Group Formation and Endogenous information Collection in Microcredit

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Abstract

This paper attempts to address the effects of different types of loan contract on a borrower's incentive for investment in information. An attempt has been made to model the trade-off that a borrower faces when she collects information about the potential of her intended projects both under individual and joint liability loan contracts. Even under limited liability, the borrower faces a trade-off at information collection stage between the cost of signal collection, and the cost of her time and effort for project execution in case the project fails. In this set-up, we also examine how the borrower's ex-ante incentive for investment in information changes with lender's objective. It has been seen that there may be situations in which individual liability lending may emerge as the dominant lending arrangement in a micro-credit market.

JEL Classification: G 21, D 82, O16
Keywords: Joint Liability, Group lending, Moral Hazard, Peer Monitoring, Group size, Social Sanction.

1 Introduction

Before undertaking a project, borrowers generally collect information about the potential of their intended projects. Undertaking a project requires the borrower to put in effort and time besides the borrowed investment which are costly and are sunk in case of project failure. Even under limited liability - where an unsuccessful borrower doesn't have to repay her loan - information about the chance of project success is welcome since this allows the borrower to save time and effort cost if she finds out that the project will fail almost surely. Moreover, this also has implications about the lender's profitability and the equilibrium rate of interest. If the borrowers only undertake projects when they get a good signal about project success, the lender loses much less borrowed capital on

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failed projects. This allows the lenders to lower the rate of interest generally leading to higher efficiency in the micro-credit market.

However, collecting information is costly for the borrowers and this cost rises with the quality of information. The trade-off a borrower faces at the information collection stage is between the cost of signal collection and his potential saving of time and effort cost in case the realized signal indicates strong probability of failure. This chapter attempts to understand this trade-off in terms of a simple model of credit market. More specifically, we examine how individual lending mechanism vis-a-vis the group based lending mechanism affects the ex-ante incentive for information collection and the welfare of the agents. We also examine how this incentive is affected by the change in lender's objective, in particular, social motive vis-a-vis profit motive.

The last point needs some clarification here. The group may be formed before the borrowers collect the information. In this case, the group members may cooperatively decide on the quality of information collected by the members. Any deviation from this cooperatively chosen quality is immediately detected and punished by the group. Alternatively, a group may be formed after the borrowers independently decide on the quality of information they would collect, but before the signal is realized. Notice that group formation at this stage does not leave any scope for group punishment based on quality of information. Finally, the group may be formed after the signal realization. In the last two cases, there will be an assortative matching in the group formation stage though quality of information is chosen independently by each borrower. But the knowledge that one can form a group with a borrower with good quality signal only if the first borrower herself has good quality information may create extra incentive to collect good quality information for the borrowers.

In the large body of microfinance literature, a significant amount has been devoted to explore the potential of group lending under joint liability. Most of the existing literature on group-lending with joint liability attempts to address the main obstacles behind the non-viability of the credit support system for the poor and then tries to resolve the problems. A number of theoretical models in the existing literature of microfinance, have identified various mechanisms including peer screening or assortative matching (Ghatak, 1999; Ghatak and Guinnane, 1999; Tassel, 1999; Morduch and Aghion, 2004), peer monitoring (Stiglitz,1990; Aghion, 1999, Conning, 1999), and peer pressure or enforcement (Besley and Coate, 1995; Aghion and Morduch, 2004) through which group lending with joint liability enables a lender to make uncollateralized loans even to the marginal people (Mukherjee, 2014). Actually, shifting a burden of default from a lender to a group gives correct incentives to use their local information and social ties for ensuring repayments of peers within the same group (Tsukada, 2012).

Microfinance markets reveal that limited borrower liability exposes lenders to levels of adverse selection and moral hazard which rely on formal collateral. The use of joint liability contracts for those borrowers who take group loans (with joint liability) creates an intricate strategic dynamic between groups and lenders, each of whom bear some risk in the extension of loans to individual members (Janvry *et al*, 2006). While group lending with joint liability has become hugely popular as an instrument for overcoming difficulties of rural credit markets, there are other mechanisms like *savings-up* mechanism, *sequential lending* mechanism, *direct lender monitoring*, *dynamic incentives*, *contingent renewal* and *progressive lending* mechanisms, *joint benefit* mechanism etc.(Sinha, 2005, 2007; Roychowdhury, 2005, 2007; Ghosh and Ray, 1997; Bhattacharya *et al*, 2008) which micro-lenders use in practice, often in conjunction with the joint liability mechanism.

In a framework that allows project returns to accrue over time, Chowdhury et al (2014) in a recent paper provide a justification for the use of frequent repayment schemes, examine the optimal choices for the MFI, demonstrate that the MFI opts for higher project sizes under group lending with limited collusion, and also provide a theory on group size. Mukherjee and Bhattacharya (2014, 2015), have dealt with that issue of optimal group size, optimal joint liability parameter, and socially efficient group size. Unfortunately the same literature on micro-credit is silent on the aspects of endogenous information collection and group formation in the micro-credit market. It attempts to examine how individual lending mechanism vis-a-vis the group lending mechanism affects the ex-ante incentive for information collection and the welfare of the agents when the group is formed after the signal realization.

It is interesting to note that the existing literature on micro-credit is almost silent on the issues of endogenous information collection and group formation in the micro-credit market. This paper attempts to provide a theoretical framework to analyze these issues. Borrowers (even with limited liability individual lending) generally collect information about the potential of their intended projects since undertaking a project requires the borrower to put in effort, time and money (borrowed investment) which are costly and are sunk in case of project failure. Information about the chance of project success allows the borrower to save these costs if she finds out that the project will fail almost surely. This also allows the lenders to lower the rate of interest generally leading to higher efficiency in the micro-credit market.

2 The Model

We consider a simple model. Output (Y) takes two values: high (Y^H) and low (Y^L), where $Y^H > Y^L \geq 0$. For simplicity, we normalize Y^L to 0 and denote Y^H by Y. Projects are indivisible, and each project requires project fund of amount 1 to be viable. For each project, the prior probability of Y being realized is $\frac{1}{2}$ i.e., $\Pr\left(Y^H = Y\right) = \Pr\left(Y^L = 0\right) = \frac{1}{2}$.

The lending institution has the resources to lend to a number of borrowers. The borrowers face limited liability in the sense that, in case of default, the lender can not seize assets that a borrower has specifically pledged as collateral. In this context, we assume that the poor has no collateral to pledge. Hence, the lender's receipt in the event of default is zero. The limited liability constraint, along with the borrowers' lack of collateral, rule out the standard instruments used by conventional lenders to overcome information and enforcement problems.

We model the information structure as follows. The borrower collects signal about the potential of the project. The quality of signal is characterized by $\theta \in \left[\frac{1}{2}, 1\right]$. $\theta = \frac{1}{2}$ indicates a completely uninformative signal, while $\theta = 1$ indicates a fully informative signal. The signal, denoted by s, has two possible realizations: $s \in \{S, F\}$ where S and F indicate success and failure respectively. We assume that $\Pr\left[S|Y^H\right] = \Pr\left[F|Y^L\right] = \theta$.

Notice that if a borrower collects a signal of quality θ, then the probability of getting a Success (S) signal is

$$\Pr[S] = \Pr\left(S|Y^H\right).\Pr\left(Y^H\right) + \Pr\left(S|Y^L\right).\Pr\left(Y^L\right) = \theta.\frac{1}{2} + (1-\theta).\frac{1}{2} = \frac{1}{2}$$

Similarly, $\Pr[F] = \frac{1}{2}$.

The posterior belief about the success of the project and realizing output Y, if the signal is S, is given by

$$\Pr(Y \mid S) = \frac{\Pr(S \mid Y).\Pr(Y)}{\Pr(S \mid Y).\Pr(Y) + \Pr(S \mid 0).\Pr(0)} = \frac{\theta.\frac{1}{2}}{\theta.\frac{1}{2} + (1-\theta)\frac{1}{2}} = \theta$$

Similarly,

$$\Pr(Y \mid F) = \frac{\Pr(F \mid Y).\Pr(Y)}{\Pr(F \mid Y).\Pr(Y) + \Pr(F \mid 0).\Pr(0)} = \frac{(1-\theta).\frac{1}{2}}{(1-\theta).\frac{1}{2} + \theta.\frac{1}{2}} = (1-\theta)$$

Let the cost of collecting a signal of quality θ be given by $c(\theta)$. We assume that $c'(.) > 0, c''(.) > 0$ and $c\left(\frac{1}{2}\right) = 0$. We also assume that $c'(1) > Y$ to ensure that an interior solution for signal quality always exists.

Finally, we assume that if the borrower implements a project, she has to incur a fixed effort cost $e > 0$. In our model, e plays a very vital role. This is what the borrowers hope to save by collecting information. If $e = 0$ then each borrower would choose $\theta = \frac{1}{2}$. Higher the value of e, the more incentive the borrowers have to collect better quality information. Throughout, we assume that $(Y > 2e)$. This assumption is necessary to ensure a positive expected value of the project ex-ante.

We consider individual lending as well as group lending. For the sake of analytical simplicity, we limit the group size to 2. In individual lending as well as group lending, a borrower's decision about taking up the project is taken only after the realization of the signal. We assume that the borrowers do not default voluntarily, i.e. they take up the loan only if they decide to take up the project. In case of group lending, the members of the group get access to the loan only if both members decide to take up the project.

We assume that group lending entails joint liability. Our focus in this chapter is on group formation and how group formation affects the incentive for signal collection. We may consider three separate information structures. Firstly, we may consider the case when the community members can observe each other's signal realizations (s), i.e., s is publicly observable within the community. However, in extension to our theoretical framework, we may consider the case when s is not publicly observable, but the quality of signal (θ) is. Finally, in a much richer model, we may take up the case when neither s nor θ are publicly observable. In each of these cases, we can examine how the process of group formation is affected by the information structure. It will be further interesting to investigate the incentive for investment in quality of signal in all these three cases, and to observe how this incentive stands in comparison to individual liability.

Here, in this chapter, we are about to explore the first case only where the community members can observe each other's signal realizations, i.e. s is publicly observable within the community.

It is to note that the timing of the game is as follows:

1. Stage 1: The lender announces the gross rate of interest ($r > 1$) that is to be charged on a loan of amount 1.
2. Stage 2: Each borrower chooses her quality of signal/information θ at cost $c(\theta)$.
3. Stage 3: The signal is realized, i.e. the borrower observes either S or F.
4. Stage 4: After observing her signal, the borrower decides whether to take up the project or not. If she decides to take up the project, she applies for loan. In case of group lending, both the partners must decide to take up the loan[2].
5. Stage 5: If the borrower takes the loan, she incurs the effort cost e and the project outcome is realized.

In joint liability loan contract, we consider groups of two people, with each group formed voluntarily. Individuals invest independently, but the loan contract involves joint liability. Under the joint liability contract, a borrower pays her own interest (r) in the event of her success, but pays an additional interest payment in case her partner fails. We assume that $\left(r < \frac{Y}{2}\right)$, i.e., a successful borrower has sufficiently large pay-off to pay even for her unsuccessful partner.

2.1 Limited Liability Individual Lending

We first consider individual liability lending. The borrower takes the decision about the project after the signal is realized. If an individual borrower takes up

[2] Notice that a borrower's decision about taking up the project depends on the signal realization. So when the group is formed and signals are not observable, even if a borrower receives an S signal, she may not have access to loan if her partner receives an F and decides against loan in a two member group. This is an inefficiency that arises when only group-based contracts are offered. A richer model should take this into account and allow the lender to offer both type of contracts at the same time. However, in this chapter, we analyze individual lending and group lending separately. That means that we do not allow the lender to offer both individual contract and group-based contract at a time.

the project after realization of the signal, S, his expected utility will be,

$$U^1(\theta|S) = \theta(Y-r) - e - c(\theta)$$

If he doesn't, then his expected utility will be,

$$U^0(\theta|S) = -c(\theta)$$

Hence, he takes up the project after S, iff

$$U^1(\theta|S) > U^0(\theta|S) \Rightarrow \theta(Y-r) - e - c(\theta) \geq -c(\theta) \Rightarrow \theta \geq \left(\frac{e}{Y-r}\right) \quad (1)$$

Similarly, he takes up the project after F, iff

$$U^1(\theta|F) > U^0(\theta|F)$$

$$\Rightarrow (1-\theta)(Y-r) - e - c(\theta) \geq -c(\theta) \Rightarrow (1-\theta) \geq \left(\frac{e}{Y-r}\right) \quad (2)$$

Notice that since $\theta \in \left[\frac{1}{2}, 1\right]$, for $r > (Y-e)$ neither (1) nor (2) is satisfied for any θ. Hence, for $r > (Y-e)$, an individual borrower would not take up the project even after a success signal. Since the project is never taken up, the borrower would not invest any amount in collecting signals and thus $\theta^* = \frac{1}{2}$.

If $r \in (Y-2e, Y-e]$, $\frac{e}{Y-r} \in (\frac{1}{2}, 1]$. Since $(1-\theta) \leq \frac{1}{2}$, a borrower would never take a loan after F. However, if the signal realization is S, the borrower takes up the loan if and only if the signal realization is S and the signal quality $\theta \geq \left(\frac{e}{Y-r}\right)$. Notice that ex-ante the probability of a success signal (S) is $\frac{1}{2}$. Hence, the borrower's ex-ante expected utility is

$$EU(\theta) = \begin{cases} -c(\theta) & \text{if } \theta < \frac{e}{Y-r} \\ \frac{1}{2}\left[\theta(Y-r) - e\right] - c(\theta) & \text{if } \theta \geq \frac{e}{Y-r} \end{cases} \quad (3)$$

For $r \in (Y-2e, Y-e]$, the borrower chooses θ to maximize (3). Notice that if the borrower chooses $\theta^* = \frac{1}{2}$, her expected utility is 0. However, as $r \rightarrow (Y-2e)$, $\left(\frac{e}{Y-r}\right) \rightarrow \frac{1}{2}$ and the borrower is ex-post better off taking up the project for any ex-ante choice of θ. Her best choice here is determined by the FOC of optimization of the expression $\left[\frac{1}{2}\left[\theta(Y-r) - e\right] - c(\theta)\right]$. We denote this θ by $\hat{\theta}$ which is determined from

$$Y - r = 2c'\left(\hat{\theta}\right) \quad (4)$$

We further assume that $c'\left(\frac{1}{2}\right) < \frac{e}{2}$. This assumption ensures that a proper interior solution, $\hat{\theta} \in \left(\frac{1}{2}, 1\right)$, exists as $r \rightarrow (Y-2e)$. The optimal choice of θ now depends on whether $\hat{\theta}$ satisfies (1) and if it does whether it generates a positive ex-ante expected utility for the borrower. Essentially, the borrower's optimal ex-ante expected utility is given by

$$\max\left\{0, \frac{1}{2}\left[\hat{\theta}(Y-r) - e\right] - c\left(\hat{\theta}\right)\right\}$$

provided $\hat{\theta}$ satisfies (1) for the corresponding rate of interest.

To economize on notation we denote

$$g_1(r) = \frac{1}{2}\left[\hat{\theta}(r)(Y-r) - e\right] - c\left(\hat{\theta}(r)\right)$$

where $\hat{\theta}(r)$ is determined from (4). By envelope theorem, we know that

$$g_1'(r) = -\frac{1}{2}\hat{\theta}(r) < 0$$

Also notice that at $r = (Y - e)$,

$$g_1(r) = \frac{1}{2}\left[\hat{\theta}|_{r=Y-e}.e - e\right] - c\left(\hat{\theta}|_{r=Y-e}\right) < 0$$

since $\hat{\theta}|_{r=(Y-e)} < 1$ while at $r = (Y - 2e)$,

$$\begin{aligned} g_1(r) &= \frac{1}{2}\left[\hat{\theta}|_{r=Y-2e}.2e - e\right] - c\left(\hat{\theta}|_{r=Y-2e}\right) \\ &= \frac{1}{2}\left[\hat{\theta}|_{r=Y-2e}.2e - e\right] - \int_{\frac{1}{2}}^{\hat{\theta}|_{r=Y-2e}} c'(\theta)\, d\theta \end{aligned}$$

$$\begin{aligned} &> \frac{1}{2}\left[\hat{\theta}|_{r=Y-2e}.2e - e\right] - \int_{\frac{1}{2}}^{\hat{\theta}|_{r=Y-2e}} c'\left(\hat{\theta}|_{r=Y-2e}\right) d\theta \\ &= \frac{1}{2}\left[\hat{\theta}|_{r=Y-2e}.2e - e\right] - \int_{\frac{1}{2}}^{\hat{\theta}|_{r=Y-2e}} \frac{Y-r}{2} d\theta \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2}\left[\hat{\theta}|_{r=Y-2e}.2e - e\right] - \left[\hat{\theta}|_{r=Y-2e}. - \frac{1}{2}\right] e \\ &= 0 \end{aligned}$$

where the inequality follows from the fact that $c''(\theta) > 0$ and in the later part of the derivation (4) is used. Since $g_1(r)$ is continuous in r there exists $r_c^I \in ((Y-2e), (Y-e))$ such that $g_1(r) \geq 0$ for $r \leq r_c^I$. Moreover, whenever $g_1(r) \geq 0$, the corresponding $\hat{\theta}(r)$ satisfies (1). We now state this result in Lemma 1.

Lemma 1 *Suppose $r \geq (Y - 2e)$. The borrower's optimal signal quality is given by the following:*

$$\theta^* = \begin{cases} \frac{1}{2} & \textit{if } r > r_c^I \\ \hat{\theta}(r) & \textit{if } r \leq r_c^I \end{cases}$$

where $r_c^I \in ((Y-2e), (Y-e))$ and $\hat{\theta}(r)$ is determined from

$$Y - r = 2c'\left(\hat{\theta}\right)$$

Now consider $r < (Y - 2e)$. Notice that for these values of r, (1) is satisfied for all values of θ. Hence, a borrower will take up the project for all values of θ after receiving S signal. However, (2) is satisfied only if θ is sufficiently low. Thus the borrower will take up the project after F signal only if the chosen signal quality is sufficiently. The ex-ante expected utility of the borrower thus can be written as

$$EU(\theta) = \begin{cases} \frac{1}{2}\left[\theta\left(Y - r\right) - e\right] - c\left(\theta\right) & \text{if } \theta > 1 - \frac{e}{Y-r} \\ \quad \frac{1}{2}\left[\theta\left(Y - r\right) - e\right] \\ +\frac{1}{2}\left[\left(1-\theta\right)\left(Y - r\right) - e\right] - c\left(\theta\right) & \text{if } \theta \leq 1 - \frac{e}{Y-r} \end{cases}$$

which can be further simplified to

$$EU(\theta) = \begin{cases} \frac{1}{2}\left[\theta\left(Y - r\right) - e\right] - c\left(\theta\right) & \text{if } \theta > 1 - \frac{e}{Y-r} \\ \frac{1}{2}\left(Y - r\right) - e - c\left(\theta\right) & \text{if } \theta \leq 1 - \frac{e}{Y-r} \end{cases} \tag{5}$$

Notice that if the borrower decides to take up the project even after signal F, ex-ante she loses all incentives to invest in signal quality and hence would choose $\theta^* = \frac{1}{2}$. Her ex-ante expected utility from doing so is denoted by

$$g_2\left(r\right) = \left(\frac{Y - r}{2}\right) - e$$

On the other hand, she is better-off by taking up the project only after S if her chosen signal quality θ exceeds $\left(1 - \frac{e}{Y-r}\right)$. In that case her ex-ante expected utility is given by $g_1\left(r\right)$. Essentially, the borrower's optimal ex-ante expected utility is given by

$$\max\left\{g_1\left(r\right), g_2\left(r\right)\right\}$$

We now define $g\left(r\right) = g_1\left(r\right) - g_2\left(r\right)$. Notice that

$$\begin{aligned} g'\left(r\right) &= g_1'\left(r\right) - g_2'\left(r\right) \\ &= -\frac{1}{2}\hat{\theta}\left(r\right) + \frac{1}{2} \\ &> 0 \end{aligned}$$

Moreover at $r = (Y - 2e)$,

$$g\left(r\right) = g_1\left(r\right) > 0$$

while at $r = 0$

$$\begin{aligned} g\left(0\right) &= \frac{1}{2}\left[\hat{\theta}\left(0\right).Y - e\right] - c\left(\hat{\theta}\left(0\right)\right) - \frac{Y}{2} + e \\ &= \left(\hat{\theta}\left(0\right) - \frac{1}{2}\right)\frac{Y}{2} - c\left(\hat{\theta}\left(0\right)\right) - \frac{1}{2}\left(\frac{Y}{2} - e\right) \end{aligned}$$

The sign of $g\left(0\right)$ depends on the level of effort cost e. If e is high, $g\left(0\right)$ is positive. If e is low, $g\left(0\right)$ is negative. Depending on the value of e, we have two possibilities. Remember that in our relevant zone $g(r)$ is monotonic. As r falls, $g(r)$ falls. If $g(0) \geq 0$, then for every $r \leq (Y - 2e)$, the borrower collects $\theta^* = \hat{\theta}(r)$ and takes up the project only if a success signal is realized. However, if $g(0) < 0$, then there exists $r_d^I \in (0, Y - 2e)$ such that the borrower collects $\theta^* = \hat{\theta}(r)$ if and only if $r > r_d^I$. Otherwise, she collects $\theta^* = \frac{1}{2}$. This is stated in our next Lemma.

Lemma 2 *Suppose $r < (Y - 2e)$ and the loan contract is individual liability contract. If e is sufficiently high such that $g(0) \geq 0$, the borrower's optimal signal quality is $\hat{\theta}(r)$ for all $r \geq 0$. On the other hand if $g(0) < 0$, the borrower's optimal signal quality is given by the following:*

$$\theta^* = \begin{cases} \hat{\theta}(r) & \text{if } r > r_d^I \\ \frac{1}{2} & \text{if } r \leq r_d^I \end{cases}$$

where $r_d^I \in (0, Y - 2e)$ and $\hat{\theta}(r)$ is determined from

$$Y - r = 2c'\left(\hat{\theta}\right)$$

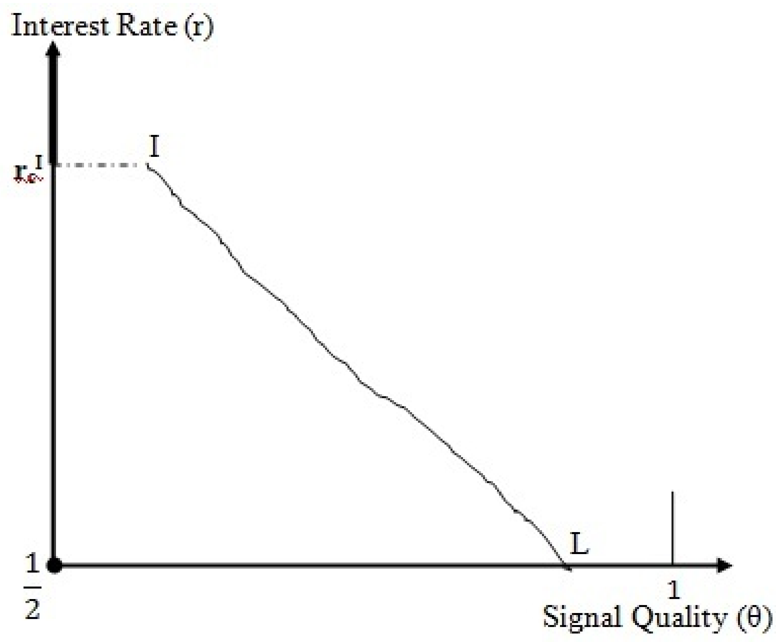

Figure 1: Borrower's choice of θ under IL Lending with $g(0) \geq 0$

The two Lemmas stated above enable us to arrive at our first proposition. If r is very high, the borrower does not take up the project irrespective of the signal realization and hence does not invest in signal quality. However, as r falls below a critical value, the borrower decides to take up the project under good signal and invests a positive amount in signal quality. The borrower's investment in signal quality rises as r falls. If the private effort cost of the project is very high, this goes on for all values of r below the critical level. Notice that in this case, the borrower takes up the project only if the realized signal is a success signal.

On the other hand if e, the cost of effort is not so high, then there is a second critical level of r below which the borrower once again stops investing in signal quality although for a completely different reason. In this case, for low values of r the borrower takes up the project irrespective of the signal and hence the signal has no value to the borrower. We state these results in the proposition below. For the ease of exposition we denote the borrower's project decision by $d(r, s) \in \{0, 1\}$ where r is the gross interest and $s \in \{S, F\}$ is the realized signal. $d = 1$ indicates that the borrower takes up the project and $d = 0$ indicates that she doesn't.

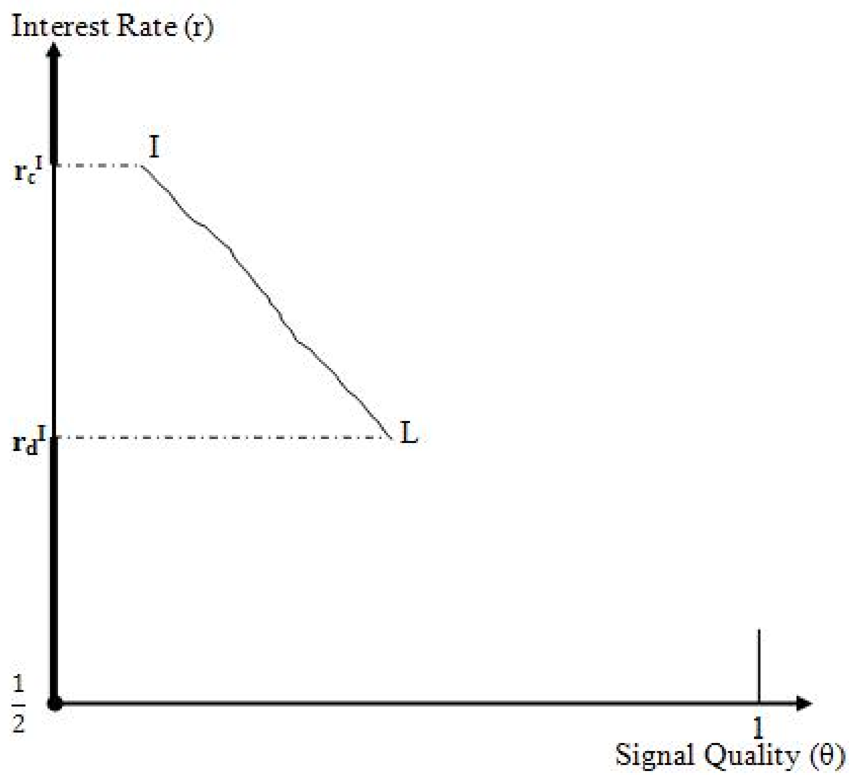

Figure 2: Borrower's choice of θ under IL Lending with $g(0) < 0$

Proposition 1 *Consider individual liability loan contracts. If e is sufficiently high such that $g(0) \geq 0$, the borrower's optimal signal quality and project deci-*

sions are given by

$$\theta^* = \begin{cases} \frac{1}{2} & \text{if } r > r_c^I \\ \hat{\theta}(r) & \text{if } r \leq r_c^I \end{cases}$$

and

$$d(r,s) = \begin{cases} 0 & \text{if } r > r_c^I \text{ and } s = S, F \\ 0 & \text{if } r \leq r_c^I \text{ and } s = F \\ 1 & \text{if } r \leq r_c^I \text{ and } s = S \end{cases}$$

If e is such that $g(0) < 0$, the borrower's optimal signal quality and project decisions are given by

$$\theta^* = \begin{cases} \frac{1}{2} & \text{if } r > r_c^I \\ \hat{\theta}(r) & \text{if } r_d^I < r \leq r_c^I \\ \frac{1}{2} & \text{if } r \leq r_d^I \end{cases}$$

and

$$d(r,s) = \begin{cases} 0 & \text{if } r > r_c^I \text{ and } s = S, F \\ 0 & \text{if } r_d^I < r \leq r_c^I \text{ and } s = F \\ 1 & \text{if } r_d^I < r \leq r_c^I \text{ and } s = S \\ 1 & \text{if } r \leq r_d^I \text{ and } s = S, F \end{cases}$$

2.2 Joint Liability Group Lending

We now consider group lending under joint liability. We restrict the size of each group to two people with groups formed voluntarily. Individuals invest independently, but the loan contract involves joint liability. Since at the time of group formation signals are publicly observable, each borrower not only knows her own signal but also observes her partner's signal.

Given this set-up, notice that if borrower i with signal quality θ_i and signal realization S partners a borrower with signal quality θ_j with signal S, her expected pay-off (in case they end up taking loans) would be

$$\begin{aligned} EU_i(\theta_i, \theta_j; s_i = S, s_j = S) &= [\theta_i\theta_j(Y-r) + \theta_i(1-\theta_j)(Y-2r)] - e - c(\theta) \\ &= \theta_i[(Y-r) - (1-\theta_j)r] - e - c(\theta) \qquad (6) \end{aligned}$$

If i's partner receives F signal her payoff is

$$\begin{aligned} EU_i(\theta_i, \theta_j; s_i = S, s_j = F) &= [\theta_i(1-\theta_j)(Y-r) + \theta_i\theta_j(Y-2r)] - e - c(\theta) \\ &= \theta_i[(Y-r) - \theta_j r] - e - c(\theta) \qquad (7) \end{aligned}$$

Since $\theta_j \geq \frac{1}{2}$,

$$EU_i(\theta_i, \theta_j; s_i = S, s_j = S) \geq EU_i(\theta_i, \theta_j; s_i = S, s_j = F)$$

Hence a borrower with signal S will always seek another borrower with signal S. Similarly, a person with F signal will also have incentive to seek a partner with S signal, but wouldn't find one since every S signal holder will seek another S signal holder. Hence, when groups are formed voluntarily, group members will have similar signals. There would be two types of groups - groups of S signal holders and groups of F signal holders.

Notice also that EU of the $i-$th borrower increases with the signal quality of her partner in both types of groups. Hence, each borrower would try to be matched with an individual with highest signal quality. If there are sufficient potential borrowers, this will lead to assortative matching - each borrower will be partnered by an individual exactly like herself - with same signal quality (θ) and same signal realization. This is stated in our next proposition.

Proposition 2 *Under joint liability loan contract with self selection of members, groups consist of homogeneous borrowers, i.e., a borrower with signal quality $\theta \in \left[\frac{1}{2}, 1\right]$ and signal realization $s \in \{S, F\}$ will be matched with a borrower with same signal quality and same signal realization.*

Once the group is formed, a borrower with a success (S) signal applies for loan only if

$$\theta\left[(Y-r)-(1-\theta)r\right]-e \geq 0 \tag{8}$$

since $c(\theta)$, the signal collection cost is already sunk. Similarly, a borrower with a failure (F) signal takes up the project only if

$$(1-\theta)\left[(Y-r)-\theta r\right]-e \geq 0 \tag{9}$$

Notice that since $\theta \geq \frac{1}{2}$, if for any r and θ (9) holds, (8) will definitely hold at the same r and θ. For $r > Y-e$, $\theta\left[(Y-r)-(1-\theta)r\right] < 0$ for every $\theta \in \left[\frac{1}{2}, 1\right]$. Hence, the borrowers will not take up projects after either signal. For these values of r, since the project is never taken up, the borrower would not invest any amount in collecting signals and thus $\theta^* = \frac{1}{2}$.

Now suppose $\frac{2}{3}(Y-2e) < r \leq (Y-e)$. Notice that the LHS of (9) is decreasing[3] in both θ and r. At $\theta = \frac{1}{2}$ and as $r \to \frac{2}{3}(Y-2e)$, the LHS of (9) becomes

$$\frac{1}{2}\left[Y-\frac{3}{2}.\frac{2}{3}(Y-2e)\right]-e=0$$

Hence for all r such that $\frac{2}{3}(Y-2e) < r \leq Y-e$, the LHS of (9) is negative. Therefore, members of the borrower groups with F signals would not take up the project for these values of r. On the other hand members of the borrower groups with signal S would take up the project only if (8) holds. Hence, in this range of r, a borrower's ex-ante expected utility (before her signal is realized) is

$$EU(\theta) = \begin{cases} -c(\theta) & \text{if } \theta\left[(Y-r)-(1-\theta)r\right] < e \\ \frac{1}{2}\left[\theta\left(Y-r-(1-\theta)r\right)-e\right]-c(\theta) & \text{if } \theta\left[(Y-r)-(1-\theta)r\right] \geq e \end{cases} \tag{10}$$

[3] It is easy to show that

$$\frac{\delta}{\delta r}(LHS) = -\left(1-\theta^2\right) \leq 0$$

for all θ and

$$\frac{\delta}{\delta \theta}(LHS) = -Y+2\theta r \leq 0$$

since $\theta \in \left[\frac{1}{2}, 1\right]$ and $r < \frac{Y}{2}$.

This expression needs some clarification. Suppose $\frac{2}{3}(Y-2e) < r \leq (Y-e)$ and a borrower decides to invest in a signal quality θ at a cost $c(\theta)$. Once the signal is realized and the borrower receives signal F (which happens with probability $\frac{1}{2}$), she will be compelled to form a group with another borrower with quality θ and signal realization F by Proposition 5.2. However, in that case she wouldn't take up the project and her investment in quality is lost. If the borrower receives the signal S, she would take up the project by incurring the effort cost only if (8) holds.

Notice that ex-ante the borrower's best choice of θ is either $\frac{1}{2}$ or is determined from the equation

$$Y - 2(1-\theta)r = 2c'(\theta) \tag{11}$$

Given our assumption that $c'(1) > \frac{Y}{2}$, we have an interior solution[4] to this expression which is denoted by $\hat{\theta}_J$. The ex-ante optimal expected utility of the borrower

$$\max\left\{0, \frac{1}{2}\left[\hat{\theta}_J\left(Y - r - (1-\hat{\theta}_J)r\right) - e\right] - c\left(\hat{\theta}_J\right)\right\}$$

Once again, to economize on notation we denote

$$g_1^J(r) = \frac{1}{2}\left[\hat{\theta}_J\left(Y - r - (1-\hat{\theta}_J)r\right) - e\right] - c\left(\hat{\theta}_J\right)$$

For any $r \leq (Y-e)$, $Y - 2(1-\theta)r \geq e$. Hence, (11) has an interior solution. However, at $r = Y - e$,

$$g_1^J(r) = \frac{1}{2}\left[\hat{\theta}_J|_{Y-e}\left(e - (1-\hat{\theta}_J|_{Y-e})r\right) - e\right] - c\left(\hat{\theta}_J|_{Y-e}\right) < 0$$

since $\hat{\theta}_J|_{Y-e} < 1$. On the other hand, at any r,

$$\begin{aligned}
g_1^J(r) &= \frac{1}{2}\left[\hat{\theta}_J\left(Y - r - (1-\hat{\theta}_J)r\right) - e\right] - c\left(\hat{\theta}_J\right) \\
&= \frac{1}{2}\left[\hat{\theta}_J\left\{Y - 2(1-\hat{\theta}_J)r - \hat{\theta}_J r\right\} - e\right] - c\left(\hat{\theta}_J\right) \\
&= \hat{\theta}_J\left\{\frac{Y - 2(1-\hat{\theta}_J)r}{2}\right\} - \frac{\hat{\theta}_J^2}{2}r - \frac{e}{2} - c\left(\hat{\theta}_J\right)
\end{aligned}$$

$$\begin{aligned}
&= \hat{\theta}_J c'\left(\hat{\theta}_J\right) - \frac{\hat{\theta}_J^2}{2}r - \frac{e}{2} - c\left(\hat{\theta}_J\right) \\
&= \left(\hat{\theta}_J - \frac{1}{2}\right)c'\left(\hat{\theta}_J\right) - \frac{\hat{\theta}_J^2}{2}r - \frac{e}{2} - c\left(\hat{\theta}_J\right) + \frac{1}{2}c'\left(\hat{\theta}_J\right) \\
&= \int_{\frac{1}{2}}^{\hat{\theta}_J}\left[c'\left(\hat{\theta}_J\right) - c'(\theta)\right]d\theta - \frac{\hat{\theta}_J^2}{2}r - \frac{e}{2} + \frac{1}{2}c'\left(\hat{\theta}_J\right)
\end{aligned}$$

[4] For the Second Order Condition, we need to assume that $c(.)$ is sufficiently convex everywhere. This can be ensured if we assume that for the relavant values of r, $c''(\theta) > r$. If we assume that $c''(\theta) > \frac{Y}{2}$ for all $\theta \in \left[\frac{1}{2}, 1\right]$, then the SOC will always be satisfied.

where the equation (11) is used in the derivation. Notice that θ is defined over $\left[\frac{1}{2},1\right]$ and $\hat{\theta}_J$ is an interior solution. This can be verified from the fact that for every $r \in (\frac{2}{3}(Y-2e),(Y-e)]$, the LHS of (11) at $\theta=\frac{1}{2}$ and $\theta=1$ are $(Y-r) \geq e > 2c'\left(\frac{1}{2}\right)$ and $Y < 2c'(1)$ respectively. Since the slope of the RHS of (11), $2c''(\theta)$, is everywhere greater than the slope of the LHS of (11), $2r$ (see Footnote 2), the solution $\hat{\theta}_J$ is unique.

Notice that since for every $\theta < \hat{\theta}_J$, the LHS of (11) is greater than $2c'(\theta)$, we can write

$$\begin{aligned}
& \int_{\frac{1}{2}}^{\hat{\theta}_J}\left[c'\left(\hat{\theta}_J\right)-c'(\theta)\right]d\theta-\frac{\hat{\theta}_J^2}{2}r-\frac{e}{2}+\frac{1}{2}c'\left(\hat{\theta}_J\right) \\
> & \int_{\frac{1}{2}}^{\hat{\theta}_J}\left[\frac{Y-2(1-\hat{\theta}_J)r}{2}-\frac{Y-2(1-\theta)r}{2}\right]d\theta-\frac{\hat{\theta}_J^2}{2}r-\frac{e}{2}+\frac{1}{2}c'\left(\hat{\theta}_J\right) \\
= & \int_{\frac{1}{2}}^{\hat{\theta}_J}\left(\hat{\theta}_J-\theta\right)rd\theta-\frac{\hat{\theta}_J^2}{2}r-\frac{e}{2}+\frac{1}{2}c'\left(\hat{\theta}_J\right) \\
= & \hat{\theta}_J\left(\hat{\theta}_J-\frac{1}{2}\right)r-\frac{1}{2}\left(\hat{\theta}_J^2-\frac{1}{4}\right)r-\frac{\hat{\theta}_J^2}{2}r-\frac{e}{2}+\frac{1}{2}c'\left(\hat{\theta}_J\right) \\
= & \frac{Y-2(1-\hat{\theta}_J)r}{4}-\frac{\hat{\theta}_J}{2}r+\frac{r}{8}-\frac{e}{2} \\
= & \frac{Y-2e}{4}-\frac{3r}{8} \\
= & 0
\end{aligned}$$

at $r=\frac{2}{3}(Y-2e)$. Hence, as $r \to \frac{2}{3}(Y-2e)$, $g_1^J(r) > 0$. Since $g_1^J(r)$ is continuous in r there exists $r_c^J \in (\frac{2}{3}(Y-2e), Y-e)$ such that $g_1^J(r) \geq 0$ for $r \leq r_c^J$. Moreover, whenever $g_1^J(r) \geq 0$, the corresponding $\hat{\theta}_J(r)$ satisfies (8). We now state this result in Lemma 3.

Lemma 3 *Suppose $r > \frac{2}{3}(Y-2e)$. The borrower's optimal signal quality is given by the following:*

$$\theta^* = \begin{cases} \frac{1}{2} & \text{if } r > r_c^J \\ \hat{\theta}_J(r) & \text{if } r \leq r_c^I \end{cases}$$

where $r_c^J \in (\frac{2}{3}(Y-2e), Y-e)$ and $\hat{\theta}_J(r)$ is determined from

$$Y-2\left(1-\hat{\theta}_J\right)r = 2c'\left(\hat{\theta}_J\right)$$

Finally, we consider the case when $r \leq \frac{2}{3}(Y-2e)$. Once again for these values of r, a borrower would take up the project after S signal for all values of θ since (8) holds for all θ. On the other hand the project would be taken up after F only if θ satisfies (9). Hence, the ex-ante expected utility of the borrower can be written as

$$EU(\theta) = \begin{cases} \frac{1}{2}\left[\theta\left\{Y - r - (1-\theta)r\right\} - e\right] - c(\theta) & \text{if } (1-\theta)\left[(Y-r) - \theta r\right] < e \\ \begin{array}{c} \frac{1}{2}\left[\theta\left\{Y - r - (1-\theta)r\right\} - e\right] \\ +\frac{1}{2}\left[(1-\theta)\left\{Y - r - \theta r\right\} - e\right] - c(\theta) \end{array} & \text{if } (1-\theta)\left[(Y-r) - \theta r\right] \geq e \end{cases} \tag{12}$$

which can be simplified to

$$EU(\theta) = \begin{cases} \frac{1}{2}\theta\left\{Y - (2-\theta)r\right\} - e - c(\theta) & \text{if } (1-\theta)\left[(Y-r) - \theta r\right] < e \\ \frac{Y-r}{2} - \theta(1-\theta)r - e - c(\theta) & \text{if } (1-\theta)\left[(Y-r) - \theta r\right] \geq e \end{cases} \tag{13}$$

Unlike in the individual liability case, even when the borrower decides to take up the project irrespective of the signal realization, he may have an incentive to invest in signal quality in joint liability lending.

Under individual liability, the signal has a single purpose. It tells the borrower the likelihood of project success under different signal realization and thus helps the borrower in her decision regarding taking up of the project. If the likelihood of project success is low, the borrower can save her own effort cost by not taking up the project. If the borrower decides to take up the project irrespective of the signal realization, there is no point in investing in signal quality under individual liability.

In case of joint liability, the choice of signal quality also determines one's partner (see Proposition 5.2). Investment in signal quality thus not only improves one's own information about the likelihood of project success, but also helps in finding a more informed partner whose success or failure in turn determines the borrower's own payoff. Thus even when the borrower decides to take up the project for both signal realizations, she may have an incentive to invest in signal quality because of this second effect. However, this incentive gets diminished as r falls since the joint liability payment falls with r. In our analysis, we assume that the cost function is such that the first effect dominates and thus if the borrower decides to take up the project irrespective of the signal realization, she does not invest in signal quality.

Let us look at a borrower's decision more closely. Suppose the borrower decides to take up the project under both signal realizations. Then she chooses θ to maximize

$$\phi(\theta) = \frac{Y-r}{2} - \theta(1-\theta)r - e - c(\theta) \tag{14}$$

subject to (9). Suppose that the solution to the above problem is denoted by $\tilde{\theta}(r)$. Since

$$\phi'\left(\frac{1}{2}\right) = -c'\left(\frac{1}{2}\right) \leq 0 \tag{15}$$

$\tilde{\theta}(r) = \frac{1}{2}$ is one solution to the problem. Notice that $\tilde{\theta}(r) = \frac{1}{2}$ satisfies (9) for all $r \leq \frac{2}{3}(Y - 2e)$. The solution is unique if $\phi(\theta)$ is concave in θ. Since $\phi''(\theta) = 2r - c''(\theta)$, to ensure concavity of $\phi(\theta)$, we need $c''(\theta) > 2r$. This definitely holds for very low values of r, but the assumption made earlier -

$c''(\theta) > \frac{Y}{2}$ - cannot guarantee the concavity of $\phi(\theta)$ for all $r \leq \frac{2}{3}(Y - 2e)$. For the sake of avoiding analytical complication, we assume that this is indeed the case[5] as discussed above.

We use the notation $g_2^J(r)$ to denote the optimized value of $\phi(\theta)$. Since by virtue of the assumption mentioned above $\tilde{\theta}(r) = \frac{1}{2}$ for all $r \leq \frac{2}{3}(Y - 2e)$, we can write

$$g_2^J(r) = \frac{Y}{2} - \frac{3r}{4} - e \tag{16}$$

Alternatively, the borrower may decide to take up the project only if she receives a success signal and in that case her expected utility is

$$g_1^J(r) = \frac{1}{2}\left[\hat{\theta}_J(r)\left(Y - r - (1 - \hat{\theta}_J(r))r\right) - e\right] - c\left(\hat{\theta}_J(r)\right) \tag{17}$$

Essentially, the borrower's optimal ex-ante expected utility under joint liability lending for $r \leq \frac{2}{3}(Y - 2e)$ is given by

$$\max\left\{g_1^J(r), g_2^J(r)\right\}$$

We now define $g_J(r) = g_1^J(r) - g_2^J(r)$. Notice that

$$\begin{aligned} g_J'(r) &= -\frac{1}{2}\hat{\theta}_J(2 - \hat{\theta}_J) + \frac{3}{4} \\ &= \frac{1}{2}\left[\frac{3}{2} - \hat{\theta}_J(2 - \hat{\theta}_J)\right] > 0 \end{aligned} \tag{18}$$

since $2 - \hat{\theta}_J < \frac{3}{2}$ and $\hat{\theta}_J < 1$. Hence as r falls $g_J(r)$ falls. As we have already shown before Lemma 3, $g_1^J(r) > 0$ at $r = \frac{2}{3}(Y - 2e)$ while $g_2^J(r) = 0$. However, at $r = 0$,

$$g_J(r) = \frac{1}{2}\left[\hat{\theta}_J(0)Y - e\right] - c\left(\hat{\theta}_J(0)\right) - \left(\frac{Y}{2} - e\right) \tag{19}$$

A comparison of (4) and (11) immediately tells us that at $r = 0$, $\hat{\theta}_J(0) = \hat{\theta}(0)$ and thus

$$\begin{aligned} g_J(0) &= g(0) \\ &= \left(\hat{\theta}(0) - \frac{1}{2}\right)\frac{Y}{2} - c\left(\hat{\theta}(0)\right) - \frac{1}{2}\left(\frac{Y}{2} - e\right) \end{aligned} \tag{20}$$

As r falls, $g_J(r)$ falls. If $g(0) \geq 0$, then for every $r \leq Y - 2e$, the borrower collects $\theta^* = \hat{\theta}_J(r)$ and takes up the project only if a success signal is realized

[5] We are not losing much by making this assumption. If it does not hold, then we may have multiple local maxima for relatively higher values of r. For any given value of r, the borrower would choose θ that gives her highest expected utility among these local maximums. However, the value function (optimized expected utility function) will be continuous in r, though the optimal choice of $\tilde{\theta}(r)$ would change discontinuously as r falls. When r is low enough, $\tilde{\theta}(r) = \frac{1}{2}$. Unless we make the assumption mentioned above, there might be a range of r, in which $\tilde{\theta}(r) > \frac{1}{2}$ and falls as r falls. Once r falls below a critical level, $\tilde{\theta}(r)$ falls to $\frac{1}{2}$ and remains there for all lower values of r.

under joint liability. However, if $g(0) < 0$, then there exists $r_d^J \in \left(0, \frac{2}{3}(Y - 2e)\right)$ such that the borrower collects $\theta^* = \hat{\theta}_J(r)$ if and only if $r > r_d^J$. Otherwise, she collects $\theta^* = \frac{1}{2}$. This is stated in our next Lemma.

Lemma 4 *Suppose $r \leq Y - 2e$ and the loan contract is joint liability contract with full liability. If e is sufficiently high such that $g(0) \geq 0$, the borrower's optimal signal quality is $\hat{\theta}_J(r)$ for all $r \geq 0$. On the other hand if $g(0) < 0$, the borrower's optimal signal quality is given by the following:*

$$\theta^* = \begin{cases} \hat{\theta}_J(r) & \textit{if } r > r_d^J \\ \frac{1}{2} & \textit{if } r \leq r_d^J \end{cases}$$

where $r_d^J \in (0, \frac{2}{3}(Y - 2e))$ and $\hat{\theta}_J(r)$ is determined from

$$Y - 2(1 - \theta) r = 2c'(\theta)$$

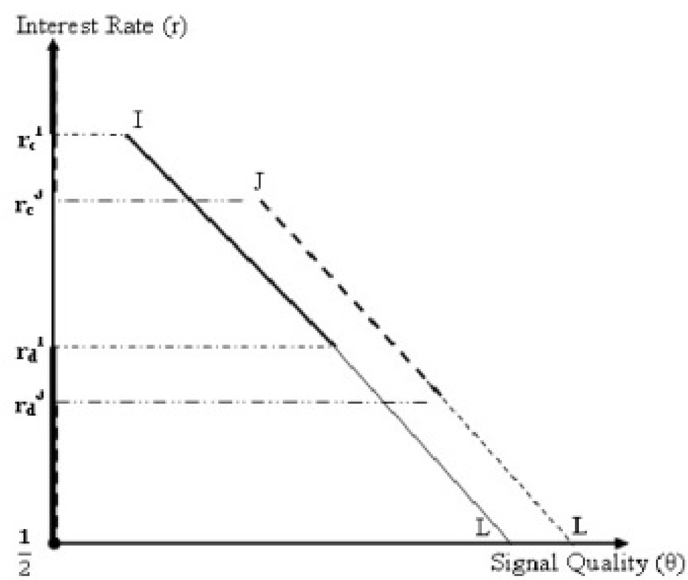

Figure 3: Borrower's choice of θ under IL Lending and JL Lending

The decision of an individual borrower regarding choice of signal quality and the project choice decision under joint liability loan contract can be summarized in the following proposition:

Proposition 3 *Consider joint liability loan contracts. If e is sufficiently high such that $g(0) \geq 0$, the borrower's optimal signal quality and project decisions are given by*

$$\theta^* = \begin{cases} \frac{1}{2} & \textit{if } r > r_c^J \\ \hat{\theta}_J(r) & \textit{if } r \leq r_c^J \end{cases}$$

and

$$d(r,s) = \begin{cases} 0 & \text{if } r > r_c^J \text{ and } s = S, F \\ 0 & \text{if } r \leq r_c^J \text{ and } s = F \\ 1 & \text{if } r \leq r_c^J \text{ and } s = S \end{cases}$$

If e is such that $g(0) < 0$, the borrower's optimal signal quality and project decisions are given by

$$\theta^* = \begin{cases} \frac{1}{2} & \text{if } r > r_c^J \\ \hat{\theta}_J(r) & \text{if } r_d^J < r \leq r_c^J \\ \frac{1}{2} & \text{if } r \leq r_d^J \end{cases}$$

and

$$d(r,s) = \begin{cases} 0 & \text{if } r > r_c^J \text{ and } s = S, F \\ 0 & \text{if } r_d^J < r \leq r_c^J \text{ and } s = F \\ 1 & \text{if } r_d^J < r \leq r_c^J \text{ and } s = S \\ 1 & \text{if } r \leq r_d^J \text{ and } s = S, F \end{cases}$$

3 Comparisons between Individual and Joint Liability Loan Contracts

We have seen from Propositions 1 and 3 that under both types of loan contracts a borrower invests in signal quality if she decides to take up the project only under S signal. If she decides to take up the project irrespective of her signal realizations, she does not invest in information. However, if a borrower decides to invest in signal quality under both types of loan contract, she invests more under joint liability. This is stated in our next proposition.

Proposition 4 *If at any r, both $\hat{\theta}(r)$ and $\hat{\theta}_J(r)$ are greater than $\frac{1}{2}$, $\hat{\theta}_J(r) > \hat{\theta}(r)$.*

Proof. Notice that $\hat{\theta}(r)$ and $\hat{\theta}_J(r)$ are determined from

$$Y - r = 2c'(\theta)$$

and

$$Y - 2(1-\theta)r = 2c'(\theta)$$

respectively. Since $Y - 2(1-\theta)r > Y - r$ for all r and $\theta \in (\frac{1}{2}, 1]$ and $c''(\theta) > 0$, the result holds. ■

Notice that at the interest rates $\left(r_c^I, r_c^J\right)$ and $\left(r_d^I, r_d^J\right)$ are critical levels at which at which the borrowers switch from *no-investment* to *investment* in signal quality or from *investment* to *no-investment* respectively under two types of loan contract. We now compare these switching rates of interest under the two regimes. We call these interest rates as the switching rates of interest.

Proposition 5 *The switching rates of interest are higher under individual liability lending than under joint liability lending, i.e. $r_c^I > r_c^J$ and when $g(0) < 0$, $r_d^I > r_d^J$.*

Proof. Notice that at $r = r_c^I$,

$$g_1(r) = \frac{1}{2}\left[\hat{\theta}(r)(Y-r) - e\right] - c\left(\hat{\theta}(r)\right) = 0$$

and

$$\begin{aligned}
g_1^J(r) &= \frac{1}{2}\left[\hat{\theta}_J(r)\left(Y - r - (1-\hat{\theta}_J(r))r\right) - e\right] - c\left(\hat{\theta}_J(r)\right) \\
&= \frac{1}{2}\left[\hat{\theta}_J(r)(Y-r) - e\right] - c\left(\hat{\theta}_J(r)\right) - \hat{\theta}_J(r)(1-\hat{\theta}_J(r))r \\
&< \frac{1}{2}\left[\hat{\theta}_J(r)(Y-r) - e\right] - c\left(\hat{\theta}_J(r)\right) \\
&\leq \frac{1}{2}\left[\hat{\theta}(r)(Y-r) - e\right] - c\left(\hat{\theta}(r)\right) \\
&= 0
\end{aligned}$$

where the last inequality follows from the fact that $\hat{\theta}(r)$ maximizes $\frac{1}{2}[\theta(Y-r)-e] - c(\theta)$. Since $g_1^J(r_c^I) < 0$, $g_1^{J\prime}(r) < 0$ and $g_1^J(r_c^J) = 0$, it must be the case that $r_c^I > r_c^J$.

Suppose $g(0) < 0$. Notice that

$$g^J(r) = \frac{1}{2}\left[\hat{\theta}_J(r)\left(Y - r - (1-\hat{\theta}_J(r))r\right) - e\right] - c\left(\hat{\theta}_J(r)\right) - \left(\frac{Y}{2} - \frac{3r}{4} - e\right)$$

and

$$g(r) = \frac{1}{2}\left[\hat{\theta}(r)(Y-r) - e\right] - c\left(\hat{\theta}(r)\right) - \left(\frac{Y}{2} - \frac{r}{2} - e\right)$$

Hence, for any r, $g^J(r) > g(r)$ if and only if

$$\begin{aligned}
&\frac{1}{2}\left[\hat{\theta}_J(r)\left(Y - r - (1-\hat{\theta}_J(r))r\right) - e\right] - c\left(\hat{\theta}_J(r)\right) - \left(\frac{Y}{2} - \frac{3r}{4} - e\right) \\
> &\frac{1}{2}\left[\hat{\theta}(r)(Y-r) - e\right] - c\left(\hat{\theta}(r)\right) - \left(\frac{Y}{2} - \frac{r}{2} - e\right)
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow\ & \frac{r}{4} - \frac{1}{2}\hat{\theta}_J(r)(1-\hat{\theta}_J(r))r \\
>\ & c\left(\hat{\theta}_J(r)\right) - c\left(\hat{\theta}(r)\right) - \left(\hat{\theta}_J(r) - \hat{\theta}(r)\right)\frac{Y-r}{2}
\end{aligned}$$

Now, the RHS of the above expression

$$\begin{aligned}
& c\left(\hat{\theta}_J(r)\right) - c\left(\hat{\theta}(r)\right) - \left(\hat{\theta}_J(r) - \hat{\theta}(r)\right)\frac{Y-r}{2} \\
=\ & \int_{\hat{\theta}}^{\hat{\theta}_J}\left[c'(\theta) - \frac{Y-r}{2}\right]d\theta \\
<\ & \int_{\hat{\theta}}^{\hat{\theta}_J}\left[c'\left(\hat{\theta}_J\right) - \frac{Y-r}{2}\right]d\theta \ \ [\text{since } c''(\theta) > 0]
\end{aligned}$$

$$
\begin{aligned}
&= \int_{\hat{\theta}}^{\hat{\theta}_J} \left[\frac{Y - 2\left(1-\hat{\theta}_J\right) r}{2} - \frac{Y-r}{2} \right] d\theta \text{ [from (11)]} \\
&= \int_{\hat{\theta}}^{\hat{\theta}_J} \frac{\left(2\hat{\theta}_J - 1\right) r}{2} d\theta \\
&= \left(\hat{\theta}_J - \frac{1}{2}\right)\left(\hat{\theta}_J - \hat{\theta}\right) r
\end{aligned}
$$

We now show that for any $r > 0$,

$$
\frac{r}{4} - \frac{1}{2}\hat{\theta}_J(1-\hat{\theta}_J)r > \left(\hat{\theta}_J - \frac{1}{2}\right)\left(\hat{\theta}_J - \hat{\theta}\right) r
$$

$$
\Leftrightarrow \frac{1}{2} > \hat{\theta}_J^2 - 2\hat{\theta}_J\hat{\theta} + \hat{\theta}
$$

which holds since $\hat{\theta}_J, \hat{\theta} \in \left(\frac{1}{2}, 1\right)$. Hence, we can conclude that for any $r > 0$, $g^J(r) > g(r)$. Since, at $r = r_d^I$, $g(r) = 0$, we can conclude that $g_J\left(r_d^I\right) > 0$. Since $g_J'(r) > 0$ and $g_J\left(r_d^J\right) = 0$, it must be the case that $r_d^J < r_d^I$. ■

The interest rate at which the borrower starts investing in signal quality is higher under individual liability.

4 Lender's choice problem

So far we have not discussed the lender's choice problem. The lender's choice of rate of interest, r, depends on lender's objective. We consider two types of monopoly lender. A benevolent lender attempts to maximize borrower's utility subject to a break-even constraint. A profit-motivated lender on the other hand tries to maximize her own profit by her choice of interest rate. We first analyze interest choice of two lender types for both individual liability as well as joint liability loan contracts. In the next section, we compare the equilibrium choices under the two types of loan contracts and their implications on borrower's as well as lender's payoffs.

Suppose the cost of one unit of loanable funds is α. First consider individual liability loan contract. If $g(0) \geq 0$, the expected profit of a lender can be written as

$$
E\Pi_I = \begin{cases} 0 & \text{if } r > r_c^I \\ \frac{1}{2}\left[\hat{\theta}(r) r - \alpha\right] & \text{if } r_c^I \geq r \end{cases} \tag{21}
$$

On the other hand if $g(0) < 0$, the expected profit of a lender is

$$
E\Pi_I = \begin{cases} 0 & \text{if } r > r_c^I \\ \frac{1}{2}\left[\hat{\theta}(r) r - \alpha\right] & \text{if } r_c^I \geq r > r_d^I \\ \frac{1}{2} r - \alpha & \text{if } r_d^I \geq r \end{cases} \tag{22}
$$

If $r > r_c^I$, no borrower would apply for loan. If $r_c^I \geq r > r_d^I$, a borrower would choose $\hat{\theta}$ as her signal quality and then would apply for loan only if she receives signal S which happens with probability $\frac{1}{2}$. A borrower with $\hat{\theta}$ signal quality and signal S succeeds with probability $\hat{\theta}$ and only then pays back the loan under limited liability.

Notice that $\left[\hat{\theta}(r)r - \alpha\right]$ is rising in r. From the borrower's FOC (4),

$$\hat{\theta}'(r) = -\frac{1}{2c''\left(\hat{\theta}\right)}$$

Hence,

$$\frac{d}{dr}\left[\hat{\theta}(r)r - \alpha\right] = \hat{\theta}(r) + r\hat{\theta}'(r) = \hat{\theta}(r) - \frac{r}{2c''\left(\hat{\theta}\right)}$$

By our assumption that $c''(\theta) > \frac{Y}{2}$ and since $r < \frac{Y}{2}$, $\left(\frac{r}{2c''(\hat{\theta})}\right) < \frac{1}{2}$. Since $\hat{\theta}(r) > \frac{1}{2}$, $\frac{d}{dr}\left[\hat{\theta}(r)r - \alpha\right] > 0$. Hence a profit-motivated lender would choose either r_c^I or r_d^I depending on which interest rate gives her higher profit. Notice that

$$\frac{1}{2}\left[\hat{\theta}\left(r_c^I\right)r_c^I - \alpha\right] > \frac{1}{2}r_d^I - \alpha$$

if and only if

$$\alpha > r_d^I - \hat{\theta}\left(r_c^I\right)r_c^I$$

On the other hand, a benevolent lender tries to maximize the borrower's utility subject to the break even constraint. Since the borrower's expected utility rises as r falls, the lender would try to set the lowest interest such that she breaks even. If $\alpha < \left(\frac{1}{2}r_d^I\right)$, the lender would choose $r = 2\alpha$. If on the other hand, $\alpha \geq \left(\frac{1}{2}r_d^I\right)$, the lender would choose the lowest interest such that she breaks even. If $\hat{\theta}\left(r_d^I\right)r_d^I \geq \alpha$ (which is a possibility even when $\alpha \geq \left(\frac{1}{2}r_d^I\right)$ since $\hat{\theta}\left(r_d^I\right) > \frac{1}{2}$), she would choose r_d^I. Otherwise, she would choose r such that $\hat{\theta}(r)r = \alpha$.

Combining all of the above, we can now express the market equilibrium under individual liability lending in the next proposition.

Proposition 6 *Under individual liability lending, the equilibrium rate of interest in different scenarios can be expressed as follows:*

1. *Suppose $g(0) \geq 0$. For every value of $\alpha \leq \hat{\theta}\left(r_c^I\right)r_c^I$, a profit motivated lender chooses $r_\pi^* = r_c^I$ while a benevolent lender chooses $r_B^* = r_0$ such that $\hat{\theta}(r_0)r_0 = \alpha$.*

2. *Suppose $g(0) < 0$. The profit motivated and benevolent lender's choice of interest rates, r_π^* and r_B^* are*

$$r_\pi^* = \begin{cases} r_c^I, & \text{if } r_d^I - \hat{\theta}\left(r_c^I\right)r_c^I < \alpha \\ r_d^I, & \text{if } \alpha \leq r_d^I - \hat{\theta}\left(r_c^I\right)r_c^I \end{cases}$$

and

$$r_B^* = \begin{cases} 2\alpha, & \textit{if } \alpha \leq \frac{r_d^I}{2} \\ r_d^I, & \textit{if } \frac{r_d^I}{2} < \alpha \leq \hat{\theta}\left(r_d^I\right) r_d^I \\ r_0, & \textit{if } \hat{\theta}\left(r_d^I\right) r_d^I < \alpha \end{cases}$$

The proof of the proposition follows from the discussion above. One interesting case arises when $\alpha \in \left(\frac{r_d^I}{2}, \hat{\theta}\left(r_d^I\right) r_d^I\right)$. Even a benevolent lender charges $r^* = r_d^I$ and makes some profit in such a scenario, though the borrower's utility rises with a fall in r. Further reduction in interest vanishes the borrower's incentive to invest in signal quality and as a result the borrower takes up the loan irrespective of signal realization. This raises the lender's risk discretely and as a result the lender can no longer break even.

Notice the case when $g(0) < 0$. If the equilibrium interest is less than or equal to r_d^I, the borrower no longer invests in signal quality. When the lender is profit-motivated, she charges r_d^I, only if $\alpha \leq \left[r_d^I - \hat{\theta}\left(r_c^I\right) r_c^I\right]$. A benevolent lender on the other hand charges an interest less than or equal to r_d^I whenever $\alpha \leq \hat{\theta}\left(r_d^I\right) r_d^I$. It is easy to show that[6] $\left[r_d^I - \hat{\theta}\left(r_c^I\right) r_c^I\right] < \left[\hat{\theta}\left(r_d^I\right) r_d^I\right]$. This leads to an important observation.

Observation Under individual liability lending, a profit-motivated lender induces the borrower to invest more often in information collection than a benevolent lender.

We now consider joint liability lending. If $g(0) \geq 0$, the expected profit of a lender now can be written as

$$E\Pi_J = \begin{cases} 0 & \text{if } r > r_c^J \\ \frac{1}{2}\left[\hat{\theta}_J(r)\left(2 - \hat{\theta}_J(r)\right) r - \alpha\right] & \text{if } r_c^J \geq r \end{cases} \tag{23}$$

On the other hand if $g(0) < 0$, the expected profit of a lender under joint liability lending is

$$E\Pi_J = \begin{cases} 0 & \text{if } r > r_c^J \\ \frac{1}{2}\left[\hat{\theta}_J(r)\left(2 - \hat{\theta}_J(r)\right) r - \alpha\right] & \text{if } r_c^J \geq r > r_d^J \\ \frac{3}{4} r - \alpha & \text{if } r_d^J \geq r \end{cases} \tag{24}$$

Once again, we can show that given our assumption[7] $c''(\theta) > 2r$, $\hat{\theta}_J(r)\left(2 - \hat{\theta}_J(r)\right) r$ is rising in r. To see this, notice that

$$\hat{\theta}_J'(r) = -\left(1 - \hat{\theta}_J\right) \frac{r}{c''\left(\hat{\theta}_J\right) - r}$$

[6] Notice that $r_d^I - \hat{\theta}\left(r_c^I\right) r_c^I < \hat{\theta}\left(r_d^I\right) r_d^I$ if and only if $\left[1 - \hat{\theta}\left(r_d^I\right)\right] r_d^I < \hat{\theta}\left(r_c^I\right) r_c^I$ which always holds since $1 - \hat{\theta}\left(r_d^I\right) < \frac{1}{2} < \hat{\theta}\left(r_c^I\right)$ and $r_d^I < r_c^I$.

[7] We have made this assumption to ensure that for all values of r, the borrower's optimization problem has a unique solution.

which can be derived from (11). Therefore,

$$\begin{aligned}
&\frac{d}{dr}\left[\hat{\theta}_J(r)\left(2-\hat{\theta}_J(r)\right)r-\alpha\right] \\
=\ & \hat{\theta}_J(r)\left(2-\hat{\theta}_J(r)\right)+2\left(1-\hat{\theta}_J(r)\right)r\hat{\theta}_J'(r) \\
=\ & \hat{\theta}_J(r)\left(2-\hat{\theta}_J(r)\right)-2\left(1-\hat{\theta}_J(r)\right)^2\frac{r}{c''\left(\hat{\theta}_J(r)\right)-r}
\end{aligned}$$

Since $\left[\frac{r}{c''\left(\hat{\theta}_J(r)\right)-r}\right]<1$,

$$2\left(1-\hat{\theta}_J(r)\right)^2\frac{r}{c''\left(\hat{\theta}_J(r)\right)-r}<2\left(1-\hat{\theta}_J(r)\right)^2$$

Now,

$$\begin{aligned}
&\left[\hat{\theta}_J(r)\left(2-\hat{\theta}_J(r)\right)\right]-\left[2\left(1-\hat{\theta}_J(r)\right)^2\right] \\
=\ & \left[6\hat{\theta}_J(r)-3\left(\hat{\theta}_J(r)\right)^2-2\right]>0
\end{aligned}$$

for all $\hat{\theta}_J(r)\in\left[\frac{1}{2},1\right]$. This proves that

$$\frac{d}{dr}\left[\hat{\theta}_J(r)\left(2-\hat{\theta}_J(r)\right)r-\alpha\right]>0$$

The above argument establishes that a profit motivated lender would choose either r_c^J or r_d^J under joint liability lending. The profit motivated lender chooses $r_\pi^*=r_c^J$ if and only if

$$\begin{aligned}
\frac{1}{2}\left[\hat{\theta}_J\left(r_c^J\right)\left(2-\hat{\theta}_J\left(r_c^J\right)\right)r_c^J-\alpha\right] &> \frac{3}{4}r_d^J-\alpha \\
\Leftrightarrow \alpha &> \frac{3}{2}r_d^J-\hat{\theta}_J\left(r_c^J\right)\left(2-\hat{\theta}_J\left(r_c^J\right)\right)r_c^J
\end{aligned}$$

Now consider a benevolent lender. For $\alpha<\frac{3}{4}r_d^J$, the benevolent lender would choose $r_B^*=\frac{4\alpha}{3}$. If $\frac{3}{4}r_d^J\leq\alpha\leq\hat{\theta}_J\left(r_d^J\right)\left(2-\hat{\theta}_J\left(r_d^J\right)\right)r_d^J$, the lender would choose the lowest interest such that she breaks even which is $r_B^*=r_d^J$. Finally if $\alpha>\hat{\theta}_J\left(r_d^J\right)\left(2-\hat{\theta}_J\left(r_d^J\right)\right)r_d^J$, $r_B^*=\theta_0^J$ such that $\hat{\theta}_J\left(r_0^J\right)\left(2-\hat{\theta}_J\left(r_0^J\right)\right)r_0^J=\alpha$.

We can now state the equilibrium interest choice of two types of lender under joint liability in our next proposition.

Proposition 7 *Under joint liability lending, the equilibrium rate of interest in different scenarios can be expressed as follows:*

1. *Suppose $g(0) \geq 0$. For every value of $\alpha \leq \hat{\theta}_J\left(r_c^J\right)\left(2-\hat{\theta}_J\left(r_c^J\right)\right)r_c^J$, a profit motivated lender chooses $r_\pi^* = r_c^J$ while a benevolent lender chooses $r_B^* = r_0^J$ such that $\hat{\theta}_J\left(r_0^J\right)\left(2-\hat{\theta}_J\left(r_0^J\right)\right)r_0^J = \alpha$.*

2. *Suppose $g(0) < 0$. The profit motivated and benevolent lender's choice of interest rates, r_π^* and r_B^* are*

$$r_\pi^* = \begin{cases} r_c^J, & \text{if } \alpha > \frac{3}{2}r_d^J - \hat{\theta}_J\left(r_c^J\right)\left(2-\hat{\theta}_J\left(r_c^J\right)\right)r_c^J \\ r_d^J, & \text{if } \alpha \leq \frac{3}{2}r_d^J - \hat{\theta}_J\left(r_c^J\right)\left(2-\hat{\theta}_J\left(r_c^J\right)\right)r_c^J \end{cases}$$

and

$$r_B^* = \begin{cases} \frac{4\alpha}{3}, & \text{if } \alpha < \frac{3}{4}r_d^J \\ r_d^J, & \text{if } \frac{3}{4}r_d^J \leq \alpha \leq \hat{\theta}_J\left(r_d^J\right)\left(2-\hat{\theta}_J\left(r_d^J\right)\right)r_d^J \\ r_0^J, & \text{if } \hat{\theta}_J\left(r_d^J\right)\left(2-\hat{\theta}_J\left(r_d^J\right)\right)r_d^J < \alpha \end{cases}$$

A comprehensive comparison of the decisions regarding the two types of loan contracts by two types of lenders is problematic at this point. We only know that $r_d^J < r_d^I$ and $r_c^J < r_c^I$ which are not enough for the purpose. However, we identify some interesting cases. If $g(0) \geq 0$ both profit motivated lender as well as the benevolent lender would opt for joint liability lending. But while the profit motivated lender chooses $r_\pi^* = r_c^J$, the benevolent lender would choose $r_B^* = r_0^J$. Notice that the profit motivated lender would choose either $r_\pi^* = r_c^I$ or $r_\pi^* = r_c^J$ in this case. The expected profit under individual liability and joint liability can be expressed as

$$E\Pi_I = \frac{1}{2}\left[\hat{\theta}\left(r_c^I\right)r_c^I - \alpha\right]$$

and

$$E\Pi_J = \frac{1}{2}\left[\hat{\theta}_J\left(r_c^J\right)\left(2-\hat{\theta}_J\left(r_c^J\right)\right)r_c^J - \alpha\right]$$

respectively. Notice that r_c^I and r_c^J are defined from

$$g_1^J\left(r_c^I\right) = \frac{1}{2}\left[\hat{\theta}\left(r_c^I\right)\left(Y - r_c^I\right) - e\right] - c\left(\hat{\theta}\left(r_c^I\right)\right) = 0$$

and

$$g_1\left(r_c^J\right) = \frac{1}{2}\left[\hat{\theta}_J\left(r_c^J\right)\left(Y - \left(2-\hat{\theta}_J\left(r_c^J\right)\right)r_c^J\right) - e\right] - c\left(\hat{\theta}_J\left(r_c^J\right)\right) = 0$$

Using these we can write

$$\begin{aligned} & E\Pi_J - E\Pi_I \\ = \; & \frac{1}{2}\left[\hat{\theta}\left(r_c^J\right)\left(2-\hat{\theta}_J\left(r_c^J\right)\right)r_c^J - \hat{\theta}\left(r_c^I\right)r_c^I\right] \\ = \; & \left[\frac{1}{2}\hat{\theta}_J\left(r_c^J\right)Y - c\left(\hat{\theta}_J\left(r_c^J\right)\right)\right] - \left[\frac{1}{2}\hat{\theta}\left(r_c^I\right)Y - c\left(\hat{\theta}\left(r_c^I\right)\right)\right] \\ = \; & \int_{\hat{\theta}\left(r_c^I\right)}^{\hat{\theta}_J\left(r_c^J\right)}\left[\frac{Y}{2} - c'(\theta)\right]d\theta > 0 \end{aligned}$$

since $\frac{Y}{2} > c'(\theta)$ for all $\theta \in \left[\hat{\theta}\left(r_c^I\right), \hat{\theta}_J\left(r_c^J\right)\right]$.

A benevolent lender is interested in maximizing the borrower's pay-off subject to the break even constraint. Notice that the lender breaks even at r_0 under individual liability and at r_0^J under joint liability. The borrower's expected utility in these cases are

$$\begin{aligned} EU_I &= \frac{1}{2}\left[\hat{\theta}(r_0)(Y - r_0) - e\right] - c\left(\hat{\theta}(r_0)\right) \\ &= \frac{1}{2}\left[\hat{\theta}(r_0)Y - \alpha - e\right] - c\left(\hat{\theta}(r_0)\right) \end{aligned} \tag{25}$$

and

$$\begin{aligned} EU_J &= \frac{1}{2}\left[\hat{\theta}_J\left(r_0^J\right)\left(Y - \left(2 - \hat{\theta}_J\left(r_0^J\right)\right)r_0^J\right) - e\right] - c\left(\hat{\theta}_J\left(r_0^J\right)\right) \\ &= \frac{1}{2}\left[\hat{\theta}_J\left(r_0^J\right)Y - \alpha - e\right] - c\left(\hat{\theta}\left(r_0^J\right)\right) \end{aligned} \tag{26}$$

since $\hat{\theta}(r_0)r_0 = \alpha$ and $\left[\hat{\theta}_J\left(r_0^J\right)\left(2 - \hat{\theta}_J\left(r_0^J\right)\right)r_0^J\right] = \alpha$. Hence,

$$\begin{aligned} & EU_J - EU_I \\ =& \left[\frac{1}{2}\hat{\theta}_J\left(r_0^J\right)Y - c\left(\hat{\theta}_J\left(r_0^J\right)\right)\right] - \left[\frac{1}{2}\hat{\theta}(r_0)Y - c\left(\hat{\theta}(r_0)\right)\right] \\ =& \int_{\hat{\theta}(r_0)}^{\hat{\theta}_J\left(r_0^J\right)}\left[\frac{Y}{2} - c'(\theta)\right]d\theta > 0 \end{aligned}$$

since $\hat{\theta}(r_0) < \hat{\theta}_J\left(r_0^J\right)$ and $\frac{Y}{2} > c'(\theta)$ for all $\theta \in \left[\hat{\theta}(r_0), \hat{\theta}_J\left(r_0^J\right)\right]$. Thus a benevolent lender would choose $r_B^* = r_0^J$. This is summarized in our next proposition.

Proposition 8 *Suppose $g(0) \geq 0$. Then both the profit motivated and the benevolent lender would choose joint liability lending over individual liability lending.*

The above result is standard and on expected line. However, there are other possibilities which cannot be ignored. Consider the case when $g(0) < 0$. In this case r_d^I and r_d^J both are positive. We cannot rule out the possibility that $r_d^I > \frac{3}{2}r_d^J$. Suppose this is the case. Since $\hat{\theta}\left(r_c^I\right)r_c^I < \hat{\theta}_J\left(r_c^J\right)\left(2 - \hat{\theta}_J\left(r_c^J\right)\right)r_c^J$,

$$r_d^I - \hat{\theta}\left(r_c^I\right)r_c^I > \frac{3}{2}r_d^J - \hat{\theta}_J\left(r_c^J\right)\left(2 - \hat{\theta}_J\left(r_c^J\right)\right)r_c^J$$

Now suppose $\alpha < \frac{3}{2}r_d^J - \hat{\theta}_J\left(r_c^J\right)\left(2 - \hat{\theta}_J\left(r_c^J\right)\right)r_c^J$. Notice that

$$\frac{3}{4} < \hat{\theta}_J\left(r_c^J\right)\left(2 - \hat{\theta}_J\left(r_c^J\right)\right)$$

for every $\hat{\theta}_J \in \left(\frac{1}{2}, 1\right)$ and $r_c^J > r_d^J$. We can now conclude that

$$\frac{3}{4} r_d^J < \hat{\theta}_J\left(r_c^J\right)\left(2 - \hat{\theta}_J\left(r_c^J\right)\right) r_c^J$$

$$\Leftrightarrow \frac{3}{2} r_d^J - \hat{\theta}_J\left(r_c^J\right)\left(2 - \hat{\theta}_J\left(r_c^J\right)\right) r_c^J < \frac{3}{4} r_d^J$$

Similarly if $\alpha < r_d^I - \hat{\theta}\left(r_c^I\right) r_c^I$, then $\alpha < \frac{r_d^I}{2}$.

Hence, if $r_d^I > \frac{3}{2} r_d^J$ and $\alpha < \frac{3}{2} r_d^J - \hat{\theta}_J\left(r_c^J\right)\left(2 - \hat{\theta}_J\left(r_c^J\right)\right) r_c^J$, under individual liability a profit motivated lender chooses $r_\pi^* = r_d^I$ while a benevolent lender chooses $r_B^* = 2\alpha$. Under joint liability, a profit motivated lender chooses $r_\pi^* = r_d^J$, and a benevolent lender chooses $r_B^* = \frac{4\alpha}{3}$.

Thus, for a profit motivated lender, the expected profit under individual liability is $E\Pi_I = \left(\frac{1}{2} r_d^I - \alpha\right)$ while the same under joint liability is $E\Pi_J = \left(\frac{3}{4} r_d^J - \alpha\right)$. If $r_d^I > \frac{3}{2} r_d^J$, the expected profit under individual liability exceeds that under joint liability. For a benevolent lender however, the expected utility of the borrower is same under two types of loan contract which can be seen from

$$EU_I = \frac{Y}{2} - \alpha - e = EU_J$$

We now summarize this in our next proposition.

Proposition 9 *Suppose $g(0) < 0$, $r_d^I > \frac{3}{2} r_d^J$ and $\alpha < \frac{3}{2} r_d^J - \hat{\theta}_J\left(r_c^J\right)\left(2 - \hat{\theta}_J\left(r_c^J\right)\right) r_c^J$. A profit motivated lender would choose individual liability loan contract while the benevolent lender is indifferent between the two types of loan contract.*

Our last proposition shows that in our framework some situations may arise where individual liability loan contract may arise as the dominant form of loan contracts used by the lenders particularly the profit motivated ones. In the microcredit literature the recent move towards the individual liability lending by the micro-lenders is presently being discussed and analyzed by many researchers. We feel that our framework provides a structure that has potential to deal with that issue.

5 Concluding Remarks

This paper attempts to provide a theoretical framework to analyze these issues of endogenous information collection and group formation in the micro-credit market. Borrowers generally collect information about the potential of their intended projects since undertaking a project requires the borrower to put in effort, time and money which are costly and are sunk in case of project failure. Information about the chance of project success allows the borrower to save these costs if she finds out that the project will fail almost surely. This also allows the lenders to lower the rate of interest generally leading to higher efficiency in the micro-credit market. However, cost of collecting information rises with the quality of information. The borrower faces a trade-off at this stage of information collection: whether to bear the cost of signal collection, or to bear the cost of time and effort in case the realized signal indicates strong probability of failure. This paper attempts to understand this trade-off in terms of a simple model of credit market. Further, we have examined how individual lending mechanism vis-a-vis the group lending mechanism affects the ex-ante incentive for information collection and the welfare of the agents when the group is formed after the signal realization.

Here, both under limited liability individual lending and joint liability group lending, we find that if interest rate is very high, the borrower(s) does not take up the project irrespective of the signal realization and hence does not invest in signal quality. However, as interest rate falls below a critical value, the borrower(s) decides to take up the project under good signal and invests a positive amount in signal quality. The investment in signal quality rises as interest rate falls. If the private effort cost of the project is very high, this goes on for all values of interest rate below the critical level. In this case, the borrower(s) takes up the project only if the realized signal is a success signal. On the other hand, if effort cost is not so high, then there is a second critical level of interest rate below which the borrower(s) once again stops investing in signal quality. In this case, for low values of interest rate, the borrower(s) takes up the project irrespective of the signal and hence the signal has no value to the borrower(s) - she does not invest in information. However, we interestingly notice that if a borrower decides to invest in signal quality under both types of loan contract, she invests more under joint liability.

Comparing the two types of lending contracts, we further observe that the switching rates of interest are higher under individual liability lending than under joint liability lending. We further observe that under individual liability lending, a profit-motivated lender induces the borrower to invest more often in information collection than a benevolent lender. Our model, in conventional line, finds that there occurs assortative matching in the group formation stage (though quality of information is chosen independently by each borrower); and in general, both the profit motivated lender and the benevolent lender would prefer to choose joint liability lending over individual liability lending. However, under certain conditions, we find that a profit motivated lender would choose individual liability loan contract while the benevolent lender is indifferent between the two types of loan contract. It shows that some situations may arise where individual liability loan contract may arise as the dominant form of loan contracts used by the lenders particularly the profit motivated ones. In the microcredit literature the recent move towards the individual liability lending by the micro-lenders is presently a highly discussed issue. We have tried to develop a theoretical framework to deal with that important issue.

It is here to admit that there is an ample scope of improvement, and extension of the model. We may also examine how the ex-ante incentive for information collection and the welfare of the agents is affected by group formation in different stages under group lending. We have not discussed the cases where the group members may cooperatively decide on the quality of information collected by the members; or, a group that is formed after the borrowers independently decide on the quality of information they would collect, but before the signal realization. Interestingly, group formation at this stage does not leave any scope for group punishment based on quality of information. We are interested to explore these cases in our future researches either using the present theoretical framework or by using a much richer one.

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