Intangible Capital and the Rise in Wage and Hours Volatility

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Abstract

In a standard real business cycle model extended to include intangible capital (IC) I show that a rise in the income share of IC in the production function, in line with data can account for a significant share of the increase in real wage volatility (both absolute and relative to income) and labor input volatility (relative to income) observed in the U.S. since the mid 1980’s even as volatility of output declined. Intangible capital accumulates stochastically and similar to final goods requires physical capital, intangible capital and labor to produce. Under these conditions an increase in the share of IC in production makes the IC-specific shock more important relative to the standard technology shock, increasing (absolute and relative) wage and labor input volatility in the process. The rising importance of the IC shock also accounts for the large decline in the procyclicality of labor productivity (relative to both output and labor) observed during this period.


Keywords: Intangible capital, business cycles, wage volatility, measured labor productivity, Great Moderation.

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1 Introduction

Recent literature documented substantial changes in the dynamics of key labor market aggregates that accompanied the large drop in output volatility in the post-1984 period in the U.S. These changes are:

i) Volatility of the real average wage rate both relative to output and in absolute terms increased markedly (Ohanian and Raffo, 2012; Champagne and Kurmann, 2013; Gali and Van Rens, 2014),

ii) Volatility of labor input relative to output increased (Ohanian and Raffo, 2012; Gali and Van Rens, 2014) and

iii) The procyclicality of labor productivity relative to output and labor declined significantly with the latter turning strongly negative (Stiroh, 2009; Gali and Gambetti, 2009; Gali and Van Rens, 2014).

I argue in this paper that a rise in the importance of intangible capital in production in recent decades, along with the additional employment volatility associated with the production of such investments, can jointly account for the observed shifts in labor market dynamics along with the decline in aggregate output volatility that characterized the so called Great Moderation of this period.

I first document that employment in IC related occupations as a share of total employment increased substantially in recent decades and show that such employment is significantly more volatile than aggregate employment at business cycle frequencies. Then, in a standard Real Business Cycle (RBC) framework extended to include an intangible capital (IC) producing sector, I introduce a productivity shock to IC (called the IC shock interchangeably throughout this paper) which generates additional volatility in the employment of IC sector. In such an environment, the stock of IC acts as a constraint on the production of the representative firm. In a standard RBC model without IC, when a positive technology shock hits final goods, hours rise immediately while the capital stock adjusts more slowly. In the current setting however, investment in IC requires labor input as well, and the stronger the need for IC in production, the larger the increase in
labor in the IC sector relative to final goods and lower the response of final goods to the productivity shock. The lower increase in output causes the wage rate and hence labor supplied to rise less as well. Therefore, output, wages and hours, all vary less in response to a productivity shock as IC becomes more important for production. Note here that IC investments are different from investments in physical capital in that the latter is produced from final output one for one while the former requires physical capital, labor and IC to produce.

With an increase in the importance of IC, however, the firm responds more to the IC shock since it relaxes the IC constraint by raising the productivity of IC investments. Investment and labor demand in the IC sector increase more, driving up real wage and employment growth in the economy. Thus as IC becomes more important for production, the IC shock plays a larger role in driving fluctuations in wages and hours causing these variables to become more volatile. To the extent that output of the IC sector is unmeasured, the productivity shock continues to be the main influence on the volatility of measured output in the economy and since the increasing usage of IC causes the relative importance of the productivity shock to decline, measured output volatility falls.

It is relevant to emphasize here that I treat intangible investments as unmeasured in my model, since by definition, and from the substantial evidence in the literature (Corrado, Haltiwanger, and Sichel (2005); Atkeson and Kehoe (2005); McGrattan and Prescott (2010); and Corrado, Hulten, and Sichel (2009)), intangible investments are largely left out in official GDP estimates for the U.S., especially for the period(s) under consideration in this paper. Therefore the terms, output of final goods, measured output or simply output, mean the same thing here unless otherwise specified. By extension labor productivity, measured productivity or simply productivity are used interchangeably in this paper.

The IC shock generates a strong negative correlation between labor input and labor productivity since final output falls upon impact of the IC shock as resources reallocate from

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1The Bureau of Economic Analysis (BEA), has recently made great strides in including important categories of intangible investments into national accounts tables (eg., software, R&D etc.,), but much intangible investment still remains unmeasured (eg., advertising, marketing research etc.,)
the final goods to IC sectors, while total employment rises due to the higher IC sector employment. Thus when both shocks are present, the correlations of measured productivity with output and labor are positive (due to the dominant effect of the productivity shock) but lower (due to the IC shock’s influence) than in standard RBC models. An increase in the role of the IC shock that occurs with the rise in IC, however, causes these correlations to decline significantly, with the correlation relative to labor turning strongly negative as observed in the data.

Intangible capital, as defined in McGrattan and Prescott (2012) is ”accumulated know-how from investing in research and development, brands, and organizations, which is for the most part expensed by companies rather than capitalized”. Hall (2000, 2001) attributes the majority of the increase in the valuation of corporations in the 1990’s to what he calls e-capital. Nakamura (2001) reports, using different estimates of intangible investments that the rate of such investments, and its economic value, accelerated significantly beginning around 1980 such that US private gross investment in intangibles was at least $1 trillion by the end of 1999, same as business investment in traditional, tangible capital. This finding is matched by Corrado, Hulten, and Sichel (2005). Corrado et al. (2009) find that IC’s share in income increased from 9.4% in the period 1973-1995 to 14.6% in 1995-2003 while Corrado et al (2009, 2010), Falato et al (2014) and Dottling and Perotti (2015) show that the ratio of intangible to tangible assets increased from 20% in the pre-1980 period to around 60% in 2010.

As expected, the share of employment in occupations that are predominantly associated with production of intangibles, as a fraction of total employment also increased substantially during this period. Using occupation data from the Department of Labor’s March Current Population Survey (CPS), I split workers into two groups (a) workers engaged in the creation of innovative property like engineers, architects, scientists, artists, entertainers and IT workers, and (b) organizational workers namely managers, marketers and human resource specialists all of whom are associated with developing economic competencies. I term the total employment of the two groups together as IC employment.

This is similar to the classification of IC related employment in Nakamura (2001). McGrattan and Prescott (2012) also use occupation data to show the shift in employment to IT sectors, that occurred in the 1990’s. Here I focus more broadly on the intangible capital revolution that began in the years leading up to the Great Moderation, and which includes but is not limited to the IT sector\(^3\).

Figure 1: Employment shares in IC related occupations. Group (a)-engineers, architects, scientists, artists, entertainers and IT workers; Group (b)-managers, marketers and human resource specialists. Data is from Occupation series, 1990 basis (occ90) from IPUMS-CPS

\(^3\)Nakamura (2001) gives an account of the reasons for the increased use of intangible capital from around this period.
Figure 2: IC employment is the sum of employment in group (a) and group (b). Both IC employment and aggregate employment series are logged and HP-filtered using a value of $\lambda = 6.25$ for annual data following Ravn and Uhlig’s (2002) suggestion.

Figure 1 plots the evolution of employment of the two groups of IC workers in recent decades. Between 1970-2010, employment of group (a) doubled while group (b)’s employment increased by 65%. Total IC employment rose by a marked 80% during this period. Moreover, from Figure 2 IC related employment is significantly more volatile than aggregate employment over the business cycle as shown in. Taking logs and HP-filtering both series, I find that employment in IC related occupations is on average five times more volatile than the aggregate.

Motivated by the evidence presented in this section I allow, (i) the income share of IC to increase in my model in such a way that the steady state employment share in IC rises as in the data, and, (ii) an IC specific productivity shock which generates additional volatility of employment in the IC sector in line with Figure 2.

Related Literature
Papers trying to explain the shift in labor market dynamics attribute these changes to a rise in US labor market flexibility around this time. Gali and Van Rens (GVR from now on) (2014), show that all three of the above changes can be caused by a reduction in hiring costs arising from an increase in labor market turnover. Champagne and Kurmann (2013) and Nucci and Riggi (2013) argue that a shift towards performance-pay contracts played an important role in the changing U.S. labor market dynamics. The former use micro-data to establish the empirical evidence and show that changes in workforce composition did not contribute to the rising wage volatility. Nucci and Riggi (2013) use a DSGE model to show that this structural change alone, can account for a sizeable drop in the volatility of output along with an increase in the relative and absolute volatility of real wages. They also qualitatively account for the decline in procyclicality of labor productivity during this period. Comin, Groshen and Rabin (2008) associate the higher wage volatility with a general increase in firm level (profit-to-sales ratio or the growth rate of sales, employment or sales per worker) volatility. They too rule out any role played by compositional changes of the workforce and observe that the relationship between sales and wage volatility at the firm level is stronger since the 1980’s and for services rather than manufacturing firms. None of these papers however, focus on the link between intangible capital and the shift in labor market dynamics. To my knowledge, this is the first paper to do so.

McGrattan and Prescott (2010, 2012) and Gourio and Rudanko (2014) on the other hand, include intangible capital in standard RBC models to shed light on otherwise puzzling labor market behavior. McGrattan and Prescott (2010) use such a framework to generate the observed boom of the 1990’s while without IC, their model predicts a depressed economy in the 90’s. Mcgrattan and Prescott (2012), using the same model, reassess the Great Recession of 2008-2009 and the slow recovery period from 2009-2011 and show that the inclusion of intangible capital and nonneutral technology change in the production of final goods and services can account for the fact that labor productivity rose during the Great Recession even as GDP crashed. Thus Mcgrattan and Prescott (2012) are the first in my knowledge to attribute the fall in the procyclicality of labor productivity to a rise in the productivity of intangible capital. However, they do not consider the role of an IC shock and they focus on the productivity boom of the 1990’s whereas my focus
is on the period generally associated with the Great Moderation beginning in the mid 1980’s. I choose this break date following the common practice in the literature (Gali and Gambetti (2009), Barnichon (2010), Champagne and Kurmann, 2012 and GVR (2014)) of dating the changes in labor market dynamics, including the vanishing procyclicality of productivity, from the start of the Great Moderation, regarding the timing of which there is some consensus in the literature (McConell and Perez-Quiros, 2000; Stock and Watson, 2003).

Finally, Gourio and Rudanko (2014) account for the countercyclical and highly volatile labor wedge (ratio of the marginal rate of substitution of households and the marginal product of labor of firms) when they incorporate complementary IC-production into an otherwise simple RBC framework.

The rest of the paper is organized as follows, Section 2 provides a summary of the changes in labor market dynamics documented in the literature for the pre and post-84 periods, Section 3 presents the model with IC and highlights the key channels through which labor market and other aggregates are affected, Section 5 discusses the impact of a rising share of intangible capital in the model economy, Section 6 tests the sensitivity of the model generated results to changes in parameter values and Section 6 concludes.

2 Changes in labor market dynamics

In this section I review the evidence provided in the literature of the key changes in labor market dynamics in the post-1984 period. Different authors using varied data sets, lengths of time series and filtering methods find largely similar and statistically significant changes in key labor market moments. I especially focus on and compare my model generated results to GVR (2014) since their empirical study jointly focuses on the three main changes in labor market dynamics that I seek to understand in this paper. In this section however, I discuss results from a wide range of studies in the literature, all of whom report similar changes in labor market trends in post-84 U.S. data.
The rising relative volatility of labor input For BP filtered log data, Gali and Gambetti (2009), using an estimated structural vector autoregression (SVAR) with time-varying coefficients and stochastic volatility, report an increase in hours volatility relative to output from 0.79 in the pre-84 to 1.10 in the post-84 period. GVR (2014) report labor market moments for both the private sector and the total economy. The former uses data from the BLS labor productivity and cost program (LPC) while the latter uses an unpublished series of economy-wide hours constructed by the BLS, also used in Francis and Ramey (2009). For BP filtered data, GVR (2014) find an increase in relative hours volatility for the private sector from 0.86 to 1.06 for the pre and post-84 periods while for the total economy volatility increased from 0.71 to 0.76. When using HP filtered data they find relative hours volatility increased from 0.80 to 1.20 or by 50% for the private sector and from 0.70 to 0.89 or by 27% for the total economy. They also report slightly smaller but statistically significant increases in relative employment volatility for the same time periods for the different filtering methods.

The rising volatility of real wage GVR (2014) find that volatility of compensation per hour for the private sector in the National Income and Product Accounts (NIPA) increased from 0.71 to 1.38 in absolute terms and from 0.30 to 0.88 relative to GDP from their pre-84 to post-84 sample using BP filtered data. For HP-filtered data wage volatility increased from 0.85 to 1.03 or by 21% (absolute) and 0.35 to 0.86 or by 46% (relative to GDP) between the pre and post-84 periods. Combining NIPA and the unpublished economy-wide series for hours constructed by BLS, they report volatilities of compensation per hour for the total economy as well. For this measure of the wage rate, volatility increased from 0.84 to 0.95 or by 13% in absolute terms and 0.34 to 0.80 or by 35% relative to GDP for the HP filtered series. They also report (smaller) increases in absolute and relative volatility of earnings per hour using a slightly smaller data set from the Current Employment Statistics (CES) across the different filtering methods. Gourio (2007) and Champagne and Kurmann (2013) find similar increases in absolute and relative real wage volatility between the pre-84 and post-84 periods.

The fall in procyclicality of labor productivity Stiroh (2009) reports that correlation of labor productivity growth and hours growth declined substantially during the
period after the mid-80s. Gali and Gambetti (2009) find that the unconditional correlation of labor productivity and output (logged and BP-filtered) fell close to zero in the post-84 period from a high of 0.61 in their pre-84 sample whereas unconditional correlation between labor productivity and hours went from 0.18 to -0.46. When a first difference transformation of the data is used instead of a BP-filter they find a similar although weaker (and statistically significant) change in the correlations of these variables. GVR (2014) report similar declines in correlations of labor productivity across their alternative definitions of variables and filtering methods. Specifically, for the private sector, using HP filtered data and hours as the measure of labor input they find the correlation of labor productivity with GDP fell from 0.61 in pre-84 to 0.04 in post-84 or by 57%. Correlation of labor productivity with hours went from 0.17 to -0.56, a fall of 73% in the same period.

3 Model

The model is a two-sector variant of the standard Real Business Cycle framework with a final goods and an intangible capital sector. A representative firm combines physical capital, intangible capital and labor to produce final goods and IC. Both final goods and IC sectors are subject to productivity shocks. The firm accumulates physical and intangible capital while labor is supplied by a representative household.

Firm

The firm solves the following problem,

$$\text{Max } E_t \sum_{t=0}^{\infty} M_{0,t} [y_t - w_t l_t - x_{k,t}], \quad (1)$$

subject to,
\[ y_t = A_t k_{y,t}^\alpha z_t^\gamma (l_{y,t})^{1-\gamma} - \zeta \left( \frac{x_{ky,t}}{k_{y,t}} \right) k_{y,t}, \]  
\( (2) \)

\[ x_{z,t} = B_t k_{z,t}^\alpha z_t^\gamma (l_{z,t})^{1-\gamma} - \zeta \left( \frac{x_{kz,t}}{k_{z,t}} \right) k_{z,t}, \]  
\( (3) \)

\[ k_{t+1} = (1 - \delta) k_t + x_{k,t}, \]  
\( (4) \)

\[ z_{t+1} = (1 - \phi) z_t + x_{z,t}. \]  
\( (5) \)

where \( M_{0,t} \) is the stochastic discount factor, in equilibrium equal to the marginal rate of substitution of households. \( y_t \) is total output in the final goods sector and \( l_t \) is the total labor employed by the representative firm. \( k_{i,t}, z_t \) and \( l_{i,t} \) where \( i = \{ y, z \} \), are the physical capital, intangible capital and labor inputs in the final good and IC sectors respectively in period \( t \). As in Mcgrattan and Prescott (2010, 2012) I assume that the level of IC available for production in both final goods and IC sectors are the same. \( x_{i,t} \) are investments in physical capital and IC with \( \delta \) and \( \phi \) their respective depreciation rates. \( x_{ki,t} \) are physical capital investments in the final good and IC sectors such that \( x_{ky,t} + x_{kz,t} = x_{k,t} \) and \( k_{y,t} + k_{z,t} = k_t \). Equation (2) is the production function for final goods, while equation (3) gives the production function for IC investment. Production in both sectors are subject to convex adjustment costs in physical capital specified by the function \( \zeta(\cdot) \).

\( A_t \) is a productivity shock in the final goods sector. It follows a first order autoregressive process,

\[ \log A_t = \rho_A \log A_{t-1} + e_t^A, \]

where \( e_t^A \) are zero-mean, i.i.d. innovations. \( B_t \) is a productivity shock to the IC investment sector, which also follows an AR(1) process given by

\[ \log B_t = \rho_B \log B_{t-1} + e_t^B \]

such that \( e_t^B \) are zero-mean, i.i.d. innovations. Finally equations (4) and (5) give the
laws of motion for physical and intangible capital accumulation respectively.

My aim in this paper is to study the effects of an increase in the share of intangible capital \( \gamma \) relative to physical capital and labor in the production process. Corrado et al (2005) show in their empirical work that IC’s share in income increased from 9.4% in the period 1973-1995 to 14.6% in 1995-2003. Particularly for 2000-2003, the share of income earned by the owners of intangible capital reached 15%, while the owners of physical capital received 25%; the remaining 60% was absorbed by labor. Their calculations are complemented by results from other studies. For example, Karabarbounis and Neiman, (2014) show that labor’s share in output has declined substantially since the early 1980’s from around 67% to 60% in 2012 and demonstrate that the decline can be explained by a fall in the relative price of investment goods. They note that advances in information technology and the computer age, induced firms to shift away from labor and toward capital to such a large extent that the labor share of income declined.

I therefore assume that the majority of the increase in IC’s income share comes from a decline in labor’s share in income and, a smaller fraction from the income share of physical capital, \( \alpha \). This methodology is similar to Giglio and Severo (2012) and essential for maintaining constant returns to scale in production. Note that this means, while labor’s share in income declines with a rise in \( \gamma \), there is an overall increase in the income share of capital, \( \alpha + \gamma \), in the model. Specifically, I assume that an increase in \( \gamma \) causes \( \alpha \) to change in the following way,

\[
\alpha_1 = \alpha_0 - \tau (\gamma_1 - \gamma_0),
\]

where the subscripts 0 and 1 refer to the pre-84 and post-84 income shares of factor inputs respectively and \( \tau < 1 \) is the fraction of the increase in \( \gamma \) that is deducted from \( \alpha \). The remaining, \( 1 - \tau \), is then deducted from the income share of labor. Thus an increase in \( \gamma \) leads to a less than proportionate decline in the income shares of both labor and physical capital.

The first order condition for physical capital in the final goods sector is,
The right hand side is the marginal cost of having an extra unit of $k_{t+1}$ which is one unit of output (given up today) plus the associated adjustment cost of the added unit of investment, $\zeta'(\cdot)$. The left hand side gives the marginal benefit of an additional unit of $k_{t+1}$ which is composed of the discounted marginal product of physical capital, the value to the firm of undepreciated future capital and the contribution of the new unit of capital to the marginal decline in installation costs in the future.

The first order condition with respect to physical capital in the IC sector is similarly given by,

$$M_{t+1}E_t \left( \alpha y_{t+1} + 1 - \delta - \zeta \left( \frac{x_{ky,t+1}}{k_{y,t+1}} \right) + \zeta' \left( \frac{x_{ky,t+1}}{k_{y,t+1}} \right) \frac{x_{ky,t+1}}{k_{y,t+1}} \right) = M_t(1 + \zeta' \left( \frac{x_{ky,t}}{k_{y,t}} \right)). \quad (7)$$

Here $\lambda$ is the lagrange multiplier associated with the law of motion for IC given by equation (5) which acts as a constraint on the firm’s production. Similar to the final goods sector the marginal benefit of an additional unit of $k_{t+1}$ in IC on the left is equated to its marginal cost on the right. However, unlike the final goods sector, the marginal product of an extra unit of $k_{t+1}$ in the IC sector on the left hand side, is weighted by $\lambda_{t+1}$, the future value of the lagrange multiplier associated with the IC constraint. That is, the contribution to marginal revenue generated from an additional unit of $k_{t+1}$ in the IC sector depends on the expected value to the firm of its future IC-constraint. The rest of the terms in equation (8) are similar in meaning to the corresponding terms in equation (7).

The firm’s optimality condition with respect to IC is given by,

$$M_{t+1}E_t \left( \frac{y_{t+1}}{k_{y,t+1}} + 1 - \delta - \zeta \left( \frac{x_{kz,t+1}}{k_{z,t+1}} \right) + \zeta' \left( \frac{x_{kz,t+1}}{k_{z,t+1}} \right) \frac{x_{kz,t+1}}{k_{z,t+1}} + \alpha \frac{x_{z,t+1}}{k_{z,t+1}} \right) = M_t(1 + \zeta' \left( \frac{x_{kz,t}}{k_{z,t}} \right)). \quad (8)$$

The firm’s optimality condition with respect to IC is given by,
λ measures the "shadow value" of the IC-constraint to the firm and equation (9) gives an intuitive expression for it. λ equals the expected discounted value of the marginal benefit from having an extra unit of $z_{t+1}$ which is the sum of two components: IC’s contribution to an increase in output of final goods by the amount of its discounted marginal productivity and, the change in the expected shadow value of the IC constraint due to a rise in IC investments by the amount of its marginal productivity in the IC sector along with the undepreciated amount of IC.

Finally, labor demand in the final goods and IC sectors are given by the respective sectoral first order conditions with respect to labor,

\[(1 - \alpha - \gamma) A_t k^\alpha_t l^{\alpha - \gamma}_{y,t} = w_t, \quad (10)\]
\[\lambda_t (1 - \alpha - \gamma) B_t k^\alpha_t l^{\alpha - \gamma}_{z,t} = w_t. \quad (11)\]

In both equations (10) and (11) the firm equates the marginal cost of employing an additional unit of labor, or the real wage, on the right hand side, to its marginal benefit on the left. In the final goods sector in equation (10), the marginal benefit of an extra unit of labor is simply its marginal product. In the IC sector however (equation (11)), the marginal benefit of any additional labor internalizes its effect on the value of the IC-constraint to the firm given by $\lambda_t$.

It is straightforward to see from equation (10) that the responsiveness of real wage to the productivity shock $A_t$ declines as $\gamma$ rises. Intuitively, as IC becomes more important in production and the importance of labor declines $(1 - \alpha - \gamma)$ falls), firms vary their labor input less in response to the productivity shock. That is, labor demand in final goods rises less in response to the productivity shock causing the wage rate to become less sensitive to this shock at higher $\gamma$.

In the IC sector (equation 11), there are two opposing effects on the wage rate of an increase in $\gamma$. Similar to equation (10), there is a fall in the response of real wage to the IC-shock, $B_t$ due to the lower importance of labor in production as labor’s income
share falls. There is however, a more direct and hence larger effect of a change in $\gamma$ on the wage rate through $\lambda_t$. From equation (9), an increase in $\gamma$ directly raises $\lambda_t$ because the expected discounted marginal productivity of IC is now higher in both sectors. In other words, when IC is more important in production, any investment in IC today has greater future returns because the marginal product of the future IC stock in both sectors is higher. This direct effect of an increase in $\gamma$ dominates its dampening effect working through a lower labor share in equation (11), implying, the responsiveness of real wage to the IC-shock rises as $\gamma$ increases.

Another way to think about the dominant effect of an increase in $\gamma$ in equation (11) is the following - as the importance of IC in production rises, firms become more IC-constrained since production in both sectors now require higher stocks of IC. The value ($\lambda_t$) to the firm of relaxing this constraint therefore rises. An IC-shock raises the current productivity of IC investments and relaxes the constraint causing the firm’s labor demand in the IC sector to increase more (than in the case of a lower $\gamma$). The wage rate rises on the back of a higher labor demand in the IC sector implying, when $\gamma$ is higher, the wage rate changes more in response to the IC sector shock as denoted by equation (11). Thus, from equations (10) and (11), an increase $\gamma$ makes the IC-shock the more important driver of fluctuations in real wage while lowering the importance of the productivity shock.

**Households**

The representative household maximizes consumption,

$$\text{Max} \quad E_t \sum_{t=0}^{\infty} \beta^t (c_t - \psi t^{\frac{1+\eta}{\eta}}),$$

subject to the following budget constraint,

$$c_t = w_t l_t,$$

where $c_t$ is the household’s consumption and $l_t$ is total labor supplied by the household. $\psi$ represents the disutility derived from working and $\eta$ is the Frisch elasticity of labor.
supply. The first order condition with respect to labor supply is then given by,

\[ l_t = \left( \frac{w_t}{\psi} \right)^{1/\eta}. \]  

(13)

We assume preferences of the form described in Greenwood, Hercowitz and Hoffman (1988) (hereafter GHH) in this section. As is well known in the literature these preferences do not take into account the wealth effect of a change in the wage rate on labor supply as represented by the household’s optimality condition above. From equation (13), labor supply is a function of the wage rate alone (and not of household consumption). From equations (10) and (11), we know that an increase in \( \gamma \) leads to a decrease in the responsiveness of the real wage to the productivity shock and an increase in its responsiveness to the IC shock. From equation (13) it becomes further clear that any change in the wage rate causes labor input to change in the same direction. Thus equations (10), (11) and (13) together imply, an increase in \( \gamma \) increases the responsiveness of real wage and hours to the IC shock while decreasing their responsiveness to the productivity shock.

In Section 6 I consider log preferences and show that the results of the model remain qualitatively unchanged. However, the responsiveness of labor supplied changes more than the wage rate does with an increase in \( \gamma \), under that specification, due to the wealth effect. I discuss this in more detail later, however, empirically, the change in labor input volatility is lower than that of the wage rate while under GHH preferences the changes are of a similar magnitude. I therefore consider GHH preferences in this section and present results with log preferences in Section 6.

**Definition of equilibrium**

An equilibrium in this economy is defined in the usual way. That is, an equilibrium is a sequence of wages, \( \{w_t\}_{t=0}^\infty \), and corresponding labor inputs in the two sectors \( \{l_{y,t}, l_{z,t}\}_{t=0}^\infty \) such that (i) firms maximize profits subject to equations (2)-(5) and households maximize their utility subject to equation (12) taking as given the exogenous and endogenous states \( \{A_t, B_t\}, \{k_{y,t}, k_{z,t}, z_t\} \) and the price sequence \( \{w_t\}_{t=0}^\infty \) for labor, and (ii) the capital, labor and goods markets clear as follows:
\[ k_{y,t} + k_{z,t} = k_t, \quad (14) \]
\[ l_{y,t} + l_{z,t} = l_t, \quad (15) \]
\[ c_t + x_t = y_t, \quad (16) \]

where \( k_t \) is the aggregate physical capital stock in the economy.

4 The impact of a rise in IC

Using steady state versions of equations (9) and (3) gives us \( \lambda \) as a function of \( \frac{y_z}{y} \) at steady state,

\[ \lambda = \left( \frac{\beta \gamma}{\phi(1 - \gamma)} \right) \frac{y}{z} \quad (17) \]

Substituting (17) into the optimality condition for labor in the IC sector (equation 11) and using steady state equation (3) once again, we arrive at the following condition for steady state employment in the IC sector,

\[ w = \left( \frac{\beta \gamma (1 - \alpha - \gamma)}{1 - \gamma} \right) \frac{y}{l_z} \quad (18) \]

The optimality condition for employment in final goods sector or equation (10) similarly gives us the following steady state expression,

\[ w = (1 - \alpha - \gamma) \frac{y}{l_y} \quad (19) \]

Equating (18) and (19) above we get \( l_y \) as a function of \( l_z \) at steady state,
\[ l_y = \frac{1 - \gamma}{\beta \gamma} l_z, \]  

(20)

implying a total labor supply \((l_y + l_z)\) of,

\[ l = \frac{1 - \gamma(1 - \beta)}{\beta \gamma} l_z, \]  

(21)

and an employment share of IC given by,

\[ \frac{l_z}{l} = \frac{\beta \gamma}{1 - \gamma(1 - \beta)}. \]  

(22)

In (22) the employment share of IC is a positive function of \(\gamma\), the income share of intangible capital in the production function. Intuitively this is straightforward, since an increasing share of IC in the production process implies a larger emphasis on production of IC investments and hence greater employment in the IC sector. Thus an increase in \(\gamma\) in the model is directly associated with rising intangible sector employment share.

I next calibrate the model and examine the interaction between final goods and the IC sector quantitatively with the aim to understand the aggregate consequences of an increase in IC and the IC specific shock in the economy, with particular focus on changes in the labor market. I compare the effects of the IC shock to those arising from fluctuations in the pure productivity shock and examine the sensitivity of my findings to changes in the model’s key parameters.

### 4.1 Calibration

The model is calibrated to the U.S. economy with the time period \(t\) representing a quarter. I set the discount factor of the households, \(\beta = 0.99\) corresponding to a quarterly interest rate of 1%. I assume \(\eta = 3\) for the Frisch elasticity of labor supply which lies in between
the range of 2 to 4 typically estimated by macroeconomic studies. The results of the model remain unchanged for reasonably higher or lower values of \( \eta \). The disutility of labor parameter \( \psi \) is a constant and is set to equal 2.5 in order that total steady state hours worked is \( 1/3 \) or \( l_t = 0.3 \) in the pre-84 period in the model. Like all parameters of the model (except \( \gamma \)), I do not allow \( \psi \) to change when I consider a higher value of \( \gamma \), however, unlike other parameters of the model \( \psi \) has no quantitative impact on the model’s results.

The quarterly depreciation rate of physical capital, \( \delta \) is set to the standard value of 0.025 which gives a yearly depreciation rate of 10%. The depreciation rate of intangible capital, \( \phi \) is more difficult to calibrate. Corrado et al (2009) use limited information available for different types of IC to compute the annual depreciation rate for each type. The corresponding quarterly depreciation rates for the different types of IC are 5% for scientific and non-scientific R&D, 8.25% for computerized information other than software, 10% for firm-specific resources and 12% for brand equity. A simple average yields a depreciation rate for IC of around 8%. Mcgrattan and Prescott (2009, 2012) assume benchmark annual depreciation rates for IC between 0-7%, which imply quarterly depreciation rates of between 0-1.75%. I assume a benchmark depreciation rate for IC of 5%, which lies in between the values reported by Corrado et al (2009) and those used by Mcgrattan and Prescott (2009, 2012). I report results for both higher and lower values of \( \phi \) used in the literature, in Section 6.

The convex adjustment cost function for investment in physical capital is of the form, 
\[
\zeta = \frac{b}{2} \left( \frac{x_{k,t}}{k_t} - \delta \right)^2 k_t,
\]
such that the cost of adjustment depends on the ratio of investment to capital and scales up with the level of capital. \( b \), the capital adjustment cost parameter is chosen to match a volatility of investment in physical capital that is about three times that of output. \( \gamma \), the income share of IC and the parameter of interest in the model, requires values for the periods before and after the Great Moderation. I allow the value of \( \gamma \) to shift in a way that causes the share of employment in the IC sector to go from a targeted pre-84 value in the occupation data analyzed in Section 1, to a post-2010 target. In the dataset the employment share of IC in 1968 is 7% rising to 16% in 2016. From steady state equation (22), there is a direct relationship between \( \gamma \) and the IC
employment share implying, the above targets yield pre- and post-84 values for \( \gamma \) of 0.07 and 0.16 respectively. In line with these model-implied values for \( \gamma \), Corrado et al (2005) estimate that the income share of IC rose from an average 9.4% in the period 1973-1995, to an average of 14% in 1995-2003.

\( \alpha_0 \), which is the Pre-84 value of physical capital’s income share in equation (10), is set to 0.28 such that the total elasticity of the two types of capital taken together is equal to 0.35 in the Pre-84 period. This implies that labor’s income share is 0.65 to begin with, which is in line with the findings of Karabarbounis and Neiman (2014) for the pre-84 period. Given the strong evidence in favor of a significant decline in labor’s income share that accompanied the period of increase in IC’s income share in the literature (Karabarbounis and Neiman, 2014; Corrado et al, 2009), I set \( \tau \), the percentage increase in \( \gamma \) that is deducted from \( \alpha \) (in equation (6)), equal to 30% implying 70% of the rise in IC’s income share is deducted from the share of labor. Labor’s income share thus declines to 59% in the post-84 period in the model, similar to the estimates of Karabarbounis and Neiman (2014) causing the total income share allocated to (intangible and physical) capital to rise to 41%.

I set the standard deviation of innovation to the productivity shock in final goods to 0.009 with a persistence of 0.95 following standard business cycle literature. The standard deviation of innovations to the IC shock, \( e_b \), is set to match the excess volatility of IC sector’s employment in Figure 2 while the persistence of this shock is set to \( \rho_b = 0.8 \), to match the pre-84 correlation between labor productivity and labor input in GVR (2014). As I discuss in Section 6 below, the persistence of the IC shock relative to the productivity shock matters for the level of the model generated correlations between measured labor productivity, and employment and output but not for the changes in these correlations brought about by a rising \( \gamma \). Therefore I assign a value to \( \rho_b \) in this section, that gives rise to the empirically observed correlation of labor input with labor productivity in the pre-84 period. This way, the model generated decline in the procyclicality of labor productivity, due to a rise in \( \gamma \), can be more easily compared to the observed decline in the data. Finally, I assume the shocks to be highly correlated in this section, with the correlation parameter \( \chi \) set to 0.7. I present results with uncorrelated shocks in Section
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.99</td>
<td>Quarterly interest rate=0.01</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Labor supply elasticity</td>
<td>3</td>
<td>Literature</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Disutility of labor parameter</td>
<td>2.6</td>
<td>Hours worked=0.3</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate of $k$</td>
<td>0.025</td>
<td>Literature</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Depreciation rate of IC</td>
<td>0.05</td>
<td>Mcgrattan and Prescott (2012)</td>
</tr>
<tr>
<td>$b$</td>
<td>Capital Adjustment parameter</td>
<td>5</td>
<td>Rel. investment volatility=3</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>Income share of $k$ ($\alpha$ pre-84)</td>
<td>0.28</td>
<td>Literature</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>Pre-84 IC income share</td>
<td>0.07</td>
<td>Pre-84 IC to capital ratio</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>Post-84 IC income share</td>
<td>0.16</td>
<td>Post-2010 IC to capital ratio</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Persistence of prod. shock</td>
<td>0.95</td>
<td>Literature</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>St. dev. of prod. shock</td>
<td>0.009</td>
<td>Literature</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>Persistence of IC-shock</td>
<td>0.80</td>
<td>Pre-84 Corr(LP,employment) in GVR (2014)</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>St. dev. of IC shock</td>
<td>0.04</td>
<td>Excess volatility of IC employment</td>
</tr>
<tr>
<td>$\text{corr}(e,e_b)$</td>
<td>Correlation of shocks</td>
<td>0.7</td>
<td>Av. corr. of sectoral employment</td>
</tr>
</tbody>
</table>

Model Determined

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>Post-84 income share of $k$</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 1: Calibrated parameter values

### 4.2 Response of labor, real wage and output to a productivity shock

In this section, I study the response of key labor market aggregates to a one standard deviation shock to the productivity of final goods alone. In other words, the innovation to the IC-shock is set to zero throughout this section. I start the model at steady state and simulate it for one thousand periods. I drop the first two hundred observations and HP-filter the model generated time series, before presenting the results in Figure 2.
Figure 3: The figures show impulse responses to a "pure" (i.e. they leave the innovation to the IC shock unaffected) productivity shock. The responses are percent deviations from steady state.

As expected, a positive productivity shock increases final output, employment, labor productivity and real wage upon impact as in standard RBC models. Unlike the standard RBC framework however, labor input volatility relative to output, in response to the productivity shock, is higher in this model. This is because, unlike in the standard setting, an increase in final good’s productivity here, increases the productivity of IC, which causes labor demand and consequently labor input in the IC sector to rise in addition to the original increase in labor input in final goods. Thus aggregate hours are
more volatile relative to output in this framework, making this extension an improvement over the standard RBC framework which is known to generate too little volatility of employment. This increased responsiveness of labor input to a technology shock due to the inclusion of IC in an otherwise standard RBC model is also highlighted in Gourio and Rudanko (2014). Finally, in Figure 3, there is a strong positive correlation between labor productivity and both output and labor in response to the productivity shock. These correlations are about 0.99 (see Table 2), as is once again, standard in simple RBC models.

<table>
<thead>
<tr>
<th></th>
<th>Low $\gamma$</th>
<th>High $\gamma$</th>
<th>Relative</th>
</tr>
</thead>
<tbody>
<tr>
<td>$vol(y)$</td>
<td>2.28</td>
<td>2.08</td>
<td>0.91</td>
</tr>
<tr>
<td>$vol(l)$</td>
<td>1.7</td>
<td>1.53</td>
<td>0.9</td>
</tr>
<tr>
<td>$vol(w)$</td>
<td>0.55</td>
<td>0.5</td>
<td>0.91</td>
</tr>
<tr>
<td>$vol(l)/vol(y)$</td>
<td>0.75</td>
<td>0.74</td>
<td>0.98</td>
</tr>
<tr>
<td>$vol(w)/vol(y)$</td>
<td>0.24</td>
<td>0.24</td>
<td>0.99</td>
</tr>
<tr>
<td>$Corr(lp, y)$</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$Corr(lp, l)$</td>
<td>0.99</td>
<td>0.99</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: The table reports moments of model implied output, hours, wages, and labor productivity in response to a pure (i.e. they leave the innovation to the IC shock unaffected) productivity shock. All series are HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments.

In Table 2, output, employment and real wage volatilities fall as $\gamma$ rises\(^4\). At higher $\gamma$ the IC-constraint faced by the firm is stronger, implying, it is more important for the firm to raise their investment in IC for them to increase the production of final goods in response to a productivity shock to the latter. Thus more labor gets diverted to production of IC (than when $\gamma$ is lower) as the productivity shock hits final goods causing hours and hence output to rise less, therefore lowering measured output volatility as $\gamma$ increases. Volatilities of wages and hours decline in Table 2 because, from our discussions of equations (10) and (12) in Section 3, an increase in $\gamma$ makes the real wage and hence

\(^4\)Volatility of a variable $x$ in the model, is measured by its coefficient of variation, such that $vol(x) = \frac{|\text{var}(x)|^{1/2}}{\text{mean}(x)}$. 

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total labor input, less responsive to the productivity shock.

4.3 Response of labor, real wage and output to a IC shock

In this section, I repeat the simulation exercise from above with an IC shock alone. That is, I shut down the productivity shock to final goods and allow only for a one standard deviation shock to the IC sector. As before, I simulate the model for one thousand periods, drop the first two hundred observations, HP-filter the model generated time series and report the impulse responses for both high and low values of $\gamma$. 
Figure 4: The figures show impulse responses to a pure IC shock. The responses are percent deviations from steady state.

Figure 4 presents the impulse responses of labor input, wages, output and productivity to a pure IC shock. A positive IC shock causes reallocation of labor from the final goods to the IC sector upon impact as the productivity of the latter increases relative to the former. Thus final good’s output falls upon impact of the IC shock while IC investment rises. The latter drives up labor demand in IC and hence the wage rate causing total labor input to rise in turn. Thus measured output and aggregate hours move in opposite directions as the IC shock hits, causing measured labor productivity to fall upon impact in
Figure 4. This generates a negative correlation between labor input and measured labor productivity. Measured output and labor productivity are, however, positively correlated since both fall upon impact of the IC shock. As the initial impact of the IC shock passes, the stock of IC in the economy increases, final good’s output rises and employment in the IC sector falls causing the wage rate and total employment to climb back down while labor productivity recovers.

<table>
<thead>
<tr>
<th></th>
<th>Low $\gamma$</th>
<th>High $\gamma$</th>
<th>Relative</th>
</tr>
</thead>
<tbody>
<tr>
<td>$vol(y)$</td>
<td>0.41</td>
<td>0.73</td>
<td>1.78</td>
</tr>
<tr>
<td>$vol(l)$</td>
<td>0.43</td>
<td>0.95</td>
<td>2.21</td>
</tr>
<tr>
<td>$vol(w)$</td>
<td>0.14</td>
<td>0.31</td>
<td>2.21</td>
</tr>
<tr>
<td>$vol(l)/vol(y)$</td>
<td>1.05</td>
<td>1.30</td>
<td>1.24</td>
</tr>
<tr>
<td>$vol(w)/vol(y)$</td>
<td>0.34</td>
<td>0.2</td>
<td>1.24</td>
</tr>
<tr>
<td>$Corr(lp, y)$</td>
<td>0.82</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>$Corr(lp, l)$</td>
<td>-0.84</td>
<td>-0.89</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: The table reports moments of model implied output, hours, wages, and labor productivity in response to a pure IC shock. All series are HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments.

When $\gamma$ is higher, the IC constraint is more important for the firm and there is greater reallocation from final goods to the IC sector due to the IC shock, causing measured output volatility to rise as seen in Table 3. Wage volatility increases because we know, from equation (11), an increase in $\gamma$ raises the responsiveness of the real wage to the IC-shock and consequently, from equation (13), the volatility of labor input rises as well. Thus output, real wage and labor input volatilities rise with an increase in $\gamma$ in response to the IC shock.

The higher reallocation from final goods to IC, when $\gamma$ is higher, implies there is a larger fall in measured output due to the IC shock. This is accompanied by larger employment growth in IC causing total employment growth to be higher as well, at higher $\gamma$. Thus the negative correlation between labor input and measured labor productivity is stronger when $\gamma$ is higher, as reported in Table 3. Finally, note from Table 3 that although the
level of volatility generated by the IC shock is much lower compared to the productivity shock in Section 4.2, the changes in these volatilities brought about by a rise in $\gamma$ are substantial.

### 4.4 Effect of intangible capital when both productivity and IC shocks are present

In this section, I allow for an increase in $\gamma$ to occur when both the productivity and IC shocks operate simultaneously. The model is solved similarly to the above two sections. As emphasized earlier, the aim here is to investigate if the increase in IC’s importance in production in recent decades can move several macroeconomic moments in the direction observed in the data. Table 4 presents the correlations generated by the complete model and compares them to the different empirical results discussed in Section 2.

<table>
<thead>
<tr>
<th>Data</th>
<th>Pre-84</th>
<th>Post-84</th>
<th>Relative</th>
<th>Pre-84</th>
<th>Post-84</th>
<th>Relative</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. (GVR)</td>
<td>0.61</td>
<td>0.04</td>
<td>-0.57</td>
<td>0.17</td>
<td>-0.56</td>
<td>-0.73</td>
</tr>
<tr>
<td>1949-2007, HP filtered</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. (OR)</td>
<td>0.77</td>
<td>0.67</td>
<td>-0.10</td>
<td>0.27</td>
<td>-0.03</td>
<td>-0.30</td>
</tr>
<tr>
<td>1960-2007, BP filtered</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. (CK)</td>
<td>0.65</td>
<td>0.01</td>
<td>-0.64</td>
<td>0.21</td>
<td>-0.50</td>
<td>-0.71</td>
</tr>
<tr>
<td>1964-2006, HP filtered</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>0.43</td>
<td>0.08</td>
<td>-0.35</td>
<td>0.17</td>
<td>-0.45</td>
<td>-0.62</td>
</tr>
</tbody>
</table>

Table 4: Correlation Productivity. GVR=Gali and VanRens (2014), OR=Ohanian and Raffo (2012) and CK=Champagne and Kurmann (2012)

Firstly, note that the model generated correlation of measured labor productivity with output is much lower in Table 4 than in the sections immediately above. This is because, as shown in Sections 4.2 and 4.3, a positive productivity shock increases output and
labor input while a positive IC-shock reduces output and increases labor input. The productivity shock has the more dominant effect causing both final output and labor input to rise, but due to the influence of the IC shock, output rises less, while labor rises more than in the case of a pure technology shock. Measured labor productivity therefore rises much less than in the case of the pure technology shock causing procyclicality of measured productivity with respect to output to be lower. As $\gamma$ increases, the impact of the IC-shock rises implying, the increase in output is even lower while the rise in hours accompanying it is higher still. Thus the procyclicality of measured productivity with respect to output falls substantially in Table 4 with an increase in $\gamma$.

From Sections 4.2 and 4.3, we also know that labor input and measured productivity move strongly positively in response to a pure technology shock and strongly negatively in response to a pure IC-shock. Therefore, when both shocks are present, the (positive) correlation between hours and productivity generated by the model is lower as seen in Table 4. Recall that the model generated correlation between measured labor productivity and hours for the lower value of $\gamma$ is targeted in Section 4.1 to match its pre-84 counterpart in GVR (2014) for HP-filtered data. As $\gamma$ rises to its post-84 value in the model, the correlation between measured productivity and labor input declines significantly and becomes strongly negative as observed in the data. This happens because at higher $\gamma$ the IC shock becomes more important and the negative correlation between labor input and measured productivity, generated by this shock becomes much stronger. Another way to think about it is that an increase in $\gamma$ causes larger increases in labor input to be associated with smaller increases in measured output leading to a (sharp) decline in the correlation of measured productivity and hours.
Table 5: The table reports volatilities of model implied quantities of output, hours, wages, and labor productivity. All series were HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments. Moments of HP-filtered (total economy) data for sample period 1949-2007 is from Gali and Van Rens (2014).

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th></th>
<th>Model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard Deviation</td>
<td>Relative standard deviation</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1) Pre-84</td>
<td>2) Post-84</td>
<td>3) Post-84/Pre-84</td>
<td>4) Pre-84</td>
</tr>
<tr>
<td>(\sigma(y))</td>
<td>2.47</td>
<td>1.19</td>
<td>0.48</td>
<td>1</td>
</tr>
<tr>
<td>(\sigma(l))</td>
<td>1.71</td>
<td>1.06</td>
<td>0.62</td>
<td>0.70</td>
</tr>
<tr>
<td>(\sigma(w))</td>
<td>0.84</td>
<td>0.95</td>
<td>1.14</td>
<td>0.34</td>
</tr>
<tr>
<td>(\sigma(y))</td>
<td>2.19</td>
<td>2.05</td>
<td>0.93</td>
<td>1</td>
</tr>
<tr>
<td>(\sigma(l))</td>
<td>2.02</td>
<td>2.3</td>
<td>1.14</td>
<td>0.92</td>
</tr>
<tr>
<td>(\sigma(w))</td>
<td>0.68</td>
<td>0.77</td>
<td>1.14</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Table 6: The table reports moments of other key variables implied by the model. \(x_k\) is investment in physical capital, \(c\) is consumption, \(l\) is total labor supplied and \(y\) is total output. All series are HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments.

As argued by Stiroh (2009) and Gali and Gambetti (2009), a substantial fraction of the decline in output volatility characterizing the Great Moderation can be explained by the sizeable decline in the correlation between labor productivity and hours. From Table 4 an increase in \(\gamma\) does indeed lead to a large decline in the correlation of hours and productivity in the model as output responds less and labor responds more to a combination of technology and IC shocks. Thus output volatility declines while labor input volatility rises with \(\gamma\) in the model.

Real wage volatility increases with \(\gamma\) in Table 5 both absolutely and relative to output. Both the effects described in equations (10) and (11) are now at work together, that is, the responsiveness of real wage to the IC shock increases while its responsiveness to the productivity shock declines. The fall in measured output volatility with a rising \(\gamma\),
also contributes to the rise in relative wage volatility in the model. Labor input volatility responds similarly to the wage rate following equation (13), implying labor input volatility rises with $\gamma$ as well. Note here that while the model predicts an increase in the absolute volatility of labor input, both GVR (2014) and Gali and Gambetti (2009) report a small decline in the volatility of this variable in the data during this period. However, the decline in absolute volatility of labor input in the data is a small one and models trying to explain one or more of these labor market features often find an increase in absolute labor input volatility. For instance, Nucci and Riggi (2013) and Champagne and Kurmann (2012), both demonstrating the role played by the increasing importance of performance pay in generating shifts in key labor market dynamics also fail to generate a fall in labor input volatility over this period. However, labor input volatility relative to output increased substantially in the data as it does in the model. The model generated 22% increase in labor input volatility relative to income represents over 80% of the increase in relative hours volatility documented in GVR (2014).

Table 6 presents some other key business cycle statistics generated by the benchmark model with IC. Except for the relative volatility of investment in the first column, none of the other moments were targeted in the calibration process. It is clear from the table that the model does a good job of reproducing standard business cycle moments. Moreover, it generally improves labor market related results along several lines, like generating a higher volatility of hours relative to output and a more realistic positive correlation between measured productivity and output. Thus the model provides a framework within which the recent shifts in labor market dynamics arise jointly as a result of the rising importance of IC, without sacrificing, and often improving upon, key business cycle moments.

4.5 On the nature of the IC shock

In this section, I briefly compare the IC shock of my model with both investment specific technology (IST) shocks of Greenwood, Hercowitz and Krusell (1997, 2000) (or GHK) and neutral or multifactor productivity shocks (MFP). This is especially important because, on the surface, some effects of an IC shock resemble that of an IST shock, but upon
careful analysis it becomes clear that the mechanisms involved are in fact more similar to an MFP shock to the IC investment sector. In this sense, the setting of the IC shock in this paper can be likened to the MFP shock to investment in physical capital in Guerrieri et al (2014)\textsuperscript{5}.

Similar to an MFP shock, the IC shock in my model raises output, investment (in both types of capitals) and employment in the IC sector. A differentiating feature of IC is that intangible investment is not measured, implying, an increase in the production of IC investment brought about by the IC shock does not raise aggregate (measured) output. In fact, measured output falls as resources reallocate from the final goods to the IC sector in response to the IC shock. Consumption falls with output but investment in both types of capital increase due to the higher productivity of the IC sector which uses both types of capital in its production. Thus the IC shock, working alone, generates a positive correlation between measured output and consumption but a negative correlation between measured output and investment giving rise to a negative correlation between consumption and investment (of both types). It is well established in the literature that an IST shock generates negative comovement between consumption and investment. Guerrieri et al (2014) show for instance that expansionary MFP shocks boost consumption in every period, whereas expansionary IST shocks cause consumption to fall substantially for many periods generating the negative correlation between investment and consumption commonly associated with IST shocks. This happens because IST shocks make consumption more expensive relative to investment causing agents to substitute in favor of investment and away from consumption. Unlike this mechanism for IST shocks, however, the negative correlation arises, due to an IC shock, in my model because IC investments are unmeasured. Hence although IC investments rise, measured output and hence consumption fall in response to the IC shock. Had IC investments been measured, the IC shock would raise total output, total labor supplied, investment and consumption similar to an MFP shock and generate the observed positive correlations between these variables.

A second effect of the IC shock that resembles that of an investment specific shock is

\textsuperscript{5}The authors provide a good account of the conditions under which an aggregate IST shock can approximate an MFP shock to the investment sector (in physical capital).
that both shocks cause labor productivity to fall upon impact. In case of an investment shock however, productivity falls because hours rise immediately but investment takes time to adjust. In case of the IC shock, productivity falls due to the drop in measured output upon impact although there is also an immediate increase in hours. Thus under an IC shock the negative impact on measured productivity is stronger than under an investment specific shock - but as before, if IC investments were measured, both output and hours would rise, reversing the effect of the IC shock on labor productivity and making it resemble a neutral productivity shock to the IC (investment producing) sector.

In sum, there are similarities between the effects of the IC shock of my model and the IST shocks of GHK, however, the likeness does not stem from the similar nature of the two shocks, but from the assumption that investment in intangibles are unmeasured. In combination with the standard productivity shock, the IC shock seems to account for a number of otherwise puzzling phenomenon, in case of this paper, with regards to observed shifts in the evolution of labor market aggregates, which are more accurately measured.

5 Sensitivity Analysis

In this section I test the sensitivity of my model's results to some key parameters. The results discussed are presented in Tables 7 and 8.

Log preferences: I first substitute the GHH preferences of Section 3 with a standard or log utility function as follows: $u = \log(ch,t) - \psi \frac{1 + \frac{1}{\eta}}{(1 + \frac{1}{\eta})}$. As expected, the wealth effect of a wage change now comes into play under these preferences. Recall that with GHH preferences, the household’s first order condition with respect to labor (equation (13)) gives labor as an increasing function of the wage alone. Under log preferences, the same first order condition becomes, $l_t = \left( \frac{w_t}{c_t} \right)^{1/\eta}$, that is, changes in $w_t$ now also affect current consumption, $c_t$.

The substitution effect of a wage change causes labor supplied by households to rise in response to an increase in wage rate as leisure becomes more costly. The wealth effect on the other hand, implies labor supplied falls with wage increases due to an
increase in household’s consumption (including leisure). The lower the incentive to save or accumulate capital in an economy, the higher is the increase in consumption and lower is the rise in labor supply due to wage increases.

In the context of my model, an increase in $\gamma$ raises the importance of (intangible) capital in the economy. Thus the incentive to save and invest in IC in an economy with higher $\gamma$, is higher. This greater saving motive causes agents to increase their current labor supply more in response to an increase in the wage rate. Only, since there is no actual saving by households in the model, the households ensure higher consumption tomorrow in a high-$\gamma$ economy by supplying more labor to the IC investment sector today since marginal productivity of future IC stock is higher implying higher wages and hence higher consumption next period. Thus, in a high-$\gamma$ scenario, an increase in the wage rate causes $c_t$ to rise less and $l_t$ to rise more leading to larger changes in labor input for a given change in the wage rate. That is, under log preferences, the increase in labor input volatility due to a rise in $\gamma$, is heightened while the rise in wage volatility is subdued as seen in Panel 1 of Table 7. Under the GHH preferences of Section 3, both hours and real wage volatility rise by similar amounts as $\gamma$ increases (Table 5).

Correlations of measured labor productivity with respect to output and labor are much higher than the benchmark model, especially for the lower value of $\gamma$. At the lower $\gamma$, the influence of the productivity shock is higher, implying most of the increase in labor input generates an increase in measured output. Since labor increases more under log preferences, output also increases more, causing measured productivity and measured output to move more in the same direction thus giving rise to much stronger procyclicality of labor productivity with respect to both labor and output in Table 8. As $\gamma$ increases however, the contribution of the productivity shock falls and labor increases mainly in the IC sector in response to the IC shock. Now as labor increases more strongly (due to log preferences), the larger increase in output occurs in the IC sector which is not measured. This causes labor productivity and measured output to become even less positively correlated at higher $\gamma$ than with GHH preferences.

In sum, the results of the benchmark model go through under standard preferences, and
in some cases results are stronger - for example, we get larger increases in labor input volatility and sharper declines in procyclicality of productivity. However, the rise in wage volatility is less pronounced than under GHH preferences.

**IC’s depreciation rate ($\phi$):** As explained in Section 4.1, the depreciation rate of IC is a difficult parameter to pin down given the dearth of empirical estimates and the wide range of values used in the literature. In Section 4, I use a quarterly depreciation rate of 5% (20% annual depreciation rate). In this Section I experiment with values of $\phi$ both above and below the benchmark.

I first consider a higher value of $\phi$ - an annual depreciation rate of 30% or a quarterly rate of $\phi = 0.075$. From equation (9), a higher $\phi$ lowers $\lambda_t$, the value of the IC constraint to the firm. A lower value of the IC constraint implies the firm responds more strongly to a productivity shock in final goods. Additionally, a higher $\phi$ also increases the firm’s response to the IC shock, since firms want to increase their IC investments more today, to make up for a larger amount of IC depreciation tomorrow. This causes greater reallocation of resources from final goods to the IC sector when the IC shock hits. These two effects of a higher $\phi$ together imply that in the presence of both shocks, the volatilities of output, labor input and real wage are higher across the different values of $\gamma$ as reported in Panel 2 of Table 7.

An increase in $\gamma$ has the same qualitative effect as in Section 4.4, that is, output volatility falls while hours and real wage volatility rises. However, at a higher $\phi$ these changes are less pronounced, that is output volatility falls less and wage and employment volatility relative to output increases less than before. This happens because, as $\gamma$ increases, as before the effect of the IC shock rises. As the firm becomes more sensitive to the IC shock they increase their production of IC investments more today in order to make up for the larger depreciated IC shock. The increased reallocation from final goods to IC makes output volatility rise more with $\gamma$ due to the IC shock. Thus when both shocks are present, output volatility falls much less causing the relative volatilities of hours and real wage to rise less in turn, in Panel 2 of Table 7.

Correlations of measured productivity with labor and output in Table 8 are slightly higher.
in case of the higher $\phi$, again because a higher $\phi$ makes the IC-constraint less important for the firm and firms respond less to the IC-shock causing final output and labor to move less in opposite directions upon impact. Thus the effect of the IC shock on the procyclicality of measured labor productivity in the model is lower giving rise to higher correlations of productivity with respect to both output and labor for both values of $\gamma$, relative to the benchmark case. The effect of an increase in $\gamma$, however, is similar to the benchmark, with the magnitude of decline in the procyclicality of measured productivity with labor and output falling by similar amounts as the benchmark. Thus $\phi$ affects (slightly) the level of the correlations but not the magnitude of their change as $\gamma$ rises.

Panel 3 of Table 7 presents the results of using a lower value of $\phi$ relative to the benchmark. Following McGrattan and Prescott (2009, 2012), who use annual depreciation rates for IC between 0 and 7%, I set $\phi = 0.01$ reflecting an annual depreciation rate of 4%. A lower $\phi$, as expected, generates stronger model results. That is, a rise in $\gamma$ now generates a larger fall in output volatility and a stronger increase in hours and wage volatility relative to the benchmark. Also, correlations of measured productivity with hours and output are much lower than when $\phi$ is higher, since the IC shock plays a larger role. However, as before the magnitude of the decline in correlations, with an increase in $\gamma$, is not much affected by a lower $\phi$.

Correlation of shocks ($\chi$): In Section 4, I assumed a correlation of $\chi = 0.70$ between the shocks. In this section I first allow the shocks to be uncorrelated ($\chi = 0$) and then raise the correlation further to $\chi = 0.9$ to test how the model’s results vary.

From Tables 7 and 8 in both cases of higher and lower $\chi$, the qualitative results remain unchanged, with labor and wage volatility increasing, output volatility declining and the procyclicality of measured productivity falling significantly. The quantitative strength of these results wane however with declining correlations between the shocks. The model generated results are one of the weakest for the uncorrelated shock with labor and wage volatility rising only 8% and output falling only 4% as $\gamma$ rises.

Thus a rising $\gamma$ has a larger effect on labor market dynamics, when the two shocks are more strongly correlated since the higher correlation between the shocks implies the
productivities of the two sectors increase almost simultaneously, causing the reallocation effect (from final goods to IC) to be stronger. Recall from our discussions above, that as $\gamma$ increases the IC constraint becomes more important for the firm. At a higher $\gamma$ therefore the firm depends on and responds more to the IC-shock. When the shocks are more strongly correlated, there is a tighter tradeoff between the two sectors and the firm’s response due to the IC shock affects the economy more strongly.

When the shocks are less correlated, the firms face less of a tradeoff in choosing to reallocate resources between the two sectors. They accumulate as much IC stock as possible while productivity in the IC sector is high (due to the IC shock), in order to be able to increase production of final goods as much as possible when the shock to the latter sector hits. Thus the effects of the IC shock does not have a strong influence on how the firm reacts to the productivity shock in final goods and all the general effects of the IC shock discussed in Section 4.3, become less pronounced. That is, output volatility falls less and labor input and wage volatilities increase less (panel 5, Table 7). The degree of procyclicality generated, between measured labor productivity and both labor and output in Table 8 are also much higher for uncorrelated shocks. This is once again due to the IC shock, which moves measured output and labor input in opposite directions, having a lower impact, when uncorrelated to the productivity shock.

Importantly however, when the shocks are uncorrelated, the decline in procyclicality of measured productivity with an increase in $\gamma$ is much lower with respect to output than with respect to labor with the latter being of a similar magnitude to the benchmark model. This happens because at the low-$\gamma$ scenario, the IC shock already matters less, and, when the shocks are uncorrelated its influence is further lowered giving rise to the higher procyclicality numbers in Table 8. As $\gamma$ increases, the IC shock’s importance rises - however, uncorrelated shocks imply the IC shock affects the IC sector alone with no spillover to the final goods sector. Thus an increase in $\gamma$ which increases the importance of the IC shock increases labor and production of the IC sector (without lowering final good’s output as before). While the rise in IC’s labor is measured, its output is not implying procyclicality of measured productivity with labor falls substantially with the rising $\gamma$. Along the same lines, an increase in the importance of the IC shock with $\gamma$ now
has much lower impact on final goods, that is, final output does not fall upon impact of the IC shock, causing measured productivity to fall less as well. Thus as $\gamma$ rises procyclicality of measured labor productivity with output declines much less than when the shocks are more highly correlated.

Persistence of shocks $(\rho, \rho_b)$: The persistence of the IC shock was set to match the pre-84 degree of procyclicality between labor productivity and labor input in GVR (2014) which implied $\rho_b = 0.8$. Here I experiment with higher and lower values of persistence of the IC shock ($\rho_b$) relative to the persistence of the productivity shock ($\rho$).

I first raise $\rho_b$ to 0.95 making both shocks equally persistent. Compared to the benchmark specification, this generates lower output volatility and higher relative volatilities of wages and hours across both values of $\gamma$ in Panel 6 of Table 7. Recall from Section 4.2 that the IC shock increases final output’s volatility due to reallocation of resources from final goods sector to IC as the IC shock hits. A higher persistence of the IC shock implies the IC sector enjoys higher productivity longer and therefore the incentive to immediately reallocate resources from final goods to IC, due to the IC shock, is lower. In the presence of both shocks, this implies that final output volatility is lower and consequently, relative wage and labor input volatilities are higher for both values of $\gamma$. More importantly, however, the magnitude of the decline in output volatility and the increase in labor and wage volatilities caused by a rising $\gamma$ in Panel 6, are higher relative to the benchmark model. This can be understood by looking at equation (9). Here a more persistent IC shock implies a higher $x_{z,t+1}$ and therefore a larger $\lambda_t$, which is the value to the firm of the IC constraint. A higher $\lambda_t$ implies, from equation (11), that an increase in $\gamma$ causes the sensitivity of the wage rate to the IC shock to rise more causing the increase in volatility of the wage rate due to a rising $\gamma$ to be even higher. It follows from equation (13) then that labor input volatility also increases more with $\gamma$. When the IC shock is more persistent relative to the productivity shock, an increase in the importance of IC with a rising $\gamma$ also puts less pressure on the resources of final goods sector, causing final output to respond less to the IC shock. In the presence of both shocks, what this means is, final good’s volatility falls more as $\gamma$ increases (that is the volatility-lowering effect of the productivity shock is relatively stronger). Thus a more persistent IC shock generates
stronger increases in labor market volatility and larger declines in output volatility in the model.

In Table 8, the level of positive correlation between measured labor productivity and both output and labor are higher in the low-\(\gamma\) case. This is because final goods output now falls less upon impact of the IC shock as less labor is reallocated immediately to the IC sector and IC sector employment itself now rises less upon impact causing aggregate employment to rise less in turn. These imply that measured productivity falls less upon impact of the IC shock which combined with the lower fall in output and a lower increase in hours causes procyclicality of labor productivity to be higher with respect to both output and labor at the lower \(\gamma\).

As \(\gamma\) rises, the procyclicality of measured productivity, with respect to both output and labor input falls more than in the benchmark specification. This is once again due to the higher increase in sensitivity of labor input to the IC shock as outlined above, implying an increase in \(\gamma\) now causes a larger increase in IC’s sector’s employment and hence aggregate employment rises more, reducing in turn the correlations of measured productivity with both output and labor more strongly than the benchmark. Thus higher persistence of the IC shock generates stronger declines in the procyclicality of labor productivity in the model.

Finally, in Panel 7, I allow the IC shock to be more persistent than the productivity shock by keeping \(\rho_b\) at 0.95 and lowering \(\rho\) to 0.9 and show that the model generated shifts in labor market dynamics, due to an increase in \(\gamma\), are even stronger. In the presence of highly correlated shocks, firms need to respond more to the IC shock as \(\gamma\) rises since a larger \(\gamma\) makes the IC constraint stronger for the firms. But from our discussions above, a more persistent IC shock implies an increase in \(\gamma\) has a stronger impact on the wage and labor input of the economy causing the increase in wages and hours volatility to be even higher and the decline in procyclicality of labor productivity to be even steeper.
<table>
<thead>
<tr>
<th>Panel</th>
<th>Description</th>
<th>1) Pre-84 (Low $\gamma$)</th>
<th>2) Post-84 (High $\gamma$)</th>
<th>3) Post-84/Pre-84</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel 1: Log preferences</td>
<td>$\sigma(y)$</td>
<td>1.35</td>
<td>1.3</td>
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<td>Panel 2: $\phi = 0.075$</td>
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<td></td>
<td>$\sigma(l)/\sigma(y)$</td>
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<td>$\sigma(w)/\sigma(y)$</td>
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<tr>
<td>Panel 3: $\phi = 0.01$</td>
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<td>1.27</td>
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<tr>
<td>Panel 4: $\text{corr}(e, e_b) = 0$</td>
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<td>0.28</td>
<td>1.08</td>
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<tr>
<td>Panel 5: $\text{corr}(e, e_b) = 0.9$</td>
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<tr>
<td>Panel 6: $\rho_b = 0.95$</td>
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<td>Panel 7: $\rho = 0.9, \rho_b = 0.95$</td>
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<td></td>
<td>$\sigma(w)/\sigma(y)$</td>
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<td>0.39</td>
<td>1.27</td>
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</table>

Table 7: Sensitivity analysis of the effect of IC output and labor market volatilities.)
6 Conclusion

I study the effects of a rise in the importance of intangible capital in the production process since the mid 1980’s, on labor market dynamics. I show that an increase in the share of IC in production where IC accumulation is subject to additional volatility causes wage and labor input volatility to rise, both absolutely and relative to income while measured output volatility falls as observed during this period. Further, there is an increase in the importance of the IC sector shock relative to the productivity shock in the model which generates a significant decline in the procyclicality of measured labor productivity relative to both output and labor, also observed during this period.

The main effect of an increase in intangible capital in the model, is to lower the responsiveness of wages and hours to the productivity shock in final goods while raising their sensitivity to the shock to intangible investments causing volatility of both wage and labor input to rise as the importance of IC and hence the IC shock rises in production. Output volatility, however, falls because the rise in intangible investments remains unmeasured, and (measured) output of the final goods sector rises more slowly as the share of IC in income increases, because more intangible investments need to be produced before final output can rise in response to the productivity shock.

The fact that measured output increases less in the presence of an IC-shock while labor input increases more gives rise to a lower procyclicality of measured labor productivity relative to both output and labor input compared to standard RBC models. As IC’s importance rises and the IC shock plays a larger role, the procyclicality of productivity
declines further, with the correlation of productivity relative to labor turning strongly negative as observed in the data.
References


