Trade openness, uncertainty shocks and monetary policy

Sargam Gupta*

ISI Delhi

September, 2018

Abstract

Motivation: When an economy opens up its trade to the world, it also exposes itself to the worldwide events. There is an extensive literature discussing the effect of trade openness on growth and development in an economy. There is also a literature showing the destabilizing effects (higher variability of the macro variables) of trade on an economy. Most of the results in this literature does the ex-post welfare analysis of the shocks. But it is also the case that the economies especially emerging markets and developing economies (EMDEs) become vulnerable to world wide shocks as their trade openness increases. This fact is very well recognized in the literature. But the study discussing the ex-ante risks and its associated outcomes due to the opening up of the economy is extremely limited. This paper attempts to fill this gap. We believe that the question of understanding the vulnerability to external shocks associated with trade openness of a country can be better understood in a model setup as it allows us to create a counterfactual scenarios more easily.

Research Questions: How does uncertainty shocks (both demand and supply side) present outside the domestic country affect the levels of the macroeconomic variables of the domestic country? What factors and characteristics affect the extent of the this vulnerability to external shocks as the economy opens up? Since a macro monetary model is used for this analysis, is there a role for the monetary policy in stabilizing the economy?

Methodology: To do this, we build a two country heterogenous new Keynesian dynamic stochastic general equilibrium model with complete asset markets and introduce uncertainty shocks to productivity (supply side) and preference (demand

*Corresponding Author: Economics and Planning Unit, Indian Statistical Institute, New Delhi – 110016, India. Tel: 91-11-4149-3942. E-mail: sargamgupta.6@gmail.com.
side) in the foreign economy. In macro models the uncertainty shocks are modelled as shocks to second moments to the variables. We do third order approximation of the model using Dynare and MATLAB as needed to get the complete effect of the uncertainty shocks as already discussed in literature. We simulate the data for 400 quarters to get the empirical moments to understand the long run impact of the uncertainty shocks. To understand the short term transmission of the shock across the economy we simulate the impulse response functions.

Results so far: In the baseline model it is observed that the uncertainty shock external to the country does affect the level of the macroeconomic variables under free trade in the long run as well as the short run as the domestic economy opens up its trade. One of the most important feature that determines whether the real variables of the domestic economy get affected is the flexibility of prices. Using the baseline model we observe that the real variables do not get affected when the prices are completely flexible as all nominal variables adjust completely here. On the other hand when the prices are sticky the real variables adjusts. When the agents in the domestic country anticipates an uncertainty shock they reduce their consumption, increase there precautionary savings, and the output contracts.

Work in progress: To make the model more close to reality, we are working on a version of the model where domestic producers are not price makers but price takers. How does this change of market power affect the welfare of an economy?

• What is the role of monetary policy in this? Assuming that the foreign country has a zero lower bound on the interest rates, does this changes the response of domestic monetary policy? Does the role of monetary policy vary with the change of exchange rate determination from floating to fixed exchange rate regime?

• We are also trying to understand the role of important parameters like coefficient of risk version and elasticity of demand on the welfare results.

Please find attached the presentation along.
Trade openness, Uncertainty shocks and Monetary policy

Sargam Gupta\textsuperscript{1}

ISI-Delhi

14 September, 2018

\textsuperscript{1}Indian Statistical Institute, Delhi
The literature relating to trade openness is extremely vast.


2. Negative effects of trade openness on poverty and inequality, even in the long run (Goldberg & Pavcnik (2004), Lundberg & Squire (2003), McCulloch, Winters, & Cirera (2001), Winters et al. (2004)).
3. "Destabilizing effect" with trade openness: High degree of trade openness leads to higher volatility on a wider set of outcome variables (aggregate income, consumption, employment, salaries, and prices), especially in developing countries (di Giovanni & Levchenko (2009), Easterly, Islam, & Stiglitz (2001), Loayza & Raddatz (2007), Haddad, Lim, & Saborowski (2010), Hnatkovska & Loayza (2005), Loayza et al. (2007), Winters (2002)).

Higher volatility has been shown to have a negative relation with long-run growth and welfare (Ramey and Ramey (1995), Hnatkovska & Loayza (2005))

All the papers mentioned here do ex-post analysis of shocks present due to trade openness. The literature on trade openness, shocks, and uncertainty is not very well studied. Magrini, Montalbano and Winters (2018) recently show that there are ex-ante risks due to trade exposure in Vietnam and these risks affect the consumption growth.

- The first contribution of this paper is to fill this gap in literature.
Recently macro literature has introduced uncertainty shocks as a shock to second moment of the variable. It has been shown that these shocks to second order moments effects the levels of macro real variables.

- Fernandez-Villaverde (2011) — Effect of uncertainty in real interest rate on the levels of macroeconomic variables in emerging markets.
- Basu and Bundick (2017) — Effect of uncertainty in demand on the levels of macroeconomic variables.
Research Questions

- Using a two country new Keynesian macro monetary model we try to answer following questions, (which I think can be answered best in a theoretical setup)
- Does the uncertainty shocks (both supply side and demand side) present outside an economy affect the domestic country level macro variables and welfare as the degree of trade openness increases.
- What are the domestic country specific structures, factors and characteristics that can affect there degree of ”vulnerability” to these uncertain exogenous shocks.
- Role of monetary policy
Features of the basic model

1. Two country open economy NK-DSGE model. Let us assume a continuum of households in the world with domestic sector $D$ and foreign sector $F$ from $[0, 1]$. The a continuum of domestic sector exist over $[0, n]$ and that of foreign sector from $[n, 1]$. (see Benigno(2010))
   - Domestic households exist from $[0, n]$ produce good $D$
   - Foreign households exist from $[n, 1]$ produce good $F$

2. Heterogeneity of preference and demand structure between domestic and foreign households

3. Productivity and preference is characterized by time-varying volatility shocks
Households (Domestic)

The domestic households maximize following utility function (see Fernandez-Villaverde et al. (2011))

$$\max U(C_t, H_{D,t}) = \frac{Y_{D,t}(C_t)^{1-\nu_D}}{1-\nu_D} - \omega_{D} \frac{(H_{D,t})^{1+\eta_D}}{1+\eta_D}$$ (1)

subject to constraint,

$$W_{D,t} H_{D,t} + \text{profit}_t = P_t C_t + B_t - E_t \{ B_{t+1} M_{t,t+1} \}$$ (2)

where $M_{t,t+1}$ is the stochastic discount factor. In above equation, $C_t$ is the aggregate basket of goods a domestic household consumes,

$$C_t = \left[ (\mu_D)^{1/\xi_D} (C_{D,t})^{\xi_D-1} + (1-\mu_D)^{1/\xi_D} (C_{F,t})^{\xi_D-1} \right]^{\frac{\xi_D}{\xi_D-1}}$$ (3)

Assuming complete asset markets using Chari et. al. (2001).
Similarly, the foreign households maximize following utility function

\[
\max U(C^*_t, H^*_F, t) = \frac{Y_{F,t} \left( C^*_t \right)^{1-\nu_F}}{1 - \nu_F} - \omega_F \frac{(H^*_F, t)^{1+\eta_F}}{1 + \eta_F}
\]  

subject to constraint,

\[
W^*_F, t H^*_F, t + \text{profit}^*_t = P^*_t C^*_t + B^*_t - E_t \{ B^*_t M^*_t, t+1 \}
\]  

where \( M^*_{t, t+1} \) is the stochastic discount factor. In above equation, \( C^*_t \) is the aggregate basket of goods a foreign household consumes,

\[
C^*_t = \left[ \left( \mu_F \right)^{1/\xi_F} \left( C^*_{D, t} \right)^{\xi_F - 1/\xi_F} + \left( 1 - \mu_F \right)^{1/\xi_F} \left( C^*_F, t \right)^{\xi_F - 1/\xi_F} \right]^{\xi_F/\xi_F - 1}
\]

Following Benigno (2010),

\[
1 - \mu_D = (1 - n) \chi \quad ; \quad \mu_F = n \chi
\]

where \( \chi \in [0, 1] \) is the parameter capturing degree of openness. \( \chi = 0 \) means autarky and \( \chi = 1 \) means free trade.
Consumption bundles

The consumption bundles $C_{D,t}$, $C_{F,t}$, $C_{D,t}^*$ and $C_{F,t}^*$ is an aggregate over variety $i$

$$C_{D,t} = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n (C_{D,t}(i))^{\frac{\sigma-1}{\sigma}} \, di \right]^{\frac{\sigma}{\sigma-1}}; \quad (8)$$

$$C_{F,t} = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 (C_{F,t}(i))^{\frac{\sigma-1}{\sigma}} \, di \right]^{\frac{\sigma}{\sigma-1}}; \quad (9)$$

$$C_{D,t}^* = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n (C_{D,t}^*(i))^{\frac{\sigma-1}{\sigma}} \, di \right]^{\frac{\sigma}{\sigma-1}}; \quad (10)$$

$$C_{F,t}^* = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 (C_{F,t}^*(i))^{\frac{\sigma-1}{\sigma}} \, di \right]^{\frac{\sigma}{\sigma-1}}. \quad (11)$$
First order conditions

- **Domestic households**

\[
\lambda_{D,t} = Y_{D,t} (C_t)^{-(\nu_D)}
\]

\[
\lambda_{D,t} = \omega^D \left( \frac{H_{D,t}}{w_{D,t}} \right)^{\eta_D} \frac{T_{D,t}}{w_{D,t} T_{D,t}}
\]

\[
\lambda_{D,t} E_t \{ \pi_{t+1} \} = \beta (1 + R_{D,t}) E_t \{ \lambda_{D,t+1} \}
\]

where \( T_{D,t} = \frac{P_{D,t}}{P_t} \).

- **Foreign households**

\[
\lambda_{F,t} = Y_{F,t} (C_t^*)^{-(\nu_F)}
\]

\[
\lambda_{F,t} = \omega^F \left( \frac{Q_t}{w_{F,t}} \right)^{\eta_F} \frac{Q_t}{w_{F,t} T_{F,t}}
\]

\[
\lambda_{F,t} E_t \{ \pi_{t+1}^* \} = \beta (1 + R_{F,t}) E_t \{ \lambda_{F,t+1} \}
\]

where \( T_{D,t} = \frac{P_{F,t}}{P_t} \) and \( Q_t = \frac{X_t P_t^*}{P_t} \) (\( Q_t \) is real exchange rate and \( X_t \) is the nominal exchange rate)
Demand Functions I

Aggregate demand for $C_{D,t}$, $C_{F,t}$, $C^*_{D,t}$ and $C^*_{F,t}$ are,

\[ C_{D,t} = \mu_D \left( \frac{P_{D,t}}{P_t} \right)^{-\xi_D} C_t \] (18)

\[ C_{F,t} = (1 - \mu_D) \left( \frac{P_{F,t}}{P_t} \right)^{-\xi_D} C_t \] (19)

\[ C^*_{D,t} = \mu_F \left( \frac{P^*_{D,t}}{P^*_t} \right)^{-\xi_F} C^*_t \] (20)

\[ C^*_{F,t} = (1 - \mu_F) \left( \frac{P^*_{F,t}}{P^*_t} \right)^{-\xi_F} C^*_t \] (21)

where aggregate prices are,

\[ P_t = \left[ \mu_D (P_{D,t})^{1-\xi_D} + (1 - \mu_F) (P_{F,t})^{1-\xi_D} \right]^{\frac{1}{1-\xi_D}} \] (22)

\[ P^*_t = \left[ \mu_F (P^*_{D,t})^{1-\xi_F} + (1 - \mu_F) (P^*_{F,t})^{1-\xi_F} \right]^{\frac{1}{1-\xi_F}} \] (23)
Demand for each variety of consumption good by domestic households and foreign households,

\[ C_{D,t}(i) = \left( \frac{1}{n} \right) \left( \frac{P_{D,t}(i)}{P_{D,t}} \right)^{-\sigma} C_{D,t}; \quad (24) \]

where

\[ P_{D,t} = \left[ \left( \frac{1}{n} \right) \int_0^n (P_{D,t}(i))^{1-\sigma} \, di \right]^{\frac{1}{1-\sigma}}; \quad (25) \]

\[ C_{F,t}(i) = \left( \frac{1}{1-n} \right) \left( \frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\sigma} C_{F,t}; \quad (26) \]

where

\[ P_{F,t} = \left[ \left( \frac{1}{1-n} \right) \int_n^1 (P_{F,t}(i))^{1-\sigma} \, di \right]^{\frac{1}{1-\sigma}}. \quad (27) \]
Continued..

\[
C_{D,t}^*(i) = \left( \frac{1}{n} \right) \left( \frac{P_{D,t}^*(i)}{P_{D,t}} \right)^{-\sigma} C_{D,t}; \quad (28)
\]

where \( P_{D,t}^* = \left[ \left( \frac{1}{n} \right) \int_0^n (P_{D,t}^*(i))^{1-\sigma} \, di \right]^{\frac{1}{1-\sigma}} \); \quad (29)

\[
C_{F,t}^*(i) = \left( \frac{1}{1-n} \right) \left( \frac{P_{F,t}^*(i)}{P_{F,t}} \right)^{-\sigma} C_{F,t}; \quad (30)
\]

where \( P_{F,t}^* = \left[ \left( \frac{1}{1-n} \right) \int_n^1 (P_{F,t}^*(i))^{1-\sigma} \, di \right]^{\frac{1}{1-\sigma}} \). \quad (31)

Note: \( P_{D,t}(i) = X_t P_{D,t}^*(i) \) and \( P_{F,t}(i) = X_t P_{F,t}^*(i) \).
Complete asset market condition

Complete asset market condition, here it is assumed that there are state-contingent bonds denominated in home currency. Both the domestic as well as the foreign households hold these bonds denominated in home currency.

\[
\frac{U'_{C,t+1}}{U'_{C,t}} \frac{P_t}{P_{t+1}} = \frac{U'_{C^*,t+1}}{U'_{C^*,t}} \frac{X_t}{X_{t+1}} \frac{P^*_t}{P^*_{t+1}}
\]

Writing recursively it we get,

\[
Q_{t+1} = X_0 \frac{U'_{C,0}}{U'_{C^*,0}} \frac{P^*_0}{P_0} \frac{U'_{C^*,t+1}}{U'_{C,t+1}}
\]

\[
Q_{t+1} = \kappa \frac{(C^*_{t+1})^{-\nu_F}}{(C_{t+1})^{-\nu_D}}
\]

Note, \( Q_{t+1} = \frac{P^*_{t+1}X_{t+1}}{P_{t+1}} \) (implying that the real exchange rate depends on the ratios of marginal utilities) and \( \kappa \) is the initial condition, the ratio of marginal utilities at the beginning.
Some price relations

Terms of trade, \( T_t = \frac{P_{F,t}}{P_{D,t}} \) \hfill (32)

\[
\pi_t^* = \pi_{F,t}^* \frac{\left[ \mu_F (T_t)^{\xi_F} - 1 + (1 - \mu_F) \right]^{\frac{1}{1-\xi_F}}}{\left[ \mu_F (T_{t-1})^{\xi_F} - 1 + (1 - \mu_F) \right]^{\frac{1}{1-\xi_F}}} \]
\hfill (33)

\[
T_{F,t} = \frac{1 - \mu_D (T_{D,t})^{1 - \xi_D}}{1 - \mu_D} \] \hfill (34)

Real exchange rate,

\[
Q_t = \frac{\left[ \mu_F + (1 - \mu_F) \left( T_t \right)^{1 - \xi_F} \right]^{\frac{1}{1-\xi_F}}}{\left[ \mu_D + (1 - \mu_D) \left( T_t \right)^{1 - \xi_D} \right]^{\frac{1}{1-\xi_D}}}, \hfill (36)
\]
Production (Domestic households)

Assume each households produce all the varieties $i$ of good $D$

$$Y_{D,t}(i) = A_{D,t}H_{D,t}(i)$$  \hspace{1cm} (37)

where,

$$A_{D,t} = (1 - \rho_D) \bar{A}_D + \rho_D A_{D,t-1} + u_{D,t-1} \epsilon_{D,t}$$  \hspace{1cm} (38)

$$u_{D,t} = \left( 1 - \rho_{\sigma_D} \right) \bar{u}_D + \rho_{\sigma_D} u_{D,t-1} + \nu_D \zeta_{D,t}$$  \hspace{1cm} (39)

$$\max \sum_{k=0}^{\infty} \alpha_D^k M_{t,t+k} \left( P_{D,t}(i) Y_{D,t+k}(i) - MC_{D,t+k} Y_{D,t+k}(i) \right)$$  \hspace{1cm} (40)

where $Y_{D,t+k}(i) = \left( \frac{P_{D,t}(i)}{P_{D,t+k}} \right)^{-\sigma} Y_{D,t+k}$  \hspace{1cm} (41)
Production (Foreign households)

\[ Y_{F,t}(i) = A_{F,t} H_{F,t}(i) \]  \hspace{1cm} (42)

where, \[ A_{F,t} = (1 - \rho_F) \bar{A}_F + \rho_F A_{F,t-1} + u_{F,t-1} \epsilon_{F,t} \]  \hspace{1cm} (43)

\[ u_{F,t} = (1 - \rho_{\sigma_F}) \bar{u}_F + \rho_{\sigma_F} u_{F,t-1} + \kappa_F \zeta_{F,t} \]  \hspace{1cm} (44)

\[ \max \sum_{k=0}^{\infty} \alpha_F M_{t,t+k}^* \left( P_{F,t}^* Y_{F,t+k}(i) - M_{C,t+k} Y_{F,t+k}(i) \right) \]  \hspace{1cm} (45)

where \[ Y_{F,t+k}(i) = \left( \frac{P_{F,t}^*(i)}{P_{F,t+k}^*} \right)^{-\sigma} Y_{F,t+k} \]  \hspace{1cm} (46)
Optimal price setting: Calvo pricing

- **Domestic firms: Aggregate prices**

\[
P_{D,t}(i) = \left[ \alpha_D \left( P_{D,t-1}^*(i) \right)^{-\sigma} + (1 - \alpha_D) \left( P_{D,t}(i) \right)^{-\sigma} \right]^{-\frac{1}{\sigma}} \tag{47}
\]

where,

\[
P_{D,t}(i) = \frac{\sigma}{\sigma - 1} \frac{\sum_{k=0}^{\infty} \alpha_D M_{t,t+k} M_{C,D,t+k} Y_{D,t+k}(i)}{\sum_{k=0}^{\infty} \alpha_D M_{t,t+k} Y_{D,t+k}(i)} \tag{48}
\]

- **Foreign firms: Aggregate prices**

\[
P_{F,t}^*(i) = \left[ \alpha_F \left( P_{F,t-1}^*(i) \right)^{-\sigma} + (1 - \alpha_F) \left( P_{F,t}^*(i) \right)^{-\sigma} \right]^{-\frac{1}{\sigma}} \tag{49}
\]

where,

\[
P_{F,t}^*(i) = \frac{\sigma}{\sigma - 1} \frac{\sum_{k=0}^{\infty} \alpha_F M_{t,t+k}^* M_{C,F,t+k} Y_{F,t+k}(i)}{\sum_{k=0}^{\infty} \alpha_F M_{t,t+k}^* Y_{F,t+k}(i)} \tag{50}
\]
For domestic firms,

\[ \pi_{D,t} = \frac{\sigma}{(\sigma - 1)} \pi_{D,t} \frac{X_{D,t}}{Z_{D,t}} \]  

(51)

\[ X_{D,t} = \lambda_t Y_{D,t} T_{D,t} mc_{D,t} + \alpha_D \beta (\pi_{D,t+1})^\sigma X_{D,t+1} \]  

(52)

\[ Z_{D,t} = \lambda_t Y_{D,t} T_{D,t} + \alpha_D \beta (\pi_{D,t+1})^{\sigma+1} Z_{D,t+1} \]  

(53)

\[ \pi_{D,t} = \left[ \alpha_D + (1 - \alpha_D) (\pi_{D,t})^{1-\sigma} \right] \left( \frac{1}{1-\sigma} \right) \]  

(54)
Recursive form price setting II

For foreign firms,

\[
\pi_{F,t} = \frac{\sigma \pi^*_{F,t}}{(\sigma - 1) \pi_{F,t}} \frac{X_{F,t}}{Z_{F,t}} \tag{55}
\]

\[
X_{F,t} = \lambda_t Y_{F,t} \frac{T_{F,t}}{Q_t} mc_{F,t} + \alpha_F \beta \left( \pi^*_{F,t+1} \right)^{\sigma} X_{F,t+1} \tag{56}
\]

\[
Z_{F,t} = \lambda_t Y_{F,t} \frac{T_{F,t}}{Q_t} + \alpha_F \beta \left( \pi^*_{F,t+1} \right)^{\sigma+1} Z_{F,t+1} \tag{57}
\]

\[
\pi^*_{F,t} = \left[ \alpha_F + (1 - \alpha_F) \left( \pi_{F,t} \right)^{(1-\sigma)} \right]^{\left( \frac{1}{1-\sigma} \right)} \tag{58}
\]
Goods market equilibrium,

Domestic country:

\[ Y_{D,t} = (T_{D,t})^{-\xi_D} \left[ \mu C_t + \mu^* \left( \frac{1 - n}{n} \right) (T_{D,t})^{\xi_D - \xi_F} (Q_t)^{\xi_F} C_t^* \right] \]  \hspace{1cm} (59)

Foreign country

\[ Y_{F,t} = (T_{F,t})^{-\xi_D} \left[ (1 - \mu) \left( \frac{n}{1 - n} \right) C_t + (1 - \mu^*) (T_{F,t})^{\xi_D - \xi_F} (Q_t)^{\xi_F} C_t^* \right] \]  \hspace{1cm} (60)
Labour Market equilibrium,

- Domestic country:

\[ H_{D,t} = Y_{D,t} \frac{Disp_{D,t}}{A_{D,t}} \]  \hspace{1cm} (61)

where, \( Disp_{D,t} = \alpha_D \left( \pi_{D,t} \right)^\sigma Disp_{D,t-1} + (1 - \alpha_D) \left( \frac{\pi_{D,t}}{\pi_D} \right)^{-\sigma} \)  \hspace{1cm} (62)

- Foreign country

\[ H_{F,t} = Y_{F,t} \frac{Disp_{F,t}}{A_{F,t}} \]  \hspace{1cm} (63)

where \( Disp_{F,t} = \alpha_F \left( \pi_{F,t} \right)^\sigma Disp_{F,t-1} + (1 - \alpha_F) \left( \frac{\pi_{F,t}}{\pi^*_F} \right)^{-\sigma} \)  \hspace{1cm} (64)
Net Exports

Net export = Export - Import,
$NX_{D,t}$ and $NX_{F,t}$ are the real net export for domestic country and foreign country respectively.

$$
NX_{D,t} = T_{D,t} C_{D,t}^* - T_{F,t} C_{F,t}
$$

$$
NX_{F,t} = \frac{T_{F,t}}{Q_t} C_{F,t} - \frac{T_{D,t}}{Q_t} C_{D,t}^*
$$

Preference shock,

$$
\Upsilon_{D,t} = \left( 1 - \rho_{\Upsilon_D} \right) \bar{\Upsilon} + \rho_{\Upsilon_D} \Upsilon_{t-1} + \nu_{D,t} \omega_{D,t}
$$  \hspace{1cm} (65)

$$
\nu_{D,t} = \left( 1 - \rho_{\nu_D} \right) \bar{\nu} + \rho_{\nu_D} \nu_{D,t-1} + \varsigma_{\Upsilon_D} \xi_{D,t}
$$  \hspace{1cm} (66)

$$
\Upsilon_{F,t} = \left( 1 - \rho_{\Upsilon_F} \right) \bar{\Upsilon} + \rho_{\Upsilon_F} \Upsilon_{t-1} + \nu_{F,t} \omega_{F,t}
$$  \hspace{1cm} (67)

$$
\nu_{F,t} = \left( 1 - \rho_{\nu_F} \right) \bar{\nu} + \rho_{\nu_F} \nu_{F,t-1} + \varsigma_{\Upsilon_F} \xi_{F,t}
$$  \hspace{1cm} (68)
Interest rate rule

- Domestic country

\[
R_{D,t} = \left( \frac{1}{\beta} \right) \left( \frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left( \frac{Y_{D,t}}{Y_{D,t}^{fp}} \right)^{\phi_y} - 1
\]  

(69)

- Foreign country

\[
R_{F,t} = \left( \frac{1}{\beta} \right) \left( \frac{\pi_t^*}{\bar{\pi}^*} \right)^{\phi_\pi} \left( \frac{Y_{F,t}}{Y_{F,t}^{fp}} \right)^{\phi_y} - 1
\]  

(70)

- Welfare

\[
Welfare_{D,t} = U(C_t, H_{D,t}) + \beta Welfare_{D,t+1}
\]  

(71)

\[
Welfare_{F,t} = U(C^*_t, H_{F,t}) + \beta Welfare_{F,t+1}
\]  

(72)

\(U_{D,t}\) and \(U_{F,t}\) are (1) and (4) respectively.
Autarky: Degree of openness is zero

Under autarky $\chi = 0$, when no trade of goods is possible following conditions hold,

- $C_{F,t} = 0$ and $C_{D,t}^* = 0$, implying $C_{D,t} = C_t$ and $C_{F,t}^* = C_t^*$, since $\mu_F = 1$ and $\mu_F = 0$

- $T_{D,t} = 1$, $T_{F,t} = T_t$ (Terms of trade) = $Q_t$ (Real exchange rate)
Steady state

- For $\chi \neq 0$: The steady state is solved numerically for the variables (in MATLAB), $C_{t}, C_{t}^{*}, Y_{D,t}, Y_{F,t}, T_{t}, T_{D,t}, T_{F,t}, Q_{t}, \lambda_{t}, \lambda_{t}^{*}, b_{t}, b_{t}^{*}$, the rest have an analytical solution.

- For $\chi = 0$ there exist an analytical solution of the steady state.
Third order approximation for volatility shocks

- First order cannot capture the dynamics due to volatility shocks as the policy functions would only depend on level shocks.
- The second order only captures the partial effects of the volatility shocks through interactions with other shocks.
- Third order approximation of the model will fully capture the effect of rise in volatility.

Solution

*Very difficult to solve it analytically therefore we need to solve it numerically in MATLAB*
## AUTARKY

### Uncertain productivity shock

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sticky price</th>
<th>Flexible prices</th>
<th>Flexible prices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SS</td>
<td>SSS</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.359708</td>
<td>0.999846</td>
<td>0.994234</td>
</tr>
<tr>
<td>Labour</td>
<td>0.999588</td>
<td>0.999846</td>
<td>0.994234</td>
</tr>
<tr>
<td>Nominal Interest rate</td>
<td>0.622633</td>
<td>0.010101</td>
<td>-0.07272</td>
</tr>
<tr>
<td>Wages</td>
<td>0.353667</td>
<td>0.85755</td>
<td>-3.96079</td>
</tr>
<tr>
<td>Welfare</td>
<td>-20081.2</td>
<td>-20000.1</td>
<td>-20078.7</td>
</tr>
<tr>
<td>Output</td>
<td>0.359708</td>
<td>0.999846</td>
<td>0.994234</td>
</tr>
<tr>
<td>Inflation</td>
<td>1.613188</td>
<td>1</td>
<td>0.947209</td>
</tr>
</tbody>
</table>

### Uncertain demand shock

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sticky price</th>
<th>Flexible prices</th>
<th>Flexible prices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SS</td>
<td>SSS</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.213337</td>
<td>0.999846</td>
<td>0.994561</td>
</tr>
<tr>
<td>Labour</td>
<td>0.999281</td>
<td>0.999846</td>
<td>0.994561</td>
</tr>
<tr>
<td>Nominal Interest rate</td>
<td>0.770456</td>
<td>0.010101</td>
<td>-0.01242</td>
</tr>
<tr>
<td>Wages</td>
<td>-0.03859</td>
<td>0.85755</td>
<td>-3.68018</td>
</tr>
<tr>
<td>Welfare</td>
<td>-20077.6</td>
<td>-20000.1</td>
<td>-20074.7</td>
</tr>
<tr>
<td>Output</td>
<td>0.213337</td>
<td>0.999846</td>
<td>0.994561</td>
</tr>
<tr>
<td>Inflation</td>
<td>1.759765</td>
<td>1</td>
<td>0.986896</td>
</tr>
</tbody>
</table>
## Uncertain productivity shock

<table>
<thead>
<tr>
<th>Variable</th>
<th>Both Flexible prices</th>
<th>Both Sticky price</th>
<th>Foreign flex +</th>
<th>Foreign sticky +</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% Δ ss</td>
<td>% Δ sss</td>
<td>Std. Dev.</td>
<td>% Δ ss</td>
</tr>
<tr>
<td>Consumption</td>
<td>-0.05469</td>
<td>-0.05469</td>
<td>0.110997</td>
<td>-123.991</td>
</tr>
<tr>
<td>Labour</td>
<td>0.000767</td>
<td>0.000767</td>
<td>0.000114</td>
<td>-0.18991</td>
</tr>
<tr>
<td>Nominal Interest rate</td>
<td>-84.4085</td>
<td>-113.463</td>
<td>0.097077</td>
<td>24975.95</td>
</tr>
<tr>
<td>Wages</td>
<td>3.5E-13</td>
<td>3.5E-13</td>
<td>1.64E-13</td>
<td>-3.34188</td>
</tr>
<tr>
<td>Consumption_dom</td>
<td>0.000767</td>
<td>0.000767</td>
<td>5.69E-05</td>
<td>-231.644</td>
</tr>
<tr>
<td>Consumption_for</td>
<td>-0.09577</td>
<td>-0.09577</td>
<td>0.111016</td>
<td>-16.3252</td>
</tr>
<tr>
<td>Welfare</td>
<td>-0.00399</td>
<td>-3.7E-05</td>
<td>0.189374</td>
<td>-0.82579</td>
</tr>
<tr>
<td>Output</td>
<td>0.000767</td>
<td>0.000767</td>
<td>0.000114</td>
<td>0.235.284</td>
</tr>
<tr>
<td>Inflation</td>
<td>-0.67116</td>
<td>0.77789</td>
<td>0.063387</td>
<td>243.6204</td>
</tr>
<tr>
<td>Nominal Exchange Rate change</td>
<td>-2E-13</td>
<td>-1E-13</td>
<td>1.9E-13</td>
<td>240.5116</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>0.03006</td>
<td>0.03006</td>
<td>0.002456</td>
<td>-2.12391</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
<td>0</td>
<td>0</td>
<td>1.07E-13</td>
<td>0</td>
</tr>
<tr>
<td>Marginal Costs</td>
<td>3.5E-13</td>
<td>3.5E-13</td>
<td>1.73E-13</td>
<td>-3.34188</td>
</tr>
<tr>
<td>Inflation_dom_good</td>
<td>-0.6761</td>
<td>0.773771</td>
<td>0.06254</td>
<td>273.38</td>
</tr>
<tr>
<td>Net Exports</td>
<td>8.61E+14</td>
<td>1.46E+15</td>
<td>0.109971</td>
<td>-1.9E+18</td>
</tr>
</tbody>
</table>
## FREE TRADE

### Uncertain demand shock

<table>
<thead>
<tr>
<th>Variable</th>
<th>Both Flexible prices</th>
<th>Both Sticky price</th>
<th>Foreign flex + Domestic sticky</th>
<th>Foreign sticky + Domestic flex</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% Δ ss</td>
<td>% Δ sss</td>
<td>Std. Dev.</td>
<td>% Δ ss</td>
</tr>
<tr>
<td>Consumption</td>
<td>-0.00148</td>
<td>-0.00148</td>
<td>0.000122</td>
<td>-451.81</td>
</tr>
<tr>
<td>Labour</td>
<td>1.48E-06</td>
<td>1.48E-06</td>
<td>1.22E-07</td>
<td>-0.7093</td>
</tr>
<tr>
<td>Nominal Interest rate</td>
<td>-0.00161</td>
<td>0.147309</td>
<td>0.000102</td>
<td>77110.8</td>
</tr>
<tr>
<td>Consumption_dom</td>
<td>1.48E-06</td>
<td>1.48E-06</td>
<td>6.09E-08</td>
<td>-821.01</td>
</tr>
<tr>
<td>Consumption_for</td>
<td>-0.00297</td>
<td>-0.00297</td>
<td>0.000122</td>
<td>-82.603</td>
</tr>
<tr>
<td>Welfare</td>
<td>-7.8E-06</td>
<td>-7E-08</td>
<td>0.000202</td>
<td>-1.9138</td>
</tr>
<tr>
<td>Output</td>
<td>1.48E-06</td>
<td>1.48E-06</td>
<td>1.22E-07</td>
<td>-821.01</td>
</tr>
<tr>
<td>Inflation</td>
<td>-1.1E-05</td>
<td>0.000981</td>
<td>6.75E-05</td>
<td>787.75</td>
</tr>
<tr>
<td>Nominal Exchange Rate change</td>
<td>4.37711</td>
<td>2.593517</td>
<td>0.153507</td>
<td>676.181</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>2.97E-05</td>
<td>2.97E-05</td>
<td>2.45E-06</td>
<td>-7.3842</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
<td>0</td>
<td>0</td>
<td>1.07E-13</td>
<td>0</td>
</tr>
<tr>
<td>Inflation_dom_good</td>
<td>-1.1E-05</td>
<td>0.00098</td>
<td>6.67E-05</td>
<td>1179.75</td>
</tr>
<tr>
<td>Net Exports</td>
<td>2.65E+13</td>
<td>1.96E+12</td>
<td>0.000121</td>
<td>-7E+18</td>
</tr>
</tbody>
</table>
Autarky
IRFs for one period uncertain world productivity shock II
Autarky
IRFs for one period uncertain world demand shock I

![Graphs showing the impulse response functions (IRFs) for various economic indicators: Bond Savings, Consumption, Labour, Output, Bond Savings fp, Consumption fp, Labour fp, Output fp. The graphs illustrate the effects of a shock on these indicators over time.](image-url)
Autarky
IRFs for one period uncertain world demand shock II

[Graphs showing the impact of wages, nominal rate of interest, inflation, and welfare loss in response to uncertain shocks.]
Free Trade
IRFs for one period uncertain world productivity shock I

Consumption
Labour
N. Rate of Interest
Wages

Consumption fp
Labour fp
N. Rate of Interest fp
Wages fp

(ISI-Delhi)
Uncertainty
14 September, 2018
Free Trade
IRFs for one period uncertain world productivity shock II
Free Trade
IRFs for one period uncertain world productivity shock III

Output, NER change, ToT, RER, Net Exports

Output, NER change fp, ToT fp, RER fp, Net Exports fp

Uncertainty
14 September, 2018 35 / 40
Free Trade
IRFs for one period uncertain world demand shock I
Free Trade
IRFs for one period uncertain world demand shock II

![Graphs showing the impact of a demand shock on various economic indicators such as consumption (C-dom), consumption for foreign (C-for), inflation, welfare loss, and bond savings. The graphs indicate the changes over time, with axes ranging from 0 to 20 periods.]
Free Trade

IRFs for one period uncertain world demand shock III

- Output
- NER change
- ToT
- RER
- Net Exports

- Output fp
- NER change fp
- ToT fp
- RER fp
- Net Exports fp
Uncertainty shock external to the country does affect the level of the macroeconomic variables under free trade in the long run as well as the short run as the domestic economy opens up its trade.

One of the most important feature that determines whether the real variables of the domestic economy get affected is the flexibility of prices.

Using the baseline model we observe that the real variables do not get affected when the prices are completely flexible as all nominal variables adjust completely here.

When the prices are sticky the real variables adjusts and the welfare losses are higher.
A model version where domestic producers are not price makers but price takers as assumed here.

Assuming that the foreign country has a zero lower bound on the interest rates, does this changes the response of domestic monetary policy?

Does the role of monetary policy vary with the change of exchange rate determination from floating to fixed exchange rate regime?

We are also trying to understand the role of important parameters like coefficient of risk version and elasticity of demand on the welfare results.