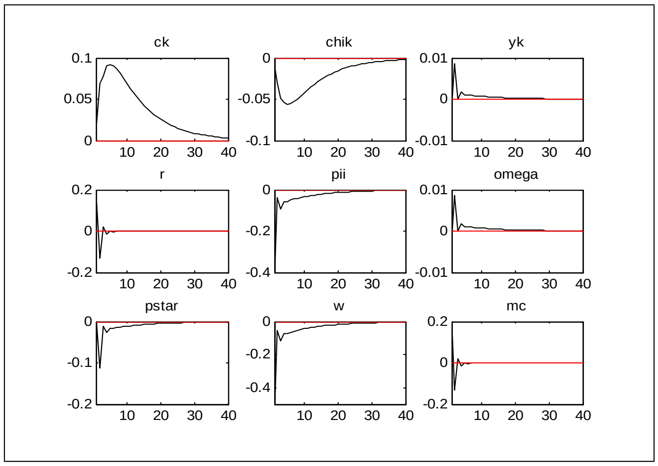


Figure 7: Effect of an IST shock

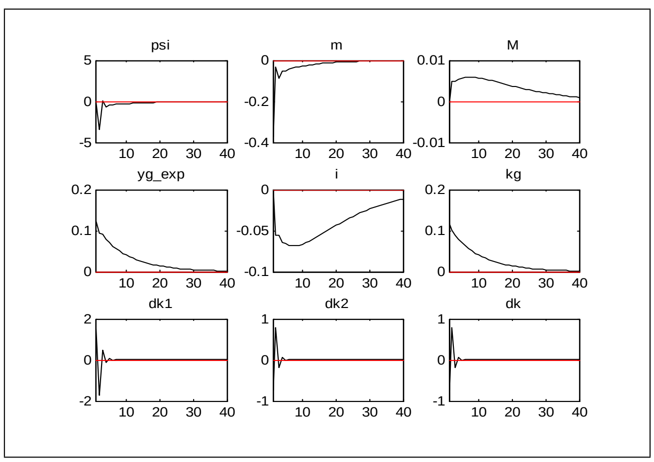


Figure 8: Effect of an IST shock

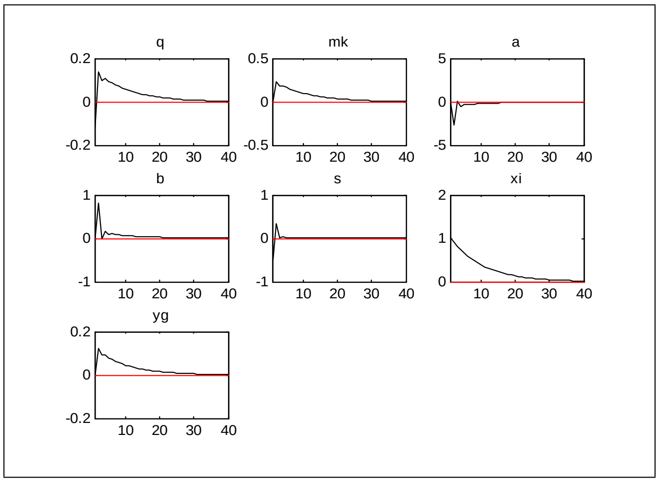


Figure 9: Effect of an IST shock

Figures 10 through 12 represent the effect of a moniary policy shock i.e. a shock to the nominal interest rate i_t on the above mentioned variables. From Figure 12, it is clear that due to a monitary policy shock both market capitalisation (mk) and growth (yg) fall. This is because with an increase in nominal interest rate, saving in stocks and in physical capial tend to decrease, thereby bringing down market capitalisation and growth (both current and expected) respecively.

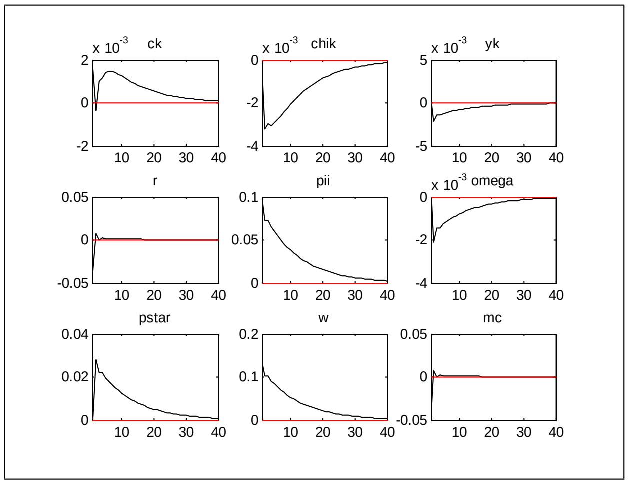


Figure 10: Effect of a Monitary Policy shock

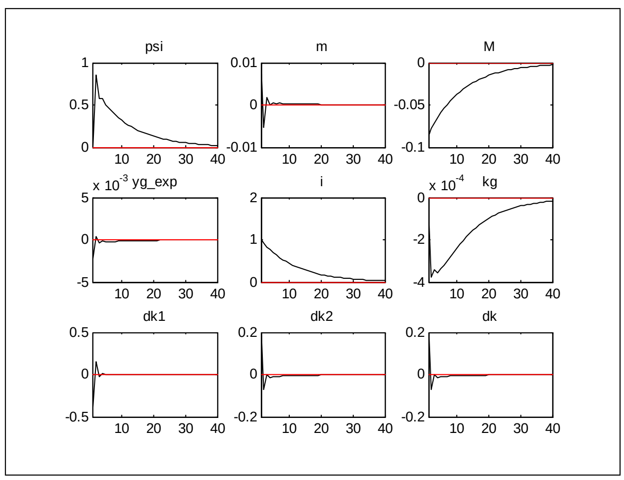


Figure 11: Effect of a Monitary Policy shock

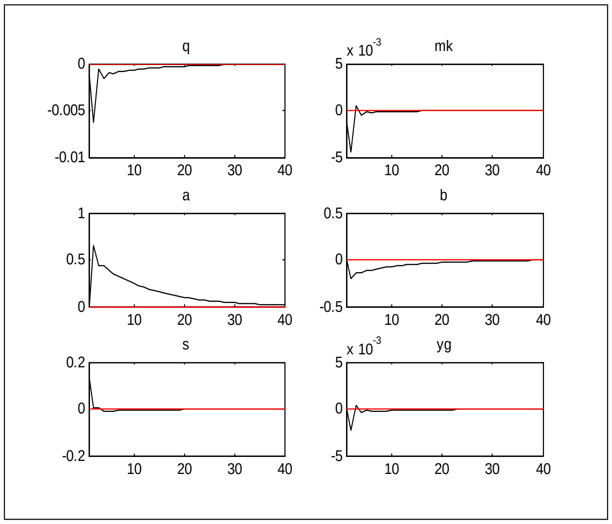


Figure 12: Effect of a Monitary Policy shock

Figures 13 through 15 represent the effect of a shock to Capial Quality (CQ) i.e. measure of depriciation δ_t on the above mentioned variables. From Figure 15, it is clear that due to a bad CQ shock, i.e. increase in depriciation (δ_t going up) both market capitalisation (mk) and output growth (yg) undergo a fall. Thus, a good CQ shock (fall in δ_t) would suggest both market capitalisation and growth to increase.

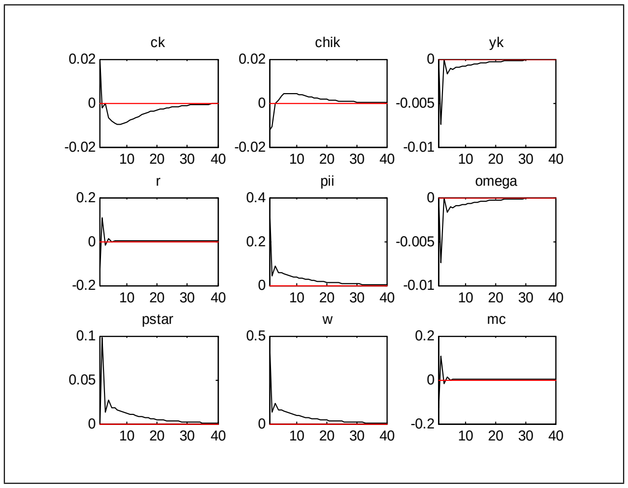


Figure 13: Effect of a Capial Quality shock

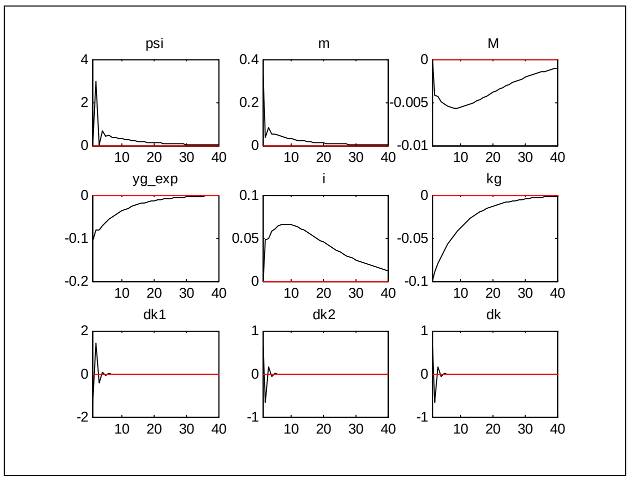


Figure 14: Effect of a Capial Quality shock

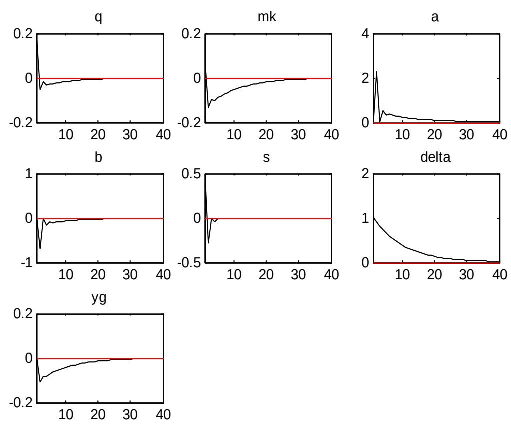


Figure 15: Effect of a Capial Quality shock

The impact effect of the 4 shocks on market capitalisation and growth is summarised in Table 3.

Table 3: Impact Effect of Shocks

	mk	yg
TFP	+	+
IST	+	+
MP	-	-
CQ	+	+

It is clear that both market capitalisation and growth are augmented by a positive TFP, IST and CQ shock. Only a positive MP shock has an adverse effect on the two variables (a rise in nominal inerest rate brings down investment in both physical and financial capital), although both move in the same direction when an MP shock is realised.

5 Conclusion

There exists a vast literature relating GDP or its growth to the levels of stock market activities, the latter being a measure of financial deepening of a country. In this paper I have looked into the relationship between financial deepening and growth using market capitalisation to GDP ratio as an indicator of financial deepening. Yearly data on market capitalisation and growth for four of the current leading emerging economies suggest a positive significant correlation between financial deepening and GDP growth. Using an endogenous growth model with nominal rigidities in the form of price stickiness and imperfect inflation indexation, I can replicate this empirical finding for a low value of the price stickiness parameter. This theoretical framework also has long run implications on growth and welfare; due to partial inflation indexation, a higher long run inflation gives rise to opposing effects on welfare via positive growth effects upto a threshold level of inflation and negative price distortionary effects. Also in the long run, upto the threshold level of inflation, both market capialisation and growth will rise due to a rise in inflation.

6 Appendix

6.1 Derivation of the intermediate good's general demand function and the price aggregator

The objective function of the F firm is

$$\begin{aligned} Max &: P_t y_t - \int_0^1 P_{it} x_{it} di \\ st &: y_t = \left(\int_0^1 x_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

Of course since the firm is competitive, profis will end up being equal to zero. Hence the problem the F firm will solve in time period t is

$$Max : P_t \left(\int_0^1 x_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} - \int_0^1 P_{it} x_{it} di$$

and this resuls in the first order condition of

$$P_t \left(\int_0^1 x_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{1}{\sigma-1}} x_{it}^{-\frac{1}{\sigma}} = P_{it}$$

which simplifies to a demand function of the ith intermediate good as

$$x_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\sigma} y_t$$

represented in equation (2).

Putting this demand for the ith intermediate good into the aggregate production function gives

$$y_t = \left(\int_0^1 \left(\left(\frac{P_{it}}{P_t} \right)^{-\sigma} y_t \right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} = y_t \left(\int_0^1 \left(\frac{P_{it}}{P_t} \right)^{1-\sigma} di \right)^{\frac{\sigma}{\sigma-1}}$$

which can be written as

$$\frac{1}{P_t} = \left(\int_0^1 \left(\frac{1}{P_{it}} \right)^{\sigma - 1} di \right)^{\frac{1}{\sigma - 1}}$$

or as a final goods pricing rule of

$$P_t = \left(\int_0^1 (P_{it})^{1-\sigma} \, di \right)^{\frac{1}{1-\sigma}}$$

represented by equation (3).

6.2 Derivation of the price dispersion recursion (equation (6)) from the price dispersion term (equation (5))

From equation (5) I have

$$\begin{aligned}
s_t &= \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\sigma} di \\
&= \int_0^{\theta} \left(\frac{P_{it-1}}{P_t} \right)^{-\sigma} di + \int_{\theta}^1 \left(\frac{P_t^*}{P_t} \right)^{-\sigma} di \\
&= \theta \int_0^1 \left(\frac{\Pi^{\gamma} P_{it-1}}{P_{t-1}} \right)^{-\sigma} \left(\frac{P_{t-1}}{P_t} \right)^{-\sigma} di + (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{-\sigma} \\
&= \Pi_t^{\sigma} \Pi^{-\gamma\sigma} \theta \int_0^1 \left(\frac{P_{it-1}}{P_{t-1}} \right)^{-\sigma} + (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{-\sigma} \\
&= \theta \Pi_t^{\sigma} \Pi^{-\gamma\sigma} s_{t-1} + (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{-\sigma}
\end{aligned}$$

which is equation (5).

6.3 Derivation of $(\frac{P_t^*}{P_t})$ (eqn (13)) and recursion for $(\frac{P_t^*}{P_t})$ (eqn (15))

The demand function with imperfect indexation (γ) is same for all flex price firms and is given by:

$$x_{t+k|t} = \left(\frac{\Pi^{\gamma k} P_t^*}{P_{t+k}}\right)^{-\sigma} y_{t+k}$$

where the general price level in the economy at time $t+k$ is given by P_{t+k} and $P_{t+k} = \Pi_{t,t+k} P_t$

I define $\Pi_{t,t+k} = \Pi_{t,t+1}.\Pi_{t+1,t+2}...\Pi_{t+k-1,t+k}$ as the level of general inflation between time period t and time period $t+k$.

Therefore I have

$$\frac{\partial x_{t+k|t}}{\partial P_t^*} = -\sigma \frac{\Pi^{-\sigma\gamma k}}{P_{t+k}^{-\sigma}} y_{t+k} P_t^{*-\sigma-1}$$

Objective function becomes:

$$\text{max: } E_t \sum_{k=0}^{\infty} \theta^k D_{t,t+k} (\Pi^{\gamma k} P_t^* \left(\frac{\Pi^{\gamma k} P_t^*}{P_{t+k}}\right)^{-\sigma} y_{t+k} - TC_{t+k|t}(x_{t+k|t}))$$

First Order Condition with respect to P_t^* gives:

$$E_t \sum_{k=0}^{\infty} \theta^k D_{t,t+k} (\Pi^{\gamma k(1-\sigma)} (1-\sigma) \left(\frac{P_t^*}{P_{t+k}}\right)^{-\sigma} y_{t+k} - MC_{t+k|t} \frac{\partial x_{t+k|t}}{\partial P_t^*} = 0$$

Plugging the value of $\frac{\partial x_{t+k|t}}{\partial P_t^*}$ in the above I have

$$E_t \sum_{k=0}^{\infty} \theta^k D_{t,t+k}(\Pi^{\gamma k(1-\sigma)}(1-\sigma)\left(\frac{P_t^*}{P_{t+k}}\right)^{-\sigma} y_{t+k} - MC_{t+k|t}\left(\frac{\Pi^{\gamma k}P_t^*}{P_{t+k}}\right)^{-\sigma} y_{t+k} = 0 \tag{90}$$

Using $P_{t+k} = \pi_{t,t+k}P_t$ and also $MC_{t+k|t} = P_{t+k}mc_{t+k|t}$ (where $mc_{t+k|t}$ is the real marginal cost at time $t+k$) in equation (90) I have

$$E_t \sum_{k=0}^{\infty} \theta^k D_{t,t+k}(\Pi^{\gamma k(1-\sigma)}(\Pi_{t,t+k}P_t)^{\sigma} P_t^* y_{t+k} - \left(\frac{\sigma}{\sigma-1}\right) mc_{t+k|t}^{-\sigma\gamma k}\Pi^{-\sigma\gamma k}(\Pi_{t,t+k}P_t)^{\sigma+1} y_{t+k} = 0$$

=>

$$E_t \sum_{k=0}^{\infty} \theta^k D_{t,t+k}(\Pi^{\gamma k(1-\sigma)}(\Pi_{t,t+k})^{\sigma}\left(\frac{P_t^*}{P_t}\right) y_{t+k} - \left(\frac{\sigma}{\sigma-1}\right) mc_{t+k|t}^{-\sigma\gamma k}\Pi^{-\sigma\gamma k}(\Pi_{t,t+k})^{\sigma+1} y_{t+k} = 0$$

From this I can derive equation (13) as

$$\frac{P_t^*}{P_t} = \left(\frac{\sigma}{\sigma-1}\right)\frac{E_t \sum_{k=0}^{\infty}(\theta\Pi^{-\sigma\gamma})^k M_{t,t+k}\Pi_{t,t+k}^{\sigma+1} mc_{t+k|t}\left(\frac{y_{t+k}}{y_t}\right)}{E_t \sum_{k=0}^{\infty}(\theta\Pi^{\gamma(1-\sigma)})^k M_{t,t+k}\Pi_{t,t+k}^{\sigma}\left(\frac{y_{t+k}}{y_t}\right)}$$

Let

$$w_t = E_t \sum_{k=0}^{\infty}(\theta\Pi^{\gamma(1-\sigma)})^k D_{t,t+k}\Pi_{t,t+k}^{\sigma}\left(\frac{y_{t+k}}{y_t}\right) \tag{91}$$

=>

$$w_t = 1 + (\theta\Pi^{\gamma(1-\sigma)})D_{t,t+1}\Pi_{t,t+1}^{\sigma}(\frac{y_{t+1}}{y_t})E_t \sum_{k=1}^{\infty}(\theta\Pi^{\gamma(1-\sigma)})^{k-1} D_{t+1,t+k}\Pi_{t+1,t+k}^{\sigma}\left(\frac{y_{t+k}}{y_{t+1}}\right)$$

=>

$$w_t = 1 + (\theta\Pi^{\gamma(1-\sigma)}) E_t D_{t,t+1} \Pi^{\sigma}_{t,t+1} (\frac{y_{t+1}}{y_t}) E_{t+1} \sum_{s=0}^{\infty} (\theta\Pi^{\gamma(1-\sigma)})^s D_{t+1,t+s+1} \Pi^{\sigma}_{t+1,t+s+1} \left(\frac{y_{t+s+1}}{y_{t+1}}\right)$$

(where $k-1=s$)

=>

$$w_t = 1 + (\theta\Pi^{\gamma(1-\sigma)}) E_t D_{t,t+1} \Pi^{\sigma}_{t,t+1} \left(\frac{y_{t+1}}{y_t}\right) w_{t+1} \tag{92}$$

From equation (13) I have

$$\frac{P_t^*}{P_t} = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1}{w_t}\right) mc_t + \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1}{w_t}\right) E_t \sum_{k=1}^{\infty} (\theta\Pi^{-\gamma\sigma})^k D_{t,t+k} mc_{t+k} \Pi^{\sigma+1}_{t,t+k} \left(\frac{y_{t+k}}{y_t}\right)$$

=>

$$\begin{aligned} \frac{P_t^*}{P_t} = & \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1}{w_t}\right) mc_t + \\ & \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1}{w_t}\right) (\theta\Pi^{-\gamma\sigma}) E_t D_{t,t+1} \Pi^{\sigma+1}_{t,t+1} \left(\frac{y_{t+1}}{y_t}\right) \\ & E_{t+1} \sum_{s=0}^{\infty} (\theta\Pi^{-\gamma\sigma})^s D_{t+1,t+s+1} mc_{t+s+1} \Pi^{\sigma+1}_{t+1,t+s+1} \left(\frac{y_{t+s+1}}{y_{t+1}}\right) \end{aligned}$$

=>

$$\begin{aligned}\frac{P_t^*}{P_t} &= \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1}{w_t}\right) mc_t + \\ &\left(\frac{1}{w_t}\right)(\theta\Pi^{-\gamma\sigma})E_t D_{t,t+1}\Pi_{t,t+1}^{\sigma+1}\left(\frac{y_{t+1}}{y_t}\right) E_{t+1}\sum_{s=0}^{\infty}(\theta\Pi^{\gamma(1-\sigma)})^s D_{t+1,t+s+1}\Pi_{t+1,t+s+1}^{\sigma}\left(\frac{y_{t+s+1}}{y_{t+1}}\right) \\ &\left(\frac{\sigma}{\sigma-1}\right) E_t\left(\frac{E_{t+1}\sum_{s=0}^{\infty}(\theta\Pi^{-\gamma\sigma})^s D_{t+1,t+s+1} mc_{t+s+1}\Pi_{t+1,t+s+1}^{\sigma+1}\left(\frac{y_{t+s+1}}{y_{t+1}}\right)}{E_{t+1}\sum_{s=0}^{\infty}(\theta\Pi^{\gamma(1-\sigma)})^s D_{t+1,t+s+1}\Pi_{t+1,t+s+1}^{\sigma}\left(\frac{y_{t+s+1}}{y_{t+1}}\right)}\right)\end{aligned}$$

=>

$$\frac{P_t^*}{P_t} = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1}{w_t}\right) mc_t + \left(\frac{1}{w_t}\right)(\theta\Pi^{-\gamma\sigma})E_t D_{t,t+1}\Pi_{t,t+1}^{\sigma+1}\left(\frac{y_{t+1}}{y_t}\right) w_{t+1}\left(\frac{P_{t+1}^*}{P_{t+1}}\right) \tag{93}$$

From (92) I have

$$(\theta\Pi^{-\gamma\sigma})E_t D_{t,t+1}\Pi_{t,t+1}^{\sigma+1}\left(\frac{y_{t+1}}{y_t}\right) w_{t+1} = \Pi_{t,t+1}\Pi^{-\gamma}(w_t - 1) \tag{94}$$

Using the relation from equation (94) in equation (93) I get

$$\frac{P_t^*}{P_t} = w_t^{-1}\left(\frac{\sigma}{\sigma-1}\right) mc_t + (1-w_t^{-1})\Pi^{-\gamma}\left(\Pi_{t,t+1}\left(\frac{P_{t+1}^*}{P_{t+1}}\right)\right)$$

which is equation (15)

6.4 Derivation of equation (43)

Refer to equation (1) where

$$y_t = \left(\int_0^1 x_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

This implies

$$\begin{aligned} y_t &= \left(\theta x_{1t}^{\frac{\sigma-1}{\sigma}} + (1-\theta) x_{2t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \\ &= \left(\theta \left(\epsilon_t k_{1t} \right)^{\frac{\sigma-1}{\sigma}} + (1-\theta) \left(\epsilon_t k_{2t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

Plugging in the value of k_{1t} and k_{2t} from (19) and (20) I have

$$y_t = \Omega_t \epsilon_t k_t$$

where

$$\Omega_t = \left[\theta \left(\frac{\psi_t}{1-\theta+\theta\psi_t} \right)^{\frac{\sigma-1}{\sigma}} + (1-\theta) \left(\frac{1}{1-\theta+\theta\psi_t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

6.5 Figures demonstrating growth effect of inflation, price distortionary effect of inflation and welfare effect of inflation

Figures 1, 2 plot the growth (G) and the price distortionary (Ω) effects of inflation for $\theta = 0.855$, $\gamma = 0.758$ and $A = 0.194$. Growth rises and tapers off after 4% inflation while price distortionary effect lowers the welfare as seen by a decline in Ω. Figure 3 plots the overall welfare effect of inflation after factoring the growth and price distortionary effects. The welfare maximizing inflation rate is 2%.

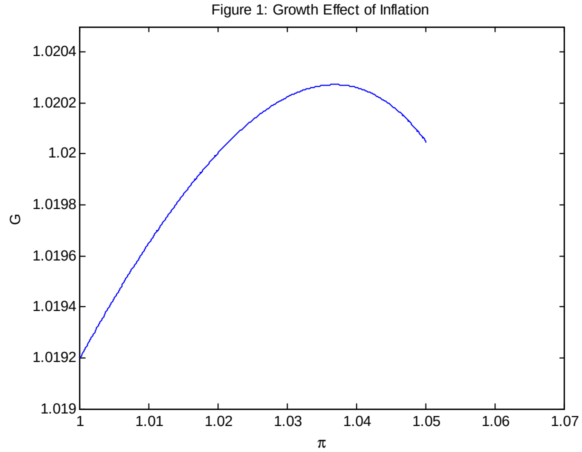


Figure 1: Growth Effect of Inflation

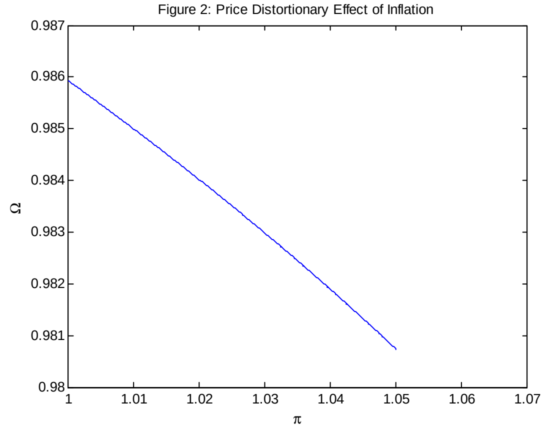
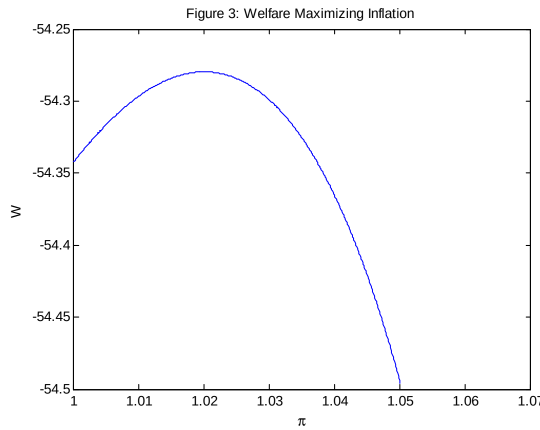


Figure 2: Price Distortionary Effect of Inflation



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