Contributing to public defense in a contest model

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Abstract

I augment the standard Tullock contest by adding a first stage in which each of the potential contestants has the option of contributing some resources to a public defender or government. In the subsequent subgame, if one of the contestants attacks the other, then the government contributes its resources to the defence of the agent that is attacked. I show that, if the resource distribution is not too unequal, agents make positive contributions to government in equilibrium and there is no fighting. The deterrence equilibria are pareto superior to the corresponding equilibria of the pure Tullock contest. The Rawlsian criterion yields the most efficient equilibrium for each given resource distribution, hence progressive taxation is efficient in this model. Finally, for a range of very unequal resource distributions, the equilibrium size of government is too large.

Note: All lemmas, propositions etc. are proven, but typed proofs are not included in this copy.

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1. Introduction

The question of how stable property rights emerge out of a state of anarchy has exercised social thinkers from the very earliest times.

In contemporary economic literature, a construct that has been widely used to investigate this question is the rational contest model, in which agents can use resources in their possession to engage in production, or to wrest away resources from other agents. The contest model is similar to constructs that been used to analyse lobbying contests and patent races. Consequences of verious formulations of the nature of contest and the form of the contest success function are explored in a number of papers by Hirshleifer (1991) Hirshleifer (1995), Skaperdas (1992), Grossman and Kim (1995), as well as several others.

Many of these papers investigate conditions under which, in the absence of an external enforcer, the potential cotestants will enter into active conflict, and conditions under which they will coexist in peace. In Hirshleifer's formulation resources that are devoted to conflict can be used both for aggression and defence, thus an investment to dissuade the adversary may also tun out to provide incentive for aggression. Grossman and Kim consider investments that are earmarked for aggression (e.g., cannons) or defence (e.g., fortification) and obtain equilibria in which peace may sometimes prevail.

One conclusion that emerges from most of these models is that conflict is more likely when there is high inequality between the agents, and in these cases the poorer agent is more likely to be the aggressor.

Surprisingly, however, very few contributions explore the possibility that the potential contestants, in anticipation of the possible destructiveness of conflict, may enter into cooperation to create such an enforcement mechanism as a public good. An exception is McBride, Milante, and Skaperdas (2011), who explore a model in which contestants can invest in a state, which is able to protect from conflict a fraction of all resources; the fraction being determined by the total investment (see also McBride and Skaperdas, 2007).

In this paper I use a simpler construction. As in McBride, Milante, and Skaperdas (2011) the two potential contestants choose to make contributions to enable a public defender. In the subsequent subgame each contestant has a choice to attack the other. If one of the contestants chooses to be an offender (and the other does not), then the defender contributes its resources to the defence of the victim.

I find that peace prevails (though at a cost) except in cases where inequality is extreme, when agents no longer contribute to public defence in equilibrium. For a large range of parameter values there are multiple equilibria, with the richer agent contributing a larger or smaller fraction of the public defence. With appropriate investments, peace becomes incentive compatible for two reasons; first, resources inveted in public defence are no longer available as conflict payoffs to the contestants, and secondly the same defence investment reduces the expected conflict payoff for both contestants.

Two additional results are of interest. First, when there are multiple equilibria, the most efficient equilibrium is always the one in which the richer agent makes the largest contribution consistent with equilibrium. If we interpret these contributions as taxes determined by a participatory government, then the efficient taxation scheme is the most progressive scheme that is consistent with peace. Secondly, we find that there is a range wher inequality is high (but not sufficiently extreme for government to break down) where a contest would in fact be more efficient than a peace equilibrium. An interpretation is that, when inequality is high, government is inefficiently large.

Beviá and Corchón (2010), which is in some ways close to this paper, consider the possibility that the richer agent may transfer some of her wealth to the poorer in order to avoid conflict. Such transfers reduce inequality and therefore the likelihood of conflict. However, when we introduce this option in the present model, we find that contributions to public defence is more attractive to the richer agent than transfers to the poorer.

The next section lays out the canonical context model in its simplest form, and derives the equilibrium outcome. Section 3 describes the model with investment in public defence. Section 4 establishes the equilibria. Section 5 discusses efficiency concerns to find the most efficient equilibria, and also to show that the worst peace equilibria are more efficient than the pure contest outcomes. The main results are summarised in this section. In conclusion, Section 6 lists some further questions that can be addressed using this model.

2. Background: pure contest

2.1. Setting

We adopt a simple version of the standard model (e.g., Hirschleifer). There is one unit of resources distributed between two agents, 1 and 2.

$$R_1 + R_2 = 1, \qquad 0 < R_1 \leq R_2$$

Each agent i can devote some or all of his resources $x_i \leq R_i$ as arms to fight. Investments are made simultaneously.

If at least one agent chooses to fight (or attack) then they fight. If they fight then the remaining resources are redistributed between the agents in proportion to their arms

$$\Pi_i(R, x, \text{war}) \;=\; \frac{x_i}{x_i + x_j}[1 - (x_i + x_j)]$$

If neither agent chooses to fight then each retains his remaining resources

$$\Pi_i(R, x, \text{peace}) \;=\; R_i - x_i$$

.

Each agent maximizes his payoff Π_i.

2.2. Solution

SPNE is the natural solution concept.

In the last stage, i will attack if

$$\Pi_i(R, x, \text{war}) \;>\; \Pi_i(R, x, \text{peace}) \quad \Rightarrow \frac{x_i}{R_i} > \frac{x_j}{R_j}$$

If i attacks, he will choose x_i to maximize

$$\max_{x_i} \Pi_i^{\text{war}} \quad \Rightarrow \quad x_i = \min\{\sqrt{x_j} - x_j, R_i\}$$

Similarly, to defend j will choose $x_j = \min\{\sqrt{x_i} - x_i, R_j\}$

Note that $\sqrt{x_j} - x_j$ reaches a maximum of $\frac{1}{4}$ when $x_j = \frac{1}{4}$, which is also a fixed point of $y = \sqrt{x} - x$.

First, note that in equilibrium each player invests positive amounts in arms.

- If $\frac{1}{4} \leq R_1 \leq R_2$, then each invests $x_i = \frac{1}{4}$ and gets payoff $\Pi_1 = \Pi_2 = \frac{1}{4}$.
- If $R_1 < \frac{1}{4} < R_2$ then investments are $x_1 = R_1$, $x_2 = \sqrt{R_1} - R_1$, and payoffs are $\Pi_1 = \sqrt{R_1}(1 - \sqrt{R_1})$, $\Pi_2 = (1 - \sqrt{R_1})^2$.

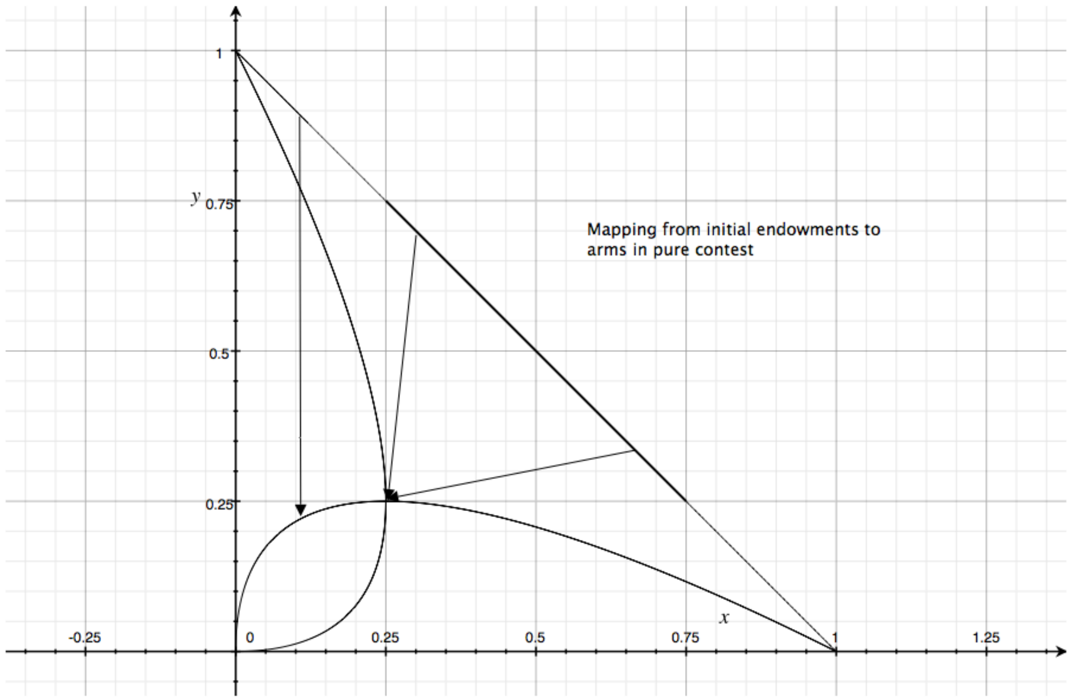

Figure 1: Optimal attack and defence in pure contest

- There is war except in the case $R_1 = R_2$.

We will denote the equilibrium outcomes of pure contest by the superscript C, i.e., $x_1^C, x_2^C, \Pi_1^C, \Pi_2^C$.

3. Investing in public defence

We augment this game by adding a first decision stage before the players choose their investments in arms.

- First, each player simultaneously chooses to invest an amount g_i to endow a public defender ("government").
- Next investments in private arms are chosen, and then attack decisions are made.
- If both attack, then the government stands aside. A pure contest occurs using only private arms to divide the remaining resources.
- If neither attacks then there is peace and each consumes his remaining resources.

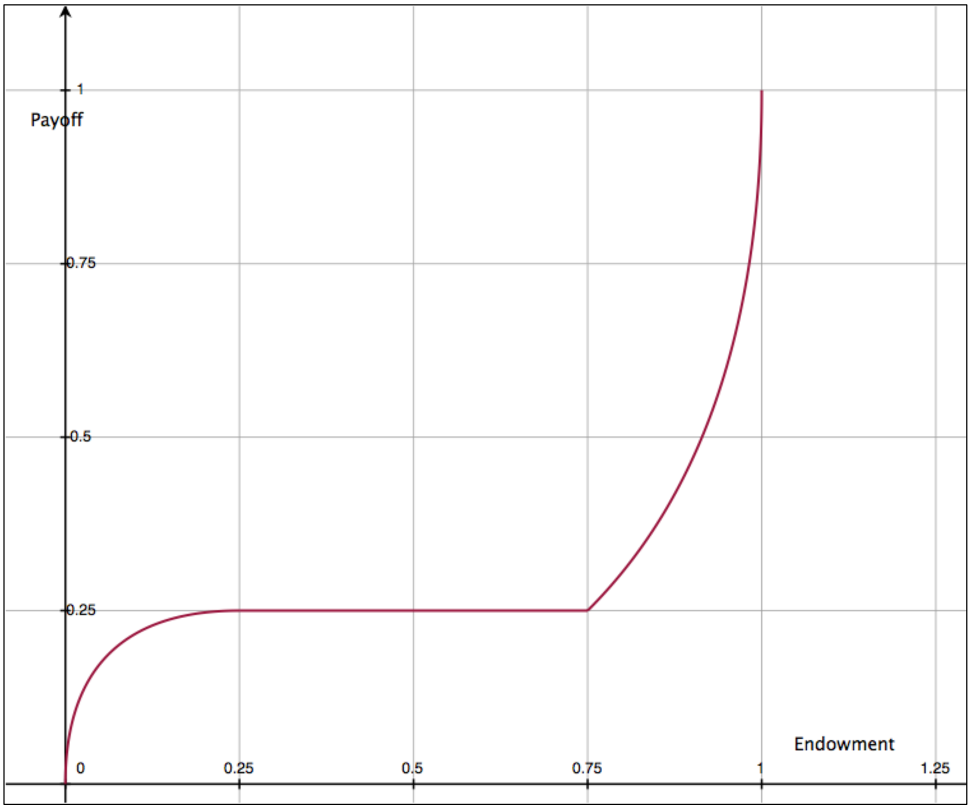

Figure 2: Contest payoffs plotted against endowment

- However, if agent i chooses to attack and agent j does not, then the government adds its resources to the defence of j.

3.1. The game

We start with $R_1 + R_2 = 1 \quad 0 < R_1 \leq R_2$.

- Stage 1 (game Γ): Agents simultaneously choose the amount g_i each will contribute to public defence, subject to $g_i \leq R_i$.
  - A pair (g_1, g_2) is a *contribution profile* (or *contribution*).
  - Let $g = g_1 + g_2$ and $\mathbf{g} = (g_1, g_2)$.
  - Define $w_i \ = \ R_i - g_i$, and $\mathbf{w} = (w_1, w_2)$.
- Stage 2 (subgame Γ_2): Agents observe $\mathbf{g}$ and simultaneously choose their arms investments $x_i \leq w_i$.
  - A pair (x_1, x_2) is an *arms profile* (or *arms*).
  - Let $x = x_1 + x_2$, and $\mathbf{x} = (x_1, x_2)$.

- Stage 3 (subgame Γ_3): Agents observe $\mathbf{x}$. Then they simultaneously choose $a_i \in \{0,1\}$. [0 is "defend", 1 is "attack".]
  - A pair (a_1, a_2) is an *attack profile*. Let $\mathbf{a} = (a_1, a_2)$.

We use $z = [\mathbf{g}, \mathbf{x}, \mathbf{a}]$ to denote the sequence of decisions in a play of the game.

3.2. Payoffs

- If $(a_1, a_2) = (0, 0)$, then
$$\Pi_i(z) = R_i - g_i - x_i, \quad i = 1, 2.$$
- If $(a_1, a_2) = (1, 1)$ then
$$\Pi_i(z) = \frac{x_i}{x_i + x_j}[1 - x - g]$$
- If $a_i = 1$ and $a_j = 0$, then
$$\begin{aligned} \Pi_i(z) &= \frac{x_i}{x_i + x_j + g}[1 - x - g] \\ \Pi_j(z) &= \frac{x_j + g}{x_i + x_j + g}[1 - x - g] \end{aligned}$$

3.3. Aggression and deterrence

We say that a player i is *aggressive* in stage 2 if, given g and $x_j = 0$, his payoff is higher when he invests optimally in arms and attacks than when he does not attack.

$$w_i < \begin{cases} (1 - \sqrt{g})^2 & \text{if } \ w_i \geq \frac{1}{4} \\ \dfrac{1}{2} - g & \text{if } \ w_i < \frac{1}{4} \end{cases} \tag{1}$$

To see this, note that given g and $x_j = 0$, if player i attackes his optimal choice of x_i is $\min\{\sqrt{g} - g, w_i\}$. This follows from the optimal choice of arms in pure contest discussed in Section 2. The resulting payoff to i conditional on w_i appears on the right-hand-side of (1). The player will prefer to attack if his wealth in stage 2 is less than his expected gain from attacking optimally.

A contribution profile $\mathbf{g}$ is a *full deterrent* if neither player is aggressive in the subgame following $\mathbf{g}$.

It follows that to ensure full deterrence it is sufficient to deter the player who has the smaller remaining resource endowment after contributions.

Lemma 1 *A contribution profile* $\mathbf{g}$ *is a full deterrent if* $g \geq \hat{g}(\mathbf{w})$*, where*

$$\hat{g}(\mathbf{w}) = \begin{cases} (1 - \sqrt{\min\{w_1, w_2\}})^2 & \textit{if } \ \min\{w_1, w_2\} \geq \frac{1}{4} \\ \dfrac{1}{2} - \min\{w_1, w_2\} & \textit{if } \ \min\{w_1, w_2\} < \frac{1}{4} \end{cases}$$

4. Deterrence equilibria

We look for subgame-perfect Nash equilibria of the game Γ. The following observations are self-evident:

Observation 1 *:*

(i) Let $\mathbf{g}$ *not be full deterrent. Then in the equilibrium of the subgame* Γ_2 *we must have* $\mathbf{x} \neq 0$ *and* $\mathbf{a} \neq 0$.

(ii) If z^* *is an equilibrium outcome with* $g^* > 0$, *then* $\mathbf{a}^* \neq (1,1)$.

(iii) If z^* *is an equilibrium outcome with* $a_i^* = 1$, *then* $g_i^* = 0$.

Proposition 1 *If* z^* *is an equilibrium outcome, then either (i)* $\mathbf{g}$ *is a full deterrent with* $\mathbf{x} = (0,0)$ *and* $\mathbf{a}^* = (0,0)$, *or (ii)* $\Pi(R, z^*) = \Pi^C(R, x^C, war)$.

Intuition: If $\mathbf{g}$ is full deterrent, then neither player has an incentive to arm and attack if the other does not. If $\mathbf{g}$ is not full deterrent, then one of the agents will attack in the subgame, hence he will not contribute. The other agent will at most contribute the resources he would spend in defence.

4.1. Minimal full deterrence investment

From Proposition 1 it follows that in equilibrium, agents will either together contribute enough to ensure full deterrence, or they will not invest in public defence at all. In the former case, we must have $g = \hat{g}(\mathbf{w})$, the minimum contribution required for full deterrence. Additional contribution is costly to the contributor and does not produce additional payoff.

By Lemma 1 the minimum full-deterrence contribution g is uniquely determined by the smaller of the two remaining resource endowments. Hence we can identify the vectors $\mathbf{w}$ that are compatible with full deterrence. Corresponding to Lemma 1 there are two cases:

Observation 2 *(i) Let* $\min\{w_1, w_2\} = w_1 \geq \frac{1}{4}$. *Then we must have* $g = \hat{g}(w_1) = (1 - \sqrt{w_1})^2$. *Hence* w_2 *must equal*

$$w_2 = 1 - [w_1 + (1 - \sqrt{w_1})^2] = 2(\sqrt{w_1} - w_1)$$

It can be checked that $w_2 \geq w_1$ *provided* $w_1 \leq \frac{4}{9}$. *When* $w_1 = \frac{4}{9}$, *we have* $w_1 = w_2$, *and* $\hat{g}(w_1) = \frac{1}{9}$.

(ii) If $w_1 < \frac{1}{4}$ *then* $g = \dfrac{1}{2} - w_1$ *and* $w_2 = \dfrac{1}{2}$.

This defines the full-deterrence frontier, summarized in the following proposition and mapped in Figure 3.

Proposition 2 *Consider the subgame* Γ_2 *with initial post-contribution allocation* (w_1, w_2) *and associated total public defence contribution* $1 - (w_1 + w_2)$. *If the following conditions hold, then the equilibrium in this subgame is* $(\mathbf{x}, \mathbf{a}) = (0,0)$, *i.e., peace with no expenditure on private arms. Wlog let* $w_1 = \min\{w_1, w_2\}$.

$$w_2 \leq \begin{cases} \frac{1}{2} & \text{if} \quad w_1 < \frac{1}{4} \\ 2(\sqrt{w_1} - w_1) & \text{if} \quad w_1 \in [\frac{1}{4}, \frac{4}{9}] \end{cases} \tag{2}$$

There are no peace equilibria in subgame Γ_2 *when* $\min\{w_1, w_2\} > \frac{4}{9}$.

Figure 3 shows the consumption pairs that are attainable with full deterrence. The line joining $(1,0)$ and $(0,1)$ plots the possible distributions of initial resources. We restrict attention to the section of this line lying above the 45-degree line, where $R_1 \leq R_2$. The analysis of the complementary segment is symmetrical.

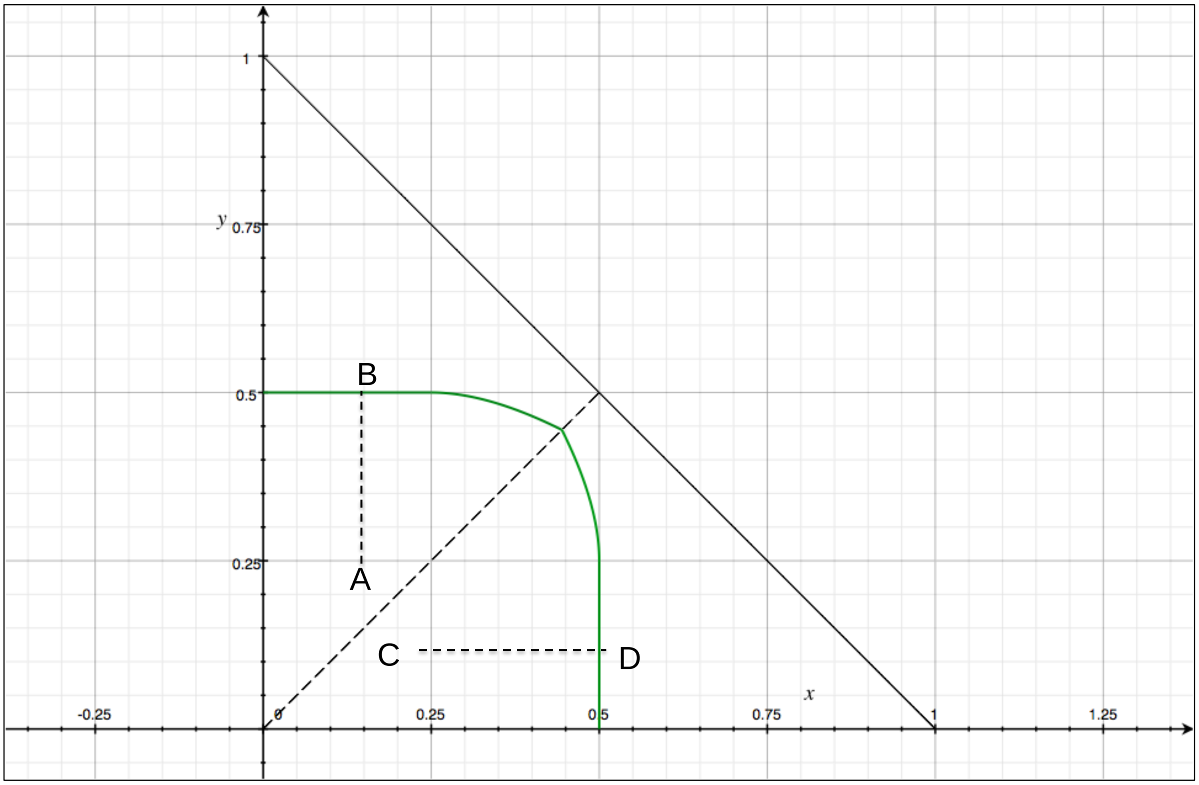

Figure 3: Payoff frontier with full deterrence

The curved frontier is the limit of the consumption pairs (w_1, w_2) that are consistent with full deterrence.[1] To see that allocation below the frontier also induce full-deterrence, note that in an allocation such as A, the public contribution is larger than in B, but $\min\{w_1, w_2\} = w_1$ is unchanged. Thus since B is compatible with full-deterrence so is A. A similar argument applies to C relative to D.

4.2. Deterrence contributions

Next we determine the vectors $\mathbf{g}$ that are candidates for contributions in equilibria with full deterrence. Note that this implies minimal full deterrence, i.e., $g = \hat{g}(\mathbf{w})$.

Proposition 3 *Let $Z(\mathbf{R})$ be the set of minimal full-deterrence outcomes corresponding to initial resource allocation $\mathbf{R}$. then the set attainable consumptions for Player i in outcome $z \in Z(\mathbf{R})$*

[1]Since full-deterrence implies $\mathbf{x} = 0$ in the subgame, w_i is indeed the consumption of i in the equilibrium of the subgame.

are:

$$w_i \begin{cases} = R_i & if \quad R_i \leq \frac{1}{4} \\ \in [\frac{1}{4}, R_i] & if \quad R_i \in (\frac{1}{4}, \frac{1}{2}] \\ \in [\frac{1}{2}\{1 + \sqrt{(2R_2 - 1)}\}, \frac{1}{2}] & if \quad R_i \in (\frac{1}{2}, \frac{5}{9}] \\ \in [2\{\sqrt{1 - R_2} - (1 - R_2)\}, \frac{1}{2}] & if \quad R_i \in (\frac{5}{9}, \frac{3}{4}] \\ = \frac{1}{2} & if \quad R_i > \frac{3}{4} \end{cases} \tag{3}$$

In figure 4 we plot the lower and upper bounds for the payoffs for player 2 that remain after contributions that are compatible with minimal full deterrence (with complementary contributions by player 1), corresponding to each endowment of resources. Note that the curvature of the full deterrence frontier in the range $R_1 \in (\frac{1}{4}, \frac{1}{2})$ implies that the contributions of the two players are imperfect substitutes; a reduction in g_2 must be compensated by a more than equal increase in g_1.

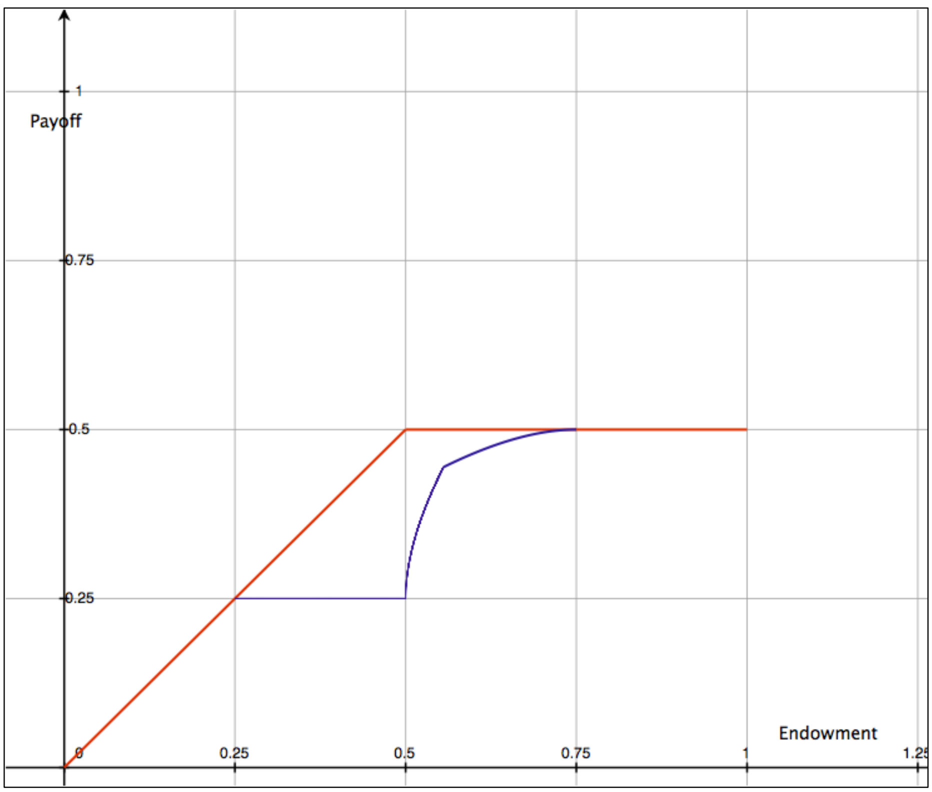

Figure 4: Maximum and minimum payoffs with full deterrence

5. Equilibria and efficiency

Proposition 3 describes the contribution profiles that are candidates for full detrrence equilibria. Observe that the richer player must always contribute to a full deterrence outcome. The poorer player may not contribute, and will indeed not contribute at all when his initial resource endowment is less than $\frac{1}{4}$. In order for a contribution profile that results in full deterrence to be an equilibrium outcome, it is necessary that each player that contributes has a payoff under full deterrence that is no less than the payoff he would obtain under pure contest.

Figure 5 superimposes the full deterrence payoffs on the pure contest payoffs for a given player. The pure contest payoffs are strictly greater than full deterrence payoffs for $R_i \in (0, \frac{1}{4})$, and in $R_i \in (\sqrt{2} - \dfrac{1}{2}, 1]$. In the lower range, player i cannot decide on full deterrence, only the richer player contributes. But in the upper range, it is the richer player that makes the entire contribution, hence the choice between conflict and deterrence is his to make. It follows that if $\max\{R_1, R_2\} \in (\sqrt{2} - \dfrac{1}{2}, 1]$, then the richer player will not invest in deterrence, and the equilibrium outcome will be pure conflict. For $\max\{R_1, R_2\} \in (\dfrac{1}{2}, \sqrt{2} - \dfrac{1}{2}]$, on the other hand, deterrence is weakly preferred if the richer player makes the maximum contribution, and strictly preferred if the poorer player makes any contribution at all, hence full deterrence is the equilibrium outcome.

This estalbishes the equilibria corresponding to the different resource endowments, summarised in the following proposition.

Proposition 4 *If $R_1, R_2 \in [\frac{3}{2} - \sqrt{2}, \sqrt{2} - \dfrac{1}{2}]$ then all equilibria are full-deterrence. If initial endowments are outside these limits then in the equilibrium outcome there is war, and payoffs are equal to the pure contest payoffs for those endowments.*

Each equilibrium is pareto-optimal, since under minimal full deterrence the contributions of the two players are (imperfect) substitutes for each other. However, for a given initial distribution of resources, the total consumption in the economy in an equilibrum differs with the allocation of contributions between the two players. A possible measure of aggregate efficiency is total consumption in the economy:

$$c = 1 - g - x.$$

We can compute c in the pure conflict outcome corresponding to each distribution of resources. In full deterrence equilibria $x = 0$, so $c = 1 - g$, hence the most efficient equilibrium is the one that minimizes g. But since $g = \hat{g}(\min\{w_1, w_2\}$, this is equivalen to maximizing $\min\{w_1, w_2\}$. This can be restated as:

Proposition 5 *For resource distributions that accommodate multiple full deterrence equilibria, the Rawlsian criterion provides the most efficient allocation of public defense contributions.*

Another interpretation is that, for these distributions, efficiency requires the richer agent to make the maximum contribution consistent with full deterrence. If contributions were allocated as taxes by a public authority, then Proposition 5 leads to the following:

Corollary 2 *Suppose that when full deterrence is mutually incentive compatible, a public authority raises public defense contributions through taxes. Then the most efficient taxation scheme is one that is most progressive subject to incentive-compatibility.*

Specifically, the efficient contribution profiles are given by:

- For $\frac{4}{9} < \ R_1, R_2 \ < \frac{5}{9}$ the efficient equilibrium outcome is $w_1 = w_2 = \frac{4}{9}, \ g = \frac{1}{9}$.

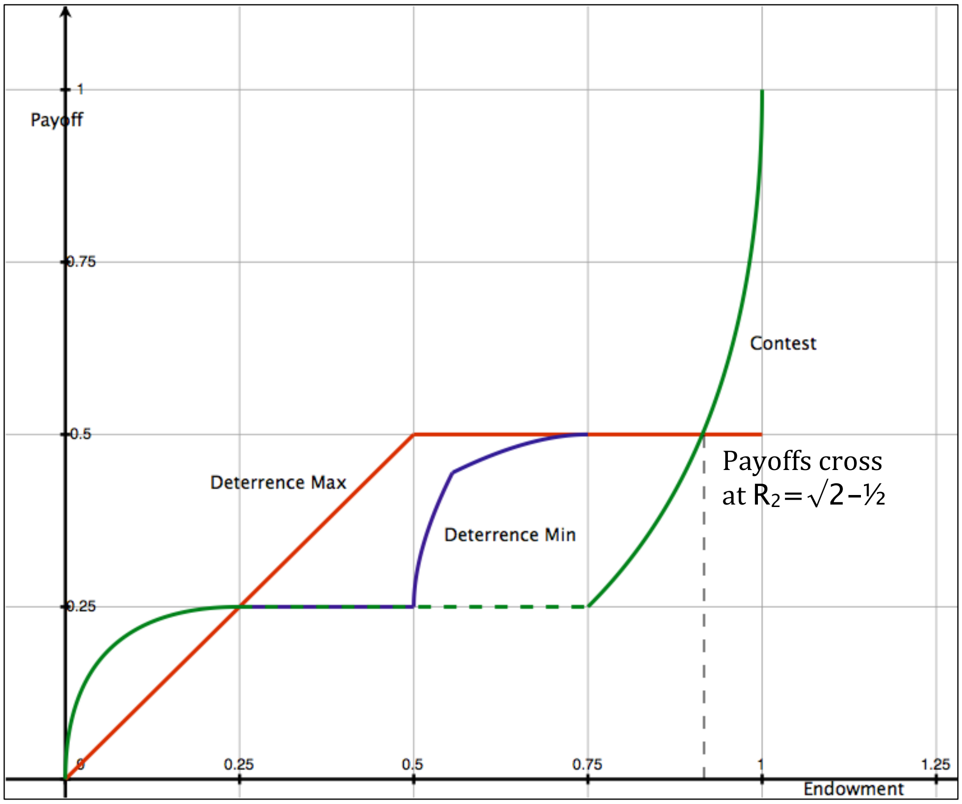

Figure 5: Comparison of payoffs under pure contest and full deterrence

- For $\frac{5}{9} \leq R_2 \leq \frac{3}{4}$ the efficient equilibrium outcome is $w_1 = R_1,\ g = (1-\sqrt{w_1})^2,\ w_2 = R_2 - g$.
- For $\frac{3}{4} \leq R_2 \leq \sqrt{2} - \dfrac{1}{2}$, the unique equilibrium outcome is $w_1 = R_1,\ g = \dfrac{1}{2} - R_1,\ w_2 = \dfrac{1}{2}$.

Finally we note that full deterrence is not efficient over the entire range in which it is an equilibrium. there is a range to the left of $R_2 = \sqrt{2} - \dfrac{1}{2}$ where the pure contest outcome is more efficient than the equilibrium outcome, but the equilibrium is full deterrence. This is because in this range the richer player unilaterally pays for deterrence, and for him the deterrence payoff is larger than the conflict payoff.

Proposition 6 *In the range $R_1 \in (\frac{3}{2} - \sqrt{2}, 1 - \frac{\sqrt{3}}{2})$, the equilibrium is full deterrence where conflict would yield a more efficient outcome.*

The proof follows by comparing the sum of the equilibrium consumptions with those that would obtain under pure conflict, as can be found in Section 2.

Thus when income distributions are very unequal (but not sufficinetly unequal for public defense to become non-viable) the equilibrium outcome is deterrence through public defense, but this is inefficient. In other words, for resource distributions in this range, the government is too large.

6. Conclusion

To be written.

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A. Proofs

Proof of Proposition 3.

We focus on the case $R_1 \leq R_2$. The analysis for the complementary case is symmetrical.

First suppose that $R_1 < \frac{1}{4}$. Then the post-contribution allocation must have $\min\{w_1, w_2\} < \frac{1}{4}$, and hence full deterrence requires $g = \frac{1}{2} - \min\{w_1, w_2\}$. Hence contributions by i s.t. $w_i = \min\{w_1, w_2\}$ do not alter the contribution required by $j \neq i$, so the only incentive compatible contribution from i is $g_i = 0$, and j must contribute $g_j = R_j - \frac{1}{2}$. It follows that when $R_1 < \frac{1}{4}$, the only contribution profile that is a candidate for equilibrium is $(g_1, g_2) = (0, R_2 - \frac{1}{2})$, which yields the consumption profile $(R_1, \frac{1}{2})$.

Next consider $\frac{1}{4} \leq R_1 \leq \frac{1}{2} \leq R_2$. For each R_1 In this range there are multiple configurations $\mathbf{g}$ that are consistent with minimal full deterrence. Recall that in this case the equilibrium conflict payoff for each player is $\frac{1}{4}$, which is the upper bound on the payoff that either player can attain in subgame Γ_2 if public contributions in stage 1 do not attain full deterrence. It therefore follows that the maximum contribution i is willing to make is $g_i \leq (R_i - \frac{1}{4})$.

First consider $R_1 \in [\frac{1}{4}, \frac{4}{9}] \;\; \Rightarrow \;\; R_2 \in [\frac{5}{9}, \frac{1}{4}]$. We know from Observation 2 that player 2 can unilaterally ensure full deterrence by contributing $(1 - \sqrt{R_1})^2$, which leaves him with consumption $w_2 = 2(\sqrt{R_1} - R_1) \geq w_1$. Since 1 does not contribute, $w_1 = R_1 = 1 - R_2$, the resultant consumption vector is $(R_1, 2[\sqrt{1 - R_2} - (1 - R_2)])$.

For $R_1 \in (\frac{4}{9}, \frac{1}{2}]$, if player 2 contributes sufficiently to deter player 1, this leaves him with $w_2 < w_1$. Hence to ensure full deterrence with no contribution from player 1, he must deter himself. This implies $g_2 = (1 - \sqrt{w_2})^2$. Since $g_2 + w_2 = R_2$, This leaves player 2 a consumption of $[\frac{1}{2}\{1 + \sqrt{(2R_2 - 1)}\}]^2$, which ranges from $w_2 = \frac{4}{9}$ when $R_2 = \frac{5}{9}$ to $w_2 = \frac{1}{4}$ when $R_2 = \frac{1}{2}$.

The payoffs for the complementary range can be found symmetrically. ■