Aspiration and Development

Dyotona Dasgupta∗ Anuradha Saha†

September 17, 2018

Abstract

We build an overlapping generations model where adults develop priors about their children’s ability from their social groups. Here aspiration is captured as a belief on one’s child’s abilities. Depending on the adult’s neighbourhood, some individuals may be overtly optimistic or pessimistic about their children’s future. This paper highlights that “unfair” aspirations may give rise to inequality in a society. We show that bad luck or low ability in one generation may have long lasting effects.

Keywords: Inequality, Probabilities, Aspiration, Social Learning

JEL Codes: I3, D8

∗Ashoka University, Sonepat - 131029. E-mail: dasgupta.dyotona@gmail.com, dyotona.dasgupta@ashoka.edu.in
†Ashoka University, Sonepat - 131029. E-mail: anuradha.saha@ashoka.edu.in.
1 Introduction

Socio-economic inequality has been an active area of research across several disciplines like economics, anthropology (Appadurai, 2004). While physical constraints, like credit market imperfections or non-convexities in technology (see Banerjee and Newman (1993), Galor and Zeira (1993) for example) have been the primary focus of research, of late there has been growing interest to investigate how psychological constraints like present-bias (see Bernheim et al. (2015), Banerjee and Mullainathan (2010) for example), or lack of aspiration (see Genicot and Ray (2017) for example), or may give rise to poverty trap.

This paper aims to contribute to this recent strand of literature. We build an overlapping generations model to show the role of “limited” social learning in giving rise to “unfair” aspirations which in turn increase inequality in the society.

In an empirical work based on India, Goel and Deshpande (2016) show individuals have different perceptions of self-worth depending on their social identity, captured by caste. In this paper, we provide a theoretical model to encapsulate a similar idea. We assume that the agent cannot observe the entire society, but only his neighbours or members of his community. In absence of perfect information, a rational individual forms some prior beliefs on the returns of a “risky” investment such an investment. He uses his own experience and his neighbours’ experiences to form his beliefs. He may allocate different weights to these experiences – maximum weight on his own experience and least on the experiences of far located neighbours or members of different communities. He may put a substantial weight on the experiences of his close neighbours or members in his community. In this set-up, we show that this kind of social learning may widen differences across communities. Here we do not refer to network effects of the type where networks give access to better quality of information. In our model, a network effect is a psychological boost (or bump) where individuals from different networks or communities look at an opportunity with different perspectives.

This paper is related to many papers which have postulated different sources of economic inequality. One of earliest papers in the field, Loury (1981), studies the effects of parental income on income distribution over time. Banerjee and Newman (1993) and Galor and Zeira (1993) show that inequality may persist in the long run if production technology is non convex. In a series of papers, Mookherjee and Ray (2002a,b, 2003) show that inequality may persist with rational agents in an economy with multiple occupations, each with increasing costs of participation.

We highlight how pessimism or lack of aspiration can give rise to inequality in the society. There are several papers in the existing literature which capture aspiration as a target level of income, as in Dalton et al. (2016), Besley (2017), Genicot and Ray (2017). Households get a utility perk from crossing the goal but inability to cross the goal may lead to frustration. Dalton et al. (2016) finds that in these settings households are stuck in a behavioural trap – poorer households set lower targets, as poorer households get lower returns for the same level of aspirations.

1This paper also provides a behavioural explanation of poverty trap and relates to the works of Ashraf et al. (2006) and Duflo et al. (2011).
We model aspiration differently. Each household have two members: an adult and a child. Each agent lives for two periods. The adult makes decisions for the household. The adult may be a skilled or an unskilled worker, but the former would require that the adult must have received education when he was a child. The adult gets utility from household’s consumption and from his child’s wealth. However there is uncertainty in his child’s ability to earn skilled or unskilled wages. Different households perceive this uncertainty differently and hence may or may not invest in their child’s schooling. A household may belong to either an optimistic or a pessimistic socio-economic environment, depending on which the household may be optimistic or pessimistic about his child’s ability to get skilled or unskilled wages. Depending on their prior beliefs, household may invest or not in their child’s education which ultimately affects the child’s future wage earnings. Thus, two persons with same initial wealth could make different decisions for their children depending on whether they identify with the optimistic or the pessimistic community.

Note, here aspiration is formed from interacting with persons in one’s ‘community’. It is not about a target wealth, but whether people have hopes for a better future or not. The question is how do differences in environment settings affect this hope differently. What is the role of social-learning in aspiration formation? How does “limited” social learning give rise to “unfair” aspiration? What is its effect on economic outcomes like wealth and skill distribution?

2 Model

In a discrete time framework, we build an overlapping generations model. At each time period, there are $N$ households. Let us normalize $N$ to 1. Each household has two individuals – one adult and one child. The adult is born with an ability ($a_t$), a certain education level ($e_t$) and has received a bequest ($f_t$). An ability of a person could be either low or high, \{L, H\}. He could be either educated or not, so $e_t$ takes a value 0 or 1. Depending on his ability, education level and an element of chance, he get employment in either a skilled sector or an unskilled sector. Thus, the parent has from two sources of wealth: wages and bequests received. The adult spends his wealth on current consumption of parent and child, his child’s education and gives bequests to his child. We assume that the bequest is saved for one period and yields an exogenous rate of interest of return.

Within each generation, there is a probability $\alpha$ that a person is of high ability, $Pr(a_t = H) = \alpha$. In the initial time period, we assume that a fraction $\eta_1$ of low ability parents are educated while a fraction $\eta_2$ of high ability parents are educated. Thus,

$$Pr(e_0 = 1|a_0 = L) = \eta_1, \quad Pr(e_0 = 1|a_0 = H) = \eta_2.$$

In our economy, education is necessary but not sufficient for getting skilled jobs.\footnote{In equilibrium, a skilled job pays higher wages than unskilled job, $w_{st} \geq w_{ut}$.} A low ability
An educated person has a probability $\beta$ of getting a skilled job. However, a high ability educated person has a probability $\gamma$ for getting skilled job, where $\gamma > \beta$. Thus, for educated persons the probability of getting a high skilled job is higher for those with high ability person than for those with low ability person. A low ability uneducated person will always be employed in unskilled job. For simplicity, we assume that a high ability uneducated person will also be always employed in unskilled job. In this sense, investment in education signals an aspiration for working in the skilled sector.

$$Pr(w_t = w_{st}|a_t = L, e_t = 1) = \beta, \quad Pr(w_t = w_{st}|a_t = L, e_t = 0) = 0,$$
$$Pr(w_t = w_{st}|a_t = H, e_t = 1) = \gamma, \quad Pr(w_t = w_{st}|a_t = H, e_t = 0) = 0,$$

Based on this, a parent in time period 0 has six possible histories:

<table>
<thead>
<tr>
<th>$a_0$</th>
<th>$e_0$</th>
<th>$w_1$</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>0</td>
<td>$w_{u1}$</td>
<td>$(1 - \alpha)(1 - \eta_1)$</td>
</tr>
<tr>
<td>$L$</td>
<td>1</td>
<td>$w_{u1}$</td>
<td>$(1 - \alpha)\eta_1(1 - \beta)$</td>
</tr>
<tr>
<td>$L$</td>
<td>1</td>
<td>$w_{s1}$</td>
<td>$(1 - \alpha)\eta_1\beta$</td>
</tr>
<tr>
<td>$H$</td>
<td>0</td>
<td>$w_{u1}$</td>
<td>$\alpha(1 - \eta_2)$</td>
</tr>
<tr>
<td>$H$</td>
<td>1</td>
<td>$w_{u1}$</td>
<td>$\alpha\eta_2(1 - \gamma)$</td>
</tr>
<tr>
<td>$H$</td>
<td>1</td>
<td>$w_{s1}$</td>
<td>$\alpha\eta_2\gamma$</td>
</tr>
</tbody>
</table>

**Table 1:** Initial History of a Parent in Time Period 0.

In initial time period, the number of skilled workers in the economy is $N_{s1} = [(1 - \alpha)\eta_1\beta + \alpha\eta_2\gamma]$, and the number of low skilled workers is $N_{u1} = 1 - N_{s1}$.

In the benchmark case, we assume that a parent can perfectly observe his child’s ability. Next we relax the assumption of complete information on the ability of the child. Investment in child’s education then depends on the parent’s belief’s on returns to education.

### 2.1 Benchmark Case: Child Ability is Known

An adult realizes her profession in time period $t$. Once realized, he could have six possible histories as noted in Table 1. Consider a parent with history $i$ who has received bequests and earns wages, $I \equiv f_{it} + w_{it}$. As a child becomes an adult in the next period, we denote a child’s characteristics with subscript $t+1$. Parents know the ability of his child, $e_{t+1} = \{H, L\}$. The adult gets utility from the household’s present consumption and his child’s expected wealth. The adult’s utility function is

$$U = \frac{x_{it}^\sigma}{\sigma} + \delta \frac{EW_{t+1}^\sigma}{\sigma}, \quad \sigma > 0$$ (1)
where $x_{it}$ is the present consumption of the household, $\delta$ is the time independent discount factor, and $EW_{t+1}$ is the expected wealth of the child. The utility function is defined for $x_{it} > 0$ and $EW_{t+1} > 0$. The utility function is isoelastic, so the adult has a constant relative risk aversion, captured by $\sigma$. The adult saves a part of his wealth as bequests for his child so that the expected wealth of the child is

$$EW_{t+1} = (1 + r_t)b_{it} + Ew_{t+1}$$

where $r_t$ is the rate of return on savings, $b_{it}$ is the bequest for his child, and $Ew_{t+1}$ is the expected wages of the child. Note, as every parent leaves bequest for the child, $f_{it} = (1 + r_{t-1})b_{jt}$ where the parent of adult $i$ is denoted by $j$. We assume that the parent cannot borrow from his child, that is

$$b_{it} \geq 0.$$  

(3)

The child would earn different wages depending on her ability and education level. The household budget is

$$x_{it} + b_{it} + s(e_{t+1}) \leq W_{it}$$

(4)

where $s(\cdot)$ is the cost of education. We assume

$$s(e) = \begin{cases} 0 & \text{for } e = 0 \\ \bar{s} & \text{for } e = 1 \end{cases}$$

Therefore, a parent can invest in his child’s education only when he has enough wealth, that is $W_{it} \geq \bar{s}$. If this holds, he invests in his child’s education only when doing so gives him higher utility.

We start our analysis with the problem of a parent if she chooses to not educate his child. Irrespective of ability of the child, she would be employed in unskilled jobs and earning unskilled wages. Such a parent maximizes his utility (1) subject to (2), (4),(3), with $Ew_{t+1} = w_{ut+1}$ and $s(e_{t+1}) = 0$. The Lagrangian is:

$$L_i = \frac{x_{it}^{\sigma}}{\sigma} + \delta \frac{[(1 + r_t)b_{it} + w_{ut+1}]^{\sigma}}{\sigma} + \lambda_1 [W_{it} - x_{it} - b_{it}] + \lambda_2 b_{it}$$

In equilibrium, the budget constraint holds with equality. If not, then the parent could increase $x_{it}$ or $b_{it}$ and get higher utility without violating the constraint. Thus, the parent will spend all his budget.

Note, the utility function is non-homothetic. Marginal utility from bequests is higher for richer parents.
The optimal choices are:

\[
x_{it} = \max \left\{ \frac{W_{it} m_{1t} W_{it} + m_{2t} w_{ut+1}}{1 + m_{1t}} \right\}
\]

\[
b_{it} = \min \left\{ 0, \frac{W_{it} - m_{2t} w_{ut+1}}{1 + m_{1t}} \right\}
\]

\[
U(e_{t+1} = 0|W_{it}) = \begin{cases} 
\frac{m_{2t}^2 + \delta}{\sigma(1 + m_{1t})^\sigma} \cdot [(1 + r_t) W_{it} + w_{ut+1}]^\sigma & \text{if } W_{it} \geq C_{wt} \equiv m_{2t} w_{ut+1} \\
\frac{W_{it}^\sigma}{\sigma} + \frac{\sigma w_{ut+1}}{\sigma} & \text{otherwise.}
\end{cases}
\]

where \( m_{1t} = [\delta(1 + r_t)^\sigma]^{\frac{1}{1-\sigma}} \) and \( m_{2t} = m_{1t}/(1 + r_t) = [\delta(1 + r_t)]^{\frac{1}{1-\sigma}} \). \( C_{wt} \) is the wealth cut-off below which the household will not leave any bequests for his uneducated child.

The adult would allocate resources differently if educates a high ability or a low ability child. Let us consider the two cases separately.

### 2.1.1 High ability child

Suppose ability of his child is \( H \). If the child is educated, her expected wages as an adult is

\[
E w_{t+1} = \gamma w_{st+1} + (1 - \gamma) w_{ut+1} \equiv \bar{w}_H
\]

(5)

Here we have incorporated that an educated high ability child becomes a skilled worker with probability \( \gamma \). As next period’s skilled or unskilled workers’ wages is unknown currently, we denote the expected wages of high ability child as \( \bar{w}_H \).

Suppose, the parent has sufficient wealth to educate his child, that is \( W_{it} \geq \bar{s} \). If he chooses to educate his child, then his problem is to maximize (1) subject to (2), (3), (5) and \( s(e_{t+1}) = \bar{s} \). The Lagrangian is

\[
L_i = \frac{x_{it}^\sigma}{\sigma} + \frac{\delta [(1 + r_t) b_{it} + \bar{w}_H]^\sigma}{\sigma} + \lambda_1 [W_{it} - x_{it} - b_{it} - \bar{s}] + \lambda_2 b_{it}
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the Lagrange multipliers for the budget and bequest constraints respectively.

The first order conditions are:

\[
x_{it}^{\sigma - 1} = \lambda_1
\]

(6)

\[
\delta [(1 + r_t) b_{it} + \bar{w}_H]^{\sigma - 1} (1 + r_t) = \lambda_1 - \lambda_2
\]

(7)

\[
\lambda_2 b_{it} = 0 \text{ and } \lambda_1, \lambda_2 \geq 0.
\]

(8)
The optimal choices are

\[ x_{it} = \max \left\{ W - \bar{s}, \frac{m_1t(W_{it} - \bar{s}) + m_2t \bar{w}_{Ht}}{1 + m_1t} \right\} \]

\[ b_{it} = \min \left\{ 0, \frac{W_{it} - \bar{s} - m_2t \bar{w}_{Ht}}{1 + m_1t} \right\} \]

\[ U(e_{t+1} = 1|a_{t+1} = H, W_{it}) = \begin{cases} 
\frac{m_2t + \delta}{\sigma(1 + m_1t)^\sigma} \cdot [(1 + r_t)(W_{it} - \bar{s}) + \bar{w}_{Ht}]^\sigma & \text{if } W_{it} \geq C^H_{wt} \equiv \bar{s} + m_2t \bar{w}_{Ht} \\
\frac{(W_{it} - \bar{s})^\sigma}{\sigma} + \delta \bar{w}_{Ht}^\sigma & \text{otherwise.} 
\end{cases} \]

The cut-off \( C^H_{wt} \) is such that adults with income lower than this level would leave their educated high ability child with zero bequest.

Observe, by definition \( C^H_{wt} > \bar{s} \), and \( C^H_{wt} > C_{wt} \) whenever \( w_{st+1} \geq w_{ut+1} \). However, we do not know whether \( \bar{s} \) is higher or lower than \( C_{wt} \). There could be two possible cases: (a) \( \bar{s} \leq C_{wt} \leq C^H_{wt} \), or (b) \( C_{wt} \leq \bar{s} \leq C^H_{wt} \).

We know that a parent who can afford to educate his child would choose to do so if and only if that provides him higher utility. Let us now consider different ranges of parent’s wealth and assess under what conditions does the parent educate his high ability child.

- When \( W_{it} \geq C^H_{wt} \). This is common to both case (a) and (b). The parent will school his high ability child if and only if

\[ \frac{m_2t + \delta}{\sigma(1 + m_1t)^\sigma} \cdot [(1 + r_t)(W_{it} - \bar{s}) + \bar{w}_{Ht}]^\sigma \geq \frac{m_2t + \delta}{\sigma(1 + m_1t)^\sigma} \cdot [(1 + r_t)W_{it} + w_{ut+1}]^\sigma \]

\[ \Rightarrow \bar{w}_{Ht} - w_{ut+1} \geq (1 + r_t)\bar{s} \]

\[ \Rightarrow \gamma(w_{st+1} - w_{ut+1}) \geq (1 + r_t)\bar{s}. \quad (9) \]

Thus, if the expected gains in earnings from higher education in \( t + 1 \) is greater than effective cost of education in \( t + 1 \), a rich parent (whose wealth is no less than \( W^H \)) would educate his child.

- Consider case (a): \( \bar{s} \leq C_{wt} \leq C^H_{wt} \). In this,
1. When $C_{wt} \leq W_{it} \leq C_{H}$. The parent will school his high ability child if and only if

$$U(e_{t+1} = 1|a_{t+1} = H, W_{it}) \geq U(e_{t+1} = 0|a_{t+1} = H, W_{it})$$

$$\Rightarrow \delta \left[ \frac{(W_{it} - \bar{s})^{\sigma}}{\sigma} + \frac{\delta \hat{w}_{Ht}^{\sigma}}{\sigma} \right] \geq \frac{m_{2t}^{\sigma} + \delta}{\sigma (1 + m_{1t})^{\sigma}} \cdot [(1 + r_{t})W_{it} + w_{ut+1}]^{\sigma}$$

(10)

2. When $\bar{s} \leq W_{it} \leq C_{wt}$. The parent will school his high ability child if and only if

$$U(e_{t+1} = 1|a_{t+1} = H, W_{it}) \geq U(e_{t+1} = 0|a_{t+1} = H, W_{it})$$

$$\Rightarrow \frac{(W_{it} - \bar{s})^{\sigma}}{\sigma} + \frac{\delta \hat{w}_{Ht}^{\sigma}}{\sigma} \geq \frac{W_{it}^{\sigma}}{\sigma} + \frac{\delta w_{ut+1}^{\sigma}}{\sigma}$$

$$\Rightarrow \delta \left[ \hat{w}_{Ht}^{\sigma} - w_{ut+1}^{\sigma} \right] \geq W_{it}^{\sigma} - (W_{it} - \bar{s})^{\sigma}.$$  

(11)

3. When $W_{it} < \bar{s}$ The parent is too poor to educate or leave any bequest for his child. His utility is

$$\frac{W_{it}^{\sigma}}{\sigma} + \delta \frac{w_{ut+1}^{\sigma}}{\sigma}.$$  

(12)

Consider case (b): $C_{wt} \leq \bar{s} \leq C_{H}$. In this,

1. When $\bar{s} \leq W_{it} \leq C_{H}$. Following a similar line of reasoning as in case (a), the parent will educate his high ability child if and only if condition (10) is met.

2. When $C_{wt} \leq W_{it} \leq \bar{s}$. The parent can not afford schooling but leaves his child a bequest.

The parent’s utility is

$$\frac{m_{2t}^{\sigma} + \delta}{\sigma (1 + m_{1t})^{\sigma}} \cdot [(1 + r_{t})W_{it} + w_{ut+1}]^{\sigma}$$

3. When $C_{it} \leq C_{wt}$. As in (12), the parent neither educates the child nor leaves her a bequest.

We illustrate the two cases and their intergenerational outcomes in Figure 1

2.1.2 Low ability child

The problem is solved similarly in the case of a low ability child. If the parent school his child, the schooling cost is $\bar{s}$ else 0. If she is educated, her expected wages as an adult is

$$Ew_{t+1} = \beta w_{st+1} + (1 - \beta)w_{ut+1} \equiv \hat{w}_{Lt}$$  

(13)
where the probability of a low ability child getting skilled wages is $\beta$. If the child is not educated she becomes an unskilled worker with certainty. Except the returns on child’s education, the adult’s problem is the same as the problem of the adult with high ability child. Accordingly, we define $C_{wt}^L$ as the wealth threshold such that a parent with wealth lower that this cut-off would not leave any bequests for their low ability child, $C_{wt}^L \equiv \bar{s} + m_2 t \tilde{w}_{Lt}$.

Note, $C_{wt}^L > C_{wt}$ and $C_{wt}^H > C_{wt}^L$ whenever $w_{st+1} > w_{ut+1}$. Here too, there are two cases: (a) $\bar{s} \leq C_{wt} \leq C_{wt}^L$, or (b) $C_{wt} \leq \bar{s} \leq C_{wt}^L$. We list the conditions of educating a low ability child for the two cases and for different ranges of wealth:

- When $W_{it} > C_{wt}^L$. This is common to both cases (a) and (b). The parent will school his child if and only if
  \[ \beta (w_{st+1} - w_{ut+1}) \geq (1 + r_t) \bar{s} \quad (14) \]

- Consider case (a): $\bar{s} \leq C_{wt} \leq C_{wt}^L$. In this,
  1. When $C_{wt} \leq W_{it} \leq C_{wt}^L$. The parent will educate her low ability child if and only if
     \[ \delta \left[ \tilde{w}_{Lt}^\sigma - \frac{[(1 + r_t)W_{it} + w_{ut+1}]^\sigma}{(1 + m_1)^\sigma} \right] \geq \frac{m_2^\sigma [(1 + r_t)W_{it} + w_{ut+1}]^\sigma}{(1 + m_1)^\sigma} - (W_{it} - \bar{s})^\sigma \quad (15) \]
  2. When $\bar{s} \leq W_{it} \leq C_{wt}$. The parent will not give any bequests to his child. He will educate her low ability child if and only if
     \[ \delta (\tilde{w}_{Lt}^\sigma - w_{ut+1}^\sigma) \geq W_{it}^\sigma - (W_{it} - \bar{s})^\sigma \quad (16) \]

Figure 1: Bequests and Education outcomes for high ability child when parent has complete information on her ability.
3. When $W_{it} \leq \bar{s}$. The parent neither educates his low ability child nor leaves her any bequest.

- Consider case (b): $C_{wt} \leq \bar{s} \leq C_{wt}^L$. In this

1. When $\bar{s} \leq W_{it} \leq C_{wt}^L$. The parent will educate her low ability child if and only if condition (15) is satisfied.

2. When $C_{wt} \leq W_{it} \leq \bar{s}$. The parent will not educate his child but will give her positive bequests.

3. When $W_{it} \leq C_{wt}$. The parent neither educates his low ability child nor leaves her any bequest.

We depict the two cases and the related intergenerational transfers in Figure 2.

![Figure 2](image-url)

**Figure 2**: Bequests and Education outcomes for low ability child when parent has complete information on her ability.

### 2.2 Aspiration Formation through Clubs: Ability is not Known

Let us now consider the scenario when ability is not known. Parents have different priors on the ability of their children in entering the skilled wage sector.

Depending on the parent’s profession, an adult belongs to one of the two clubs – skilled worker’s club (say *optimists*) and unskilled worker’s club (*pessimists*). The parents do not know their children’s abilities but know that high ability children will gain more from education than low ability children. So an adult looks within his community to assess whether education is a worthy investment. The conditions which determine an adult’s beliefs are:
• An adult believes that his generation’s education-employability conditions will continue to exist for his child’s generation.

• The probability that an adult believes that an educated child from his club will get skilled jobs is equal to the fraction of educated adults in club. An educated adult values education and has seen individuals earning wages from skilled jobs only through education. He imparts this knowledge to his community. The more information one receives from his club members, the more he believes in the importance of education for skilled job wages.

\[ P_B(w_{st+1}|e_{t+1} = 1, w_{st}) = \frac{P_r(w_{st}, e_t = 1)}{P_r(w_{st})}, \quad P_B((w_{st+1}|e_{t+1} = 1, w_{ut}) = \frac{P_r(w_{ut}, e_t = 1)}{P_r(w_{ut})}, \]

where we have used subscript \( B \) for beliefs and distinguish them from actual probabilities.

• Parents know that no education implies a certainty of no skilled jobs.

\[ P_B((w_{ut+1}|e_{t+1} = 0, w_{ut}) = P_B((w_{ut+1}|e_{t+1} = 0, w_{st}) = 1 \]

As only skilled workers belong to the optimistic club and education is necessary to be a skilled worker, all optimistic adults are educated. Thus, \( P_B(w_{st+1}|e_t = 1, w_{st}) = 1 \). At each period, skilled parents would believe that educating their children guarantees them a skilled job.

In contrast, the unskilled workers could be educated or not. For the initial time period,

\[ P_B((w_{s1}|e_1 = 1, w_{u0}) = \frac{(1 - \alpha)\eta_1(1 - \beta) + \alpha\eta_2(1 - \gamma)}{(1 - \alpha)(1 - \eta_1) + (1 - \alpha)\eta_1(1 - \beta) + \alpha(1 - \eta_2) + \alpha\eta_2(1 - \gamma)} \equiv q_{u0} \]

where \( q_{u0} \) is the probability with which an unskilled parent in time period 0 believes his child will get skilled jobs on receiving education.

Let us now consider how parents in different clubs decide whether to educate their child.

### 2.2.1 Skilled club

A skilled parent believes that his child will become a skilled worker on receiving education. As in the benchmark case, the adult maximizes his utility (1) subject his budget (4) and bequest constraint (3). If the parent schools his child, then the schooling costs is \( s(e_{t+1}) = \bar{s} \) and the expected wealth of the child is \( EW_{t+1} = (1 + r_t)b_{lt} + w_{st+1} \). However, if the parent does not school his child, the school costs are zero and the expected wealth of child \( EW_{t+1} = (1 + r_t)b_{lt} + w_{ut+1} \). The optimal
choices are

\[ x_{it} = \max \left\{ W_{it} - \bar{s}, \frac{m_1W_{it} - \bar{s} + m_2w_{st+1}}{1 + m_1} \right\} \]

\[ b_{it} = \min \left\{ 0, \frac{W_{it} - \bar{s} - m_2w_{st+1}}{1 + m_1} \right\} \]

\[ U(e_{t+1} = 1|W_{it}, w_{st}) = \begin{cases} 
\frac{m_2\bar{s} + \delta}{\sigma(1 + m_1)^\sigma} 
\left[ (1 + r_t)(W_{it} - \bar{s}) + w_{st+1} \right]^\sigma & \text{if } W_{it} \geq C^{S}_{w_{it}} \equiv \bar{s} + m_2w_{st+1} \\
\frac{(W_{it} - \bar{s})^\sigma}{\sigma} + \frac{\delta w_{st+1}^\sigma}{\sigma} & \text{otherwise.} 
\end{cases} \]

As before, when the parent has enough wealth, he chooses to educate his child if and only if that provides him higher utility. As before, there are two cases: (a) \( \bar{s} \leq C_{w_{it}} \leq C^{S}_{w_{it}} \) and (b) \( C_{w_{it}} \leq \bar{s} \leq C^{s}_{w_{it}} \). There are three conditions for the different cases and different ranges of skilled parent’s income when he chooses to invest in his child’s education

\[ (17) \quad \text{When } W_{it} \geq C^{S}_{w_{it}} \text{ if and only if } w_{st+1} - w_{ut+1} \geq (1 + r_t)\bar{s} \]

In case (a) when \( C_{w_{it}} \leq W_{it} \leq C^{S}_{w_{it}} \) or in case (b) when \( \bar{s} \leq W_{it} \leq C^{S}_{w_{it}} \) if and only if

\[ (18) \quad \delta \left[ w_{st+1}^\sigma - \frac{[(1 + r_t)W_{it} + w_{ut+1}]^\sigma}{(1 + m_1)^\sigma} \right] \geq \frac{m_2\bar{s}[(1 + r_t)W_{it} + w_{ut+1}]^\sigma}{(1 + m_1)^\sigma} - [W_{it} - \bar{s}]^\sigma \]

In case (a) when \( \bar{s} \leq W_{it} \leq C_{w_{it}} \) if and only if

\[ (19) \quad \delta \left( w_{st+1}^\sigma - w_{ut+1}^\sigma \right) \geq W_{it}^\sigma - (W_{it} - \bar{s})^\sigma \]

The parent does not educate his child for the remaining cases. We summarize these decisions, at different wealth levels, along with the optimal choices in Figure 3.

### 2.2.2 Unskilled club

We solve the problem similarly for the parents in the unskilled club. If an unskilled parent schools his child, he bears a schooling cost of \( s(e_{t+1}) = \bar{s} \) and assumes the expected wages of his child to be

\[ E w_{t+1} = q_u w_{st+1} + (1 - q_u)w_{ut+1} \equiv \bar{w}_{Ut} \]. As always, if the child is not educated she is expected
to earn unskilled wages, \( Ew_{t+1} = w_{ut+1} \) and has zero schooling costs. The optimal choices are

\[
x_{it} = \max \left\{ W_{it} - \bar{s}, \frac{m_{1t}(W_{it} - \bar{s}) + m_{2t}\hat{w}_{Ut}}{1 + m_{1t}} \right\}
\]

\[
b_{it} = \min \left\{ 0, \frac{W_{it} - \bar{s} - m_{2t}\hat{w}_{Ut}}{1 + m_{1t}} \right\}
\]

\[
U(e_{t+1} = 1|W_{it}, w_{ut}) = \begin{cases} 
\frac{m_{2t}^2 + \delta}{\sigma(1 + m_{1t})^\sigma} \cdot [(1 + r_t)(W_{it} - \bar{s}) + \hat{w}_{Ut}]^\sigma & \text{if } W_{it} \geq C_{ut}^U \equiv \bar{s} + m_{2t}\hat{w}_{Ut} \\
\frac{(W_{it} - \bar{s})^\sigma}{\sigma} + \delta \frac{w_{ut+1}^\sigma}{\sigma} & \text{otherwise.}
\end{cases}
\]

As before, there are two cases: (a) \( \bar{s} \leq C_{ut} \leq C^U_{ut} \) and (b) \( C_{ut} \leq \bar{s} \leq C^U_{ut} \). The unskilled parent invests in child’s education at different wealth levels for different cases:

When \( W_{it} \geq C^U_{ut} \) if and only if \( \hat{w}_{Ut} - w_{ut+1} \geq (1 + r_t)\bar{s} \) \hspace{1cm} (20)

In case (a) when \( C_{ut} \leq W_{it} \leq C^U_{ut} \) or in case (b) when \( \bar{s} \leq W_{it} \leq C^U_{ut} \) if and only if

\[
\delta \left( \frac{\hat{w}_{Ut}^\sigma}{(1 + m_{1t})^\sigma} \cdot [(1 + r_t)W_{it} + w_{ut+1}]^\sigma - (W_{it} - \bar{s})^\sigma \right) \geq \frac{m_{2t}^2}{(1 + m_{1t})^\sigma} \cdot [(1 + r_t)W_{it} + w_{ut+1}]^\sigma
\]

In case (a) when \( \bar{s} \leq W_{it} \leq C_{ut} \) if and only if

\[
\delta \left( \frac{\hat{w}_{Ut}^\sigma - w_{ut+1}^\sigma}{(1 + m_{1t})^\sigma} \right) \geq W_{it}^\sigma - (W_{it} - \bar{s})^\sigma
\]

The parent does not invest in child’s education for the remaining cases. We depict the decisions in Figure 4.
Figure 4: Bequests and Education outcomes for a child when parent is unskilled and has no information on the child’s ability.

3 Comparison

We compare results from full information cases, sections 2.1.1 and 2.1.2, with those when parents do not know their children’s abilities, section 2.2.1 and 2.2.2, to assess how does aspiration affect children of different households differently. The hypothesis is that the parents in the skilled club would invest in their child’s education, independent of the child’s ability levels. There may be over investment in education. The parents in the unskilled club may not invest in a high ability child. There may be over- or under-investment in children’s education by parents in this club. This is work in progress.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Full Information</th>
<th>Optimist Parent</th>
<th>Pessimist Parent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case (a)</td>
<td>Cond. under which $e_{t+1} = 1$</td>
<td>Cond. under which $e_{t+1} = 1$</td>
<td>Cond. under which $e_{t+1} = 1$</td>
</tr>
<tr>
<td>Case (b)</td>
<td>Over/Under Investment</td>
<td>Over/Under Investment</td>
<td>Over/Under Investment</td>
</tr>
</tbody>
</table>

Table 2: High Ability Child

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Full Information</th>
<th>Optimist Parent</th>
<th>Pessimist Parent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case (a)</td>
<td>Cond. under which $e_{t+1} = 1$</td>
<td>Cond. under which $e_{t+1} = 1$</td>
<td>Cond. under which $e_{t+1} = 1$</td>
</tr>
<tr>
<td>Case (b)</td>
<td>Over/Under Investment</td>
<td>Over/Under Investment</td>
<td>Over/Under Investment</td>
</tr>
</tbody>
</table>

Table 3: Low Ability Child

13
References

Appadurai, A. (2004). The capacity to aspire: Culture and the terms of recognition’in vijayendra rao and michael walton (eds), culture and public action. 1, 2


Goel, D. and Deshpande, A. (2016). Identity, perceptions and institutions: Caste differences in earnings from self-employment in india. 1


