Intertemporal optimization approach to the current account in India

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Abstract

Using a simple consumption-smoothing benchmark model and an extended model, this study tries to examine the intertemporal solvency of India’s current account in the post-liberalization period. The study uses quarterly data ranging from 1996Q1 to 2014Q2 and finds that the current account in India is not only affected by internal shocks but also by external shocks. We find that although the optimal current account in both the models are able to track the actual current account movements, the extended model works fairly better over the benchmark model more formally. However, the findings also implies that the optimal current account is greater and more volatile than the actual current account which makes an interesting case for further liberalization of capital account. The policy aimed at further liberalization of capital flows, both inflows and outflows, will help agents to further smoothen their consumption to desired optimal level, allowing higher current account deficit to attain potentially higher growth without worrying about risks of insolvency.

Key words: Present value model, Current account deficits, Solvency, Consumption-smoothing, Intertemporal, India.

JEL Classification: E21, F30, F31, F42

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1 Introduction

Liberalization has been widespread and it has transformed the global economic landscape in the past three decades. The policies towards this global integration has affected almost all countries in the world. While international trade flows have risen dramatically, in some countries exports and imports have grown at very different paces. The result has been a worrying widening of trade and flow imbalances that leads to imbalances in current account and capital account (Ahearne 2007). Current account, as reflective of these policies, has become a metric of openness of a country in a globalized world. A current account balance of a country can be expressed as the difference between its national (both public and private) savings and investment. It is a measure of whether a country is a net lender or a net borrower from its trading counterparts. Therefore, the ability of a debtor country to continue running current account deficits is an issue of utmost importance given the magnitude of global imbalances in the last two decades. The debate has led to analysing several key questions. When is a debtor country insolvent? What is its sustainable current account path? Has the pattern of international capital movement optimal? (Cashin & McDermott 2002).

India as one of the emerging economies has also suffered from some serious current account imbalances. Current account balance in India has been persistently in deficit for most of the period since 1950s and have faced severe solvency issues since then. Since liberalization in the early 1990s until the mid-2000s, India experienced a period of external sector stability. However, when the crisis unfolded in 2008-09, capital flows dried up from US $106.585 million in 2007-08 to US $7.396 million in 2008-09 (where portfolio investment actually experienced a net outflow highlighting the vulnerability of such capricious capital flows to emerging market economies in times of crisis and loss in investors confidence) (Sen Gupta & Sengupta 2016). Hence, India had to draw down its reserves from US $309.72 million in 2007-08 to US $251.98 million in 2008-09 in order to finance its imports and the import cover to decrease from 14.4 to 9.8 weeks during the same period (RBI 2012). The global slowdown soon spilled over to emerging and developing economies which adversely impacted India’s trade balance too. The imports however did not slowdown due to the inelastic demand of oil and gold imports. Imports rose from 20.1% of GDP in 2007-08 to 25.2% of GDP in 2008-09 to 27.3% in 2012-13. Due to this, the CAD deteriorated during this

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1Since liberalization in the capital account is still considered risky for emerging economies and caution should be kept before moving to full capital account convertibility.
period and even reached 6.7 per cent in the third quarter of 2012-13—highest in the last five decades. As pointed out by Rangarajan & Mishra (2013) and Goyal (2014), India is in a situation of rising CAD in the midst of a slowing economy suggesting the case of a countercyclical CAD where a deficit rises when output falls and not when demand rises. The deterioration in India’s current account balance since the mid-2000s and the associated volatility in capital flows, has been accompanied by a vigorous debate questioning the reasons behind, consequences and the policy remedies. Therefore, the main aim of this study is to examine whether India’s current account is solvent and whether its borrowing post liberalization period (1996-2014) is optimal.

We use the intertemporal approach to address the above issues in this study. The intertemporal approach to the current account views the current account balance as the outcome of forward-looking dynamic saving and investment decisions. This model was developed by Sachs (1982) and is based on the permanent income hypothesis of Friedman (1957) and Hall (1978) where consumption expenditure of agents depends upon the expected permanent income. When the current income fluctuates, the current level of saving would also correspondingly fluctuate in order to maintain the level of consumption. Extended in the context of a small open economy, fluctuations in current income translates into borrowing or lending from the international markets i.e., movement of capital, to smooth out consumption. Thus, intertemporal consumption optimization behavior of agents predict the desired level of capital flows to meet the resulting current account balance. Hence, if the saving and investment decisions of agents are optimal, the resulting current account balance is also optimal and intertemporally solvent, irrespective of whether it is in deficit, surplus or balanced. In brief, the model states that the current account is the outcome of the rational expectations of forward looking economic agents, who look to smooth consumption in the face of random shocks to output and other important macroeconomic variables by borrowing and lending.

A class of models within this broad classification is the present value model of the current account (PVMCA). The PVMCA implies that a country’s current account surplus should be equal to the present value of expected future declines in output, net of investment and government. This states that the current account reflects the expectations of agents regarding future declines in net output. Thus, the current account forms the optimal forecast.

\footnote{Liberalization in India began in the early 1990s involving removal of capital controls.}
of future changes in net output, made by rational economic agents. This is captured by their saving and borrowing behavior as they seek to smooth consumption. The model was developed by Ghosh (1995) and Ghosh & Ostry (1995) by adapting the hypothesis of Campbell (1987) and Campbell & Shiller (1987) according to which saving anticipates future declines in labor income. This is the saving for a rainy day hypothesis which when adapted to the open economy implies that the country will run a current account surplus if net output or national cash flow is expected to decline in the future. If net output is expected to increase in the future, a CAD will be witnessed as agents seek to smooth consumption. Thus, the current account will reflect present value of future changes in net output.

The remainder of the paper is organized as follows. Section 2 discusses the empirical literature. Section 3 deals with the theoretical models and econometric methods followed by data issues in section 4. Section 5 discussed the empirical results and Section 6 concludes with the policy implications.

2 Review of Empirical Literature

The empirical literature on the intertemporal approach to the CA using PVM has generated mixed results with the model being rejected for many small open economies in the early 1990s in particular.

In case of developed countries, Sheffrin & Woo (1990) conducted tests of the model for Canada, Belgium, Denmark and the UK for the period 1955-85 using annual data and found its performance to be valid for Belgium and Denmark. One suggestion made by them for improving the performance of the PVMCA was relaxation of the assumption of a single good and to make a distinction, perhaps, between tradable and non-tradable goods. This would make the real exchange rate an important variable in the model. In another study, Otto (1992) found that the version of the consumption-smoothing hypothesis that was adopted failed to provide a statistically adequate explanation of the dynamic behavior of the US and Canadian current accounts. Using quarterly data for both countries, covering 1950-88, they rejected the restrictions implied by the present-value relationship for the current account. Makrydakis (1999) evaluated the intertemporal solvency of Greece over the period 1950-1995. They found that the model fails to predict the evolution of the current account imbalances implying the current account is on an unsustainable path. Agénor et al. (1999) empirically estimated a simple consumption-smoothing model to analyze the French current account for the period 1970 to 1996 using quarterly data. The results indicated that the
model is found to be valid and accurately predicts the sharp turnaround in Frances current account, especially for the period 1993-1996. Similarly, Kano (2008) also investigated the model by applying Structural VAR approach for two countries i.e., UK and Canada for the period 1960-1997 using quarterly data. They found that the present value model is not valid in explaining the current account movements in these countries. Kim et al. (2001) used quarterly data from 1982Q2 to 1999Q3 to test the external solvency in New Zealand and found evidence in favor of the intertemporal approach.

Bergin & Sheffrin (2000) extends the earlier present value tests of the current account to allow for variations in the interest rate and exchange rate. By allowing for a time-varying interest rate as well as distinction between tradable and non-tradable goods, it was found that including the interest rate and exchange rate significantly improves the fit of the model over a benchmark model which excludes them. However, the results showed that the intratemporal elements rather than the intertemporal elements, are primarily responsible for improving the fit. They found that the PVM prediction improves for all the three countries, i.e. UK, Australia and Canada, after the underlying assumptions are modified to suit the characteristics of a small-open economy.

Numerous studies on Australian current account solvency has been conducted. Milbourne & Otto (1992) employed quarterly data for the period 1959Q3 to 1989Q1 and found that the Australian current account was not consistent with the version of the permanent-income hypothesis used in their analysis. Using annual data from 1960-1995, Guest & McDonald (1998) rejected the validity of the model in the case of Australia, although they reported some improvement in its validity following the deregulation of capital markets. Cashin & McDermott (1998a,b) rejected the validity of the model for Australia based on annual data for the period 1954-94, however, found it to be valid in the case of quarterly data over the period 1984Q1 to 1998Q2. Otto (2003) found support for the model for the sub-sample of 1980-2000 over the full sample of 1960-2000 and mainly attributed this improvement in the performance due to the effective deregulation of Australian financial markets.

Japan, Germany, United Kingdom and United States) using quarterly data from 1960Q1-1988Q4. Using a quadratic utility function and a one-good model the study found some support for the PVMCA in the case of above countries.

In case of developing and emerging market economies, there have been very few studies on testing the intertemporal solvency using PVMCA. Ghosh & Ostry (1995) made the first attempt to apply the present-value approach to analyze the behavior of current account in the context of developing countries. The study estimated the consumption-smoothing model and the results indicated that across a large sample of developing countries, the validity of the PVMCA was consistent. Adedeji (2001) further augmented the model by introducing the terms of trade in a study of the PVM for Nigeria using annual data over the period 1960-97. The model was found to be valid as the optimal current account traced the movements of the actual current account closely. Landeau (2002) used the PVMCA to test the solvency condition for Chile. For the period 1960-1999, the model accounted for most of the observed imbalances in the current account. The study also finds the relevance of variable interest rates and exchange rates but found capital controls to have no effect on the solvency condition. Ogus & Sohrabji (2006) analyzed the Turkish current account within an intertemporal benchmark model. The study used quarterly data ranging from 1992 to 2004. The study estimated the consumption-smoothing model and the results indicated that Turkey breached the intertemporal solvency condition in the 1990s, while this was not true for the period following the 2001 crisis. The change in the validity could be attributed to the change in macroeconomic fundamentals that made the CAD in Turkey sustainable post-crisis period. Moccero (2008) used annual data ranging from 1885-2002 for Argentina and could not find support for the validity of the model. To date, the application of the model to the current account of small open economies has produced unambiguous but mixed results.

In the Indian context, very few studies have been conducted to test the validity of the present value model of the current account. Callen & Cashin (1999) conducted tests of the model using annual data over the period 1950-51 to 1998-99 and introduced features to capture the asymmetry of capital flows to and from India. They found that the current account is intertemporally

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3India was also a part of the sample of developing and the study found evidence in favour of validity of the PVMCA.

4Intertemporal benchmark model is referred to as the model in Ghosh (1995)
solvent over the entire sample period; however in the sub-sample from 1950-51 to 1990-91, the solvency condition was not met. The PVM (adapted to incorporate capital controls) could not be rejected for India. Khundrakpam & Ranjan (2008) used a similar model that captures asymmetry in capital flows to test the PVM for India over the period 1950-51 to 2005-06 using annual data. Their findings regarding solvency of the current account were similar to those of Callen & Cashin (1999) and the model was found to be valid in the Indian context. They also found that the optimal current account balance was larger than the actual current account balance and attributed it to the capital controls and foreign exchange restrictions prior to liberalization and reforms. Another study by Khundrakpam & Ranjan (2009), where they extend data for 2006-07 and 2007-08 to further substantiate the results from the previous study. The study highlighted the welfare implications of their findings by stating that easing of capital controls gradually can enable Indian consumers to smooth consumption and can also aid in stimulating investment and growth.

3 Theoretical Models

3.1 Benchmark Model

Although the PVMCA has been extended along several dimensions, there are two models that have been developed for empirical testing the present value model of the current account. The first model has been developed by Sheffrin & Woo (1990) and Ghosh & Ostry (1995) which is based on the intertemporal models developed by Sachs (1981) (hereafter, referred to as the “Benchmark Model”). The second model combines aspects from the models presented by Dornbusch (1983) and Bergin & Sheffrin (2000) (hereafter, referred to as the “Extended Model”). The two model differs primarily in the assumptions made about the nature of the utility function of the representative agent, the world rate of interest and in differentiating goods consumed by the economy into tradables and non-tradables.

We first discuss the benchmark model which is based on the Sachs (1981) model of intertemporal approach to the current account. The model emphasizes on the intertemporal trade implied by the divergence of savings and investment. As Sachs (1981) argues, the current account can be divided into two components. The first is the consumption tilting component, whereby a

country tilts its consumption towards the present or the future. The second is the consumption smoothing motive, which stabilizes consumption in the face of shocks to output, investment or government expenditure. It is important to note that in intertemporal model of the current account, what matters for the determination of the current account is the agents’ expectations of the shocks to the economy, rather than the shocks themselves.

The benchmark model captures agents’ expectation of future shocks to such variables as output and government expenditure. The simplest approach, in this case, would be to project government expenditure on its current and past values, but this is unlikely to be adequate since individuals, in general, have a much larger information set on which to base their expectations. Such additional information could include knowledge about political or other exogenous events that produce changes in expenditure and that, obviously, cannot be captured by merely projecting on past values of expenditure. In general, it is very difficult to re-create the information set used by individuals in making their optimal choices. However, it turns out that under the null hypothesis (that the intertemporal smoothing model is true and the capital is perfectly mobile) it is possible to include all information used by the individuals. This is shown by Campbell (1987) and Campbell & Shiller (1987) in which current account itself reflects this information. Therefore, by including the current account in the conditioning information set, agents’ expectations of shocks to output, investment and government expenditure could be captured.

Following Sachs (1981), Campbell (1987), Campbell & Shiller (1987) and Ghosh (1995) the benchmark model uses the standard model of international borrowing and lending its horizon infinite; its economy small and open. The economy is assumed to be populated by a single, infinitely-lived, representative agent whose preferences are given by:

\[ U_t = \sum_{t=0}^{\infty} \beta^t E_t[u(C_t)] \quad 0 < \beta < 1 \]  

where \( \beta \) is the subjective discount factor, \( u(.) \) the instantaneous utility function and \( c_t \) denotes the consumption of a single good. The utility function is of the quadratic form.

\[ U(C) = C - \frac{C_t}{2} \quad C_t < 1 \]
The dynamic budget constraint for the economy is given by:

\[ CA_t = B_{t+1} - B_t = Y_t + r B_t - C_t - G_t - I_t \]

\[ B_{t+1} = (1 + r)B_t + Y_t - C_t - I_t - G_t \]  \hspace{1cm} (2)

The world interest rate \( r \) is assumed to be constant and given exogenously, \( Y \) denotes the GDP for the economy. Deriving the optimal consumption, removing the tilting parameter from the actual current account (in Appendix A) and simplifying it gives the optimal current account:

\[ CA^*_t = - \sum_{i=1}^{\infty} \left( \frac{1}{1 + r} \right)^i E_t \Delta NO_{t+i} \]  \hspace{1cm} (3)

where \( \Delta NO_t \) is the net output and \( \Delta NO_t = Y_t - I_t - G_t \).

Equation (3) provides the hypothesis of the present value model of the current account. It states that the optimal current account is equal to minus the expected present discounted value of future changes in net output. It shows that transitory shocks will not lead to large movements in the current account as net changes will be expected in the future and discounted at the constant rate \((1+r)\).

To create this optimal current account series we need to calculate the expected present discounted value of changes in national cash flow, where the expectation is conditional on the information set used by individual agents. We follow the techniques of Campbell & Shiller (1987) and Ghosh (1995) and first estimate an unrestricted VAR in \( CA^*_t \) and \( \Delta NO_t \), where \( CA^*_t \) is the actual consumption-smoothing component of the current account.

Thus, the actual consumption smoothing component of the current account can be obtained as follows:

\[ CA^S_t \equiv Y_t - I_t - G_t - \theta C_t \]  \hspace{1cm} (4)

This consumption-smoothing component of the current account can be used along with \( \Delta NO_t \) to estimate an unrestricted VAR model as explained earlier. The consumption tilting parameter \( \theta \) can be estimated by regressing net output \( NO_t \) on consumption \( C_t \). The two variables are expected to be
cointegrated and this regression also provides a basis for testing solvency of
the intertemporal budget constraint.
For calibrating the expected present value of national cash flow, an unre-
stricted VAR in first differences of national cash flow and de-trended current
account can be estimated. The VAR may be written as:

\[
\begin{bmatrix}
\Delta NO_t \\
CA^S_t
\end{bmatrix} =
\begin{bmatrix}
\psi_{11} & \psi_{12} \\
\psi_{21} & \psi_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta NO_{t-1} \\
CA^S_{t-1}
\end{bmatrix} +
\begin{bmatrix}
e_{1t} \\
e_{2t}
\end{bmatrix}
\]

(5)

Or, more compactly as:

\[
X_t = \Psi X_{t-1} + e_t
\]

(6)

where \(X_t \equiv [\Delta NO_t, CA^S_t]\) and \(\Psi\) is the transition matrix of the VAR.

From (6), the k-step ahead expectation is:

\[
E(X_{t+k}) = \Psi^k X_t
\]

(7)

so that \(E_t \Delta NO_{t+k} = \begin{bmatrix} 1 & 0 \end{bmatrix} \Psi^k X_t\)

If we use the vector \(\begin{bmatrix} 1 & 0 \end{bmatrix}\) to pick off the forecast of \(\Delta NO\) then the
infinite sum in the present value model (3) can be written as:

\[
CA^*_t = -\sum_{k=1}^{\infty} \beta^k \begin{bmatrix} 1 & 0 \end{bmatrix} \Psi^k Y_t
\]

(8)

or

\[
CA^*_t = -\beta \begin{bmatrix} 1 & 0 \end{bmatrix} \Psi(I - \beta \Psi)^{-1} X_t
\]

(9)

where \(I\) is a 2 \(\times\) 2 identity matrix (for details, see Appendix B).

The variable \(CA^*_t\) is typically called the optimal current account and is
an estimate of the current account that is consistent with both the VAR(1)
model and the restrictions of the intertemporal model.

\footnote{It is simple to generalise this expression for higher order VAR by writing a pth order
VAR in first order form.}
Therefore, Eq. (3) can be expressed in terms of the VAR in (5), which can be specifically expressed as:

\[
CA^*_t = \begin{bmatrix}
1 & 0
\end{bmatrix}
\begin{bmatrix}
\Psi^* \frac{1}{(1+r)} & I - \frac{\Psi}{(1+r)}
\end{bmatrix}^{-1}
\begin{bmatrix}
\Delta NO_t \\
CA^S_t
\end{bmatrix}
\]

(10)

Testable Implications:

There are there important tests that will determine whether the evolution the current account is consistent with the intertemporal approach.

1. The Granger-causality test.
2. The Orthogonality test.

The first test is concerned with testing the hypothesis whether current account Granger-causes changes in net output. The hypothesis implies that if the present value model is valid then today’s current account will reflect the agents’ expectations about future changes in net output. This can be tested formally by running an unrestricted VAR in \( \Delta NO_t \) and \( CA^S_t \) or using the following model:

\[
\Delta NO_t = c + \alpha_1 \Delta NO_{t-1} + \alpha_2 CA^S_{t-1} + u_t
\]

(11)

According to the present value model, if today’s current account Granger-causes future changes in net output then the sign of \( \alpha_2 \) should be negative and statistically significant.

The second test is the test of orthogonality. This test implies that the present value model is valid if and only if \( E_{t-1}[CA^S_t - \Delta NO_t - (1+r)CA^S_{t-1}] = 0 \). Therefore, equality between the actual and optimal current account implies that \( R_t = CA^S_t - \Delta NO_t - (1+r)CA^S_{t-1} \) should be uncorrelated with the lagged values of \( \Delta NO_t \) and \( CA^S_t \). This restriction can be tested formally by constructing \( R_t \) and running the following regression:

\[
R_t = c + \theta_1 CA^S_{t-1} + \theta_2 \Delta NO_{t-1} + e_t
\]

(12)

and testing the null hypothesis, \( H_0 = \theta_1 = \theta_2 = 0 \). The non-rejection of the null hypothesis implies evidence in favour of the present value model.
The third test is goodness-of-fit test. This test implies that the movement in actual consumption-smoothed current account should fully reflect the movement in optimal consumption-smoothed current account. This can be tested both informally and formally. The informal test includes visual inspection of the actual and optimal current account series, correlation coefficient and testing the equality of their variances. The formal method includes testing whether the vector $K$ equals $[0\ 1]$. This is similar to testing whether the actual and optimal paths are equal.

### 3.2 Extended Model

As observed by Bergin and Sheffrin (2000), the results from empirical implementation of the benchmark model are mixed at best. While it has often been found to work fairly well for large countries, it ironically tends to fail for many small open economies for which the assumptions of the theory should be most appropriate. A likely explanation is that external shocks, which are not considered in the simplest form of the intertemporal model, strongly affect small economies. External shocks will generally affect the small open economy via movements in the interest rate and the exchange rate. As individuals may adjust consumption and saving behavior in response to changes in the real interest rates, countries may also adjust their current account response to movements in the world real interest rate (Ismail & Baharumshah, 2008; Landeau, 2002).

Hence, in our empirical work, we also considered the model of Bergin & Sheffrin (2000). The model is an extension of the benchmark model described above. It is an infinite horizon model for a small open economy consisting of a single, representative agent. The country produces traded and non-traded goods, borrows and lends with the rest of the world at a time-varying real interest rate. The representative individual solves an intertemporal maximization problem, choosing a path of consumption and debt that maximizes discounted lifetime utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t (C_{Tt}, C_{Nt}) \quad (13)$$

subject to:

$$Y_t - (C_{Tt} + P_t C_{Nt}) - I_t - G_t + r_t B_{t-1} = B_t - B_{t-1} \quad (14)$$

where $U(C_{Tt}, C_{Nt}) = \frac{1}{1-\sigma}(C_{Tt}^\alpha, C_{Nt}^{1-\alpha})^{1-\sigma}$; $\sigma > 0, \sigma \neq 1, 0 < a < 1$.
The consumption of the traded and non-traded good is denoted by \( C_{Tt} \) and \( C_{Nt} \) respectively. \( Y_t \) denotes the value of current output, \( I_t \) is investment expenditure, and \( G_t \) is government expenditure, all measured in terms of traded goods. \( P_t \) denotes the relative price of home non-traded goods in terms of traded goods. \( B_t \) denotes the initial stock of external assets and \( r_t \) denotes the net world real interest rate in terms of traded goods. The left hand side of the budget constraint given in equation (14) can be interpreted as the current account. Total consumption expenditure in terms of traded goods is given by:

\[
C_t = C_{Tt} + P_t C_{Nt}.
\]

Once the optimal consumption is derived and substituted in the intertemporal budget constraint (see Appendix C), the optimal current account may be written as:

\[
CA^*_t = -E_t \sum_{i=1}^{\infty} \beta^i (\Delta no_{t+i} - \gamma r^*_t + \gamma r_{t+i})
\]

where, \( r^* \) is a consumption-based real interest rate defined by:

\[
r^*_t = r_t + \left( \frac{1 - \gamma}{\gamma} (1 - a) \right) \Delta p_t + \text{constant}
\]

The equation (15) forms the basis for testing the present value model of the current account. It states that if net output is expected to fall the current account will rise as the agents try to smooth their consumption. But the condition also says that if we hold the changes in net output constant, a rise in the consumption-based real interest rate will improve the current account as the agents try to reduce consumption below the smoothed level as current consumption becomes more expensive in terms of future consumption foregone.

From equation (15) it becomes evident that the current account itself reflects and incorporates agents’ expectations regarding the future value of the changes in net output and the consumption-based real interest rate. This null hypothesis can be tested using an unrestricted VAR model that represents agents’ forecasts:

\[
\begin{bmatrix}
\Delta no \\
CA^* \\
r^*
\end{bmatrix}_t =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
\Delta no \\
CA^* \\
r^*
\end{bmatrix}_t-1 +
\begin{bmatrix}
u_{1t} \\
u_{2t} \\
u_{3t}
\end{bmatrix}
\]

This can be written more compactly in the form of standard VAR model as: \( z_t = A z_{t-1} + u_t \), where \( E(z_{t+i}) = A^i z_t \) and \( A^i \) is the companion matrix.
of the VAR coefficients. This can easily be generalized to higher order VAR models by writing a pth order VAR in first order form. Using (17), the restrictions on the current account in (12) can be expressed as:

\[ hz_t = -\sum_{i=1}^{\infty} \beta^i (g_1 - \gamma g_2) A^i z_t \]  

(18)

where \( g_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \), \( g_2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \) and \( h = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \). For a given \( z_t \), the right-hand side of equation (18) may be expressed as:

\[ \hat{CA}_t^* = k z_t \]  

(19)

where,

\[ k = -(g_1 - \gamma g_2) \beta A (I - \beta A)^{-1} \]

The equation (19) gives a model prediction of the current account variable consistent with the VAR model as well as the restrictions of the theory. Also, it is important to note that \( k z_t \) is not a forecast of the current account in the usual sense, but rather it captures the restrictions imposed by the model. Formally, the model restrictions can be tested using the fact that if the theory and data are consistent, such that , then the vector \( k \) should assume the values \( \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \). This implies that the model can be tested statistically by using the delta method to calculate a \( \chi^2 \) statistic for the hypothesis that \( k = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \).

4 Data and Measurement of Variables

The study uses quarterly data over the period 1996Q1 to 2014Q2. The rationale for choosing quarterly data over the annual data is for two reasons. First, the intertemporal model implicitly assumes the capital to be mobile for the representative agent to smooth consumption and therefore the period after the liberalization in the early 1990s is more valid. Second, the empirical literature shows that the present value tests using only annual data are often unable to reject the restrictions of the model.

The data on GDP (\( Y \)), Investment (\( I \)), Consumption (\( C \)) and Government expenditure (\( G \)) is taken from various publications of the RBI’s Handbook of Statistics on the Indian Economy. All variables are seasonally adjusted and used in real terms, with the common base year shifted to 2004-05=100. Then the net output (\( NO \)) is constructed as GDP less investment less government expenditure. Since, the present value model is based on a representative
agent, all series is converted to per-capita basis using annual population figures from RBI. These variables were used in log form for the extended model.

The computation of consumption-based real interest rate requires estimation of ex-ante world real interest rate and ex-ante expected change in the real exchange rate. For the first component, ex-ante world real interest rate, we do not follow the methodology adopted by Bergin & Sheffrin (2000) in which short-term nominal interest rates for the G7 nations (Canada, France, Germany, Italy, Japan, the United Kingdom and the United States of America) is used and then these rates is adjusted for expected inflation to arrive at an ex-ante real interest rate for each nation. Then, using time-varying weights based on the share of real GDP of each nation in the G7 total real GDP, average world real interest rate is calculated. Instead, we use US ex-ante real interest rate as a proxy for the world nominal interest rate because of negative nominal interest rate found in some quarters of 2012 and 2014 in case of France and Germany. The data on US 90 days T-bill rate is taken from Federal Reserve Bulletin while data on CPI is taken from IMF’s International Financial Statistics. Data on inflation based on the CPI is used to estimate expected inflation using ARMA (1,1) specification. To derive the second component of the consumption-based real interest rate, ex-ante expected change in the real exchange rate, we follow the methodology of Rogoff (1992) and Bergin & Sheffrin (2000) in using as a proxy for \( P_t \) a measure of the real exchange rate. The data is taken from the RBI on 36-currencies trade-weighted Real Effective Exchange Rate (REER) and shifted to a common base year 2004-05. An ex-ante expected appreciation of the real exchange rate is computed using an ARIMA (4,1,1) model.

With regards to the other parameters, \( \beta, \alpha \) and \( \gamma \), we use previous studies in order to deal with their values. For assigning a value to \( \beta \), we denote \( \bar{\rho} \) as the sample mean for the US real interest rate and since the model implies that \( \beta = 1/(1 + \bar{\rho}) \), equal to 0.98. Regarding the share of traded goods in consumption, \( \alpha \), we use the Kohli & Mohapatra (2007) estimate of share of traded goods at 0.25 in this study. For the intertemporal elasticity, \( \gamma \), we use the standard approach based on Hall (1988) recommendation that the intertemporal elasticity is unlikely to be greater than 0.1. This is based on the observation that consumption tends to respond very weakly to the real interest rate. Finally, the consumption-based real interest rate, \( r^*_t \), is computed using the world real interest rate and the expected exchange rate series. Since, we are interested in the dynamic implications of the intertem-
poral model, $\Delta no$, $CA^*$ and $r^*$ are all demeaned.

5 Empirical Results and Discussion

5.1 Benchmark Model

5.1.1 Unit root tests and Cointegration tests

The first step in our empirical analysis is to verify that both consumption $C_t$ and Net output $NO_t$ is integrated of order one, $I(1)$. We employ ADF and PP unit root tests to check the stationarity and the results are reported in Table 1.

<table>
<thead>
<tr>
<th>Table 1: Test for unit roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Augmented Dickey-Fuller</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>NO</td>
</tr>
<tr>
<td>Phillips-Perron</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>NO</td>
</tr>
</tbody>
</table>

The corresponding figures are the t-statistics. * represents significance at 1% level.

Results from the three unit root tests presented in Table 1 shows that $C_t$ and $NO_t$, appear as an $I(1)$ process at 1% significance levels. Therefore, the results of the unit root tests is consistent with the present value models.

As a next step, cointegration technique is adopted to determine if there exists a long-run relationship between $NO_t$ and $C_t$ and to calculate the consumption-smoothing component as the residual from the cointegrating regression of $NO_t$ on $C_t$. We applied Johansen’s (1988) cointegration technique and the results are presented in Table 2. The panel (a) of Table 2 presents the trace and maximum eigenvalue test statistic while panel (b) presents the estimated consumption-tilting parameter, $\theta$, of the cointegrating relationship. The results for the cointegration test suggests that there

\[7\] It is important to note that the null hypothesis in KPSS test is that a series is stationary
exists one cointegrating relationship between \( N_{0t} \) and \( C_t \) at the 1% significance level. The findings are in line with the present value model where \( N_{0t} \) and \( C_t \) move in the same direction in the long-run which forms a necessary and sufficient condition for satisfying the intertemporal budget constraint of the economy. The estimate of \( \theta \) from the cointegrating regression of \( N_{0t} \) on \( C_t \) is found to be 0.6731 and highly statistically significant. The estimate shows that Indias consumption is tilted towards the present as \( \theta < 1 \) and this could be seen consistent with high current account deficits in India for most of the sample period. Additionally, we formally tested and found that the parameter is significantly different from unity.

Table 2: Test for cointegration of NO and C

(a) Johansen-cointegration test

<table>
<thead>
<tr>
<th>Hypothesized no of CE(s)</th>
<th>Eigenvalue</th>
<th>Test statistic</th>
<th>5% critical value</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Trace</td>
<td></td>
<td></td>
</tr>
<tr>
<td>None*</td>
<td>0.2633</td>
<td>23.133</td>
<td>15.495</td>
<td>0.0029</td>
</tr>
<tr>
<td>Almost</td>
<td>1 0.0156</td>
<td>1.1298</td>
<td>3.8415</td>
<td>0.2878</td>
</tr>
<tr>
<td>Max-Eigen</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>None*</td>
<td>0.2633</td>
<td>22.0032</td>
<td>14.2646</td>
<td>0.0025</td>
</tr>
<tr>
<td>Almost 1</td>
<td>0.0156</td>
<td>1.1298</td>
<td>3.8415</td>
<td>0.2878</td>
</tr>
</tbody>
</table>

(b) Cointegration regression of NO on C

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimated Values</th>
<th>Standard error</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>-0.6731</td>
<td>0.0261</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

* represents significance at 1% level.

The actual consumption-smoothing component of the current account can be estimated by subtracting the consumption-tilting parameter from the actual current account:

\[
CA_t^S \equiv Y_t - I_t - G_t - 0.67C_t
\]

For robustness, we also use Dynamic OLS to estimate the \( \theta \) parameter and the value is estimated at 0.6705 which is consistent with the Johansens cointegration estimate.
It can be seen that if NO and Ct are cointegrated, the actual consumption-smoothing component will be stationary. Then $\Delta NO_t$ (first difference of net output) and $CA_t^S$ (consumption-smoothing current account) is defined in terms of deviations from their respective means (demeaned) so that only the dynamic restrictions of the theory are tested (Campbell 1987; Campbell & Shiller 1987; Ghosh 1995).

5.1.2 Granger-causality test

The first test of the model is the Granger causality test. According to the present value model, current account should in general Granger-causes changes in net output. Therefore, we estimate an unrestricted VAR in change in net output and consumption-smoothing current account and the results are summarized in Table 3. Prior to estimating the VAR, we select the lag length based on both the AIC and SIC criterion and a one-lag VAR model is chosen. The results in Table 3 indicates that $CA_t^S$ Granger-causes $\Delta NO_t$ at 1% significance levels. This finding supports the proposition that todays current account reflects agents expectations about future movements in the net output. Diagnostic checking on the estimated VAR model suggests that the model is stable and is free from residual autocorrelation.

### Table 3: Unrestricted VAR model and Granger-causality test

(a) Unrestricted VAR model of $\Delta NO_t$ and $CA_t^S$

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\Delta NO_t$</th>
<th>$CA_t^S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta NO_{t-1}$</td>
<td>-0.0652</td>
<td>-0.1340</td>
</tr>
<tr>
<td>(0.1147)</td>
<td>(0.0696)</td>
<td></td>
</tr>
<tr>
<td>$[0.5684]$</td>
<td>$[1.9257]$</td>
<td></td>
</tr>
<tr>
<td>$CA_t^S$</td>
<td>-0.8301</td>
<td>0.3649</td>
</tr>
<tr>
<td>(0.1915)</td>
<td>(0.1161)</td>
<td></td>
</tr>
<tr>
<td>$[-4.3352]$</td>
<td>$[3.1436]$</td>
<td></td>
</tr>
</tbody>
</table>

The standard errors are in parantheses and t statistics in square brackets.

(b) Granger-causality test

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>$\chi^2$-value</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CA_t^S$ does not cause $\Delta NO_t$</td>
<td>18.7939</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\Delta NO_t$ does not cause $CA_t^S$</td>
<td>3.7085</td>
<td>0.0541</td>
</tr>
</tbody>
</table>

9In order to verify, we test the actual $CA_t^S$ series for unit root and finds that the series is stationary at 1% significance level.
5.1.3 Orthogonality test

The second test of the model is the orthogonality test. Although the ability of the current account to forecast changes in the net output is consistent with the present value model however it does not provide a full test of the restrictions imposed on the data by the model. We can test the restrictions by estimating equation (12) and checking if the dependent variable is uncorrelated with the lagged values of $\Delta NO_t$ and $CA^S_t$. To implement the test, we first assume a real interest rate of 5 per cent. The results of the regression is presented in panel (a) of Table 4. The results of the orthogonality test presented in panel (b) of Table 4 shows that the combined coefficients of the lagged variables are jointly equal to zero for the sample period. Therefore, the present value model is not rejected by the data however it does not provide any evidence of how well the model actually fits the Indian data.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimated Value</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>-0.051963</td>
<td>0.359564</td>
<td>-0.144517</td>
<td>0.8855</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>0.145032</td>
<td>0.122277</td>
<td>1.186093</td>
<td>0.2397</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>-0.068755</td>
<td>0.073277</td>
<td>-0.938285</td>
<td>0.3514</td>
</tr>
</tbody>
</table>

(b) Orthogonality test

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th>Value</th>
<th>df</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$ statistic</td>
<td>0.8259</td>
<td>2, 69</td>
<td>0.4421</td>
</tr>
<tr>
<td>$\chi^2$ statistic</td>
<td>1.6518</td>
<td>2, 69</td>
<td>0.4378</td>
</tr>
</tbody>
</table>

5.1.4 Goodness-of-fit test

To examine the goodness of fit, we first need to calculate the optimal current account. The optimal current account is obtained by taking a linear combination of $\Delta NO_t$ and $CA^S_t$ (as in 24.) where the estimated weights $\Gamma_{NO}$ and $\Gamma_{CA^S}$ are non-linear functions of the VAR(1) coefficients as explained in 10. This world real rate of interest is consistent with the literature (see, Ghosh (1995), Ghosh & Ostry (1995), Khundrakpam & Ranjan (2008) and Ismail & Baharumshah (2008), among others) We, however, also found that world real interest rate of 4% and 6% do not make much difference in the results.
Appendix B. As discussed earlier, the calculated optimal current account is observed if the restrictions implied by the present value model held exactly. The point estimates of the weights on $\Gamma_{NO}$ and $\Gamma_{CA}$ are 0.106 and -1.361, respectively. The goodness of fit can be examined both through formal testing and informal testing. The results of the informal tests is presented in Table 5. We found that the correlation between the optimal and the actual consumption-smoothed current account to be highly correlated at 0.98 over the sample period. However, it is interesting to note that the variance of the optimal current account is higher than that of the actual current account and statistically different from unity.

**Table 5: Variance of $CA^s$, $CA^*$, Test of Equality and Correlation**

<table>
<thead>
<tr>
<th></th>
<th>var($CA^s$)</th>
<th>var($CA^*$)</th>
<th>Ratio</th>
<th>Prob.</th>
<th>Corr($CA^s$, $CA^*$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10.9230</td>
<td>17.9485</td>
<td>1.6753</td>
<td>0.0330</td>
<td>0.9892</td>
</tr>
</tbody>
</table>

Ratio = var($CA^*$)/var($CA^s$). The probability values are associated with the F-test for equality of variances. Correlation coefficient is Spearman’s rank correlation coefficient.

Figure 1 shows the path of the actual and optimal consumption-smoothed current account over the sample period. It can be clearly observed from the visual inspection that the actual and predicted observations track the major turning points for most of the sample period, including the Asian crisis of 1997 and the recent Global Financial crisis of 2008.

![Benchmark Model](image)

**Figure 1:** The actual and optimal consumption-smoothing current account
5.2 Extended Model

5.2.1 Unit Root Test Results

Before testing the extended present value models, we must check the assumptions that the variables, $CA^*$, $r^*$ and $\Delta no$, are stationary at level. We applied ADF and PP unit root tests and found that all the variables are stationary at the maximum 5% significance level \footnote{We do not include a constant and time trend while checking for unit root because the three series have been demeaned.}

<table>
<thead>
<tr>
<th>Variable</th>
<th>Test</th>
<th>Level</th>
<th>First difference</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Augmented Dickey-Fuller</em></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no</td>
<td>ADF</td>
<td>-11.0079*</td>
<td>-11.3640*</td>
</tr>
<tr>
<td>ca</td>
<td>PP</td>
<td>-12.4604*</td>
<td>-7.7003*</td>
</tr>
<tr>
<td>r</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Phillips-Perron</em></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no</td>
<td>ADF</td>
<td>-10.9940*</td>
<td>-40.2117*</td>
</tr>
<tr>
<td>ca</td>
<td>PP</td>
<td>-14.1731*</td>
<td>-51.7757*</td>
</tr>
<tr>
<td>r</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The corresponding figures are the t-statistics. * and ** represents significance at 1% and 5% level.

5.2.2 Results of Unrestricted VAR and Informal Tests

Before putting the extended present value model for formal and informal testing, it is essential to decide about the lag length of the VAR model. Following the standard approach, we tested the lag length criteria for two AIC and SBC and both the criteria suggest one lag for the model. The VAR model is stable and free from any autocorrelation, non-normality and heteroscedasticity. The VAR model’s estimated parameters and the informal present value tests are reported in Table 7. From the results in Table 7(b), it is evident that there exists a causality running from the current account to changes in net output and it goes in favour of the model informally. Additionally, the coefficients of the current account in response to net output and consumption-based real interest rate has the expected theoretical signs.

Next, we use the VAR model parameters given in Table 7(a) to derive the optimal current account series. It is evident from the visual inspection in
Figure 2 that the optimal current account series is able to track the actual current account very closely and outcome is relatively better than the benchmark model. Some interesting result include the ratio of the variance of the optimal current account to the actual current account has reduced significantly from 1.67 in the benchmark model to 1.1132. Also, the weights of the current account has reduced from -1.36 to -1.06 in the application of the extended model (see Table 7(c)). However, formal testing remains to be done whether the vector of the weights is theoretically equal to the restrictions \[0 1 0\].

Table 7: Unrestricted VAR model and Granger-causality test

<table>
<thead>
<tr>
<th>Variable</th>
<th>( \Delta no_t )</th>
<th>( ca_t^* )</th>
<th>( r^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta no_{t-1} )</td>
<td>-0.2549 ( (0.1184) )</td>
<td>-0.2401 ( (0.0736) )</td>
<td>-0.0107 ( (0.1883) )</td>
</tr>
<tr>
<td>( ca_t^* )</td>
<td>-0.1635 ( (0.0910) )</td>
<td>0.8533 ( (0.0566) )</td>
<td>0.21186 ( (0.1448) )</td>
</tr>
<tr>
<td>( r_t^* )</td>
<td>0.0108 ( (0.0774) )</td>
<td>-0.0191 ( (0.0481) )</td>
<td>-0.0749 ( (0.1231) )</td>
</tr>
</tbody>
</table>

The standard errors are in parantheses and \( t \) statistics in square [ ] brackets.

(b) Granger-causality test

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>( \chi^2 )-value</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ca^* ) does not cause ( \Delta no )</td>
<td>3.2261</td>
<td>0.0725</td>
</tr>
<tr>
<td>( ca^* ) does not cause ( r^* )</td>
<td>2.1412</td>
<td>0.1434</td>
</tr>
</tbody>
</table>

(c) Informal Test of the Model

<table>
<thead>
<tr>
<th>Variables</th>
<th>Weights</th>
<th>( \frac{\text{var}(ca^*)}{\text{var}(ca)} = 1.1132 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta no_t )</td>
<td>-0.0002</td>
<td></td>
</tr>
<tr>
<td>( ca_t )</td>
<td>-1.0606</td>
<td></td>
</tr>
<tr>
<td>( r_t^* )</td>
<td>0.0351</td>
<td>( \text{corr} (ca^*,ca) = 0.9978 )</td>
</tr>
</tbody>
</table>

Correlation coefficient is Spearman’s rank correlation coefficient. Share of tradables in consumption, \( a \), is 0.5 and \( \beta \) is 0.98.
6 Conclusions and Policy Implications

In this paper, we use the standard intertemporal model of Sachs (1981, 1982) to test the solvency condition for India by using the present value models of the current account developed by Ghosh (1995) and Bergin & Sheffrin (2000). While the former takes into account the effect of domestic shocks in predicting the optimal current account series while the latter extends it to account for external shocks as well. We use both the models to test the solvency since the current account behaviour of a small-open (developing) economy is not only affected by shocks in the domestic macroeconomic variables but also external shocks via movement in the interest rate and exchange rates. The study uses quarterly data over the period 1996Q1 to 2014Q2.

Through informal methods, we found that the external solvency is fully satisfied in the case of the extended model while it is only partially satisfied in the case of benchmark model. The model passes the Granger-causality test in both the models and the correlation coefficient is found to be very close to one. The variance ratio is much higher in the benchmark model as compared to the extended model where the test of equality is not rejected. The visual inspection of the predictions of the two models clearly indicates that the optimal path moves very closely to the actual path and is also able to detect the major turning points (including the recent crisis) in the current account series.

Figure 2: The actual and optimal current account
In general, the optimal current account is able to explain the movements in the actual current account. This implies that the current account is acting as a buffer for the consumers in face of any random domestic or external shocks and consumers are able to smooth their consumption. The variance ratio being statistically greater than one in the benchmark model and statistically equal to one (greater than one in absolute terms) in the extended model implies that the optimal current account is more volatile than the actual current account which is associated with the less than optimal capital flows. Thus, the above results making an interesting case for further liberalization of capital account. The policy aimed at further liberalization of capital flows, both inflows and outflows, will help agents to further smoothen their consumption to desired optimal level, allowing higher current account deficit to attain potentially higher growth without worrying about risks of insolvency.

A Appendices

A.1 Appendix A

Ghosh (1995) maximizes the utility in (1) subject to the budget constraint (2) and imposing the transversality condition leads to the optimal level of consumption:

\[ C_t^* = \left( \frac{r}{\theta} \right) \left[ B_t + \frac{1}{(1+r)} E_t \left\{ \sum_{i=0}^{\infty} \frac{1}{(1+r)^{-i}} (Y_{t+i} - I_{t+i} - G_{t+i}) \right\} \right] \]  \hspace{1cm} (20)

where \( \theta = \frac{\beta(1+r)r}{\beta(1+r)^2-1} \) is the constant of proportionality and captures the consumption-tilting parameter. The term in the parenthesis represents the country’s net productive wealth and this implies the permanent income equals wealth times the constant interest rate. Therefore, it can be seen that consumption is proportional to permanent national cash flow or net output.

The symbol \( \theta \) is reflecting the consumption-tilting dynamics that may arise if there is a difference between the world interest rate and the domestic rate of time preference (impatience). If \( \beta < 1/(1+r) \) then \( \theta < 1 \), which means that the world capital market gives the country a rate of return that fail to compensate for deferring consumption so that a country will shift consumption to the present and run current account deficits. If \( \beta > 1 \), then consumption is tilted towards the future and \( \theta = 1 \) implies the absence of the tilting component in the current account. The consumption tilting component of the current account at a permanent level of national cash flow.
Given (20), we can define the optimal consumption-smoothing current account as follows:

\[ CA^*_t \equiv Y_t - I_t - G_t - \theta C^*_t \]  \hspace{1cm} (21)

Substituting (20) into (21), gives:

\[ CA^*_t = -E_t \sum_{i=0}^{\infty} (1 + r)^{-i} \Delta(Y_{t+i} - I_{t+i} - G_{t+i}) \]  \hspace{1cm} (22)

A.2 Appendix B

To understand the restrictions imposed by the present value model more clearly, let \( \Psi(I - \beta \Psi)^{-1} \equiv \tilde{\Psi} \) then we can write (9) as

\[ CA^*_t = -\beta \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \tilde{\psi}_{11} & \tilde{\psi}_{12} \\ \tilde{\psi}_{21} & \tilde{\psi}_{22} \end{bmatrix} \begin{bmatrix} \Delta NO_t \\ CA^S_t \end{bmatrix} \]  \hspace{1cm} (23)

then doing the matrix multiplication gives:

\[ CA^*_t = -(\beta \tilde{\psi}_{11} \Delta NO_t + \beta \tilde{\psi}_{12} \Delta CA^S_t) \]  \hspace{1cm} (24)

For the optimal current account to equal the actual current account the following two restrictions must hold:

\[ -\beta \tilde{\psi}_{11} = 0 \]
\[ -\beta \tilde{\psi}_{12} = 1 \]  \hspace{1cm} (25)

that is, the weights on \( CA_t \) must equal one and that on \( \Delta NO_t \) must equal zero. For a given value for \( r^* \) and estimates of the VAR(1) model it is straightforward to compute the estimates of the two weights in (25) and (24) to derive the optimal current account.

Premultiplication by the 1 x 2 vector \([1 \ 0]\) yields \( E_t \Delta NO_t \):

\[ E_t \Delta NO_t = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \tilde{\psi}_{11} & \tilde{\psi}_{12} \\ \tilde{\psi}_{21} & \tilde{\psi}_{22} \end{bmatrix}^{s-t} \begin{bmatrix} \Delta NO_t \\ CA^S_t \end{bmatrix} \]  \hspace{1cm} (26)

where \( \tilde{\psi}_{11}, \tilde{\psi}_{12}, \tilde{\psi}_{21} \) and \( \tilde{\psi}_{22} \) as explicit functions of the parameters of the VAR model (5)). Re-writing (9):

\[ CA^* = -\beta \begin{bmatrix} 1 & 0 \end{bmatrix} \Psi(I - \beta \Psi)^{-1} X_t \]  \hspace{1cm} (27)

as
\[ CA^* = -\beta \begin{bmatrix} 1 & 0 \\ \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} \begin{bmatrix} 1 - \beta \psi_{22} & \psi_{12} \\ \psi_{21} & 1 - \beta \psi_{11} \end{bmatrix} \begin{bmatrix} \Delta NO_t \\ CA_t \end{bmatrix} \]

\[ = -\beta \begin{bmatrix} 1 & 0 \\ \tilde{\psi}_{11} & \tilde{\psi}_{12} \\ \tilde{\psi}_{21} & \tilde{\psi}_{22} \end{bmatrix} \begin{bmatrix} \Delta NO_t \\ CA_t \end{bmatrix} \]  

(28)

Solving for \( \tilde{\psi}_{11} \) and \( \tilde{\psi}_{12} \) gives:

\[ \tilde{\psi}_{11} = \frac{\psi_{11}(1 - \beta \psi_{22}) + \psi_{12} \beta \psi_{21}}{D} \]

\[ \tilde{\psi}_{12} = \frac{\psi_{12}(1 - \beta \psi_{11}) + \psi_{12} \beta \psi_{11}}{D} \]  

(29)

\[ D = (1 - \beta \psi_{11})(1 - \beta \psi_{22}) - \beta^2 \psi_{12} \psi_{21} \]

### A.3 Appendix C

Bergin & Sheffrin (2000) derives the optimal consumption profile for the agent from the first-order conditions of the maximization problem and can be expressed as:

\[ 1 = E_t \beta^\gamma (1 + \rho_{t+1})^\gamma \left( \frac{C_t}{C_{t+1}} \right) \left( \frac{P_t}{P_{t+1}} \right)^{(\gamma - 1)(1 - a)} \]  

(30)

where \( \gamma = 1/\sigma \), is the intertemporal elasticity of substitution. The intertemporal Euler equation can be written in terms of total consumption expenditure and the relative price of non-traded goods. Assuming joint log normality and constant variances and covariances, condition (30) may be written in logs:

\[ E_t \Delta c_{t+1} = \gamma E_t r_{t+1}^* \]

(31)

where \( r^* \) is a consumption-based real interest rate defined by:

\[ r_{t+1}^* = r_t + \left[ \frac{1 - \gamma}{\gamma} (1 - a) \right] \Delta p_t + \text{constant} \]  

(32)
Here $\Delta c_{t+1} = \log C_{t+1} - \log C_t$ and $\Delta p_{t+1} = \log P_{t+1} - \log P_t$. The constant term in equation (4) drops out at a later stage when the consumption-based real interest rate is demeaned.

This condition characterizes how the optimal consumption profile is influenced by the consumption-based real interest rate, $r^*$, which reflects both the interest rate, $r$, and the change in the relative price of non-traded goods, $p$. The incorporation of the consumption-based real interest rate, $r^*$, is important as it indicates that changes in this variable may prevent the representative agent from smoothing consumption all the time. Changes in the real interest rate can lead to a trade-off between current and future consumption. For example, an increase in conventional interest rate, $r$, makes current consumption more expensive in terms of future consumption foregone, and induces substitution toward future consumption with elasticity $\gamma$.

A similar intertemporal effect can result from a change in the relative price of non-traded goods. If the price of traded goods is temporarily low and expected to rise, then the future repayment of a loan in traded goods has a higher cost in terms of the consumption bundle than in terms of traded goods alone. Thus, the consumption-based interest rate $r^*$ rises above the conventional interest rate, $r$, and lowers the current total consumption expenditure by elasticity $\gamma(1 - \alpha)$.

In addition to these intertemporal effects, a change in the relative price of non-traded goods also induces intratemporal substitution. If the price of traded goods is temporarily low relative to non-traded goods, household will substitute toward traded goods by the intratemporal elasticity, which is unity under the Cobb-Douglas specification. This raises total current consumption expenditure by elasticity, $(1 - \alpha)$. This intratemporal effect will be dominated by the intertemporal effect if the elasticity, $\gamma$, is greater than unity.

Defining $R_s$ as the market discount factor for consumption on date $s$, we get:

$$ R_s = \frac{1}{\prod_{j=1}^{s}(1 + r_j)} $$  \hspace{1cm} (33) 

Using the budget constraint of the optimization problem (14), the current account may be expressed as:
\[ CA_t = Y_t - (C_{t1} + P_tC_{Nt}) - I_t - G_t + r_tB_{t-1} \]  \hspace{1cm} (34)

Or as,

\[ CA_t = NO_t - C_t + r_tB_{t-1} \]  \hspace{1cm} (35)

where net output is defined as follows: \( NO_t = Y_t - I_t + G_t \). By summing over all period of the infinite horizon and imposing the transversality condition given below:

\[ \lim_{t \to \infty} E_0(R_tB_t) = 0 \]  \hspace{1cm} (36)

We may express the intertemporal budget constraint of the representative agent as:

\[ \sum_{t=0}^{\infty} E_0(R_tC_t) = \sum_{t=0}^{\infty} E_0(R_tNO_t) + B_0 \]  \hspace{1cm} (37)

where \( B_0 \) denotes the initial stock of net foreign assets. The log-linear form of the intertemporal budget constraint can be expressed as:

\[ - \sum_{t=1}^{\infty} \beta^t \left[ \Delta n_{t+1} - \Delta c_t \Omega - \left( 1 - \frac{1}{\Omega} \right) r_t \right] = n_0 - c_0 \Omega + \left( 1 - \frac{1}{\Omega} \right) b_0 \]  \hspace{1cm} (38)

where the lower case letters denote the logs of upper case counterparts and \( \Omega \) denotes a constant slightly less than one, and \( \Omega = 1 - B / \sum_{t=0}^{\infty} R_tC_t \) where \( B \) denotes the steady state value of foreign assets.

Now, taking expectations of equation (38) and combining it with the Euler equation (39), we may write:

\[ - E_t \sum_{i=1}^{\infty} \beta^i \left[ \Delta n_{t+i} - \frac{\gamma}{\Omega} r_{t+i} - \left( 1 - \frac{1}{\Omega} \right) r_t \right] = n_0 - c_t \Omega + \left( 1 - \frac{1}{\Omega} \right) b_t \]  \hspace{1cm} (39)

The right hand side of the above equation is similar to the definition of the current account in (6), except that it is now in log form and this representation can be denoted by \( CA^* \). \[ \text{Bergin \\ & Sheffrin (2000)} \] choose the steady state in which net foreign assets are zero, implying that \( \Omega = 1 \) and thus the equation (39) may be written as:
\[ CA_t^* = -E_t \sum_{i=1}^{\infty} \beta^i (\Delta n_{t+i} - \gamma r_{t+i}^*) \] (40)

where,

\[ CA_t^* \equiv n_t - c_t \] (41)
References


