Competing Lending Platforms, Endogenous Reputation, and Fragility in Microcredit Markets

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Abstract

This paper shows that market fragility and mass default can arise in microcredit markets as a result of the strategic interaction between a microlender using a reputation-based mechanism and a traditional lender using physical collateral. In our model, borrowers solve a dynamic programming problem which induces an endogenous equilibrium distribution of reputational capital. Because the quality of each lender’s pool of borrowers is affected by both lenders’ interest rates, lender reaction curves are non-monotonic and discontinuous. This can result in knife edge equilibria and mass default on the microlender precipitated by minor parametric perturbations. Fragility is exacerbated by borrower screening and sovereign risk, but ameliorated when microlenders have social welfare goals. Our results highlight the importance of studying the entire credit market rather than microfinance in isolation.

Key Words: competing platforms, reputation, dynamic incentives, strategic default, microcredit, microfinance crisis.

JEL Codes: D82, L22, G21, O16
In this paper we seek to understand markets in which microcredit institutions using reputational mechanisms interact with local moneylenders who use traditional collateralised debt contracts. Over the past decade the performance of microcredit institutions has come under increasing scrutiny, particularly in the light of high profile market crises in a number of jurisdictions. We find that there may be intrinsic market forces that can contribute to such instability.

In our context, reputational mechanisms constitute a commitment to provide future loans if and only if the borrower or group of borrowers has an immaculate history of repayment - these contracts are in widespread use by microfinance institutions (Armendariz de Aghion, 1999). While microlenders do not generally use collateral nor other threats of punishment, they compete in the credit market with informal lenders who do have the local knowledge and ability to use these tools (Berg et al 2013, Sarap 1991). The consequences of strategic interaction between these competing lending platforms has not been studied. We model this interaction in an environment with both adverse selection and moral hazard, in a model where the borrower’s reputation is endogenous. Our key finding is that this interaction can create market fragility, characterised by non-monotic and discontinuous lender reaction curves and the potential for knife-edge equilibria.

The importance of understanding the credit markets used by the very poorest households in developing countries can hardly be overstated. Credit constraints are theorised to be a major barrier to growth for poor families, who do not have access to the range of formal financial tools enjoyed by those in the developed world (Collins et al, 2009). Microcredit was once the great hope of the fight against global poverty exactly because it proposed to fill this gap, but evidence from randomised controlled trials suggest it does not generally revolutionise the lives of the poor (Banerjee et al 2015, Crepon et al 2015, Attannasio et al 2015). The problem appears to be much more complex than a simple lack of supply, and despite the attention given to it in the literature, much remains to be understood about microcredit itself and its role in these markets (Banerjee 2013).
Moreover, there is increasing pressure on regulators to restrain the microfinance sector in the wake of high-profile crises in Bolivia in 2001, Nicaragua, Bosnia and Morocco in 2009, and the Indian state of Andhra Pradesh in 2010 (for more information see Rhyne 2001, Rhyne and Otero 2007, Bateman et al 2012 or Roodman 2012). But making policy while barely understanding these markets and the source of this apparent fragility is a risky endeavour. Our key insight is that the interaction of the formal and informal sectors appears to be important in understanding market instability. This interaction is difficult to study empirically as these informal institutions are often reluctant to be surveyed by researchers (Sarap, 1991).

In this paper, we examine this instability through the lens of competing lending platforms. When microfinance lenders enter a market, they do not exist in a vacuum. They are introduced into economies where there are already existing formal or informal lenders, with which they interact. In particular, they almost always face an incumbent informal moneylender (Collins et al 2009; Armendariz de Aghion and Morduch 2010; Berg, Emran and Shilpi 2013). These different lending institutions can be viewed as competing platforms with different intrinsic characteristics. While microcredit organizations use a variety of lending technologies, the fundamental characteristic on which we focus is that they are outside institutions with limited local knowledge. Because they interact with borrowers only through the credit market, they see only the history of their own transactions with the borrower. Hence, they can commit only to reputational mechanisms, in which future lending depends only on whether previous loans have been repaid. In contrast the traditional lender has a richer information set and a variety of enforcement mechanisms, including seizing collateral and imposing direct punishment on defaulters. However, they do not use reputational mechanisms to punish default, and they will lend to anyone who has the requisite collateral.¹

¹We assume, in the model as specified below, that the direct sanctions available to the traditional lender are already sufficient to deter any voluntary default against this lender. Weakening this assumption would complicate the model, but would not change the main conclusions.
The performance of any financial institution depends on the quality and behavior of its pool of borrowers. When there are competing platforms, borrowers will act strategically when deciding whether to default or to invest in their reputation, and the equilibrium distribution of reputation within the population is endogenous, as is the equilibrium quality of the borrowing pool facing each lender. Changing the interest rate charged by a lender has a direct effect on the behavior and profits earned from existing borrowers, but it also has an indirect effect on the quality of the pool of borrowers. We show that due to this effect reaction curves will generically be discontinuous and non-monotonic, and in some circumstances there may not exist a pure strategy Nash equilibrium. When no pure strategy Nash equilibrium exists, mixed strategy Nash equilibria and Stackelberg equilibria can generate heterogeneous outcomes in identical markets, including mass defaults on the reputational lender. We show that this mechanism can generate outcomes consistent with the stylized facts that we have outlined above.

The potential for strategic interaction between lending platforms has not been widely studied. Foundational papers have tended to focus on dynamic reputation mechanisms in themselves, and in isolation, most notably Stiglitz and Weiss 1983 and Bond and Krishnamurthy 2004. Many of these papers embed reputational lending in a context of sovereign debt (Bulow and Rogoff 1989 and Kehoe and Levine 1993). In these contexts dynamic incentive mechanisms are generally unable to support a sustainable credit market. However these papers typically assume either access to perfect savings and insurance markets, or a complete set of contingent markets, assumptions far from the credit market experiences of the global poor. There is a literature on optimal dynamic contracting, with or without group lending, but this does not address competition between platforms (Ahlin and Waters 2012, Bhole and Ogden 2009, Tedeschi 2006). There is also a literature that has focussed on a lender or lenders who are all equipped with both a reputational mechanism and a physical collateral mechanism, for example, Ghosh and Ray 2001, Bennardo, Pagano and Piccolo 2008 and Ferreira and Torres-Martinez 2009. But neither does this literature address competition between platforms. These frameworks
are very different to a world in which the two mechanisms are pitted against each other, which is what we study, and which is closer to the situation actually faced by microcredit providers. The only paper which considers the case of competing platforms as we do is McIntosh and Wydick 2005, but their model is static and reputation plays no role; furthermore, their reputational lender is a charity who maximizes social surplus. While this is true of some microcredit firms, profit maximizing microlenders are increasingly prevalent (Banerjee and Duflo, 2011).

The paper is structured as follows. In section 1 we describe the competing platforms framework that we will use, and in Section 2 we set out the formal model. The main result on the generic discontinuity and non-monotonicity of reaction curves is proved in Section 2.2.1 and Appendix A, and illustrated with two examples in Section 2.2.2. The implications for equilibrium and market stability are discussed in Section 2.3. In Sections 3.1, 3.2, and 3.3 we discuss three policy-related extensions to the basic model, exploring the implications of switching to a socially-motivated MFI, introducing borrower screening technologies, and accounting for sovereign risk.

1 Competing Platforms

We study a model with two competing lenders. One lender, whom we call the traditional lender, lends against physical collateral which may be seized to punish default. We interpret this punishment quite broadly: what matters is that the lender can take an action that harms the borrower, and this creates an incentive to repay. We do not, however, allow this lender to use reputational mechanisms: they cannot commit not to lend to anyone who has the requisite collateral. We will sometimes refer to this lender as the informal lender or the moneylender. The other lender, whom we call the reputational lender or MFI, cannot interact with borrowers except through the credit market. The only sanction that this lender can impose is not to lend again in the future. We have in mind an outside institution with only limited local knowledge and access.
The two lending institutions are in all other respects identical. In particular, they have the same cost of funds. In reality, the difference between lenders is not so starkly different. But it is useful to simplify the model in this way in order to clarify the nature of the interaction between reputational microcredit lenders and traditional markets.

We present a deliberately parsimonious model which focuses only on the most crucial components of the situation we study. Nevertheless, the model reflects important realities in rural credit markets in developing countries. Microlenders who rely on a combination of reputation and social lending mechanisms almost always compete with an incumbent local moneylender (Collins et al, 2009). Moneylenders usually demand physical assets as collateral, and sometimes incorporate the threat of property or personal damage in the case of default (Collins et al, 2009; Sarap, 1991). While microfinance institutions make use of a broad range of lending strategies including group lending, dynamic exclusionary threats are integral to these other strategies (Armendariz De Aghion, 1999). Hence, reputational contracting is fundamental to virtually all forms of microfinance lending, although the unit of reputation may be a group as well as an individual. We follow papers such as Bond and Krishnamurthy 2003, Tedeschi 2006, and Bhole and Ogden 2009 in omitting asset accumulation and savings from our model. Although the poor do save, the amount they save is extremely small compared to the amount they can borrow (Banerjee and Duflo, 2011). Furthermore, even when microcredit becomes very easy to access, it does not seem to permit income growth sufficient to escape the credit market (Banerjee, 2013).

We have overlapping generations of borrowers and long-lived institutions. Borrowers are born at a steady rate and die stochastically with a constant hazard rate. They are endowed at birth with a good reputation and a type $\theta$, which is the probability that their investment projects will succeed. Our market has both adverse selection and moral hazard: the type $\theta$ is private information, and the outcome of a project is also private. Hence, the borrower may hide at least some of their assets and strategically default if they so wish, even if they could repay. If a borrower defaults on the traditional lender
then they incur a punishment. If they default on the reputational lender then they incur the loss of their good reputation, and thus the loss of future borrowing opportunities from this lender. Borrowers face a dynamic problem in managing their reputation, and their behavior is determined by solving a dynamic programming problem. Their strategies determine the transition probabilities of a reputational Markov process and a stationary distribution of borrower reputations: this determines how many borrowers of type $\theta$ have in equilibrium a good reputation, and how many a bad reputation.

At the start of the game, the lenders each set an interest rate once and for all, taking into account the equilibrium distribution of strategies and reputations in the borrower population implied by these interest rates. Their objective is to maximize the expected present value of the stream of per-period profits, and we assume they have a temporal discount factor sufficiently close to 1, such that their optimal strategy is to maximize the per-period profit under the stationary distribution of types induced by their choice of interest rates.

We note that by restricting the strategy space to a single variable (the interest rate) we implicitly assume that the informal lender does not engage in Bayesian updating and individualistic price discrimination but sets a common interest rate for all, and that the reputational lender uses only a binary reputational variable, rather than a multi-step reputation and multidimensional pricing. We make these assumptions in the interest of realism, and in order not to overcomplicate the model. In fact MFIs do tend to use a binary or categorical reputation mechanism: instead of individual Bayesian updating or credit score indices, borrowers are blacklisted forever if they default (Armendariz de Aghion and Morduch, 2010). This may be due to real-world constraints and the potentially high costs of implementing any more detailed reputational mechanisms.
2 The Model

Time is discrete with an infinite horizon and indexed by \( t = 0, 1, 2, \ldots \). Borrowers are risk-neutral profit maximizers, each endowed with an individual type \( \theta \) drawn from a distribution \( f(\theta) \) with full support \([0, 1]\). In each period, the borrowers choose whether to borrow in order to undertake a project which matures in that period.\(^2\) The size of the project is normalized to 1, and we assume that agents must borrow this amount in order to invest. With probability \( \theta \) the project succeeds, yielding a gross return of \( m > 1 \) so that at least some borrowers should optimally invest. With probability \( 1 - \theta \) it fails, yielding 0. The borrower’s type \( \theta \) and whether the project succeeds or fails are private information, so they can choose to default strategically. All borrowers face a constant hazard rate, surviving with probability \( w \) to the end of the period; this induces a time preference (we could add an additional discount factor \( \beta \), but that would simply scale \( w \) down). To normalize the population size to 1, we set the total mass of new borrowers born every period to be \( 1 - w \).

There are two infinitely lived, risk neutral, profit maximizing lenders endowed with different contracting technologies. The reputation based lender sets a gross interest rate \( R_R \) (that is, at term she demands total repayment of \( R_R \)) and she faces a gross lending cost \( c_R \) which could include administrative costs, fixed operating costs and the cost of funds. Borrowers are born with a good reputation \( G \), and they are demoted to the bad reputation state \( B \) if they default on the reputational lender, in which case they can never borrow from her again. We may without loss of generality restrict the strategy space to \( R_R \in [0, m] \) since otherwise no borrower enters except to default, and the market collapses.

The traditional lender sets gross interest rate \( R_T \in [0, m] \) and he faces gross lending cost \( c_T \). He does not have the commitment ability to blacklist defaulters\(^3\), but can inflict a penalty \( p \in [0, \infty) \) on them by seizing their assets

\(^2\)For simplicity we refer to these agents as borrowers, rather than as potential borrowers, even though some of them may choose not to take out a loan in some periods.

\(^3\)He cannot distinguish between bad luck (the project fails) and strategic default; in
or doing physical damage. He recoups value from this equal to a fraction 
\( x \in [0, 1] \) of the penalty, either because he must liquidate the assets or because 
he implements a combination of violence and asset seizure. Without loss of 
generality we restrict the strategy space to \( R_T \in [0, \min[p, m]] \) since otherwise 
no borrower enters except to default, and the market collapses.

In the baseline version of the model, at the start of \( t = 0 \) both lenders once 
and for all set their interest rates, which are fixed for all future periods. We 
will later consider both simultaneous and sequential commitment to interest 
rates. In every period borrowers decide whether or not to take a loan: borrowers 
in state \( G \) can choose either lender, but borrowers in state \( B \) and can only 
transact with the traditional lender. At the end of the period the surviving 
borrowers see their loans mature. Borrowers whose projects fail have no choice 
but to default, losing their good reputation if they borrowed from the reputa-
tional lender or incurring penalty \( p \) if they borrowed from the traditional 
lender. Borrowers with successful projects can decide whether to be honest 
and repay their debt, or be dishonest and strategically default. All surviving 
agents then consume their income and proceed to the next period.

2.1 Borrowers

Borrowers solve a discrete choice dynamic programming problem with an infini-
tate horizon. We consider only stationary Markov strategies, so choices depend 
only on the interest rates \( R_R \) and \( R_T \), the borrower’s type \( \theta \), and their reputa-
tion \( G \) or \( B \). In each period the borrower decides whether to stay out or to 
borrow, and from which lender to borrow (the bad reputation borrower has 
no choice of lender). If they borrow then Nature chooses whether or not their 
project succeeds, which will occur with probability \( \theta \). If the project succeeds 
then the borrower decides whether to repay or to default strategically. If the 
project fails then there is no choice and the borrower is forced to default. If 
the borrower stays out, or if the project fails, then the payoff is zero. If the 
equilibrium we will find that no lender ever defaults strategically from the traditional lender).
Figure 1: Participation (dashed) and Switching (solid) Curves.

project succeeds then the payoff is either $m$ or $m - R$, depending on whether the borrower defaults or repays, where $R$ is the gross interest rate. The borrower’s reputation is unchanged unless they default on the reputational lender, in which case it changes from $G$ to $B$.

It is easy to check that a borrower with reputation $G$ will never borrow from the traditional lender (this strategy is dominated by defaulting on the reputational borrower), and that no borrower will strategically default on the traditional lender (since $p > R_T$). We can thus write the Bellman equations describing the borrower’s choice:

$$V_G = \max[w(\theta m + V_B), w(\theta(m - R_R) + V_G) + (1 - \theta)V_B]$$
$$V_B = \max[wV_B, w(\theta(m - R_T) - (1 - \theta)p + V_B)].$$

The first equation describes the good reputation borrower’s decision between honesty and dishonesty. The second describes the bad reputation borrower’s participation decision; they will always play honestly if they participate. In
fact, these equations can be solved explicitly, yielding:

\[ V_G = \max \left[ w(\theta m + V_B), \frac{w(\theta(m - R_R) + (1 - \theta)V_B)}{1 - w} \right] \quad (2.1) \]

\[ V_B = \max \left[ 0, \frac{w(\theta(m - R_T) - (1 - \theta)p)}{1 - w} \right]. \quad (2.2) \]

To see this consider, for example, the equation defining \( V_G \). The equilibrium value of \( V_G \) occurs where the 45\(^\circ\) line intercepts the envelope of the functions \( w(\theta m + V_B) \) and \( w(\theta(m - R_R + V_G) + (1 - \theta)V_B) \), considered as functions of \( V_G \). But these functions have slope less than 1. So this must be the maximum of the points where the 45\(^\circ\) line intercepts the individual functions.

Using the Bellman equations, we can segment the type space and characterize borrower strategies.

**Lemma 1 (bad reputation: participation)** Let \( \theta^* = \frac{p}{m+p-R_T} \). An agent of type \( \theta \) with reputation \( B \) will participate if and only if \( \theta \geq \theta^* \). They can borrow only from the traditional lender and they will never strategically default (that is, they will default only if their project fails).

**Proof.** This is just a restatement of the condition \( V_B = 0 \) from equation 2.2.

**Lemma 2 (good reputation: strategic default)** Let \( \theta_L = R_R/wm \) and \( \theta_H = (pw - R_R)/(w(p - R_T)) \). An agent of type \( \theta \) with reputation \( G \) will always participate and they will borrow only from the reputational lender. They will strategically default if \( \theta \leq \theta_L \) or \( \theta \geq \theta_H \), but will otherwise repay if they can.

**Proof.** This is just a restatement of the condition \( V_B = V_G \) from equation 2.1.

One can check that \( \theta^* \) is always between \( \theta_L \) and \( \theta_H \). These results are illustrated in Figure 1. The participation curve \( V_B \geq 0 \) is shown dashed,
intersecting the axis at \( \theta^* \). Types with reputation \( B \) to the right of \( \theta^* \) will participate; those to the left will not. The switching condition, from equation 2.1 can be written \( \frac{m w \theta - R_R}{1 - w} \geq V_B \); the locus of this switching value (the switching curve) is shown solid. The switching curve crosses the participation curve at \( \theta_L \) and \( \theta_H \). Types \( \theta \) to the left of \( \theta_L \) will strategically default, as will types \( \theta \) to the right of \( \theta_H \).

The intuition for the behavior of low types with \( \theta < \theta_L \) is straightforward. If their project succeeds today, they should not bother investing in a good reputation: their probability of project success next period is very low, so it is not worth giving up \( R_R \) today to try their luck investing again. These types will borrow once from the reputational lender, default and then stay out of both markets. The intuition for the behavior of types with \( \theta > \theta_H \) is more subtle. The logic becomes clear after rewriting the strategic default condition for types \( \theta > \theta^* \) as follows:

\[
R_R < w (\theta R_T + (1 - \theta)p) .
\]

A borrower repays this loan if the cost of repayment now is less than the cost of default, which is the discounted cost of repayment in the traditional market in the next period.\(^4\) Since \( p > R_T \), the right hand side is increasing in \( \theta \). Furthermore, the second term becomes negligible as \( \theta \to 1 \). For high types the risk of their project failing, and their having to incur the penalty \( p \) vanishes. Since they will never have to default and face the penalty \( p \), the highest types will look only at the relative interest rates. So if the discounted interest rate offered by the traditional lender is attractive the highest types will choose to default even though lower types, who worry about the penalty \( p \), will not choose to do so.

\(^4\)Note that \( \theta \geq \theta_H \geq \theta^* \), so this borrower knows that they will participate in the informal market next period.
2.1.1 Reputation

Borrower strategies induce a Markov process over reputation states from which we can derive the stationary distribution of borrowers in $G$ and $B$ (see Figure 2).

Let us define the probability that a borrower in state $G$ defaults and transitions to state $B$ conditional on surviving the period as $\gamma$ where

$$\gamma(\theta, R_R, R_T) = \begin{cases} 
1 & \text{for } \theta \in [0, \theta_L], [\theta_H, 1] \\
1 - \theta & \text{for } \theta \in [\theta_L, \theta_H]
\end{cases}$$

Thus, taking into account the distribution from which all borrowers are drawn $f(\theta)$ and survival probability $w$, the stationary distribution of borrowers in state $G$ is

$$f_G(\theta) = \frac{(1 - w)f(\theta)}{1 - w + \gamma(\theta, R_R, R_T)w}. \quad (2.3)$$

The stationary distribution of borrowers in state $B$ is

$$f_B(\theta) = \frac{f_G(\theta)\gamma(\theta, R_R, R_T)w}{1 - w}. \quad (2.4)$$
We will see that the distribution of types conditional on reputation can depart substantially from the underlying distribution of types $f(\theta)$. The effect is relatively complex because $\gamma(\theta, R_R, R_T)$ is both discontinuous and non-monotonic. Examples of the stationary reputation distributions are presented in Figures 4, 5 and 6. These examples will be discussed in context below.

2.2 Lenders

We now have a reasonably complete picture of the behavior of borrowers. Borrowing strategies, conditional on type and reputation, are set out in Lemmas 1 and 2. The equilibrium distribution of types, conditional on reputation, is set out in Equations 2.4 and 2.3. Together these determine the quality of the pool of borrowers facing each lender, and the profit that they will make, as a function of the interest rates.

Given the borrowers’ strategies, and taking into account the stationary distribution of reputations that this induces, each lender sets an interest rate $R_T$ or $R_R$. The lenders set rates only once. They maximize the infinite sum of per-period expected profits, and we assume they have a temporal discount factor sufficiently close to 1, such that their optimal strategy is to maximize the per-period profit under the stationary distribution of types induced by their choice of interest rates. This allows us to ignore the transition path to the stationary equilibrium.

The traditional lender faces a market of borrowers with stationary distribution $f_B(\theta, R_R, R_T)$, where every type $\theta > \theta^*(R_T, R_R)$ takes a loan. Each loan costs the lender $c_T$ and brings expected revenue $\pi_T(\theta, R_R, R_T)$, where

$$\pi_T(\theta, R_R, R_T) = \begin{cases} 
wxp & \text{if } R_T > p \\
\theta R_T + w(1 - \theta)p & \text{for } R_T < p 
\end{cases}$$

The total profit for this lender is therefore

$$\Pi_T(R_R, R_T) = \int_{\theta^*(R_T, R_R)}^{1} (\pi_T(\theta, R_R, R_T) - c_T) f_B(\theta, R_R, R_T) \, d\theta$$
Figure 3: Profit Functions (Type Distribution $U[0, 1]$)
The best response function of the traditional lender is determined by choosing \( R_T \) to maximize this expression given \( R_R \).

The reputational lender faces a market of borrowers with stationary distribution \( f_G(\theta; R_R, R_T) \), where every type takes a loan. Each loan costs the lender \( c_R \) and brings expected revenue \( \pi_R(\theta; R_R, R_T) \), where

\[
\pi_R(\theta; R_R, R_T) = \begin{cases} 
0 & \text{for } \theta \in [0, \theta_L], [\theta_H, 1] \\
w \theta R_T & \text{for } \theta \in [\theta_L, \theta_H]
\end{cases}
\]

The total profit for this lender is therefore

\[
\Pi_R(R_R, R_T) = \int_0^1 (\pi_R(\theta; R_R, R_T) - c_R) f_G(\theta; R_R, R_T) d\theta
\]

The best response function of the reputational lender is determined by choosing \( R_R \) to maximize this expression given \( R_T \).

The shape of these profit functions, for the case that the underlying distribution of types is the standard uniform distribution on \([0, 1]\), is illustrated in Figure 3. The shape of the reputational lender’s profit function is relatively straightforward; that of the traditional lender more complex. We will look at this example in more detail below.

### 2.2.1 Best Response: The Main Result

The strategic interaction between the two lenders in this model is complex. Varying the interest rate has both a direct effect of changing the behavior of individual borrowers and an indirect effect of changing the quality of the pool of borrowers. These effects tend to act in the opposite directions, so the overall effect is complicated. Simulations show that reaction curves can be discontinuous and non-monotonic. Before illustrating what can occur, and discussing the implications for equilibrium, we establish that this is in fact a general result.

In order to state the result, we introduce some terminology. We will say
that the reputational lender covers the top of the market if $\theta_L \geq 1$. In this case there will be no strategic default at the top of the market. There may be an interval of types $\theta \in [0, \theta_L]$ at the bottom of the market who will borrow once from the reputational lender and then default, but other borrowers will never voluntarily default. They will continue to repay and protect their good reputation until they are forced to default involuntarily because their project has failed, when they will fall into the $B$ population. If the reputational lender covers the top of the market then the borrowers facing the traditional lender are distributed on the interval $[\theta^*, 1]$. None of these borrowers has chosen to default strategically, since $\theta_L < \theta^*$. That is to say, any borrowers who have chosen to default on the reputational lender will choose not to participate any further in the market. The residual market facing the traditional borrower is thus distributed on the interval $[\theta^*, 1]$ with density $f_B(\theta) = \frac{w^{(1-\theta)}}{1-\theta w} f(\theta)$.

Since neither $\theta^*$ nor $f_B(\theta)$ depends on $R_R$, the $G$ market and the $B$ market are then in effect strategically uncoupled. Conditional only on the reputational lender covering the top of the market, marginal changes in $R_T$ will not change the composition or behavior of borrowers in the $G$ market, and marginal changes in $R_R$ will not change the composition or behavior of borrowers in the $B$ market. If the markets are strategically uncoupled in this way then the traditional lender is a monopolist in the residual market of borrowers whose project has failed at least once. We let $R_T^m$ be the monopoly interest rate set by the traditional lender in this residual market. The condition that the traditional lender make non-negative profit in this market is that

$$c_T < wR_T^m.$$  

We will say, in this case, that the traditional lender is viable as a residual monopolist when the reputational lender covers the top of the market.

We note that $R_T^m$ is the equilibrium interest rate set by the traditional lender in the Stackelberg equilibrium that we will discuss below. The condition $c_T < wR_T^m$ can be interpreted in that context as saying that the traditional lender prefers to accommodate the reputational lender’s leadership in that
equilibrium rather than to exit. We have not pursued any investigation of the case where the traditional lender is induced to exit, which seems far from any policy relevant scenario.

**Proposition 1** Assume that $0 < w < 1$, that $f(\theta)$ is a continuous density function fully supported on $[0, 1]$, and that $c_T < wR_T^m$. Then the traditional lender’s reaction function is discontinuous with a downwards jump.

The proof is set out in Appendix A. The intuition is as follows. If the reaction curve is continuous then, since the poaching set $[\theta_H, 1]$ depends continuously on the parameters, there must be an interest rate $R_R$ at which it collapses to the single point $\{1\}$. That is to say, poaching just the top borrower is profitable. But this is impossible since it would require reducing the interest rate to the entire set of inframarginal borrowers, which is of positive measure. A profitable attack on the reputational lender’s market thus requires a discontinuous jump in the interest rate.
2.2.2 Two Key Examples

We illustrate the nature of the strategic interaction with two examples. The first, Figure 4, is the benchmark case with a uniform $U[0,1]$ distribution of types. The second, illustrated in Figure 5 has a Beta $B[3,1]$ type distribution; that is to say, it is skewed to the right, so that good types are relatively rare.\footnote{The other parameters, in both these examples, are $m = 4, p = 4, w = .8, c_R = c_T = 1.01, x = 0.2.$} We show the reaction curves, and also the equilibrium reputation distributions. In both cases the traditional lender’s reaction curve is, as predicted by Proposition 1, is discontinuous and non-monotonic. In the first case, because of the discontinuity, the reaction curves do not intersect and there is no pure strategy Nash equilibrium. In the second they do intersect, despite the existence of the discontinuity. In the former case, even though there is no pure strategy Nash equilibrium there is a mixed strategy equilibrium in which the traditional lender mixes between setting a higher and a lower interest rate.\footnote{In the example shown the reputational lender sets $R_R = 1.67$, and the traditional lender mixes between $R_T = 1.9$ and $R_T = 2.7$ with probabilities .875 and .125} There is also a Stackelberg equilibrium where the reputational lender commits to covering the top of the market and the traditional lender responds by setting the higher rate. We defer discussion of these equilibria for the moment, and focus on the distribution of reputations at the two strategy profiles supporting the mixed equilibrium (the second of these profiles occurs also in the Stackelberg equilibrium, where the reputational lender commits to covering the top of the market).

The $U[0,1]$ case is shown in Figure 4. The type contingent distribution of reputations is shown in the second panel. We consider first the strategy profile in which the traditional lender plays aggressively, and sets a low interest rate. This is shown by a dashed curve. Since the underlying type distribution is uniform, the total density at any type $\theta$ is 1. The area under the dashed curve represents the population in state $G$, while the area above represents the population in state $B$; the area under this curve thus represents the market share of the reputational lender. Since everybody is born with a good type,
there is a baseline proportion of $1 - w$ with a good reputation, irrespective of type. Types with $\theta < \theta_L$ or $\theta > \theta_H$ will default strategically, but those with $\theta_L < \theta < \theta_H$ will repay if they can. The survival time of these types in the $G$ population, and hence the density of such borrowers, is increasing in $\theta$. So the density of types in the $G$ population is increasing in the range $\theta_L < \theta < \theta_H$. We notice that the average quality of borrowers in the $G$ population is higher than the average of the whole population, because high quality borrowers survive longer. This is mitigated by involuntary default at the top, but overall the entry of the reputational lender into the market causes the quality of the pool of borrowers facing the traditional lender to decline.

The distribution under the second strategy profile, in which the traditional lender plays high, is shown by a dotted curve. Under this interest rate profile we have $\theta_H = 1$, so there is no strategic default at the top. The traditional lender is indifferent between this profile, with a higher interest rate but a poorer quality pool of borrowers, and the profile with a lower interest rate and a better quality pool of borrowers. We note that these points on the reaction correspondence are strict local maxima, dominating any intermediate interest rate. The traditional lender is willing to randomize between these two interest
rates, but not to set an intermediate rate.

In the $B [1, 3]$ case, shown in Figure 5, the equilibrium lies to the left of the discontinuity, so a pure strategy Nash equilibrium exists despite the discontinuity. There is no strategic default at the top, and the traditional lender sets the monopoly rate $R_{T}^{m}$ in the residual market, as discussed in the preamble to Proposition 1.\textsuperscript{7} Since this residual market is determined only by attrition, the rate at which projects fail and borrowers default involuntarily, and this is not affected by the marginal behavior of the reputational lender, the two markets are effectively strategically disconnected.

There are also cases leading to uninteresting equilibria where the reaction curves intersect on the straight line region to the right, in which everybody defaults strategically, the reputational lender earns zero revenue and a negative profit, and the traditional lender is a monopolist in the residual market.\textsuperscript{8}

There are thus two modes of competition between the different lending platforms. In the "mild competition" regime there is a relatively less intense contest between lenders for the right tail of the distribution and a pure strategy equilibrium exists; there is no poaching, and the markets are strategically disconnected. In the "intense competition" regime, the quality of the traditional lender’s market has deteriorated to the point that attacking the reputational lender’s market share and poaching the best borrowers has become an attractive proposition; however, because of the impact on inframarginal borrowers this can never be a marginal move, and there is no pure strategy equilibrium. The key factor determining the nature of competition between lenders appears to be the distribution of borrower talent. The instability that we are observing is driven by relatively intense competition for the best borrowers at the top of the market, and the shape of the right tail is the major driver of the nature of this competition.

\textsuperscript{7}There will be strategic default at the bottom, but since $\theta_{L} < \theta^{*}$ any such defaulting types will not participate in the traditional market, so they can be neglected.

\textsuperscript{8}It is unclear whether there are environments where the reaction curves meet to the right of the discontinuity but without inducing strategic default by everybody. That is to say, the curves cross on the curved region of the traditional lender’s reaction curve. We have not been able to generate such scenarios in simulations.
Our results suggest this intense competition happens when the distribution of entrepreneurial talent is heavy at the top with a fat tail of highly productive borrowers. The idea that markets with a greater proportion of higher quality borrowers are more prone to fragility may be surprising. But if we grant that there is likely a positive correlation between a population’s average income and the average quality of borrower projects, and that a fat tail of very productive borrowers would produce a higher average project quality and thus average wealth, then this is actually consistent with the empirical facts. Many comparatively rich markets such as Bolivia, India and Morocco experienced crises while poorer nations such as Bangladesh, Cambodia and Laos have not. This is not merely a reflection of more lenders entering richer markets ex-ante: the latter nations were and are more saturated with lenders than the former at the times of crisis (Planet Rating, 2013). Our model suggests that poaching and strategic default on the MFI is more likely if there is a fat tail of good quality borrowers with high quality projects, for whom it is worth competing. Thus, the more fragile "intense competition" regime may be more likely to emerge in somewhat wealthier contexts, and that is indeed the pattern we see in the crises. However, it is really the presence of this tail of very productive borrowers that seems to drive the result, and not the average productivity or average wealth. To our knowledge this is the first indication that the shape of the distribution of entrepreneurial talent may itself be important, and not just the mean and variance.

2.3 Equilibrium

We now discuss the nature of equilibrium, focusing on the case where there is no pure strategy Nash equilibrium. It is useful first to discuss timing and time scales. In our model we address equilibrium in a one shot game played between two long lived lenders, subject to borrowers being in a steady state equilibrium. We seek to describe long run equilibria in which lender behavior is fixed over a time scale that is sufficient to reach a steady state in the distribution of borrower reputations. This requires that lender behavior is stable over a
reasonable time frame, either through commitment or through the nature of equilibrium.

Although there exists a mixed strategy equilibrium, we do not find it plausible in the microcredit context. There are two standard justifications for considering such equilibria. One is explicit randomization that is used in order to hide one’s action from the other player. The other is apparent randomization that reflects a pure strategy played in a related game with hidden types (Aumann 1987, Reny and Robson 2003). In our model, it is the traditional lender who will randomize in a mixed strategy equilibrium. Subject to this prior randomization, he will choose either a high or a low interest rate on a once for all basis. Randomization in this model cannot be interpreted as randomizing the interest rate period by period, as this would induce an equilibrium reputation distribution corresponding to the average interest rate, and hence is formally identical to setting an intermediate interest rate. An intermediate rate is suboptimal because of the non-linearity of the model the high and the low interest rates supporting the mixed equilibrium are strict local maxima. Since the informal lender is indifferent between high and low, there is no problem of time inconsistence in this decision. But the reputational lender, ex post, has an incentive to adapt to this decision, and it is not clear what could sustain this equilibrium in the long run. We could imagine scenarios in which a mixed strategy Nash equilibrium could be supported in a multimarket contact context, for example a single NGO interacting with many local moneylenders each setting different rates but with the NGO constrained to offer a single rate in all markets, because of either administrative constraints or lack of local knowledge in each micro-market. This equilibrium still relies on an inability to fully optimise, however, so our preferred interpretation is not mixed strategy Nash.

A Stackelberg equilibrium with the MFI as the first mover is much more plausible and relevant to the market we study. We have already observed that the two types of lender differ in their ability to commit. By its very nature, the reputational lender makes a long term commitment to encourage borrowers to repay and to invest in their reputation. In contrast the informal lender makes
frequent, confidential and idiosyncratic transactions. For these reasons the reputational lender has a first mover commitment advantage, and we find the Stackelberg equilibrium more persuasive. In this equilibrium the reputational lender chooses the highest interest rate consistent with covering the top of the market. The top type does not default but is on the margin of doing so. The traditional lender accommodates this market leadership, but is tempted to attack the reputational lender’s market. As noted at the beginning of this section, we need stable behaviour consistent with long run equilibrium in the borrower pool. We achieve this stability here through optimal commitment by the reputational lender and optimal response by the traditional lender. There is no tension in this equilibrium; no player wishes to reoptimise given their commitment constraints. ⁹

The striking feature of the Stackelberg equilibrium is its knife-edge nature, since it is located at the discontinuity in the traditional lender’s reaction curve. This knife-edge equilibrium has significant implications for the robustness of the market. Strictly speaking, the formal model is silent on disequilibrium dynamics. But we have enough information to discuss the direction of change, assuming as above that the traditional lender responds more flexibly than the reputational lender, and that the distribution of borrower reputations evolves only slowly through population dynamics. If market fundamentals unexpectedly change in a way that shifts the location of the discontinuity marginally to the right, then qualitatively there will be no change in the equilibrium. If however an unexpected shock shifts the market to the left, then we would see what looks very like the beginnings of a market crisis. The traditional lender would drop their interest rate by a substantial amount, triggering mass strategic default by all the reputational lender’s most profitable borrowers at the top of the market.

⁹In the Stackelberg equilibrium the traditional lender’s reaction is weakly optimal as they are indifferent between playing high and low. If we restrict the strategy space to a discrete grid, say to two decimal places, then with probability 1 the traditional lender has a unique best response.
3 The Policy Environment

3.1 Non-Profit Objectives

Some MFIs are motivated by social and development objectives, albeit subject to a financial sustainability objective and possibly a profit constraint. Hence MFIs may be prepared to trade off profits for an increase in the number of loans that they make and the size of the market that they service. We can extend our model to account for this by adjusting the reputational lender’s objective to be

$$\Pi_R(R_R, R_T) = \int_0^1 (\pi_R(\theta, R_R, R_T) - c_R) f_G(\theta, R_R, R_T) \, d\theta + \lambda \int_0^1 f_G(\theta, R_R, R_T) \, d\theta$$

$$= \int_{\theta_0(R_R, R_T)}^{\theta_0(R_R, R_T)} (w\theta R_R - c_R + \lambda) \frac{(1 - w)f(\theta)}{1 - w\theta} \, d\theta.$$

Here the first term is profit, as before, while the second is market share weighted by a policy parameter $\lambda$ that parametrises the relative importance of profit and market share in the MFI’s objectives. It is straightforward to calculate that $\frac{\partial^2 \Pi_R}{\partial R \partial \lambda} \leq 0$, so the reputational lender’s reaction curve shifts left as $\lambda$ increases. An increased emphasis on market share leads the reputational lender to reduce their interest rate, even at the cost of some fall in profit. Conversely, an increased emphasis on profit will shift the equilibrium to the right. If we assume that initially we are in the stable regime where there is a pure strategy Nash equilibrium, located to the left of the cusp, then a desire for more profit marked by a decrease in $\lambda$ moves the equilibrium to the right: closer to the cusp. Since the position of the cusp depends on exogenous parameters that are subject to random shocks, such as the cost of funds or, as discussed below, market confidence, this increases the likelihood of a market collapse triggered by such external shocks.

This finding is of some policy interest, given the emphasis on "mainstreaming" and a greater emphasis on profits and self-sustainability in the microcredit policy debate. Over a decade ago, policymakers voiced concerns about the in-
creasing commercialisation of the microcredit sector as a major contributer to
the Bolivian crisis (Rhyne, 2001). Mohammed Yunus himself has expressed
the view that a desire to maximise profits without considering social benefit
is responsible for the crises (Yunus, 2011). Yunus, Rhyne and others in the
policy world often suggest that increased profit-seeking leads MFIs to lend
too much at inappropriately high interest rates which then cannot be repaid,
thus leading to crisis. But logically it is unclear why a profit-seeking MFI
would lend to people they knew could not repay, as this is unlikely to lead
to high profits. However, our model illuminates a plausible mechanism which
links profit-seeking to market fragility. In the intense competition regime, the
Stackelberg equilibrium will indeed be more stable and resilient to unforeseen
exogenous shocks when the MFI is a socially motivated NGO rather than
profit-maximising, and hence locate further to the left of the cusp. Hence,
the trend to commercialising microfinance could have a destabilising effect on
these markets.

3.2 Borrower Screening

In our baseline model the reputational lender has a simple strategy space: they
observe the past repayment or default behavior of their borrowers, and they set
an interest rate. While this framework captures the essential features of MFIs
that differentiate them from information rich traditional lenders, MFIs and
other reputational lenders also use a variety of complementary mechanisms
to improve the quality of their borrowing pool (Armendariz de Aghion, 1999;
Rai, 2002; Rai and Sjostrom, 2004, Ghatak, 2000; Armendariz de Aghion and
Gollier, 2000). In particular, group lending can reasonably be expected to
trace the type distribution by screening out low quality borrowers.

We explore the implications of borrower screening by endowing the reputa-
tional lender with a perfect screening technology that can exclude borrowers
below a threshold type $\bar{\theta}$. While this is perhaps somewhat simplistic, it does
reveal that screening can interact with market equilibrium in unexpected ways.
While screening improves the quality of the reputational lender’s borrowing
pool, improving this market causes the quality of the traditional lender’s market to deteriorate. Poaching good borrowers becomes increasingly attractive, in comparison to acting as the residual monopolist covering the bottom of the market. Competition becomes more intense, and this may shift the discontinuity and create the preconditions for market instability.

We illustrate using the $B[1, 3]$ specification discussed previously. Without screening, the equilibrium lies in the mild competition regime, and there is a pure strategy Nash equilibrium as already discussed and shown in Figure 5. However, when low quality borrowers are screened out the equilibrium shifts into the knife-edge intense competition regime (Figure 6). The specification of the model shown here is identical with the specification shown in Figure 5, except that the reputational borrower screens out types $\theta < \bar{\theta}$.\textsuperscript{10} Screening is attractive to the reputational lender, as it improves profits - but it can shift the market to a more fragile configuration.

\textsuperscript{10}In this example, in order to illustrate the effect clearly, we set $\bar{\theta} = .8$. All other parameters are unchanged.

Figure 6: Reaction Curves and Reputation Distribution (Type Distribution Truncated $B[1, 3]$)
3.3 Sovereign Risk

Some policymakers and scholars argue that government intervention in microcredit markets has played a role in causing mass defaults (Yerramilli 2013, Banerjee 2013). There are several documented cases of governments becoming hostile to microcredit institutions prior to or during market crises, such as Nicaragua in 2009 and in Andhra Pradesh in 2010. In these cases, governments pursued lenders they apparently perceived to be problematic or lending practices they thought unsustainable. Within our framework, however, we can show that this government hostility can actually create the problems it purports to solve. A loss of confidence in the long-term future of the reputational lender reduces the value $V_G$ of a good reputation. This reduces the cost of poaching good borrowers to the moneylender, and the discontinuity will move to the left, reflecting increased fragility and perhaps triggering mass default.

This section examines this issue formally by introducing into the model an additional exogenous risk that the reputational lender may cease operations. In our model, reputations reside in a single lending institution, so if the lender exits the market a good borrower’s reputation would be destroyed. The possibility of this exit thus reduces the value of a good reputation and the incentive to invest in one. We model this by introducing a probability $\alpha$ that the institutional lender will continue into the next period, and a probability $1 - \alpha$ that it will exit. From the point of view of a borrower who has chosen to repay a loan and invest in a good reputation this means that the outcome will be $G$ with probability $\alpha$ and $B$ with probability $1 - \alpha$ (since in the latter case all reputations are in effect bad). We assume for simplicity that death occurs immediately after loans are repaid, and before next period’s loans are taken out. The Bellman equations thus become

$$V_G = \max\{w(\theta m + V_B), w(\theta (m - R_R + \alpha V_G + (1 - \alpha) V_B) + (1 - \theta) V_B)\}$$

$$V_B = \max\{w V_B, w(\theta (m - R_T) - (1 - \theta) p + V_B)\}.$$  

It is straightforward to verify that the switching points, which determine the
state transition probabilities, become

\[ \theta_L = \frac{1}{m} \frac{R_R}{\alpha w} \]

\[ \theta^* = \frac{p}{m - R_T + p} \]

\[ \theta_H = \frac{p - \frac{R_R}{\alpha w}}{p - R_T} \]

As \( \alpha \) decreases from 1, \( \theta_L \) increases and \( \theta_H \) decreases. The reputational lender’s market contracts from both ends. The calculation of profit functions and reaction curves proceeds as before, but using these revised switching points.

We explore this effect using the \( U [0, 1] \) specification of the model illustrated in Figure 4 and discussed previously. Reducing \( \alpha \) from 1 to 0.9 results in a leftward shift in the discontinuity from the net interest rate 0.67 to about 0.5.\(^{11}\) Thus an adverse shock to \( \alpha \) can precipitate default by the top of the market. There is, of course scope for further feedback. Market collapse could in itself be treated by borrowers as news, leading to a further fall in \( \alpha \); this could affect the cost of funds, and the cycle continues. This echoes the classic Diamond and Dybvig (1983) result: borrowers’ perceptions of the longevity of the microlender are crucial to that lender’s solvency. This also supports the idea that lender reputation in microcredit markets has an important role, as suggested in Banerjee 2013.

4 Conclusion

This paper has offered a new perspective on potential sources of instability in the credit markets that serve the poorest households in the developing world. We studied the interaction between a reputation-based microcredit institution and an informal moneylender, emphasising the dynamic of competing platforms. We find that this strategic interaction between the lenders cre-

\(^{11}\)In the interest of space, we omit the graph illustrating the equilibrium with \( \alpha = .9 \).
ates discontinuous and non-monotonic reaction functions, which can in some cases lead to fragile knife-edge equilibria. Our analysis further suggests that the skewness of the distribution of entrepreneurial talent may be important for determining the intensity of competition between these lenders, which in turn determines market stability. This result is unsettling, particularly when we also find that borrower screening technologies and government hostility to microcredit can exacerbate this fragility. However, we do find that the overall problem is ameliorated when the microlender has a social motive rather than purely maximising profits.

Overall, these results strongly suggest that microcredit institutions cannot be studied in isolation. While lack of data means we cannot be sure that the interaction between MFIs and informal moneylenders we study played a major role in the recent microcredit crises, our model shows the importance of considering this interaction both for academics and policymakers. It should be a priority for the development community to make headway on empirical studies of the entire credit landscape rather than microfinance in isolation. Without further evidence we cannot determine the appropriate regulatory approach to these markets.

The fragility of the reputational mechanism is a theme we share with the sovereign debt literature. In their classic paper, Bulow and Rogoff (1989) study a market with a single long lived reputational lender transacting with a long lived borrower who has access to perfect savings and insurance markets. They find that the only default-free equilibrium is one of zero borrowing since the borrower can arbitrage any asset buffer required to prevent default using the perfect external markets. In our model, as in the real credit markets that serve the poorest households, there is no recourse to perfect external markets. In this context the reputational mechanism can support equilibria with positive borrowing free from strategic default. But the mechanism does remain vulnerable to being undercut by external agents, and this creates fragility. Hence, while our model is tailored to the microcredit context, it does echo broader findings in the finance literature.
The results we present here are stark because the model is designed to highlight this particular aspect of the interaction. In reality, institutions and environments are more complex than can be reflected in a model. Yet the internal logic of the interaction will remain valid in a more general context: that lenders will compete for the quality of the lending pool; that the best quality borrowers with the best projects will be most likely to abandon the reputational lender; and that an attack on the reputational lender’s market can occur only through a jump, not a marginal change in interest rates. The main implication remains that policy design for microcredit markets should be sensitive to the interaction between these new institutions and existing financial markets, both formal and informal.
Figure 7: Construction of the Best Response Function

A Proof of the Main Result

To make the proof easier to read we simplify the notation in this Appendix by writing \( \tau \) for \( R_T \), and \( \rho \) for \( R_R \). We write \( \tau^m \) for \( R^m_T \) and \( \rho^m \) as an abbreviation for \( w\tau^m = wR^m_T \). The construction in the proof is illustrated in Figure 7, in which both notations are shown. We write partial derivatives in subscript notation, writing for example \( \frac{\partial \Pi_2 (\rho, \tau)}{\partial \tau} \), and we write simple derivatives in dot notation, writing for example \( \dot{\theta} (\tau) \) for \( \frac{d\theta(\tau)}{d\tau} \). Using this notation, the result to be proved becomes

**Proposition 1** Assume that \( 0 < w < 1 \), that \( f(\theta) \) is a continuous density function fully supported on \([0,1]\), and that \( c_T < \rho^m \). Then the traditional lender’s reaction function is discontinuous with a downwards jump.

**Proof.** Without loss of generality we may assume that \( 1 \leq \tau \leq p \).\(^{12}\)

\(^{12}\)Any choice of \( \tau > p \) leads to immediate default by all participating borrowers against the traditional lender. The choice of \( \tau = p \) leads to the same, or a better, outcome, as borrowers are then indifferent between default and repayment, and there is no change to the participation decision.
We note that the lender’s profit equation can be written

\[ \frac{\Pi(\rho, \tau)}{w} = \int_{\theta^*(\tau)}^{1} g(\tau, \theta) \, d\theta + \int_{\theta_H(\rho, \tau)}^{1} h(\tau, \theta) \, d\theta, \]

where for ease of notation we write

\[ g(\tau, \theta) = \left( \theta \tau + (1 - \theta) x p - \frac{c_T}{w} \right) \frac{w(1 - \theta)}{1 - \theta w} f(\theta) \]

\[ h(\tau, \theta) = \left( \theta \tau + (1 - \theta) x p - \frac{c_T}{w} \right) \frac{(1 - w) \theta}{1 - \theta w} f(\theta). \]

Since \( \theta^* \) depends on \( \tau \), and \( \theta_H \) on \( \rho \) and \( \tau \), we write these as \( \theta^*(\tau) \) and \( \theta_H(\rho, \tau) \). The first integral represents the profit in the absence of strategic default by borrowers with a good reputation, and the second integral represents the additional profit induced by strategic default. Notice that the first integral depends only on \( \tau \), not on \( \rho \). This will be useful in what follows.

The lender will choose \( \tau \) to maximize \( \frac{\Pi(\rho, \tau)}{w} \). We note for future reference that

\[ \Pi_2(\rho, \tau) = w \int_{\theta^*(\tau)}^{1} g_1(\tau, \theta) \, d\theta - w g(\tau, \theta^*(\tau)) \theta^*(\tau) \]

\[ + w \int_{\theta_H(\rho, \tau)}^{1} h_1(\tau, \theta) \, d\theta - w h(\tau, \theta_H(\rho, \tau)) \theta_H(\rho, \tau). \]

There is no strategic default if \( \rho < w \tau \), that is to say, above the diagonal line in the diagram. In that case \( \theta_H(\rho, \tau) > 1 \) and the second integral vanishes since the support of \( f(\theta) \) is \([0, 1]\). It is useful to begin the analysis in this region.

Let \( \tau(\rho) \) be the lender’s reaction curve. In the region \( \rho < w \), there will be no strategic default by any agent no matter how \( \tau \in [1, p] \) is chosen, and the traditional lender faces the residual market of bad reputation agents. Let \( \tau^m \) be the monopoly interest rate in this market. It is clear that the reaction curve is \( \tau(\rho) = \tau^m \) for \( \rho < w \). We provisionally extend this curve by setting \( \tau(\rho) = \tau^m \) for \( w \leq \rho < \rho^m = w \tau^m \). Note that \( \tau(\rho) = \tau^m \) is at least a local
maximum on this domain. This is because \( \Pi(\rho, \tau) \) does not depend on \( \rho \) in this domain, and by definition \( \tau^m \) maximizes \( \Pi(\rho, \tau) \) for small \( \rho \). This is because it lies above the default line, so the second part of the integral vanishes.

We cannot however exclude the possibility that a different global maximum \( \tilde{\tau} \) arises at some point \( \tilde{\rho} \) in this region, but if so it must be that \( \tilde{\tau} \) is low enough to induce strategic default by some agents. This requires that \( \tilde{\tau} < \frac{\tilde{\rho}}{w} < \frac{\rho^m}{w} = \tau^m \). So there must be a discontinuity and a downward jump from \( \tau^m \) to \( \tilde{\tau} \) at this point.

It remains to consider what happens if the reaction curve remains globally optimal over the whole interval \( \rho \leq \rho^m \). Assume, towards a contradiction, that the reaction curve \( \tau(\rho) \) is continuous. We examine \( \Pi_2(\rho, \tau(\rho)) \), the marginal incentive to deviate from \( \tau(\rho) \), along the curve near \( \rho^m \). \( \Pi_2(\rho, \tau(\rho)) \) is clearly continuous in \( \rho \) if \( \tau(\rho) \) is continuous, since all the quantities occurring in the explicit expression for \( \Pi_2(\rho, \tau(\rho)) \) above are continuous. It is also clear by construction that \( \Pi_2(\rho, \tau(\rho)) = 0 \) for \( \rho < \rho^m \). We estimate \( \Pi_2(\rho, \tau(\rho)) \) for \( \rho > \rho^m \) by considering the above expression for \( \Pi_2(\rho, \tau(\rho)) \) term by term, and taking the limit as \( \rho \to \rho^m \) from above.

Note first that

\[
\begin{align*}
\frac{\hat{\rho}}{w} \int_{g^*(\tau(\theta))}^{1} g_1(\tau, \theta) d\theta & - w g(\tau, g^*(\tau(\rho))) \hat{g}^*(\tau(\rho)) \\
& \to w \int_{g^*(\tau^m)}^{1} g_1(\tau, \theta) d\theta - w g(\tau^m, g^*(\tau^m)) \hat{g}^*(\tau^m) \\
& = 0.
\end{align*}
\]

The last equality holds because this is just the first order condition for the optimality of \( \rho^m \) for small \( \rho \), and this is zero by construction. We note next that

\[
\frac{\hat{\rho}}{w} \int_{h(\rho, \tau(\theta))}^{1} h_1(\tau(\rho), \theta) d\theta \to 0
\]
since $\theta_H (\rho, \tau (\rho)) \to \theta_H (\rho^m, \tau^m) = 1$ as $\rho \to \rho^m$. And finally,

$$-w h (\tau, \theta_H (\rho, \tau)) \theta_H (\rho, \tau) \to - \left( \frac{\rho^m - c_T}{p - \tau^m} \right) pf (1).$$

Thus $\Pi_2 (\rho, \tau (\rho)) \to - \left( \frac{\rho^m - c_T}{p - \tau^m} \right) pf (1) < 0$ as $\rho \to \rho^m$ from above. This is a contradiction.

So the reaction curve must be discontinuous at this point. Locally, it is clear that the incentive is for $\tau$ to jump down, but this argument is not sufficient to establish that the global maximum jumps down. However this is easy to see directly. An upward jump would take us into the no strategic default region, where we already know that $\tau^m$ is optimal.

So, we have that there is a downward jump somewhere in the region $w < \rho < \rho^m$. In all of our simulations it actually occurs at $\rho^m$.

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