Value Creation vs. Appropriation, and Evolution of Property Rights

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When property rights are imperfect, economic agents have incentive to diversify resources between creating own value and capturing others' value. At the same time, recognition of potential losses from appropriative conflicts can induce agents to work collectively towards improving property rights. We consider two countries (societies), differentially-endowed in initial resources, populated by a sequence of generations, and initially situated in anarchy. In each generation, the myopic residents of the two countries decide jointly on incremental changes in inherited property rights, and then allocate resources between productive and combative uses. We study interactions between within-generation value creation vs. appropriation (guns vs. butter) decisions, and across-generation evolution of property rights.

In every generation, there exists a 'paradox of power' in the guns vs. butter decisions: as compared to the richer country, the poorer country invests a larger fraction of its resources on guns, and in a conflict, wins a disproportionately large share of 'contested value'. As a result, the poorer country has a weaker preference for improving property rights than the richer country, and prefers a worsening of these rights when its relative poverty is significant. Due to this difference in 'derived preference over property rights', it is not the case that the countries will always want to collectively strengthen inherited property rights. Specifically, if the evolution of generational resource endowments is exogenous, then low initial resource inequality leads to eventual establishment of perfect property rights, while high initial resource inequality perpetuates anarchy; alternatively, when past consumption is the principle determinant of a country's resource growth, perfect property rights are eventually established; however, when resource growth is determined mainly by past productive investments, there is perpetuation of the state of anarchy.

Our model contrasts two scenarios – one in which 'credible Coasian bargaining given imperfect property rights' is possible, and another in which it is not. In this context, our analysis validates the intuitive conclusion that the possibility of ex post Coasian bargaining under imperfect property rights dilutes ex ante incentives to strengthen such property rights.

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"The efforts of men are utilized in two different ways: they are directed to the production or transformation of economic goods, or else to the appropriation of goods produced by others." Vilfredo Pareto [quoted in Hirshleifer (1994)]

"What is the structure of property rights in a society at a point of time? What consequences for social interaction flow from a particular structure of property rights? How has this property right structure come into being?"

Alchian and Demsetz (1973)

1. Introduction

The two issues referred to in the above quotes are intrinsically related. It is precisely in environments where property rights are *imperfect* (i.e., incompletely specified or imprecisely enforced) that economic agents have incentives to diversify their efforts and resources between creating own value and appropriating rival value. At the same time, it is the recognition of the potential economic losses caused by such *appropriative conflicts* that can encourage agents to work collectively towards improving the property rights structure under which they operate.

This paper enquires into (*a*) the nature of economic inefficiencies that arise from economic agents' strategic choices regarding value creation *vs*. appropriation in an imperfect property rights regime, and (*b*) the agents' incentives for collective action to alter the property rights regime. Our aim is to understand the interactions effects between a property rights regime that evolves over time by being 'incrementally nudged' by generations of economic agents, and the within-generation portfolio allocation between value creation and value capture between asymmetrically-endowed agents.

1.1 Overview of the Model

We model strategic interactions between two countries (societies) populated by generations of residents. In each time period, the contemporary residents of the two countries first decide to bring about incremental changes in their inherited property rights regime by mutual consent. The countries then decide on how to allocate their resource endowments (human capital) between productive uses (creating butter) and combative uses (assembling guns), given the possibility of appropriative conflicts (i.e., conflicts arising out of a country's attempt to appropriate rival's value) following each country's creation of guns and butter. The game of production and appropriation that is played out in every generation between the residents of the two countries will be subsequently referred to as the 'guns *vs*. butter game'.

In our model, the prevalent property rights regime determines the fraction of own created value that is protected from rival appropriation. It is the value of this fraction that contemporary residents of the two countries can nudge by mutual consent, before engaging in their guns *vs*. butter game. Our analysis starts from a scenario of 'pure anarchy' where value creation is completely unprotected, and where the two countries are *differentially* endowed with stocks of

initial resources.¹ The model specifies a law of motion for the basic resource endowment of each country in each generation, where the 'growth' of resources depends on the magnitude of past productive investments and past aggregate consumptions.

In a within-generation guns *vs*. butter game, we posit that each country first allocates its basic resources over guns and butter investments. Then, if an appropriative conflict takes place, each country splits its total guns output across 'offensive use' (aimed at appropriating rival butter) and 'defensive use' (aimed at protecting own butter).

Regarding the possibility of appropriative conflicts, we consider two alternative regimes. In one regime, the countries can effectively avoid conflicts by signing a 'credible' peace-treaty (in the shadow of conflict) that irreversibly commits each country's combative forces (guns) to purely defensive use; such a credible peace treaty can involve one country transferring a part of consumption output (butter) to the other as a means of winning the peace. In the other case, a peace-treaty (while it can still be attempted) will not be credible because any commitment of combative forces to defensive use can be subsequently reversed at no cost.

We assume that every generation is *myopic* – each generation's goal in nudging the property rights regime and in making guns *vs*. butter decisions is to maximize its own final consumption. Our aim is to delineate how a sequence of within-generation (short-term) decisions about value creation *vs*. capture leads to a specific evolution trajectory for the institution of property rights.

1.2 Summary of Results

As our analysis will clarify, the equilibrium relationship between within-generation tension regarding creating and capturing value, and across-generation evolution of property rights, depend crucially on two factors in our model: (*a*) the inequality in initial resource endowments between the 'richer country' and the 'poorer country',² and (*b*) the relative dependence of resource growth in each country on past productive investment and past aggregate consumption.

Our analysis begins by characterizing the within-generation guns *vs*. butter equilibria. In our model, the following phenomenon of 'paradox of power' holds *irrespective* of whether the countries can avoid conflict by signing a peace-treaty or not: as compared to the richer country, the poorer country always invests a larger fraction of its resources on guns, and in a conflict, wins a disproportionately large share of 'contested value'.³ The important consequence of this

¹ As our subsequent analysis will make clear, what will matter in equilibrium is the *ratio* of resource endowments of the two countries, and not their absolute levels. In order to focus on the impact of resource asymmetry between the two countries, we posit the countries to be identical in all other respects (including in their productivity parameters).

 $^{^{2}}$ We will refer to the country whose initial resource endowment is larger as the 'richer country', and the other country as the 'poorer country'. Our analysis in Section 3 will confirm that the initial resource ranking will be preserved in all subsequent time periods in our model; there will be no intertemporal role reversal between the richer and the poorer country.

³ This 'paradox of power' holds when there is conflict on the equilibrium path of play because peacetreaties are not credible, and also when there is only a 'shadow of conflict' on the equilibrium path of play as credible peace-treaties can be signed.

phenomenon is that a poorer country always has a *weaker preference* for strengthening an inherited property rights regime than a larger country. In fact, when a country is sufficiently poorly endowed vis-à-vis the other, then that country can prefer anarchy to any other property rights regime.

The difference in 'derived preference over property rights' between the two differentiallyendowed countries generates interesting time-trajectories of the property rights regime. It is certainly *not* the case that every generation of residents of the two countries always prefers to strengthen the inherited institution of property rights.

Specifically, the following 'steady-state' results hold in our model:

- (1) If the evolution of generational resource endowments is exogenous (i.e., do not depend on past histories of investment/consumption), then low initial resource inequality between the countries leads to eventual establishment of perfect property rights, while sufficiently high initial resource inequality perpetuates pure anarchy. Anarchy as the steady state is more likely when credible peace-treaties can be signed.
- (2) Alternatively, when past aggregate consumption is the main determinant of a country's resource growth, then resource inequality between the countries decrease over time, and the institution of perfect property rights is eventually built.
- (3) However, when past productive investment is the principle determinant of resource growth, then resource inequality between the two countries increases over time, and this leads to the eventual perpetuation of pure anarchy.

Further, the specific time-trajectories of the property rights regime (and relative resource inequality) depend not only on the law of motion of basic resources, but also on the extent of the initial resource asymmetry, and on whether or not credible peace-treaties can be signed.

Finally, our analysis also informs on the Coase Theorem. Recognize that the regime of credible peace-treaties (in which the two countries can effectively negotiate to circumvent conflict) can be considered to be an environment that permits 'Coasian bargaining under imperfect property rights'. In such a regime, the countries can foresee the outcome of Coasian bargaining and take initial collective action to strengthen (or, indeed, weaken) the property rights regime. In contrast, the regime in which credible peace-treaties cannot be negotiation precludes the possibility of such Coasian bargaining. Under which regime will the countries have a greater collective incentive to improve the institution of property rights? A little reflection should convince the reader that it is precisely when value-destroying conflicts *cannot* be avoided by credible negotiated agreements that economic agents will have the greatest incentive to strengthen property rights. And that is precisely what happens in a within-generation equilibrium in our model. While this insight is intuitive, we have not seen it emphasized sufficiently in the extant literature: The possibility of *ex post* Coasian bargaining in an imperfect property rights.⁴

⁴ In the 'credible peace-treaties regime' Coasian bargaining happens in the shadow of conflict, while in the 'non-credible peace-treaties regime' the conflict itself replaces the possibility of Coasian bargaining.

1.3 Relation to the Literature

The current paper builds on a substantial body of research that concerns itself with two related issues: (1) appropriative conflicts in anarchic societies, and (2) emergence of property rights and their impact on economic activity.⁵

The two seminal papers on the 'state of nature models involving conflict and appropriation' are Skaperdas (1992) and Grossman and Kim (1995).⁶ Our within-generation guns *vs.* butter game builds on and complements their analyses in the following ways. Skaperdas (1992) considers a static model in which agents contribute own resources in a common pool to create a consumption good, and then take part in an appropriative conflict to secure as large a portion of the consumption good as possible. In contrast, in every generation in our model, we consider a guns *vs.* butter game in which two differentially-endowed agents produce their own consumption goods, and then split their guns output between 'offense' and 'defense' in order to protect own value and to capture rival value.⁷ In this respect, our model is closer to that of Grossman and Kim [G-K] (1995).

However, our model differs from the [G-K] model with respect to the structure of the guns *vs.* butter game, and the economic good over which imperfect property rights are defined. In the production *vs.* appropriation game in [G-K], two countries first build their defensive fortifications and then allocate their remaining resources between productive butter and offensive guns. We consider the following alternative order-of-moves specification – each country first decides on guns *vs.* butter, observes the butter outputs of both countries, and then takes offense-defense decisions – as being equally plausible, if not more so.⁸ Our analysis also differs from that of [G-K] in a more fundamental way. [G-K] considers the case where property rights over a country's *resources* are imperfect and where each country tries to appropriate the total resources of the other .⁹ In contrast, our analysis considers imperfect property rights over *consumption goods* produced by each country; in our guns *vs.* butter game, each generation of residents in each country tries to protect the consumption good that it has created, and tries to appropriate a part of its rival's consumption good. Thus, according to the classification due to Barzel (1997), we are concerned with imperfections in *economic property rights regimes* that

⁵ Haavelmo (1954) and Demsetz (1967) are two early papers on these two issues.

⁶ Some of the other major papers in this area of research are Hirshleifer (1995), Grossman and Kim (1996), Neary (1997), and Skaperdas and Syropoulos (1997).

⁷ In contrast to the Skaperdas specification, it will be easy to incorporate in our model the case where the two countries produce non-identical consumption goods. Even though we do not study that scenario in our formal model, our qualitative results are likely to survive that extension.

⁸ Our order-of-moves specification will certainly be more plausible when butter production process is stochastic in each country. Then each country will want to observe the butter outputs of both countries before deciding on self-protection vs. rival appropriation. We note that our formal analysis, while carried out in a deterministic setting, can be easily extended to incorporate stochastic butter production.

⁹ In the [G-K] model, a country can attempt to appropriate *all* of its rival's resources, irrespective of what use the resources have been put to. Specifically, if a country has employed some of its resources to build fortifications, those resources can also be subsequently appropriated by the rival country. It is not clear to us as to what particular reality such a model specification captures.

influence output distributions.¹⁰ We model such imperfections by positing that only a part of a country's 'own fruits of labour' is secure against rival appropriation; it is the size of this secure part of own value creation that the two countries can change over time by mutual consent.¹¹

In contrast to many papers on appropriative conflicts under imperfect property rights, we are careful to distinguish between two regimes – one in which peace-treaties are credible and thus effective negotiations in the shadow of conflict is possible, and another in which credible peace-treaties cannot be negotiated. Our analysis clarifies the impact of these regimes on withingeneration guns *vs.* butter choices, and across-generation evolution of property rights. In doing so, we show that the phenomenon of 'paradox of power' – first noted by Hirshleifer (1991) – is present irrespective of whether appropriative conflicts occur on the equilibrium paths of play or not. We also elaborate on the Coase theorem (Coase (1960)) by studying the impact of Coasian bargaining under imperfect property rights on *ex ante* incentives to strengthen property rights.

Recent papers by Tornell (1997), Gradstein (2004), Muthoo (2004), and Gonzalez (2007) address issues of emergence of property rights from anarchy and/or study the impact of property rights on economic growth. However, none of these papers focus on the time-trajectory of property rights generated by incremental changes to historical regimes that are brought about by successive generations of economic agents through mutual consent. Specifically, Gonzales (2007) studies the impact of changes in property rights on economic growth without examining the sources of such changes, while Gradstein (2004) and Tornell (1997) posit that the property rights regime can change discretely from anarchy to perfect rights and *vice versa* depending upon the perceived societal benefits from such changes. Muthoo (2004) considers an infinitely-repeated game version of the state of nature model, and identifies conditions under which 'absence of appropriation' will be a subgame-perfect Nash equilibrium outcome.¹²

In contrast to the above-mentioned papers, and in congruence with Kumar (2008), we consider property rights to be a public good between economic agents, whose supply can be incrementally changed (either increased or decreased) over time by collective action.¹³ In this, we follow the tradition of Demsetz (1967) in viewing the evolution of property rights to be the outcome of cost-benefit calculations of successive generations of agents.¹⁴ Further, we

¹⁰ Muthoo (2004) refers to this kind of property rights as "one of the most basic of property rights, namely, the right of an individual or an organization to the fruits of its labour".

¹¹ Hafer (2006) and Boyce and Bruner (2012) also model *ex ante* imperfect property rights by positing 'partial security' of own output.

¹² One can consider the issue studied by Muthoo (2004) as one regarding the establishment of *de facto* property rights rather than *de jure* property rights. Hafer (2006) also focuses on the emergence of *de facto* property rights in a very different economic environment – where pairs of economic agents engage in wars of attrition with each other, given *ex ante* imperfect property rights over resources.

¹³ Kumar (2008) asks many of the questions addressed in this paper. But he considers a setting in which many symmetric agents interact with each other, and his focus is on symmetric equilibria. In contrast, all of our results about value creation and value capture, and about the evolution of property rights, are predicated on resource-asymmetry between the players.

¹⁴ Recently, Dixit (2004) has discussed similar cost-benefit analyses about private provision of *de facto* property rights.

recognize the *complementarity* in the production of property rights: since it is a covenant that all involved parties need to jointly uphold, we posit that property rights cannot be strengthened without mutual consent (while it can be weakened unilaterally).¹⁵

The rest of the paper (in this preliminary draft) is organized as follows. Section 2 presents the within-generation guns *vs.* butter game. Then, the evolution of property rights in an infinite sequence-of-generations model is studied in Section 3. Section 4 contains some concluding remarks. The proofs of all our results will be contained in an (as yet unprocessed) appendix.

2. Value Creation and Appropriation

We study the evolution of two countries placed in a strategic interaction scenario characterized by two features: (1) the countries function under historically-bequeathed imperfect property rights, which can be incrementally altered in every generation by collective action, and (2) in every generation, given imperfect property rights, each country has to decide how to split its basic resource investments between creating own value and appropriating rival value.

In this section, we take an imperfect property rights regime as given, and focus on the withingeneration decision in each country regarding value creation *vs.* appropriation. In what follows, we determine the possible equilibrium outcomes of the strategic interaction between the two countries with respect to their 'guns and butter' choices.

2.1 The Within - Generation Guns vs. Butter Game

We consider the following 'guns *vs*. butter game' played between the residents of the two countries -1 and 2; here, the residents belong to the same generation (say generation *t*). We ignore within-country heterogeneity of any kind, and consider each country to be a single decision-making unit.

The data of the guns *vs*. butter game are: (*a*) the basic resource endowments of the two countries 1 and 2: $R_1 > R_2 > 0$, and (*b*) the prevalent property rights regime $\pi \in [0, 1)$ under which the countries operate.¹⁶ The magnitude of π determines what amount of a country's

Furthermore, we study the guns *vs*. butter game for a given $\pi < 1$. If the value of π is unity then property rights are perfect, and the game outcome is trivial: each country will invest only in value creation.

¹⁵ In our formal analysis, we do not explicitly model the resource-cost of changing property rights regimes, mainly because we consider the case where each generation can alter historical property rights only by a miniscule amount. The introduction of arbitrarily small costs of changing property rights will not affect our qualitative results.

¹⁶ In our subsequent analysis, we will study a scenario where the guns *vs*. butter game is played in every generation *t*, for $t \in \{0, 1, 2, 3, ...\}$. What is being described here is the structure of the guns *vs*. butter game in a specific generation *t*. Here, the conditioning of all variables on *t* is being suppressed only to reduce notational burden. Regarding the data $\{R_1, R_2, \pi\}$, it will be clarified in Section 3 that (*a*) the resource base R_i for each country *i* in each generation *t* will evolve over time depending on past histories, and (*b*) the property rights regime π in each generation *t* will also evolve depending upon past histories and upon collective actions taken by each generation of residents.

productive output (butter) is secure from any attempt at appropriation by the other country. Given (R_1, R_2) , we define $R \equiv R_1 + R_2$ and $\rho \equiv [R_2/R] \in (0, \frac{1}{2}]$. Here, ρ parameterizes the extent of 'resource inequality' between the two countries.¹⁷

The guns vs. butter game is played in three stages:

<u>Stage 1</u>: Each country *i* (for *i* = 1, 2) allocates its basic resources R_i over combative investment (guns investment) and productive investment (butter investment).¹⁸ If country *i* invests an amount $G_i \in \{0\} \cup [g^0, R_i]$ (where $g^0 > 0$ is arbitrarily close to 0) in guns investment, its guns output will be G_i and its butter output will be $B_i = [R_i - G_i]$.¹⁹ In a specific property rights regime $\pi \in [0, 1)$, π fraction of the butter output B_i of any country *i* cannot be appropriated by rival country *j* under any circumstances, while the amount $[(1-\pi)B_i]$ is *contestable* by country *j*.

<u>Stage 2</u>: Once the 'guns and butter vector' { $(G_1, B_1), (G_2, B_2)$ } of the two countries is realized, the countries enter into peace negotiations of the following kind. One of the two countries is randomly chosen (with probability ¹/₂) to be a peace proposer and the other to be the responder.²⁰ A peace proposal is an offer that specifies a feasible net butter transfer to each of the countries: { b_1, b_2 } such that $b_1+b_2=0$, in exchange for each country committing its entire guns output to defensive use.²¹ Defining a 'peace treaty' to be an accepted peace proposal, we will consider two polar opposite regimes regarding the *credibility* of a peace treaty:

REGIME [C] In this regime, we will assume that a commitment to peace can never be broken – specifically, once a country commits its guns to defensive use, it cannot subsequently break the commitment and re-deploy some of its guns to launch an attack on the other country. Thus, a peace treaty will necessarily be credible in this regime.

REGIME [N] We will assume that in this regime, a commitment to peace can be broken at no cost - a country's decision to commit its guns to defense can be fully and costlessly reversed (and its guns re-deployed to attack its rival). No peace treaty will be credible in this regime.

¹⁷ The two countries operate in a specific 'credibility of peace-treaty' regime; this is explained below.

¹⁸ As a consistent story for our model, it will help to consider the basic resource of a country to be its aggregate human capital: a part of this human capital is directed towards acquiring combative skills (this is the guns investment), and the remainder of the human capital is directed towards acquiring productive skills (this is the butter investment).

¹⁹ Our substantive assumptions regarding the production processes are: (*i*) they are identical across the two countries, and (*ii*) each production process exhibits constant returns to scale, subject to the *caveat* that there exists a 'small initial indivisibility' in guns production. Our assumption that the production coefficient in the guns and butter production functions are unity is just a simplifying assumption, none of our results will change if the production coefficients are different from unity (this is immediate from our specified payoff functions; see below). In fact, our results are robust to the specification of a stochastic butter production function (that is subject to constant returns to scale).

²⁰ As will be clear from our subsequent analysis, this 'random proposer specification' implements the Nash bargaining solution in our model.

²¹ In a scenario where guns investment creates a set of skilled warriors, a peace commitment involves garrisoning those warriors in a way that prevents them from launching an attack on the rival country.

Stage 3.1: If a peace proposal is rejected by the responder, then the game enters the 'no-treaty terminal conflict' phase: Given { $(G_1, B_1), (G_2, B_2)$ } and $\pi < 1$, each country *i* decides on how to split its guns into 'defensive forces D_i ' and 'attacking forces A_i ', with $A_i + D_i = G_i$ for each country.²² If country *i* sets $A_i = G_i$ then it necessarily attacks its rival *j* for its contested butter output' $[(1-\pi)B_j]$ but does not defend its own contested butter output $[(1-\pi)B_i]$. Alternatively, if *i* sets $A_i = 0$ then it only defends its own $[(1-\pi)B_i]$. If, however, country *i* sets $A_i \in (0, G_i)$ then it concurrently defends its own $[(1-\pi)B_i]$ and attacks its rival's $[(1-\pi)B_j]$. Whenever any country *j* is attacked, its contested butter output $[(1-\pi)B_j]$ loses a fraction of its value and shrinks to $\sigma[(1-\pi)B_j]$; here, $\sigma \in (\frac{1}{2}, 1)$ is taken to parameterize the 'loss of value from conflict'. When $A_i > 0$ and $A_j = 0$, only $[(1-\pi)B_j]$ while country *j* retains $[\pi + (1 - \alpha_i(.)(1-\pi)\sigma]B_j.^{23}$ In this case, the final payoff (butter wealth) to country *i* is $W_i = [\pi B_i + (1 - \alpha_i(.)(1-\pi)\sigma B_j]$ and the final payoff (butter wealth) to country *i* is $W_i = [\pi B_i + (1 - \alpha_i(.)(1-\pi)\sigma B_j]$. In contrast, when $A_i > 0$ and $A_j > 0$, both contested butter outputs are under attack, and the conflicts generate the following final payoff (butter wealth) for each country *i*: $W_i = [\pi B_i + (1 - \alpha_i(.)(1-\pi)\sigma B_i + \alpha_i(.)(1-\pi)\sigma B_i]$.

<u>Stage 3.2</u>: If a peace proposal is accepted and a peace treaty is signed, then the proposed butter transfers $\{b_1, b_2\}$ (with $b_1+b_2=0$) are implemented, and each country *i* commits to set $D_i = G_i$. If the game is played in the 'credibility Regime [*C*]', then the committed $D_i = G_i$ for each country *i* cannot be changed subsequently, and the game ends with each country *i* retaining its butter output $[B_i + b_i]$ and earning final payoff of $W_i = [B_i + b_i]$.

In contrast, if the game is played in the 'no-credibility Regime [*N*]', then the commitment of each country *i* to set $D_i = G_i$ can be reversed at no cost. Consequently, after the signing of a peace-treaty, the game enters the 'post-treaty terminal conflict' phase. This phase is identical to the no-treaty terminal conflict phase, except that it starts from the 'post-transfer guns and butter vector' {(G_1, B_1+b_1), (G_2, B_2+b_2)}.

We complete the description of the guns *vs*. butter game by specifying the functional form of the contest success function $\alpha_i(.)$. In our formal analysis, we work with the simplest 'Tullock contest success function':

$$\alpha_i(A_i, D_j) = \begin{cases} 0 & \text{for } A_i = 0 \text{ and } D_j \ge 0, \\ [A_i/(A_i + D_j)] & \text{for } A_i > 0 \text{ and } D_j \ge 0. \end{cases}$$

In an (as yet unprocessed) appendix to this paper, we study the robustness of our results for a more general contest success function: for $A_i > 0$ and $D_j \ge 0$, $\alpha_i(.) = [\theta(A_i)^{\eta}] / [\theta(A_i)^{\eta} + (D_j)^{\eta}]$ for some $\theta \in (0, 1]$ and $\eta \in (0, 1]$.

²² We can think of A_i as being the set of warriors that country *i* deploys to attack country *j*, leaving $D_i \equiv G_i - A_i$ warriors to defend the mother/father land.

²³ The structure of the contest success function $\alpha_i(.)$ is specified below.

2.2 Continuation Equilibria in a Terminal Conflict Phase

In a situation where, given $\pi < 1$ and a realized guns and butter vector $\{(G_1, B_1), (G_2, B_2)\}$, the two countries fail to sign a peace treaty, the guns *vs*. butter game enters the no-treaty terminal conflict phase. Analogously, after a peace proposal $\{b_1, b_2\}$ (with $b_1+b_2=0$) is accepted in Regime [*N*], the game enters the post-treaty terminal conflict phase. For a given $\pi \in [0, 1)$ and $\sigma \in (\frac{1}{2}, 1)$, Result 1 specifies the equilibrium split of the guns output of each country, and the continuation equilibrium payoffs, in these terminal conflicts:

RESULT 1. (*a*) For a guns and butter vector { $(G_1, B_1), (G_2, B_2)$ } with $B_1 + B_2 > 0$ and $G_1 + G_2 > 0$, in the unique Nash equilibrium of the continuation game in the no-treaty terminal conflict, for *i*, *j* = 1, 2, and $i \neq j$: if either $B_i = 0$ or $G_j = 0$, then $D_i = 0$ and $A_j = 0$; if $B_i \times G_i > 0$ and $B_j \times G_j > 0$, then $A_i = [B_j/(B_1 + B_2)]G_i$ and $D_i = [B_i/(B_1 + B_2)]G_i$. The continuation equilibrium payoff to each country *i* in the no-treaty terminal conflict is { $\pi B_i + (1 - \pi)[G_i/(G_1 + G_2)]\sigma[B_1 + B_2]$ }.

(*b*) For a post-transfer guns and butter vector { $(G_1, B_1+b_1), (G_2, B_2+b_2)$ } (with $b_1+b_2=0$) with $B_1+B_2>0$ and $G_1+G_2>0$, in the unique Nash equilibrium of the continuation game in the post-treaty terminal conflict: if either $B_i+b_i=0$ or $G_j=0$, then $D_i=0$ and $A_j=0$; if $(B_i+b_i)\times G_i>0$ and $(B_j+b_j)\times G_j>0$, then $A_i = [(B_j+b_j)/(B_1+B_2)]G_i$ and $D_i = [(B_j+b_j)/(B_1+B_2)]G_i$. The continuation equilibrium payoff to each country *i* is { $\pi(B_i+b_i) + (1-\pi)[G_i/(G_1+G_2)]\sigma[B_1+B_2]$ }.²⁴

2.3 Guns - and - Butter Equilibria when Peace - Treaties are not Credible

We now consider overall equilibria in the guns *vs*. butter game in Regime [*N*], where no peace treaty is credible. From Result 1, note that starting from any $\pi < 1$ and $\{(G_1, B_1), (G_2, B_2)\}$ such that $G_1+G_2 > 0$, and for any $\{b_1, b_2\}$ such that $b_1+b_2=0$, the sum of continuation equilibrium payoffs to the two countries will be $\{[\pi + (1-\pi)\sigma](B_1+B_2)\}$ in either terminal conflict; and if $G_1+G_2=0$ then their sum of continuation payoffs will be (B_1+B_2) in either terminal conflict.

This clarifies that peace negotiations are futile in Regime [*N*] as there will be no 'net surplus' to bargain over. Consequently in Regime [*N*], in any continuation equilibrium in the subgame starting at Stage 2, it must be the case that $b_1=b_2=0$. As a result, given any $\pi < 1$ in Regime [*N*], if each country *i* chooses (G_i , $B_i = R_i - G_i$) in Stage 1, its unique equilibrium continuation payoff will be: { $\pi B_i + (1-\pi)[G_i/(G_i+G_j)]\sigma[B_i+B_j]$ } whenever $G_i + G_j > 0$, and B_i whenever $G_i = G_j = 0$. These facts allow us to determine the overall equilibria in Regime [*N*].

Given $\pi \in [0, 1)$ and $\sigma \in (\frac{1}{2}, 1)$, define the critical fraction:

 $\rho^{N}(\pi, \sigma) = 0.25\{[(1-\pi)\sigma]/[\pi + (1-\pi)\sigma]\} \in (0, 0.25].$

Note that $\rho^{N}(\pi, \sigma)$ falls from 0.25 to 0 as π rises from 0 to 1 for any $\sigma \in (\frac{1}{2}, 1)$. Our next result establishes that the equilibrium outcome of the guns *vs*. butter game depends on the extent of

²⁴ If $B_1 = B_2 = 0$ then there will be nothing to fight over, and if $G_1 = G_2 = 0$ then there will be nothing to fight with. In either terminal conflict, in the former case the continuation equilibrium payoff to each country will be 0, and in the latter case it will be B_i .

resource inequality between the two countries. Specifically, if the resource inequality is smaller than a critical value such that $\rho (\equiv [R_2/R])$ strictly exceeds $\rho^N(\pi, \sigma)$, then each country will 'spread' its resource use over (i.e., invest in both) guns and butter. Alternatively, if the resource inequality is substantial such that $\rho (\equiv [R_2/R])$ is less than $\rho^N(\pi, \sigma)$, then in equilibrium the poorer country will 'focus' its resource use on (i.e., invest only in) guns production.^{25, 26}

RESULT 2: Given $\pi \in [0,1)$ and $\sigma \in (\frac{1}{2},1)$, consider Regime [N] where no peace-treaty is credible. (*a*) If there is limited resource inequality between the countries such that $\rho > \rho^{N}(\pi, \sigma)$, then in the unique subgame-perfect Nash equilibrium: Each country *i* makes an identical investment in guns production $G^{NS} = [\rho^{N}(\pi, \sigma).R]$, and invests its remaining resources $B_i^{NS} = [R_i - G^{HN}]$ in butter production.²⁷ Then peace negotiations fail, and no butter output is exchanged. In the terminal conflict, each country *i* defends itself with $D_i^{NS} = [B_i^{NS}/(B_i^{NS} + B_i^{NS})].G^{NS}$ and attacks its rival with $A_i^{NS} = [B_i^{NS}/(B_i^{NS} + B_i^{NS})].G^{NS}$.

(*b*) If there is substantial resource inequality between the countries such that $\rho \leq \rho^{N}(\pi, \sigma)$, then in the unique subgame-perfect Nash equilibrium: The poorer country 2 invests all resources in gun production: $G_2^{NF} = R_2$, while country 1 invests $G_1^{NF} = (1/R_1)\{\sqrt{[4.\rho^N(\pi, \sigma).R.R_2] - R_2}\}$ in guns production and its remaining resources $B_1^{NF} = [R_1 - G_1^{NF}]$ in butter production. Then peace negotiations fail, and no butter is exchanged. In the terminal conflict, country 2 attacks its rival with $A_2^{NF} = R_2$ and country 1 defends itself with $D_1^{NF} = G_1^{NF}$.²⁸

Given Result 2, we specify the unique equilibrium payoffs of the two countries in Regime [N] of the guns *vs*. butter game as follows.

RESULT 3: Given $\pi \in [0,1)$ and $\sigma \in (\frac{1}{2}, 1)$, and letting $W_i^{[N]}(\pi, \rho)$ denote the equilibrium payoff to country *i* in Regime [*N*], we have:

$$\begin{split} W_1^{[N]}(\pi, \rho) &= \begin{cases} \{\pi(1-\rho) + [(1-\pi)\sigma]/4\}[R_1+R_2] & \text{for } \rho \in (\rho^N(\pi, \sigma), \frac{1}{2}], \\ \{\sqrt{[\pi+(1-\pi)\sigma]} - \sqrt{[(1-\pi)\sigma\rho]}\}^2[R_1+R_2] & \text{for } \rho \in (0, \rho^N(\pi, \sigma)], \end{cases} \\ W_2^{[N]}(\pi, \rho) &= \begin{cases} \{\pi \rho + [(1-\pi)\sigma]/4\}[R_1+R_2] & \text{for } \rho \in (\rho^N(\pi, \sigma), \frac{1}{2}], \\ \{\sqrt{[\pi+(1-\pi)\sigma]}[(1-\pi)\sigma\rho] - (1-\pi)\sigma\rho\}[R_1+R_2] & \text{for } \rho \in (0, \rho^N(\pi, \sigma)]. \end{cases} \end{split}$$

²⁵ Note that $\rho \equiv [R_2/R]$ greater (resp., less) than $\rho^N(\pi, \sigma)$ is equivalent to the condition that R_2 is greater (resp., less) than $[\rho^N/(1-\rho^N)].R_1$, where $[\rho^N/(1-\rho^N)]$ is a fraction between 0 and 1/3.

²⁶ Neary (1997) calls the fist kind of equilibrium (involving diversification) a 'Hobbesian equilibrium' and the second kind of equilibrium (involving specialization) a 'Banditry equilibrium'.

²⁷ Note that the guns investment amount does not depend on the resource endowment of a country (as long as it is no less than $\rho^{N}.R$). Further, even if the production coefficients for guns and butter were different from unity, resource allocation over guns and butter would be independent of their magnitudes.

²⁸ Note that since the indivisibility in guns investment has been taken to be arbitrarily small in our model (i.e., g^0 is small), the best-response to rival not investing in guns is to invest the amount g^0 in guns (and then attack the defenseless rival and appropriate its entire butter output). This precludes the possibility of a 'communal equilibrium' (as defined by Neary (1997)) in our model where each country invests all its resources in value creation even when $\pi < 1$. Note that if g^0 was significantly large, then even with $\pi < 1$ we could have such a communal equilibrium with $B_i = R_i$ for each country *i*.

Result 3 allows us to identify the following comparative statics properties of the equilibrium payoffs. Recognize that while the richer country's equilibrium payoff monotonically increases in π for all values of $\rho \in (0, \frac{1}{2}]$, that is not the case for the poorer country.²⁹ This difference is due to the phenomenon of 'paradox of power' (Hirshleifer (1991)) which operates in our model in the following form: as compared to the richer country, the poorer country always invests a larger fraction of its resources on guns production, and in a conflict, wins a disproportionately large share of 'aggregate contested butter output' (by allocating a larger fraction of its guns output for offensive use). As a consequence, the poorer country always has a *weaker preference for strengthening property rights* than the richer country: $dW_2^{[N]}/d\pi < dW_1^{[N]}/d\pi$. As we show in Section 3.2, the poorer country's preferred property rights regime can indeed be pure anarchy when resource inequality between the countries is sufficiently large. This possibility has important implications for the evolution of property rights when contemporaneous collective action by the two countries is required to alter history-dependent property rights regimes.

We would also like to highlight the following properties of the guns and butter equilibrium. Both the 'sum of payoffs' $[W_1^{[N]} + W_2^{[N]}]$ and the 'difference of payoffs' $[W_1^{[N]} - W_2^{[N]}]$ unambiguously increase in π . Taking $[W_1^{[N]} + W_2^{[N]}]$ to be a measure of economic efficiency, we note that a strengthening of property rights improves efficiency in two ways – by reducing the total amount of 'contestable value' of the two countries, and by reducing the guns investment amount of each country. Interpreting $[W_1^{[N]} - W_2^{[N]}]$ as a measure of 'final consumption inequality' (as opposed to initial resource inequality parameterized by ρ), we note that it is the phenomenon of 'paradox of power' that causes the poorer country to gain a relatively larger share of the total contested butter output. Given that, a weakening of property rights *reduces the consumption inequality* between the two countries within a generation.

2.4 Guns - and - Butter Equilibria when Peace - Treaties are Credible

We now consider overall equilibria in Regime [*C*], where any peace treaty is credible as it is impossible to re-deploy guns to offensive use once they have been committed to defense. Note that in Regime [*C*], for any $\pi < 1$ and {(*G*₁, *B*₁), (*G*₂, *B*₂)} such that *G*₁+*G*₂ > 0, if any feasible peace proposal (i.e., any offer {*b*₁, *b*₂} such that *b*₁+*b*₂=0) is rejected, the continuation equilibrium payoff to country *i* will be: { π .*B*_i + (1- π)[*G*_i/(*G*₁+*G*₂)] σ [*B*₁+*B*₂]}. So, if country *j* is the peace-proposer, it will propose to transfer *b*_i = (1- π){[*G*_i/(*G*₁+*G*₂)] σ [*B*₁+*B*₂] - *B*_i} amount of butter (this amount can be positive or negative) to country *i*, and keep (*B*_j-*b*_i) for itself. Since each country is equally likely to be the peace-proposer, the 'expected peace payoff' to each country *i* will be { $\frac{1}{2}(1-\pi)(1-\sigma)(B_1+B_2)$ } + { π .*B*_i + (1- π)[*G*_i/(*G*₁+*G*₂)] σ [*B*₁+*B*₂]}.³⁰ On the other hand, if *G*₁=*G*₂ = 0 then there will be no net surplus to bargain over and the peace payoff to each country *i* will be *B*_i.

As in Regime [N], the nature of the overall 'guns and butter equilibrium' in Regime [C] (with

²⁹ In Section 3.2, we specify in greater details as to how the equilibrium payoff of country 2 varies with the property rights parameter.

³⁰ Thus when $G_1+G_2>0$, each country's peace payoff is [½(net bargaining surplus) + own outside option].

respect to investing in both guns and butter or only in guns) will depend on the extent of resource inequality between the countries. Specifically, given $\pi \in [0, 1)$ and $\sigma \in (\frac{1}{2}, 1)$, we define the critical fraction:

$$\rho^{\rm C}(\pi,\sigma) = 0.5\{[(1-\pi)\sigma]/[1+\pi+(1-\pi)\sigma]\} \in (0, 0.5\sigma/(1+\sigma)].$$

Note that $\rho^{C}(\pi, \sigma)$ falls from 0.5 $\sigma/(1 + \sigma)$ (< 0.25 since $\sigma < 1$) to 0 as π rises from 0 to 1 for any $\sigma \in (\frac{1}{2}, 1)$. Further, $\rho^{C}(\pi, \sigma) < \rho^{N}(\pi, \sigma)$ for all $\pi < 1$. We then have the following result:

RESULT 4: Given $\pi \in [0,1)$ and $\sigma \in (\frac{1}{2},1)$, consider Regime [*C*] where peace-treaties are credible. (*a*) If there is limited resource inequality such that $\rho > \rho^{C}(\pi, \sigma)$, then in the unique subgameperfect Nash equilibrium: Each country *i* invests identical amount of resources in gun production $G^{CS} = [\rho^{C}(\pi, \sigma).R]$ and its remaining resources $B_i^{CS} = [R_i - G^{CS}]$ in butter production. Then in the successful peace negotiations, the randomly chosen peace-proposer country *j* transfers $b_i^{CS} = (1-\pi)\{[G_i^{CS}/(G_1^{CS}+G_2^{CS})]\sigma[B_1^{CS}+B_2^{CS}] - B_i^{CS}\}$ amount of butter to country *i*, and keeps $[B_j^{CS} - b_i^{CS}]$ for itself. Concurrently, countries commit their guns to defense and the game ends. (*b*) If there is substantial resource inequality between the countries such that $\rho \le \rho^{C}(\pi, \sigma)$, then in the unique subgame-perfect Nash equilibrium: the poorer country 2 invests all its resources in gun production: $G_2^{CF} = R_2$, while country 1 invests $G_1^{CF} = (1/R_1)\{\sqrt{[4,\rho^{C}(\pi,\sigma).R.R_2] - R_2}\}$ in gun production and its remaining resources $B_1^{CF} = [R_1 - G_1^{CF}]$ in butter production. Then in the successful peace negotiations, the randomly chosen peace-proposer country *j* transfers $b_i^{CF} \equiv$ $(1-\pi)\{[G_i^{CF}/(G_1^{CF} + G_2^{CF})]\sigma.B_1^{CF} - B_i^{CF}\}$ amount of butter to *i* and keeps $[B_j^{CF} - b_i^{CF}]$ for itself, where $B_2^{CF} = 0$; concurrently countries commit their guns to defense and the game ends.

Our next result specifies the equilibrium payoffs of the two countries in Regime [C]:

RESULT 5: Given $\pi \in [0,1)$ and $\sigma \in (\frac{1}{2}, 1)$, and letting $W_i^{[C]}(\pi, \rho)$ denote the equilibrium payoff to country *i* in Regime [*C*], we have:

$$W_1^{[C]}(\pi, \rho) = \begin{cases} \pi(1-\rho) + [(1-\pi)/2][1+(1-\sigma)\pi]/[1+\sigma+(1-\sigma)\pi] \} [R_1+R_2] & \text{for } \rho \in (\rho^C(\pi, \sigma), \frac{1}{2}], \\ \{\sqrt{\frac{1}{2}}[(1+\pi+(1-\pi)\sigma] - \sqrt{[(1-\pi)\sigma\rho]}\}^2 [R_1+R_2] & \text{for } \rho \in (0, \rho^C(\pi, \sigma)], \end{cases}$$

$$W_2^{[C]}(\pi, \rho) = \begin{cases} \pi \rho + [(1-\pi)/2][1+(1-\sigma)\pi]/[1+\sigma+(1-\sigma)\pi] \} [R_1+R_2] & \text{for } \rho \in (\rho^C(\pi, \sigma), \frac{1}{2}], \\ \{ [\pi+(1-\pi)\sigma]\sqrt{[2(1-\pi)\sigma\rho]/[1+\pi+(1-\pi)\sigma]}] + [(1-\pi)(1-\sigma)/2] - [(1-\pi)\sigma\rho] \} [R_1+R_2] & \text{for } \rho \in (0, \rho^C(\pi, \sigma)]. \end{cases}$$

Result 5 confirms that in Regime [*C*], a change in the property rights parameter π has similar effects on the equilibrium payoffs $\{W_1^{[C]}, W_2^{[C]}\}$ of the two countries as in Regime [*N*]. In Regime [*C*], $dW_1^{[C]}/d\pi > dW_2^{[C]}/d\pi$, and while $W_1^{[C]}$ monotonically rises in π for all values of $\rho \in (0, \frac{1}{2}]$, that is not the case with $W_2^{[C]}$. Specifically, $W_2^{[C]}$ falls in π when the resource inequality between the countries is large. Further, while economic efficiency as measured by $[W_1^{[C]} + W_2^{[C]}]$ rises in π , so does final consumption inequality as measured by $[W_1^{[C]} - W_2^{[C]}]$. It is instructive to note that the phenomenon of 'paradox of power' is also present in Regime [*C*],

even though there is only a 'shadow of conflict' on the equilibrium path of play in this regime of credible peace-treaties.³¹ In the next section, we study the implications of these properties of the guns and butter equilibrium payoffs for the evolution of property rights in a dynamic sequence-of-generations model.

3. Evolution of Property Rights across Generations

In this section, we take the two countries in our model to be inhabited by an infinite sequence of generations. In each generation, the contemporary residents of the two countries (where each country is a homogeneous decision-making unit) first decide on a 'mutually agreed upon incremental change' in the property rights regime that they inherit from their 'parents', and then play the guns *vs*. butter game given their generational resource endowments (which also evolve over time). We assume that each generation in each country is myopic in that its goal is to maximize own final butter consumption. We aim to determine how the series of short-term (i.e., within-period) decisions by the infinite sequence of generations in the two countries generate specific evolution trajectories of the property-rights regime.

3.1 The Sequence - of - Generations Model

There is an infinite sequence of generations in each of the two countries 1 and 2, with one generation in each period *t*, where $t \in \{0, 1, 2, ..., \infty\}$. In every time period, the contemporary generations of the two countries interact with each other in a manner described below.

The initial conditions of our dynamic model are as follows: Two countries that are asymmetrically-endowed in initial basic resources, with $R_1(t = 0) > R_2(t = 0) > 0$, start their sequential interaction from a state of 'pure anarchy' with $\pi^e(t = 0) = 0$, given a time-invariant 'loss of value from conflict' parameter $\sigma \in (\frac{1}{2}, 1)$. Across successive generations, two sets of variables evolve over time: the per-period resource endowment of each country, and the property rights regime under which the two countries function in any given period. Their evolution processes are specified below.

In any period *t*, let country *i*(*t*) refer to country *i* inhabited by its period *t* generation. The resource endowments of the two countries 1(t) and 2(t) are denoted by $R_1(t)$ and $R_2(t)$. We posit that for every $t \in \{1, 2, ..., \infty\}$:

 $R_i(t) = R_i(t-1) + x [B_i(t-1)] + y [W_i(t-1)], \text{ where } x \ge 0 \text{ and } y \ge 0.$

Recognize that $B_i(t-1)$ and $W_i(t-1)$ are the aggregate butter production and butter consumption amounts in country *i* in the preceding period. Thus, our main assumptions about the 'law of motion' governing the evolution of resource endowments are as follows: We assume that the resource completely depreciates at the end of a period, while the amount of 'new' resources that are 'born' at the beginning of the following period depends positively on the extent to which

³¹ Even in Regime [C] it is the case that the poorer country invests a larger fraction of its resource endowment on guns production than the richer country.

previous period's resources were put to productive use, and on the aggregate consumption in the previous period.³²

In any period *t*, along with receiving their resource endowments $\{R_1(t), R_2(t)\}$, the two countries 1(t) and 2(t) also inherit the property rights regime of their parents' generation as their *ex ante* property rights regime: $\pi^e(t) = \pi(t-1)$. Then they negotiate about incrementally changing that regime. We assume that within any time period, inherited property rights can be changed by at most an amount μ , where μ is taken to be a small positive number. Thus, the generation *t* population of the two countries negotiate to chose any $\pi(t) \in [\pi(t-1) - \mu, \pi(t-1) + \mu]$, with the restriction that $\pi(t)$ must belong in [0, 1]. We assume (quite plausibly) that strengthening property rights (i.e., raising π) requires the *collective will* of both countries, while weakening such rights (i.e., lowering π) can be achieved *unilaterally*. So, defining $\pi^{\mu}_i(t)$ to be the optimal π for country i(t) in the interval $[max\{0, \pi(t-1) - \mu\}, min\{\pi(t-1) + \mu, 1\}]$, we posit that negotiations at the beginning of time-period.^{33, 34}

After countries 1(t) and 2(t) agree on the property rights regime under which they will operate, they play the within-generation guns *vs*. butter game given their resource endowments $R_1(t)$ and $R_2(t)$ (as described in Section 2). We make the substantive assumption that every generation is *myopic*: in making all its decisions (including its choice of property rights) the sole aim of each generation in each country is to maximize its own final butter wealth. In this, our aim is to determine the evolution trajectory of property rights (starting from a state of pure anarchy) in a scenario where each generation is cognizant of the impact of prevailing property rights on contemporary strategic choices regarding value creation *vs*. appropriation, but ignores the future impact of the property rights regime that it bequeaths to its progeny.³⁵

3.2 Optimal Property - Rights - Regime for the Poorer Country

As we have indicated in Section 2, the equilibrium payoffs from the guns *vs*. butter game within a generation have the following properties: $dW_2^{[N]}/d\pi < dW_1^{[N]}/d\pi$ and $dW_2^{[C]}/d\pi < dW_1^{[C]}/d\pi$.

 $^{^{32}}$ Again, it helps to view the basic resource as human capital. The aggregate human capital of a generation of residents in any country *i* is born at the time of birth of the generation and dies with the generation's death. However, within a country, the magnitude of a generation's total human capital depends positively on the total amount of productive skills acquired by its 'parent generation', and on the parents' aggregate consumption.

³³ Note that this outcome will obtain if $\pi(t)$ is determined in a *descending auction* where the auctioneer lowers the 'bid' π from $min{\pi(t-1) + \mu, 1}$ to $max{0, \pi(t-1) - \mu}$, and $\pi(t)$ is set at that bid at which *both* bidders are 'in' for the first time (thus indicating mutual acceptance of the proposed bid π).

³⁴ In our formal analysis, we ignore resource costs of negotiations as well as resource costs of changing the value of π within any period *t*. Intuition suggests that our qualitative results will remain unchanged for 'small' resource costs of negotiations and/or of incrementally changing regimes.

³⁵ Intuition suggests that our qualitative results will remain valid even when a generation cares 'a little' about the welfare of its progeny. In related literature, the assumption of 'generational myopia' is made in Maxwell and Reuveny (2005), Reuveny *et al.* (2011), and Bakshi and Dasgupta (2015).

These facts, along with our position that within-generation negotiations over the property rights parameter leads to the establishment of the smaller of the choices made by the two countries, implies that in every generation *t*, it is the preferences of the poorer country 2 over the interval $[max\{0, \pi(t-1) - \mu\}, min\{\pi(t-1) + \mu, 1\}]$ that will determine the *ex post* property rights parameter $\pi(t)$ for that generation.

As a prelude to determining the sequence of $\pi(t)$ that will be chosen by the successive generations of residents of the poorer country, we address the following question in this subsection: In any generation *t*, if the residents of the poorer country 2 could choose their 'optimal property rights regime' in a manner *unconstrained by history* – i.e., over the entire interval [0, 1] – what regime would it choose? Given $\sigma \in (\frac{1}{2}, 1)$, our next result answers that question depending on whether peace-treaties will be credible or not (in stating the result, we suppress the dependence of the variables on time for notational convenience).

RESULT 6: In Regime [N]: (i) if $\rho \in [\sigma/4, \frac{1}{2})$ then $\pi = 1$ is optimal for country 2, (ii) if $\rho \in (0, (1 - \{1/(2\sigma)\})^2]$ then $\pi = 0$ is optimal for country 2, and (iii) if $\rho \in ((1 - \{1/(2\sigma)\})^2, \frac{\sigma}{4}]$ then $\pi = \pi_2^{[N]} \equiv \{1/[2-2\sigma(1-\rho) - \{2\rho\sigma(2-2\sigma(1-\rho))\}^{1/2}]\} - \{\sigma/[1-\sigma]\} \in (0, 1)$ is optimal for country 2.³⁶ In Regime [C]: (i) if $\rho \in [\frac{1}{2} - \frac{\sigma}{4}, \frac{1}{2})$ then $\pi = 1$ is optimal for country 2, (ii) if $\rho \in (0, \frac{1}{2} - \frac{\sigma}{4}, \frac{1}{2})$ then $\pi = 1$ is optimal for country 2, (ii) if $\rho \in (0, \frac{1}{2} - \frac{\sigma}{4}, \frac{1}{2})$ then $\pi = 1$ is optimal for country 2, (ii) if $\rho \in (0, \frac{1}{2} - \frac{\sigma}{4}, \frac{1}{2})$ then $\pi = 1$ is optimal for country 2, (ii) if $\rho \in (0, \frac{1}{2} - \frac{\sigma}{4}, \frac{1}{2})$ then $\pi = 0$ is optimal for country 2, and (iii) if $\rho \in (\frac{1}{2} - \frac{\sigma}{4}, \frac{1}{2} - \frac{\sigma}{4})$ then $\pi = \pi_2^{[C]} = [\{\sigma/(\frac{1}{2} - \rho)\}^{1/2} - (1+\sigma)]/[1-\sigma] \in (0, 1)$ is optimal for country 2.

The above result establishes that the poorer country's preferred property rights regime is pure anarchy for a much larger set of $\{\rho, \sigma\}$ values in Regime [*C*] than in Regime [*N*]. On reflection, this is just what we should expect. A country will have the greatest incentive to strengthen property rights precisely when value-destroying terminal conflicts *cannot* be avoided by credible peace treaties. In our model, the possibility of *ex post* Coasian bargaining in an imperfect property rights environment dilutes *ex ante* incentives to strengthen property rights.

Further, note that if there is pure anarchy in any time period ($\pi(t) = 0$ in any period *t*), then whenever there is enough resource inequality such that $\rho < \frac{1}{4}$, the poorer country will focus only on guns production. Focusing on Regime [*N*], Result 6 states that when located in pure anarchy in this regime, specializing in guns production – and thus becoming a 'pure predator' – is a necessary condition for the poorer country to prefer to continue to be in pure anarchy. However, when we shift focus to Regime [*C*], we realize that such a conclusion is not warranted. The ability to avoid conflicts reduces incentives to strengthen property rights to such an extent that in Regime [*C*], the poorer country can prefer a 'move down' to pure anarchy even when it is located in a 'positive' property rights regime in which it *does not* specialize in guns production.

3.3 Time - Trajectories of Property Rights and Resource Inequality

As noted at the beginning of this section, our dynamic model starts from the initial conditions: $\pi^{e}(t=0) = 0$ and $\rho(t=0) \equiv [R_{2}(t=0)] / [R_{1}(t=0) + R_{2}(t=0)] = \rho^{0} \in (0, \frac{1}{2})$. Then the resource

³⁶ For $\rho = \sigma/4$, any π in the interval $[(1 - \sigma)/(2 - \sigma), 1]$ is optimal for country 2.

endowment of each country *i* evolves over time according to the law of motion:

$$R_i(t) = R_i(t-1) + x [B_i(t-1)] + y [W_i(t-1)], \text{ with } x \ge 0 \text{ and } y \ge 0.3^{3/2}$$

In what follows, we characterize the evolution of the property rights parameter $\pi(t) \in [0, 1]$ and the resource inequality parameter $\rho(t) \equiv [R_2(t)] / [R_1(t) + R_2(t)]$ for the following mutually exclusive cases: $\{x = 0, y = 0\}$; $\{x > 0, y = 0\}$; and $\{x = 0, y > 0\}$.³⁸

RESULT 7: Given $\sigma \in (\frac{1}{2}, 1)$ and $\mu > 0$, suppose that $\{x = 0, y = 0\}$. In this case, $\rho(t) = \rho^0$ for all t > 0 in Regimes [N] and [C], and the following statements about the evolution of $\pi(t)$ are true. In Regime [N]: (i) if $\rho^0 \in [\sigma/4, \frac{1}{2})$ then $\pi(t)$ monotonically increases in t (in increments of μ) from 0 to 1, (ii) if $\rho^0 \in (0, (1-\{1/(2\sigma)\})^2]$ then $\pi(t) = 0$ for all t > 0, and (iii) if $\rho^0 \in ((1-\{1/(2\sigma)\})^2)$, $\sigma/4$] then $\pi(t)$ increases in t from 0 to $\pi_2^{[N]} \in (0, 1)$ and then remains stationary at $\pi_2^{[N]}$. In Regime [C]: (i) if $\rho^0 \in [\frac{1}{2} - \frac{\sigma}{4}, \frac{1}{2})$ then $\pi(t)$ increases in t from 0 to 1, (ii) if $\rho^0 \in (0, \frac{1}{2} - {\{\sigma/(1+\sigma)^2\}}]$ then $\pi(t) = 0$ for all t > 0, and (iii) if $\rho^0 \in (\frac{1}{2} - \frac{\sigma}{4})$ then $\pi(t)$ increases in t from 0 to $\pi_2^{[C]} \in (0, 1)$ and then remains stationary at $\pi_2^{[C]}$.

Result 7 considers the special case where the resource endowment of each generation in each country is exogenous and identical. In this case, the relative resource sizes of the two countries remain unchanged over time. As a result, the unconstrained preferences of the poorer country 2 (as described in Result 6) completely determine the eventual resting point of the property rights regime. The impact of the magnitude of initial resource inequality between the two countries on the evolution of property rights is brought out most clearly in this special case: a small resource inequality generates a monotonic improvement in property rights leading to the eventual establishment of perfect property rights; a large resource inequality leads to perpetuation of pure anarchy; and an intermediate resource inequality causes a partial improvement in the property rights structure. Further, Result 7 demonstrates the dilution of incentives to improve property rights when countries are able to circumvent conflict by Coasian bargaining: given $\sigma \in (\frac{1}{2}, 1)$ and $\mu > 0$, perpetual anarchy results for a larger set of initial values of ρ^0 in Regime [*C*] than in Regime [*N*].

In contrast to the special case of exogenous and stationary intertemporal resource endowments, the evolution of property rights is very different when per-period resource endowments of countries depend on investment and consumption histories. As our next results show, it all depends on whether the resource inequality between the two countries increase over time or decrease over time. Irrespective of whether peace-treaties are credible or not, in the former case

³⁷ The following generalization of the law of motion for resource endowments " $R_i(t) = z \cdot R_i(t-1) + x \cdot [B_i(t-1)] + y \cdot [W_i(t-1)]$ with $z \ge 1$, $x \ge 0$, and $y \ge 0$ " will not change our qualitative results. Such a generalization will admit an exogenous growth component to $R_i(t)$.

³⁸ Analytical results are difficult to establish in the general case where *x* and *y* are both strictly positive. In future, we intend to carry out simulation exercises to determine time-trajectories of $\pi(t)$ and $\rho(t)$ when *x* is sufficiently larger than *y*, and *vice versa*.

there is 'eventual perpetuation of pure anarchy', while in the latter case there is 'eventual establishment of perfect property rights'.

RESULT 8: Given $\sigma \in (\frac{1}{2}, 1)$ and $\mu > 0$, suppose that $\{x > 0, y = 0\}$. In this case, there exists $\underline{x} > 0$ such that for $x > \underline{x}$, the following statements are true: (i) $\rho(t+1) < \rho(t)$ for all $t \ge 0$ in Regimes [N] and [C]; (ii) there exists $t^{[NX]} > 0$ such that $\pi(t) = 0$ for all $t > t^{[NX]}$ in Regime [N]; and (iii) there exists $t^{[CX]} > 0$ such that $\pi(t) = 0$ for all $t > t^{[CX]}$ in Regime [C].

RESULT 9: Given $\sigma \in (\frac{1}{2}, 1)$ and $\mu > 0$, suppose that $\{x = 0, y > 0\}$. In this case, there exists y > 0 such that for y > y, the following statements are true: (i) $\rho(t+1) \ge \rho(t)$ for all $t \ge 0$ in Regimes [*N*] and [*C*]; (ii) there exists $t^{[NY]} > 0$ such that $\pi(t) = 1$ for all $t > t^{[NY]}$ in Regime [*N*]; and (iii) there exists $t^{[CY]} > 0$ such that $\pi(t) = 1$ for all $t > t^{[CY]}$ in Regime [*C*].

Result 8 clarifies that when past productive investment is the substantive determinant of the size of current resource endowment of a country, then resource inequality between the two countries increases over time irrespective of whether peace-treaties are credible or not. This is due to the 'paradox of power': in each generation, the poorer country invests a smaller amount of resources in value creation (butter production). As resource inequality increases, the state of anarchy becomes increasingly attractive to the poorer country – and this leads to the eventual perpetuation of pure anarchy in this scenario. Note that the overall dynamics generated by the myopic choices of successive generations of residents of the poorer country is intertemporally disastrous for the country – in the long-run the country vanishes asymptotically.³⁹

In contrast to Result 8, Result 9 identifies a scenario in which perfect property rights are eventually established in both Regimes [N] and [C]. This happens when past aggregate consumption is the substantive determinant of a country's current resource endowment. In this case, the 'paradox of power' under imperfect property rights causes final consumption inequality in each generation to decrease over time. That, in turn, causes resource inequality between the countries to decrease over time. Sufficient decrease in resource inequality eventually makes the perfect property rights regime the preferred choice for the poorer country.⁴⁰

Results 8 and 9 characterize 'long-run steady state' values of property rights and resource inequality, and highlight that these steady-state values depend primarily on the law of motion of resource endowments of the two countries.⁴¹ However, the actual time-trajectories of $\pi(t)$ and

³⁹ Under the hypotheses of Result 8, $\rho(t)$ is a strictly decreasing Cauchy sequence that asymptotically approaches 0. While this result is established in a scenario where each generation of a country's residents are completely myopic, such a result can also hold in the case where each generation cares 'just a little bit' about the welfare of their progeny.

⁴⁰ Under the hypotheses of Result 9, $\rho(t)$ increases at a decreasing rate and asymptotes to a value strictly less than ¹/₂. Thus, the country which is initially poorer remains poorer forever.

⁴¹ Results 8 and 9 do not consider the possibility that the law of motion determining the evolution of each country's resource endowments might depend on whether the countries can engage in credible Coasian bargaining or not. However, recognize that conflicts occur on the within-generation equilibrium path-ofplay in Regime [N] and not in Regime [C]. As a result, the dependence of the evolution of resource

 $\rho(t)$ also depend on the extent of initial resource inequality between the two countries, and on whether credible peace-treaties can be signed or not. We conclude our analysis by recording how $\pi(t)$ and $\rho(t)$ evolve over time in the different parameter regimes of our model:

(*a*) Consider the case where the hypotheses of Result 8 holds (where past productive investment is the substantive determinant of a country's current resources), so that $\rho(.)$ monotonically falls over time and asymptotes to zero. In Regime [N], $\rho(t)$ decreases at a decreasing rate over time, and the evolution of $\pi(t)$ depends on the magnitude of ρ^0 : if the initial resource inequality is small such that $\rho^0 > (1-\{1/(2\sigma)\})^2$, then $\pi(t)$ starts increasing from 0 from the initial period, reaches a 'peak' (smaller than 1) and then fall back to zero in finite time; on the other hand, if the initial resource inequality is large such that $\rho^0 < (1-\{1/(2\sigma)\})^2$ then $\pi(t)$ remains at 0 for all *t*. The time-trajectory of $\pi(t)$ is similar in Regime [C] in that either $\pi(t)$ initially rises and subsequently fall to zero, or remains at zero forever. However, the initial resource inequality magnitudes that generate either of these time-paths, the maximal value that $\pi(t)$ attains, and the exact length of time before $\pi(t)$ returns to zero are different in the two regimes.

(*b*) Next, consider the case where the hypotheses of Result 9 holds (where past aggregate consumption is the substantive determinant of a country's current resources), so that $\rho(.)$ is non-decreasing in *t*. In Regime [*N*], $\rho(t)$ increases at a decreasing rate, and the evolution of $\pi(t)$ depends on the magnitude of ρ^0 : if initial resource inequality is small with $\rho^0 > (1-\{1/(2\sigma)\})^2$, then $\pi(t)$ increases in increments of μ from 0 to 1 starting from t = 0; on the other hand, if initial resource inequality is large with $\rho^0 < (1-\{1/(2\sigma)\})^2$, then $\pi(t)$ initially remains at zero for some periods, and then monotonically increases from 0 to 1. The time-trajectory of $\pi(t)$ is similar in Regime [*C*] except that the number of initial time-periods that $\pi(t)$ remains at 0 depends on the parameter values in a different way than in Regime [*N*].

4. Concluding Remarks

This paper has studied strategic interactions between two countries in a scenario where (*a*) they operate under historically imperfect property rights that can be incrementally nudged in every generation by mutual consent, and (*b*) they engage in a production-and-appropriation game in every generation. Our primary query has been: Does the recognition of economic losses due to imperfect property rights *necessarily* induce the successive generations of myopic residents in the two countries to monotonically strengthen the property rights regime over time?

We have answered this question in the negative, and have identified the critical features of our sequence-of-generations model that lead to perpetuation of the state of anarchy. That is the case when either the initial resource inequality between the two countries is significant, or the magnitude of past productive investments is the substantive determinant of the growth of a

endowments on the magnitude of past 'productive investment' might be more substantial in Regime [N] than in Regime [C]. If that is the case, then anarchy as the long-run steady state outcome will be more likely in Regime [N] than in Regime [C].

country's generational resource-endowments, or both.

The principle reason behind our result is that in a scenario where mutual consent is needed in every generation to nudge the property rights regime, it is the poorer country that will determine how property rights will evolve. Further, in every generation, the poorer country will be 'more of an appropriator' than the richer country – this is the 'paradox of power'. And being more of an appropriator, the poorer country will have a weaker preference for improvement in property rights than the richer country. In fact, in any generation in which the poorer country's relative resource-endowment is sufficiently small, its residents will prefer anarchy over any other property rights regime. Greater initial resource-inequality and substantial dependence of resource growth on past productive investments cause the resource inequality in every generation to be large and growing. As a result, over time, the myopic residents of the poorer country will inexorably drag both countries into a state of anarchy, and itself into oblivion.

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