Network Structure and Credit Rating

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Abstract

We consider a model in which productive bilateral links are formed between heterogeneous agents who differ in their innate productivity. Local information is complete but an outside planner can observe only network properties. We ask if consistent credit rating – where agents' ratings are increasing in their productivity – is possible using network characteristics alone. The key to our results is that the network structure is endogenous since the use of agents' network neighborhood properties in generating ratings also impacts their incentives for link formation. Network structure and credit scores are therefore determined jointly in equilibrium. We show that if the cost of link formation is not too low, there is a pairwise stable equilibrium under credit rating where the network structure is a connected nested split graph (CNSG) with neighborhood size increasing in type. This is also the unique equilibrium if we consider a class of "truthtelling" equilibria (that is, equilibria in which the network structure separates types, enabling consistent credit rating). Further, among networks that separate types, this specific CNSG constitutes precisely the optimal structure.

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1 Introduction

Lack of access to credit is a crucial aspect of the problem of financial exclusion faced by a significant proportion of the world's population. According to the World Bank's Findex database over 1.7 billion adults worldwide do not have access to a basic bank account. Even those with a basic bank account often do not have access to services such as credit or insurance. A critical factor impeding extension of credit is the absence of suitable rating systems to assess the creditworthiness of individuals. Traditional credit rating systems rely on credit histories, but these are typically non-existent for financially excluded individuals.

In recent years, a host of startup lenders such as Lenddo and InVenture have looked to fill this gap by using data gleaned from social networks to construct credit scores for individuals without a traditional credit rating. Lenddo bases part of its rating on the ratings of "friends" in a social network, while InVenture considers frequency and duration of mobile phone usage data to judge the number and strength of relationships (a proxy measure of network position). The International Committee on Credit Reporting has drawn attention to the use of information contained in social networks.[1] The challenge in creating any such system is to gather and process credit-relevant information, especially when that information is not directly observable by the credit rating body.

This paper explores theoretically the possibility of credit rating based on observed network connections. We consider a population of heterogeneous agents who differ in their innate productivity. We refer to the innate productivity of an agent as the "type" of that agent. Local information is assumed to be complete - each agent is able to observe the types of all other agents. A network link captures a productive partnership between two agents: the joint output of any bilateral link – shared equally between the two agents – displays complementarity in that it is increasing in the types of the linked agents. An additional link is worth forming if and only if the marginal utility from the additional output exceeds the cost of forming that link, for both agents.

A planner observes the network that forms from these mutually-beneficial links but lacks direct information on individual productivity. Since local information drives the formation of the network, it is indeed seeded with information relevant to credit rating. The

[1] In the 2018 report on the 'Use of Alternative Data to Enhance Credit Reporting to Enable Access to Digital Financial Services by Individuals.'

natural question, then, is whether it is possible for the planner to exploit the local information embedded in the network, to rank individuals according to their productivity purely through its observation of network characteristics. We say that a credit rating is consistent if more productive individuals are rated higher. The outcome is complicated by the fact that the assigned credit ratings provide benefits to individuals, which affects the incentives to form network connections. Here the network structure and ratings are jointly determined: the mapping from network characteristics to credit rating affects incentives for link formation, which in turn affects ratings. So the questions are, given any network-based rating scheme, what equilibrium network-architecture emerges, whether consistent network-based rating is possible, and what the efficiency properties of such networks are. We take a first step towards answering such questions.

We begin by showing through an example that, in general, observing the network that emerges from forming mutually beneficial links does not allow consistent inference about the innate productivity of agents. This is due to the fact that for many parametric settings agents with distinct productivity may find it beneficial to connect to the same subset of agents. In these cases, we say that the equilibrium network does not separate types.

We then explore how the introduction of network-based credit ratings can alter the outcome. Since credit ratings provide benefits to individuals, their assignment changes the incentives to form links. We present our analysis in two steps. In the first step we regard the benefits provided by credit ratings as pure non-negative transfers based on inference of agent types from the network. We devise a type assignment procedure that the designer follows to assign types to agents based on their network characteristics. The assignment of types is then used to determine transfer values for agents. We solve backwards: knowing the type assignment procedure and the transfer scheme, agents form links. Based on the links formed, the actual assignment and transfers are determined.

We then ask what pattern of transfers can sustain an equilibrium network structure that enables inference about types. Our notion of equilibrium requires pairwise stability of links. We find that as long as the cost of link formation is not too low, transfers can implement a particular network architecture – a connected nested split graph (CNSG) – in which the degree of the agents is strictly increasing in their types. We show that any optimal separating network is a CNSG in which the neighborhoods are nested and higher types have larger neighborhoods. We design a set of transfers under which there is an equilibrium network structure that coincides with the optimal separating network.

The intuition for our central result – that the most productive agents will end up with largest neighborhoods – is as follows. Suppose there are five types, ranging from the highest θ_1 to the lowest θ_5. In a connected nested-split graph we want the highest type θ_1 to connect to all other types, while θ_2 connects to only other agents of types θ_1 to θ_4 and so on. This structure separates types. But why might this be incentive compatible? If identifying as type θ_1 confers the greatest benefit through transfers, why doesn't type θ_2 make additional connections to imitate the higher type? Here the condition on the cost of link formation is crucial. When this cost is not too low, the marginal link for θ_1, namely the link between θ_1 and the neighbor of the lowest type, θ_5, generates a loss for the linked agents. By design, the transfer scheme compensates the agents precisely for this loss. But forming a link with θ_5 generates an even greater loss for the less-productive type θ_2, and by forming such links purely to identify as type θ_1, they would receive inadequate compensation for the loss incurred by connecting to θ_5. This prevents lower types from imitating higher types. But if marginal links are unprofitable then why do higher types do not imitate lower types? Again, the design of our transfers prevents this from happening. Transfers are recursive, so that an agent of type θ_1 receives an amount weakly greater than an agent of type θ_2 and so on. We show that this implies that imitating a lower type is never strictly beneficial.

Further, restricting attention to a class we call "truthful equilibria" - essentially equilibria that allow for type separation by observing network connections (similar in spirit to truthtelling equilibria when agents report types directly) - we show that the optimal separating network is also the unique equilibrium network.

We then show how the required pattern of transfers can arise as benefits associated with a credit rating system. Higher credit rating lead to improved access to formal finance by, for instance, allowing agents to borrow more and/or on better terms. For any type, the benefits of improved access can be regarded as a transfer, with some natural constraints: notably that benefits cannot be negative (anyone can withdraw from the rating system, so those facing a negative transfer can choose to be unrated) and must be strictly increasing (a higher rating must confer greater benefits). We show that the benefits from a credit rating system can mimic the transfers (in a sense we make precise), implying that a credit rating scheme can implement an optimal separating network. Thus a credit rating scheme based purely on network characteristics can indeed harness local information in a way that allows for consistency in ratings and at the same time improves the network structure so that it attains constrained efficiency.

To the best of our knowledge, this is the first paper proposing an indirect mechanism to elicit types based on individual choices to form links. While others have analyzed mechanisms which make use of social network structure, in these works the network is generally assumed to be exogenously fixed. In contrast, we highlight possible challenges arising when credit ratings based on network alter incentives to rewire connections. In such cases, rating and network structures are jointly determined. We propose a mechanism which induces a truthful equilibrium where individuals reveal their productivity through their link-choices and where the equilibrium network generates the highest total output.

2 The model

2.1 Types and rating system

There are $n > 2$ agents. We denote the set of agents by N. Agents draw types according to some distribution from $\Theta \equiv (\theta_1, \ldots, \theta_m)$ where $\theta_1 > \ldots > \theta_m > 0$. For any $k \in \{1, \ldots, m\}$, let Θ_k denote the subset of agents with type θ_k and $n_k = |\Theta_k|$ its cardinality. Clearly, $\sum_{k=1}^{m} n_k = n$. Finally, let θ_k^i denote agent i of type θ_k.

Agents can observe each others' types and form connections using this information. The connection forming process and the network payoffs obtained are described in the next section. A planner can observe the network formed by agents but does not know the type of any agent. Based on information inferred from observing the network, the planner must assign a credit rating to each agent.

A *credit rating* is a real number belonging to a finite interval $[r_{\min}, r_{\max}]$ where $r_{\max} > r_{\min} > 0$, with higher numbers conveying greater creditworthiness.

In a standard direct mechanism, agents submit reports of types and a planner's allocation decision depends on the vector of reports. Here, choice of network position is a "report" and based on observed network characteristics (which we make precise later), the planner assigns a rating to each individual. Of course, such a rating system is useful only if ratings reflect underlying productivity. With this in mind, define a credit rating to be *consistent* if for any pair of agents $i, j \in N$ where i has type θ_k and j has type θ_q, ratings follow the same ranking as productivity: $r_i \gtreqless r_j$ as $\theta_k \gtreqless \theta_q$. Note that the consistency requirement is similar to truthtelling in a direct mechanism.

Finally, consider payoffs from credit rating. Receiving a rating $r_q \in [r_{\min}, r_{\max}]$ confers a benefit to an agent. For instance, those with high ratings may have better access to formal sector credit, allowing them access to capital for profitable investment. And, of course, these benefits might also vary directly with an agent's type.[2] Let $B(r_q, \theta_k)$ denote the benefit received by an agent of type θ_k who receives a rating $r_q \in [r_{\min}, r_{\max}]$. We assume $B(\cdot, \cdot)$, positive, strictly increasing in both arguments, and bounded.

We now set out the notation to analyze network formation.

2.2 Network notation

A network $G(N, L)$ is composed of a set N of nodes and a set L of links. We deal with undirected networks, for which $ij \in L$ if and only if $ji \in L$ for any pair $i, j \in N$. We call $\mathcal{N}_i = \{j \in N : ij \in L\}$, the subset of agents connected to $i \in N$, as i's *neighborhood*. For any node i, the degree refers to the cardinality of its neighborhood: $d_i \equiv |\mathcal{N}_i|$.

A *path* between two nodes i and j is a sequence of nodes $i_1, i_2, ..., i_k$ such that $i_1 = i$ and $i_k = j$, and $ii_2, i_2i_3, ..., i_{k-1}j \in L$. Two nodes are connected if there exists a path between them. A network is connected if there exists a path between every pair of nodes. A *clique* is a fully connected subset of three or more nodes, or each pair of nodes in this subset is connected.[3] A network is *complete* if $ij \in L$ for any pair $i, j \in N$.

A *nested split graph*, or NSG, is a network where if $ij \in L$ and $d_k \geq d_j$, then $ik \in L$. In other words, the neighborhoods of a NSG are nested. NSG architectures are such that agents with degree d are connected to every agent with degree $d' > d$. A *connected nested split graph*, or CNSG, is a NSG that is also connected.

Figure 1 presents and example of a CNSG with ten agents. Two agents (labelled (a)) are connected to all others (each has degree nine). Four agents labelled (b) are connected to all but those labelled (d), so each (b) agent has degree seven. Note that each agent

[2] For example, suppose higher credit rating gives an agent access to a formal sector loan of higher size (loan size $L(r_k)$ increasing in r_k), and the success of the project in which the loan is invested depends on the agent's type, so that the net return per unit of investment is $\rho(\theta)$ where $\rho' > 0$. This implies a benefit $\rho(\theta)L(r_k)$ which is clearly increasing in both rating score and agent's own type.

[3] Although it is common to refer to a pair of connected nodes as a clique of size 2, here we refer to cliques only if composed by three or more nodes.

labelled (a) or (b) are linked to all other agents of those two labels. Agents labelled (a) and (b) therefore form a clique. Next, agents labelled (c) are connected to only (a) and (b) agents (degree six), and those labelled (d) are connected only to agents (a) (degree two). Agents labelled (c) and (d) are only connected to those in the clique, and therefore form an independent set.

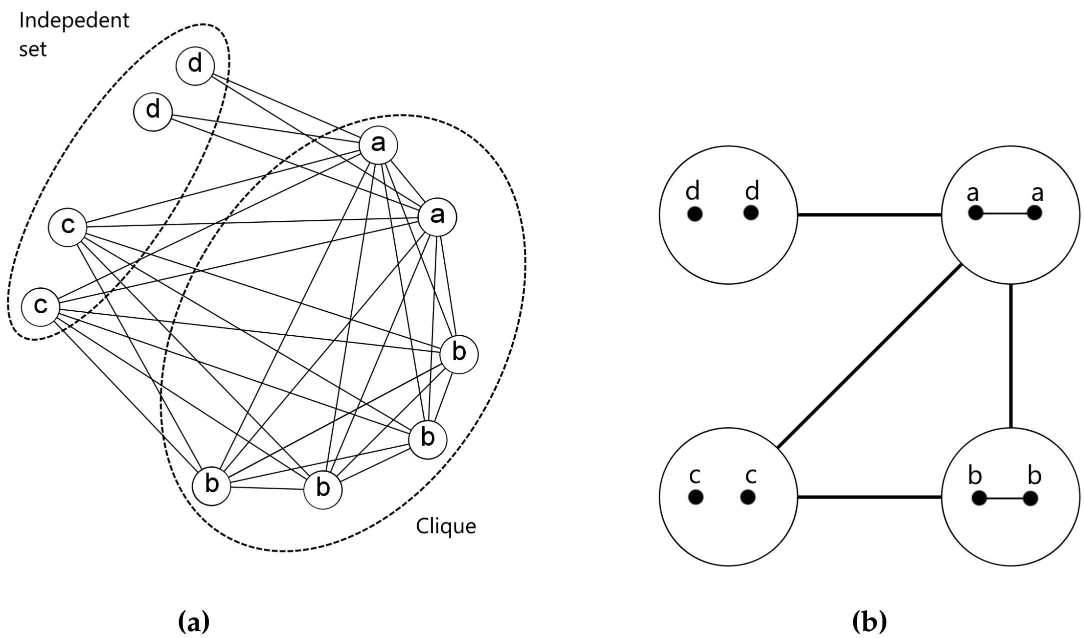


Figure 1: Two alternative representations of the same CNSG structure. In (a), nodes are labeled by degree classes and lines denote links between nodes. The circles enclose nodes that form a clique and an independent set of nodes that are connected only to some nodes in the clique. Panel (b) offers a schematic representation of the same structure. Each circle indicates a subset of nodes belonging to the same degree class. If a subset of types forms a clique (fully connected subset), the two nodes inside the circle are connected by a link. A link between two circles indicates that the two subsets of nodes are connected to each other.

2.3 Neighborhoods, payoffs, and network equilibrium

As noted previously, agents can form connections with other agents at cost $c > 0$ per link. A connection between an agent i of type θ^i_k and an agent j of type θ^j_q creates output $g(\theta^i_k, \theta^j_q)$ for each agent involved, thus the net benefit of a link ij is $g(\theta^i_k, \theta^j_q) - c$ for each agent.[4] The output generated is positive and increases in the productivity of either agent, i.e. $g(\cdot, \cdot)$ is strictly positive and strictly increasing and symmetric in both arguments.

Suppose an agent of type θ_k connects to η_1 agents of type $\theta_1, \ldots, \eta_m$ agents of type θ_m. We write the neighborhood of the agent of type θ_k as a vector $\mathcal{N}_k$, which is given by

$$\mathcal{N}_k = \{\eta_1, \ldots, \eta_m\}$$

The vector of neighborhoods $(\mathcal{N}_1, \ldots, \mathcal{N}_m)$ defines an (undirected) network G.

The total payoff to an agent i of type θ_k^i of being connected to $\mathcal{N}_i$ agents is given by

$$v(\theta_k^i, \mathcal{N}_i) = \sum_{j \in \mathcal{N}_i} g(\theta_k^i, \theta_q^j) - cd(\mathcal{N}_i) + B(r_s, \theta_k^i) \tag{2.1}$$

where the first two elements define the network payoff, or the net benefit from being connected to $\mathcal{N}_i$ excluding possible returns from a credit rating, while the third element is the benefit from rating r_s assigned by the designer to agent i. We normalize the payoff for an isolated agent to zero, i.e. $v(\theta_k^i, \varnothing) = 0$ for any θ_k^i. As we discuss in the next section, the links may also create a positive social externality.

To assess which links are formed, we adopt a modified version of the *pairwise stability* (Jackson and Wolinsky, 1996) as the notion of network equilibrium.

Definition 1. *A network $G(N, L)$ is pairwise stable (PWS) if*

(i) *for all $ij \in L$, $v(\theta_k^i, \mathcal{N}_i) \geqslant v(\theta_k^i, \mathcal{N}_i \setminus \{j\})$ and $v(\theta_q^j, \mathcal{N}_j) \geqslant v(\theta_q^j, \mathcal{N}_j \setminus \{i\})$, and*

(ii) *for all $ij \notin L$, if $v(\theta_k^i, \mathcal{N}_i \cup \{j\}) \geqslant v(\theta_k^i, \mathcal{N}_i)$, then $v(\theta_q^j, \mathcal{N}_j \cup \{i\}) < v(\theta_q^j, \mathcal{N}_j)$*

The first condition requires that no agent strictly benefits from cutting an existing link; the second condition says that any link that weakly benefits one agent without making the other agent strictly worse off must be active – i.e. no pair of agents can both benefit (weakly or strictly) from activating an additional link between them.

Note that this assumes links between two agents are formed even if both are merely indifferent to their formation. The reason for this modeling choice is as follows. Later in the paper we design transfers that induce agents to form particular links. For some links, the

[4]For expositional simplicity, we assume that c is uniform across all agents and links.

transfer value is just enough to make the agent indifferent between forming and not forming a link, in which case our equilibrium definition breaks indifference towards forming the link. We could, of course, break indifference the other way, in which case the transfers would have to add a small sum to make agents strictly prefer the connection. This would only complicate the algebra without altering our results.

2.4 Social benefit from links

We allow the possibility that a link between two nodes, apart from creating value for the participants, also generates a positive externality. In practical settings, these wider spillover benefits may be manifested in various forms: the provision of employment benefits for other (unmodelled) economic actors from bilateral projects, or by adding to social learning to improve future production processes.

Define the total output produced in a network G, ignoring the cost of link formation, as

$$T_G \equiv \sum_{i \in G} \sum_{j \in \mathcal{N}_i} g(\theta_k^i, \theta_q^j)$$

Let function $e : \mathbb{R}_+ \to \mathbb{R}_+$ measure the positive externality created by the total output generated by the nodes in G. This function maps a positive real number from a given output level. We assume $e(\cdot)$ strictly increasing with respect to its argument. The social benefit generated by a network G is then computed as

$$V(G) = \sum_{i \in N} v(\theta_k^i, \mathcal{N}_i) + e(T_G) \tag{2.2}$$

Let E_{iq} be the marginal externality created by the link iq. This is given by

$$E_{iq} \equiv e\left(T_G\right) - e(T_{G \setminus \{iq\}}) \tag{2.3}$$

We make the following assumption on the cost per link c.

Assumption 1. *The cost per link c is such that $c \leq g(\theta_k^i, \theta_q^j) + \dfrac{E_{ij}}{2}$ for all $i, j \in N$.*

Recall that a link ij generates benefit $g(\theta_k^i, \theta_q^j)$ for each agent involved and a marginal externality E_{ij}. If so, the shadow value of the per capita benefit of link ij is $g(.,.) + E_{ij}/2$. When Assumption 1 holds, each possible link between two agents in N generates shadow benefits greater than its costs.[5]

[5] This simplifies the exposition that follows. Without introducing externalities and making assumption 1,

3 The separating benchmark

Our interest lies in ranking unobservable types by observing their network connections. This is possible only if there is some dissimilarity in the neighborhood structure across agents of different types. We define a separating network as follows.

Let Θ^i denote the set of types agent i connects to. Further, let "type class" denote any set of agents of the same type.

Definition 2. ***(Separating network)*** *A network G is said to* ***separate*** *types if any two agents of different types are connected to different numbers of type classes. Consider any two agents of different types: i of type θ_k and j of type θ_q. A network G separates types if the following holds:*

$$\theta_k \neq \theta_q \iff |\Theta^i| \neq |\Theta^j|$$

Note that this definition allows the possibility that two agents considered similar (i.e. two agents connected to the same number of type classes) may have different degrees, because agents could be connected to some but not all agents belonging to the same type class (see Figure 2a for an example). When the network structure is separating and each agent i is connected to all agents with types in the set Θ^i, we say that the network is separating-dense.[6]

Definition 3. ***(Separating-dense network)*** *A network is* ***separating-dense*** *if it separates types, and if agent i is connected to agent j with type θ_k, then i is also connected to any other agent $z \neq j$ with type θ_k, if any.*

we would have to decide whether socially suboptimal links (where cost exceeds output) that help to separate agents (crucial for consistent rating) were worthwhile - in other words, we would have to put weights on the value of ranking and the social loss from links with lower types and decide whether to sacrifice rating or social value. Depending on the weight on rating and social value, we could obtain coarser rankings where types close to each other would not be ranked. This would complicate the analysis without making any qualitative change to the nature of network outcome for the agents actually ranked, so we make the simplifying assumption.

[6]The term "density" has multiple definitions in graph theory. Here, we use this term to highlight separating structures where, whenever an agent is connected to some agents in a specific type class, he must be connected to all agents in that class.

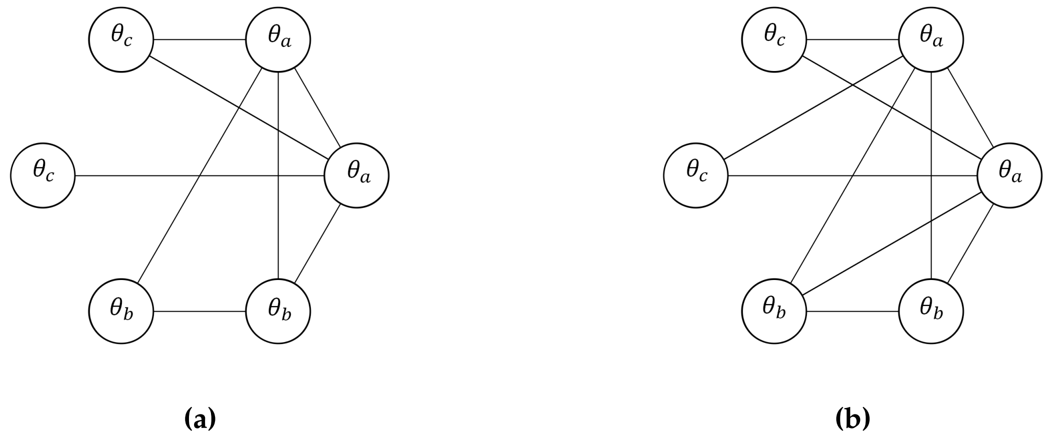


Figure 2: Both structures are separating since pairs of distinct types are connected to different number of type-classes: each type θ_a is connected to agents with types $\{\theta_a, \theta_b, \theta_c\}$, each type θ_b to types $\{\theta_a, \theta_b\}$, and each type θ_c to types $\{\theta_a\}$. In panel (b), two extra links connecting a type θ_a to types θ_c and θ_b make the network separating-dense.

A separating-dense structure is therefore a network where any non-active link, if activated, would compromise separation. Clearly, if a network is separating-dense, two agents have the same degree if and only if they are not separated in the network (see Figure 2b). In the following Lemma we characterize the architecture of a connected separating-dense network.[7]

Lemma 1. *A network is connected and separating-dense only if it is a CNSG.*

Let $\mathcal{G}_n^m$ be the set of all separating-dense networks of n nodes and m types. We say that a link ij is more *efficient* than zt if it has greater shadow value in the sense that $g(\theta_k^i, \theta_q^j) + E_{ij}/2 > g(\theta_x^z, \theta_y^t) + E_{zt}/2$. Within the class of networks $\mathcal{G}_n^m$, we define the structures that involve the most efficient links.

Definition 4. ***(Optimal separating network)*** *A network G is an* ***optimal separating*** *structure if it is separating-dense and $g(\theta_k^i, \theta_q^j) + E_{ij}/2 > g(\theta_x^z, \theta_y^t) + E_{zt}/2$ for all $ij \in L$ and $zt \notin L$.*

In other words, a network is optimal separating if by replacing any existing link the total output decreases and by activating a new link we would compromise the separation of types.

[7]When not in the main body of the paper, proofs are in the Appendix.

It is intuitive to see that an optimal separating network would have more links for higher types. Moreover, a link with a higher type is always more profitable than a link with a lower type. This implies that an optimal network must be a NSG. Proposition 1 formalizes this intuition.

Proposition 1. *The optimal separating network is a CNSG where $\Theta^j \subset \Theta^i$ for all i and j such that $\theta_k^i > \theta_q^j$.*

Note that with m types, and without connections to own type-class (e.g. if own type-class had only 1 member), there can be at most $m-1$ values of $|\Theta^i|$, implying that at most $m-1$ agents can be separated. With connections to own class also available, there are m values of $|\Theta^i|$, so all types can be separated. For example, with $m = 5$, and with connections to own class available, there are 5 possible type classes with cardinalities $5, 4, \ldots, 1$. The latter case makes the analysis clearer (as we do not have to keep track of which types are not separated). To ensure this is always true, we assume that there are at least 2 agents of any type:

Assumption 2. *For any k, $n_k \geqslant 2$.*

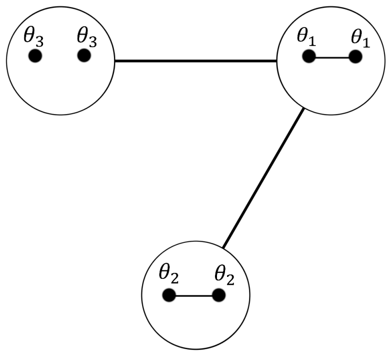
Figure 3 presents examples of separating and non-separating networks assuming two agents of each type.

Finally, our main results concern an optimal separating network, let us describe here the neighborhood structure in this network. Let $\ell = \lfloor m/2 \rfloor$, which is the smallest integer greater than or equal to $m/2$. By being a CNSG, a separating network involves a clique of the top ℓ types, $\{\theta_1, \theta_2, \ldots, \theta_l\}$ and the rest form an independent set. Members of the clique are connected to each other, so, for the types in the clique, the neighborhood structure is as follows

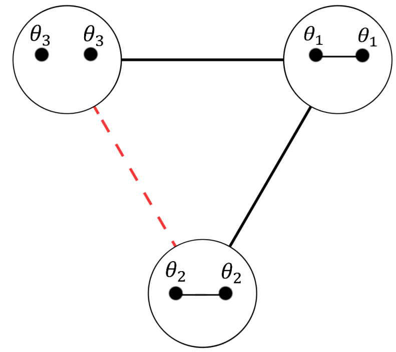
$$\mathcal{N}_1^* = \{n_1 - 1, n_2, \ldots, n_m\}, \ldots, \mathcal{N}_\ell^* = \{n_1, \ldots, n_\ell - 1, 0, \ldots, 0\}$$

Next, the neighborhoods for types below θ_ℓ are already implied by the above. These types form an independent set with each type connected only to a subset of types in the clique.

$$\mathcal{N}_{\ell+1}^* = \{n_1, \ldots, n_{(\ell-1)}, 0, \ldots, 0\}, \ldots, \mathcal{N}_m^* = \{n_1, 0, \ldots, 0\}$$



(a) Optimal separating network. No links can be added without violating separation. No links can be replaced by one that generates a higher value.



(c) A network that fails to separate all types. The extra (dashed) links prevent separation of types θ_2 and θ_1.

Figure 3

4 General design of incentives

Our objective is to design a credit rating system that relies on observed network information, and to analyze the equilibrium network structure that emerges under the rating system. To approach the problem, we split the analysis in two parts. First, we simply design a system of non-negative transfers based on network characteristics to achieve separation in the equilibrium network - i.e. an equilibrium network where different types have different neighborhoods. Next, we show how benefits arising from a credit rating system can serve the same role as transfers.

Before we analyze the design of incentives, it is instructive to note that equilibrium networks that arise in the absence of any such interventions (or, if agents were unaware that their network connections matter for rating) do not generally allow for consistent inference about agent types. Figure 4 illustrates the problem.

We now turn to the design of incentives. The analysis proceeds as follows. In this section, we present the design of transfers. Section 5 then defines our equilibrium concept and derives some of its properties. Section 6 then derives the equilibrium and characterizes the equilibrium network that obtains under the transfers described in this section. Finally, section 7 shows how to translate the transfers as benefits from having a credit rating.

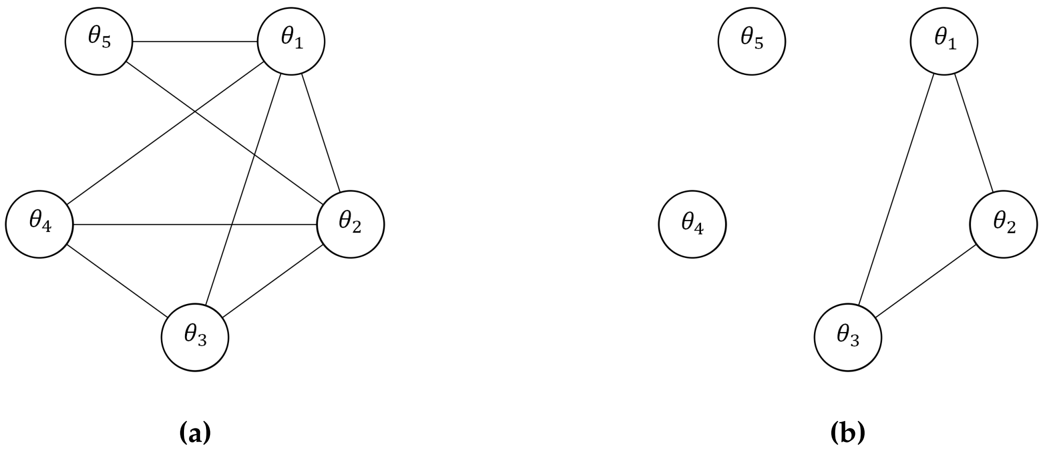


Figure 4: Pairwise stable networks arising without designer intervention. Consider the type set $\Theta = \{\theta_1 = 10, ..., \theta_5 = 6\}$ with one agent of each type. Let $g(\theta^i_k, \theta^j_q) = \frac{(\theta^i_k \theta^j_q)^{1-\gamma}}{1-\gamma}$ with $\gamma \neq 1$. For figure (a), $c = 49$ and $\gamma = 0$. Note that the pairs $\{\theta_1, \theta_2\}$ and $\{\theta_3, \theta_4\}$ cannot be separated. For figure (b), $c = 12$ and $\gamma = 0.7$. Note that in this case we cannot separate the top three types in the clique and the bottom two isolated types.

4.1 The structure of moves

Consider the interaction between agents and a mechanism designer. The sequence of moves is as follows.

1. The mechanism designer announces a system of transfers as a function of observable network characteristics.
2. Knowing this system of transfers, agents then choose their connections.

We consider the resulting pairwise-stable network and analyze its properties in terms of separation and efficiency. It is worth noting that inference based on network characteristics is complicated because when an agent deviates and connects like another type, the entire network structure changes, potentially changing the designer's network-based inference about all types. Identifying an equilibrium then requires ruling out the profitability of such network effects of any deviation.

4.2 The transfer function

We now specify a function that the mechanism designer uses to assign non-negative transfers to agents in any observed network. First, define a marginal type for any type θ_k.

Definition 5. ***(Marginal type)*** *For any $k \in \{1, \ldots, m\}$, let $\mathcal{M}(\theta_k^i)$ be the lowest type among the agents connected to agent i of type θ_k. The type $\mathcal{M}(\theta_k^i)$ is then called the marginal type for agent i. If $\mathcal{M}(\theta_k^i)$ is the same across all agents of type θ_k, we denote it as $\mathcal{M}(\theta_k)$.*

In the optimal separating network described in the section above, the lowest type that θ_1 connects to is θ_m, so $\mathcal{M}(\theta_1) = \theta_m$. Further, $\mathcal{M}(\theta_2)$ is the type θ_{m-1} and so on.

To aid exposition, we introduce a slight abuse of notation. Let $\mathcal{M}(k)$ denote the index for the marginal type of θ_k. In other words,

$$\theta_{\mathcal{M}(k)} \equiv \mathcal{M}(\theta_k)$$

Wherever possible, we denote the marginal type of θ_k by $\mathcal{M}(\theta_k)$, but the $\theta_{\mathcal{M}(k)}$ form is useful when adding over indices. For any agent, we refer to a link with a marginal type as a "marginal link".

Next, define an infra-marginal type as follows.

Definition 6. ***(Infra-marginal type)*** *For any $k \in \{1, \ldots, m\}$, let $\mathcal{M}(\theta_k^i)$ be the marginal type of agent i. Any type $\theta_q > \mathcal{M}(\theta_k^i)$ such that $\theta_q \in \Theta^i$ is called infra-marginal type for i.*

Put simply, an infra-marginal type for agent i is any type connected to i which is not a marginal type. Clearly, any such type must be a higher type compared to the marginal type. We refer to a link with an infra-marginal type as an "infra-marginal link".

We now proceed to construct transfers. The value of transfers depends on the types assigned, which are determined by the observed degree classes. We begin by describing the process of assignment of types.

4.2.1 Type assignment

We now describe the procedure for type assignment and illustrate with examples.

Step 1

Let d_1 be the highest degree observed in the network, a value that may be achieved by multiple agents. Assign type θ_1 to all such agents. Now consider the set of agents who are connected only to types assigned θ_1. If this set is non-empty (we describe the procedure in the other case in step 2 below), assign all such agents the type $\mathcal{M}(\theta_1)$. Recall that these agents represent marginal connections for those assigned type θ_1. Further, if the network has any single-agent degree-classes with degree lower than d_1 with their singleton member connected to any of agents assigned type $\mathcal{M}(\theta_1)$, assign type θ_1 to that member as well. Formally, $\mathcal{M}(\mathcal{M}(\theta_1)) = \theta_1$.

Next, consider the remaining degree classes (i.e. the degree classes not already assigned type θ_1). Let d_2 denote the highest degree observed among these. Assign type θ_2 to all such agents. Now consider the set of agents who are connected only to types assigned θ_1 and θ_2. Suppose this set is non-empty. Assign all such agents type $\mathcal{M}(\theta_2)$. Further, if the network has any single-agent degree-classes lower than d_1 with their singleton member connected to any agents assigned type $\mathcal{M}(\theta_2)$, assign type θ_2 to that member as well.[8]

In this way assign types until either degree classes are exhausted or the number of types available (m) is exhausted.

Step 2

Next, suppose agents of degree class d_k are assigned type θ_k, but there are no agents that are uniquely connected to agents assigned θ_k and any higher types. Then assign type θ_k to agents of degree d_k as well as to agents with the next highest degree, d_{k+1}. Now repeat the above procedure. If still no marginal connections can be found, assign type θ_k to all agents of degree d_k to d_{k+2} and continue. This process stops when we do find marginal connections or degree classes are exhausted. In the latter case, assign $\mathcal{M}(\theta_k) = \theta_k$.

Step 3

There is now a list of S assigned type classes: S_1 types $\theta_1, \ldots, \theta_{S_1}$ (where $S_1 \leqslant m$ - if S_1 reached m the process would stop there) and S_2 marginal types, where $S_1 + S_2 = S$. S

[8]Note that any such agent would not also be connected to any agent assigned type $\mathcal{M}(\theta_1)$. Otherwise they would already be assigned type θ_1 and not be considered in this step.

might be lower than, equal to, or greater than m. Assign types θ_1 to θ_m to the m top classes if $S \geqslant m$. If $S > m$, exclude from the mechanism any agents of the bottom $S - m$ classes. If $S < m$, simply assign types θ_1 to θ_S.

Let n_k denote the number of agents assigned type θ_k, where $k \in \{1, \ldots, m\}$.

4.2.2 Two examples illustrating type assignment

Example 1 The following example illustrates how the type allocation process works. In this first example, we start with an optimal separating network with five types and assign types.

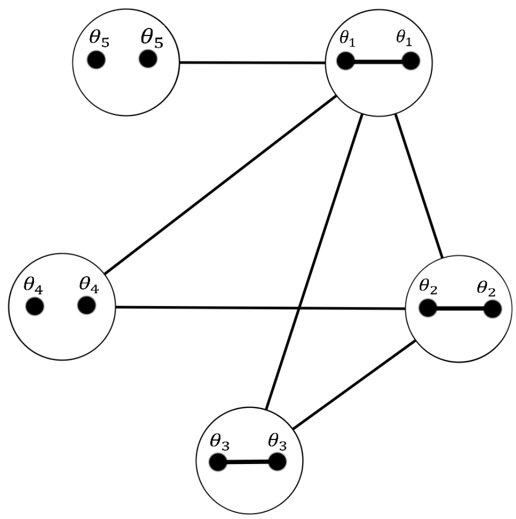


Figure 5

Suppose there are five types in the population, θ_1 to θ_5, with two agents of each type.[9] Figure 5 shows schematically the optimal separating network in this case. The marginal type for θ_1 is θ_5, for θ_2 is θ_4 and for θ_3 is θ_3 itself. Here θ_3 is the lowest type in the clique. Type θ_4 is only connected to types θ_1 and θ_2, and type θ_5 only to type θ_1.

An agent of type θ_1 is then connected to all other agents, with degree 9. This is the highest degree class, so type θ_1 would indeed be assigned the correct type. At the same time, type θ_5 would be assigned $\mathcal{M}(\theta_1)$.

An agent of type θ_2 is connected to all other agents of types θ_1 to θ_4 (so the observed

[9]There would be no qualitative change if we had more agents in each type-class.

degree is 7). Since this is the second-highest degree, the type assignment is correct again, and type θ_4 is assigned $\mathcal{M}(\theta_2)$.

Finally, an agent of type θ_3 is connected to other agents of types θ_1 to θ_3, resulting in observed degree 5, the third highest degree class, assigned type θ_3 (and it is its own marginal class).

The mechanism designer now has a list of types $\theta_1, \theta_2, \theta_3, \mathcal{M}(\theta_2), \mathcal{M}(\theta_1)$. So the final assignment is θ_1 to θ_5.

The following table shows the true type classes, assigned type classes and associated degrees for the two agents of each type.

True type	Observed degree classes	Assigned types and marginals	Final type assignment
θ_1	9,9	θ_1	θ_1
θ_2	7,7	θ_2	θ_2
θ_3	5,5	θ_3	θ_3
θ_4	4,4	$\mathcal{M}(\theta_2)$	θ_4
θ_5	2,2	$\mathcal{M}(\theta_1)$	θ_5

Example 2 Now suppose one agent of type θ_1 drops a single connection to its marginal type θ_5. Let us assign types in the resulting network.

True type	Observed degree classes	Assigned types and marginals	Assigned types and marginals	Final type assignment
θ_1	9	θ_1		θ_1
θ_1	8		θ_1	θ_1
θ_2	7,7		θ_2	θ_2
θ_3	5,5		θ_3	θ_3
θ_4	4,4		$\mathcal{M}(\theta_2)$	θ_4
θ_5	2	$\mathcal{M}(\theta_1)$		θ_5
θ_5	1	$\mathcal{M}(\theta_1)$		θ_5

Initially, the agent with the highest degree (degree 9) would be assigned type θ_1. Then the agents with degrees 2 and 1 would be identified as being uniquely connected to the agent with assigned type θ_1. These agents would be assigned type $\mathcal{M}(\theta_1)$. Next, we look for any single-agent class also linked to those assigned $\mathcal{M}(\theta_1)$. This now includes the deviating agent with degree 8. This agent would also then be assigned type θ_1. Other types are then assigned as before.

Thus given our procedure, the deviation does not change the final type assignment.

4.2.3 Constructing the transfer function

Next define functions to capture the gain for a type from forming a connection with the type marginal to it, that is the gain to any type θ_q of connecting to type $\mathcal{M}(\theta_q)$. Let MG_q denote this marginal gain for type θ_q.

$$\mathrm{MG}_q = g(\theta_q, \mathcal{M}(\theta_q)) - c$$

Note that this is the marginal gain ignoring any transfers, so MG_q could be positive or negative.

Now construct the transfer functions for agent i of type θ_k as follows. For any agent i of type θ_k, let $n^i_q(\theta_k)$ denote the number of connections to agents assigned type θ_q.

$$t_k^i = \sum_{k=1}^{\mathcal{M}(k)} n_q^i(\theta_k) \max\left[0, -\mathrm{MG}_q\right] \tag{4.1}$$

To see how this is constructed, note that while connections to type $\mathcal{M}(\theta_k)$ are the marginal connections for θ_k, connections to types $\theta_r > \mathcal{M}(\theta_k)$ are infra-marginal connections. For each infra-marginal connection with type $\theta_r > \mathcal{M}(\theta_k)$, the transfer function compensates agent i for the loss incurred by type θ_r in connecting to that type's marginal type (i.e. loss from a connection between θ_r and $\mathcal{M}(\theta_r)$). The loss could be zero or negative (a gain) in which case the transfer received is zero. Otherwise the transfer is strictly positive. Finally, for own marginal connection, θ_k is compensated for own loss from connecting to the marginal type. Multiplying each loss by the relevant number of connections made by θ_k and adding over the resulting expressions we get the total transfer to an agent i of type θ_k.

We note that in an optimal separating structure, all marginal and infra-marginal links are present. The first result below shows that in such cases, transfers can be written in a simple recursive form. This is useful since later we identify an equilibrium with precisely this property.

Lemma 2. *If all marginal and infra-marginal links are formed, the transfer to any agent of type θ_k can be written in the following recursive form.*

$$t_k = A_k + t_{k+1} \tag{4.2}$$

where $A_k \geqslant 0$ and $t_{m+1} = 0$. Specifically, let $\mathbb{1}_{k<\ell}$ be an indicator function which is equal to 1 for $k < \ell$ and 0 for $k = \ell$.

$$A_k = \begin{cases} (n_{\mathcal{M}(k)} - 1) \max\left[0, -MG_k\right] + \mathbb{1}_{k<\ell} \max\left[0, -MG_{k+1}\right] & \text{for } k \leqslant \ell \\ n_{\mathcal{M}(k)} \max\left[0, -MG_k\right] & \text{for } k > \ell \end{cases}$$

The intuition for the result above is as follows. Recall that the transfer function is constructed so that for each marginal connection, agent i of type θ_k is compensated for own loss from connecting to the marginal type. Further, for each infra-marginal connection with type $\theta_r > \mathcal{M}(\theta_k)$, the transfer function compensates agent i for the loss incurred by type θ_r in connecting to that type's marginal type. In all cases, the transfer is zero if the loss is negative.

Consider the transfer received by an agent i of type θ_{m-1}. This agent connects to all agents that θ_m connects to, but also makes an additional $n_{\mathcal{M}(m-1)}$ marginal connections. Hence t_{m-1} exceeds t_m by $n_{\mathcal{M}(m-1)} \max\left[0, -\mathrm{MG}_{m-1}\right]$.

Next, consider the transfer received by an agent i of type θ_1. Now, the agent of type θ_1 receives the loss from marginal connections for $n_{\mathcal{M}(1)}$ marginal connections. The agent also receives the same loss for connections with $n_1 - 1$ other agents of type θ_1. But any agent of type θ_2 connects to all agents of type θ_1 and receives the same loss for n_1 connections to type θ_1. This explains the first term in the expression for A_1. Next, the agent of type θ_1 connects to n_2 agents of type θ_2, while each agent of type θ_2 only connects to $n_2 - 1$ agents of type θ_2. This explains the second term in the expression for A_1.

5 Truthful equilibrium

A truthful equilibrium is a pairwise-stable network where the neighborhood choice of agents reveals the type of each agent truthfully. The specific definition is as follows.

Definition 7. *(Truthful equilibrium) A network constitutes a truthful equilibrium if it is pairwise-stable, connected (i.e. no isolated component) and separating (as defined in Definition 2).*

The rest of this section derives two preliminary results that clarify the structure of equilibria and are useful in proving later results. The next section then presents our main results, which together show that the optimal separating structure constitutes the unique truthful equilibrium given the specified type assignment procedure and transfers, and given a certain condition on the cost of link formation.

The first preliminary result notes a property of marginal types in any truthful equilibrium. Marginal types must be weakly decreasing in types.

Lemma 3. *In any truthful equilibrium, for any pair of types $\theta_k > \theta_q$, we must have $\mathcal{M}(\theta_k) \leqslant \mathcal{M}(\theta_q)$.*

Proof: The proof is by contradiction. Suppose, to the contrary, there is a pairwise-stable equilibrium for which $\mathcal{M}(\theta_k) > \mathcal{M}(\theta_q)$ for some pair of types $\theta_k > \theta_q$. Consider a deviation in which type θ_k chooses to form additional links with type $\mathcal{M}(\theta_q)$. For each such link, there is direct payoff $g(\theta_k, \mathcal{M}(\theta_q)) - c$ and the deviating agent also receives a transfer $\max[0, -MG_q]$ where, recall that $MG_q \equiv g(\theta_q, \mathcal{M}(\theta_q)) - c$. Note that as the return $g(.,.)$ is increasing in each of its arguments, we have $g(\theta_k, \mathcal{M}(\theta_q)) - c > g(\theta_q, \mathcal{M}(\theta_q)) - c = MG_q$ for $\theta_k > \theta_q$. We consider two cases. In cases where $MG_q \geq 0$, the transfer is zero, but the additional link is profitable for type θ_k as $g(\theta_k, \mathcal{M}(\theta_q)) - c > MG_q$. In cases where $MG_q < 0$, the additional link leads to a positive transfer $-MG_q$ and $g(\theta_k, \mathcal{M}(\theta_q)) - c - MG_q > 0$. In either case a profitable deviation exists, so that the original configuration could not have been pairwise stable.$\|$

The second preliminary result shows that agents form all infra-marginal links in any truthful equilibrium.

Lemma 4. *In any truthful equilibrium, each agent i of any type θ_k connects to all other agents of any infra-marginal type $\theta_q > \mathcal{M}(\theta_k)$.*

Proof: Consider a link between an agent of type θ_k and its infra-marginal type θ_q, i.e. $\theta_q > \mathcal{M}(\theta_k)$. The total payoff, including the transfer that agent of type θ_k receives for this link, is $g(\theta_k, \theta_q) - c + \max\Big[0, -\mathrm{MG}_q\Big]$.

If $g(\theta_k, \theta_q) - c > 0$, the expression above is strictly positive, so forming that infra-marginal link with θ_q is clearly advantageous.

Next, consider the case $g(\theta_k, \theta_q) - c \leqslant 0$. From Lemma 3, we know that in any truthful equilibrium, higher types must have weakly lower marginal types: therefore, for $\theta_q > \mathcal{M}(\theta_k)$, we have $\mathcal{M}(\theta_q) \leqslant \mathcal{M}\Big(\mathcal{M}(\theta_k)\Big) = \theta_k$. Since $\mathcal{M}(\theta_q) \leqslant \theta_k$, and $g(\theta_k, \theta_q) - c \leqslant 0$, it follows that $g(\theta_q, \mathcal{M}(\theta_q)) - c \leqslant 0$. Then the payoff including the transfer is $g(\theta_k, \theta_q) - g(\theta_q, \mathcal{M}(\theta_q)) \geqslant 0$.

It follows that for any agent of any type, the total payoff from an infra-marginal link is non-negative. Therefore, if we suppose that there is a truthful equilibrium in which some infra-marginal links are missing, we would immediately reach a contradiction.||

Together these results imply that higher types have (weakly) higher degree in any truthful equilibrium. This follows from the fact that in equilibrium all infra-marginal links are formed (Lemma 4) and higher types have lower marginal types (Lemma 3). As we show in the next set of results below, in the unique truthful equilibrium degree is in fact strictly increasing in type.

6 The main results

We now present the first of our main results. We suppose that assumptions 1 and 2 hold throughout the analysis.

Proposition 2. *There is $c^* > 0$ such that for $c > c^*$, under the transfer function given by equation (4.1), there is a truthful equilibrium in which the equilibrium network is the optimal separating network.*

We prove this in two steps. First, in section 6.1 we show that no agent can benefit by cutting any links starting from the optimal structure. Second, in section 6.2 we clarify the condition on the cost of link formation under which no agent has an incentive to add any links. Section 6.3 presents a proof of Proposition 2. Finally, a further result in section 6.4 shows the uniqueness of the equilibrium featuring the optimal separating network.

6.1 The incentive to maintain links

In this section we show that starting from the optimal separating network, no agent can benefit by dropping any link. From lemma 4 we know that all infra-marginal links are formed. It remains to show that all agents make all marginal connections.

Before we prove that all marginal links as in the optimal separating network are formed in equilibrium under the transfers specified, we go through an example to show this. The example also serves to illustrate the type assignment mechanism and clarify the way transfers sustain the desired equilibrium, by making unprofitable the deviations.

6.1.1 An example

We now illustrate how the type allocation process and the associated transfers make deviations that drop links starting from the optimal separating network unprofitable.

Consider the same example as in Section 4.2.2. Example 1 considers type assignment in an optimal separating network with five types, θ_1 to θ_5, with two agents of each type. Section 4.2.2 shows the type assignment in this case.

We now explore agents' incentives to deviate from the links in the optimal separating network, and consider various cases.

Case1. Suppose one agent of type θ_1 drops a single connection to its marginal type θ_5.

This is exactly the case considered in Example 2 in Section 4.2.2. Recall that the deviation does not change final type assignments.

Note (from the table in Example 2 in Section 4.2.2) that the deviating agent simply has one fewer marginal connection, which makes the agent weakly worse off (if the connection produced a positive benefit, its loss makes the deviation unprofitable; and if the connection produced a loss, the transfer would have compensated, so deviation makes no difference). The same argument shows that for an agent of any type, deviating by dropping a fraction of the marginal connections is unprofitable.

Case 2. Next, suppose an agent of type θ_1 drops all marginal connections (those with type θ_5). This changes two things. First, the agent of type θ_1 is now classified as θ_2. If θ_1 obtained a benefit from connecting to θ_5, that would be lost. If θ_1 incurred a loss from the connection, the transfer to θ_1 would have compensated for that loss. In any case, losing marginal connections cannot produce a strict benefit for type θ_1. Second, each type θ_5 now receives a weakly lower transfer since each of them only have a single marginal connection now (with the single agent classified as type θ_1).[10] Since agents with types higher than θ_5 also receive the transfer given to θ_5 (Lemma 2 clarifies the recursive

[10] If θ_5 has a strict benefit from connecting to θ_1, the transfer to θ_5 is zero in all cases. If, on the other hand, θ_5 has a loss from connecting to θ_1, the transfer would be positive for each connection to an agent of type θ_1. In this latter case the transfer to type θ_5 would strictly decrease after the deviation by the agent of type θ_1.

structure of transfers under the optimal separating structure), the deviating agent of type θ_1 would receive a weakly lower transfer now.

Once again, it cannot be profitable for an agent of type θ_1 to drop all its connections to its marginal types.

A similar argument shows that any agent of type θ_2 cannot benefit from dropping marginal connections.

True type	Observed degree classes	Assigned types and marginals	Final type assignment
θ_1	9	θ_1	θ_1
θ_1 θ_2	7 7,7	θ_2	θ_2
θ_3	5,5	θ_3	θ_3
θ_4	4,4	$\mathcal{M}(\theta_2)$	θ_4
θ_5	1,1	$\mathcal{M}(\theta_1)$	θ_5

Case 3. Finally, consider a deviation by the lowest type in the clique, type θ_3. Since there are only 2 agents of type θ_3 and the type is its own marginal type, each agent of type θ_3 has just one marginal connection. In this case, if one of these agents drop the marginal connection, all agents of type θ_3 would now have degree 4, thus a whole degree class is eliminated. There are now 4 degree classes, and type θ_3 is still identified as θ_3, but so are agents of type θ_4. Type θ_5 is now identified as θ_4. The following table shows the type assignment following the deviation.

True type	Observed degree classes	Assigned types and marginals	Final type assignment
θ_1	9,9	θ_1	θ_1
θ_2	7,7	θ_2	θ_2
θ_3 θ_4	4,4 4,4	$\mathcal{M}(\theta_2)$	θ_3
θ_5	2,2	$\mathcal{M}(\theta_1)$	θ_4

To show that the deviation cannot be beneficial, consider first the connection between θ_3 and θ_1. With or without the deviation, the payoff from each such link ($g(\theta_3, \theta_1) - c$)

remains the same. However, without the deviation, the type marginal to θ_1 is θ_5. So θ_5 and all higher types (including θ_3) receive any loss incurred by θ_5 in connecting to each agent of θ_1:

$$\max\left[-\Big(g(\theta_5,\theta_1)-c\Big),0\right]$$

But under the deviation, the marginal type of θ_1 is the higher type θ_4, so that type and all higher types receive (for each connection between θ_5 and θ_1) the weakly lower transfer

$$\max\left[-\Big(g(\theta_4,\theta_1)-c\Big),0\right]$$

Thus θ_3 is weakly worse off under the deviation in relation to connections with the infra-marginal type θ_1.

Next consider connections of type θ_3 with type θ_2 with payoff $g(\theta_3,\theta_2)-c$. If this is positive, it is the same with or without deviation. Next, suppose $g(\theta_3,\theta_2)-c<0$. With the deviation, the loss is compensated so the payoff is 0. However, without the deviation, θ_3 receives the loss of type θ_4 in connecting to θ_2 which is higher than the loss $c-g(\theta_3,\theta_2)$, making the overall payoff in connecting to θ_2 strictly positive. (This is simply a reflection of the general principle that any type receives a strictly positive payoff overall from infra-marginal connections). Therefore, again, θ_3 is weakly worse off under the deviation in relation to connections with type θ_2.

Finally, without the deviation θ_3 also receives $\max\left[\Big(g(\theta_3,\theta_3)-c\Big),0\right]$, which is not received under the deviation.

The arguments above show that the deviation cannot lead to a strict gain for type θ_3.

6.1.2 Maintaining links: a formal result

The example above shows that starting from the optimal separating network, no agent can benefit by dropping any links. The following result states the general result. The arguments for the proof (which is in the appendix) are similar to those used in the example.

Lemma 5. *Under the specified transfers, starting from the optimal separating network, no agent wants to cut any link(s).*

6.2 The incentive not to form further links

Suppose type θ_1 strictly prefers to connect to type θ_m. In this case $\mathrm{MG}_{\mathcal{M}(1)} > 0$. Suppose type θ_m would also strictly benefit from connecting to type θ_2, and vice versa. In this case, given our type assignment, types θ_1 and θ_2 can no longer be separated. So for the optimal separating structure to be a PWS equilibrium, it must be that such strict benefits do not arise.

We now derive a sufficient condition for full separation to be achieved through non-negative transfers. Essentially, this requires that the initial network is relatively fragmented, which would be the case if the cost of connections is not too small. In this case the initial network does not have some of the links which are active in the optimal separating structure, and credit-rating can implement those links. Note that these links generate a negative payoff for the nodes involved (otherwise they would have formed the links in the first place). Our result banks on these negative payoffs, as the links can then be made incentive compatible through positive transfers.

Lemma 6. *There exists $c^* > 0$ such that, starting from the optimal separating network, no agent wants to add any links if $c > c^*$.*

Proof: Suppose all links in the optimal separating structure are formed. We need to rule out the possibility that a type $\theta_q < \theta_k$ does not make a strict loss by connecting to the marginal type of θ_k. (Note that θ_k already makes all the connections that θ_q make so there is no deviation to be considered in the other direction.)

From Lemma 2 we know that in this case, for any $k \leqslant \ell$

$$t_k = t_{k+1} + (n_{\mathcal{M}(k)} - 1)\max\left[0, -\mathrm{MG}_k\right] + \mathbb{1}_{k<\ell}\max\left[0, -\mathrm{MG}_{k+1}\right]$$

where $\mathbb{1}_{k<\ell} = 1$ if $k < \ell$ and 0 if $k = \ell$.

Suppose $\mathrm{MG}_k < 0$. In this case, the transfer to type θ_k just compensates for the loss in making marginal connections. Clearly, any lower type θ_q making the same connections (and receiving the same transfer) will therefore make a strict loss since the compensation for each such link is $c - g(\theta_k, \mathcal{M}(\theta_k))$ while the loss is $c - g(\theta_q, \mathcal{M}(\theta_k))$, where the latter is higher since $g(\theta_k, \mathcal{M}(\theta_k)) > g(\theta_q, \mathcal{M}(\theta_k))$ for $\theta_k > \theta_q$. This shows that no type in the clique would want to add a connection.

The only remaining possibility is that two types in the independent set wants to add a connection. Note that all types in the independent set are marginal types for some type in the clique. Therefore if, for all types in the clique, each marginal connection generates a loss, no type θ_q can benefit by connecting to the marginal type of a higher type θ_k.

Let

$$c^* \equiv \max_{k \leqslant \ell} \{g(\theta_k, \mathcal{M}(\theta_k))\} \tag{6.1}$$

Clearly, $c > c^*$ is sufficient to guarantee $\mathrm{MG}_k < 0$ for types in the clique. This completes the proof.$\|$

6.3 Proof of Proposition 2

We are now ready to prove Proposition 2. Lemma 5 shows that no agent benefits strictly from cutting any link(s) starting from the optimal separating network. Lemma 6 then shows that for $c > c^*$, there is no profitable deviation by forming any link not already present. The two results together complete the proof.$\|$

6.4 Uniqueness

Next, we present the second main result, which shows that the optimal separating structure is in fact the unique truthful equilibrium.

Proposition 3. *For $c > c^*$, under the transfer function given by equation (4.1), the optimal separating network is the unique truthful equilibrium.*

Proof: From Lemma 3 we know that if $\theta_k > \theta_q$, we must have $\mathcal{M}(\theta_k) \leqslant \mathcal{M}(\theta_q)$ in any truthful equilibrium. Lemma 4 shows that in any truthful equilibrium all infra-marginal links are formed.

Given this, and given the network is connected and separating, it is clear that we must have a connected nested split graph in a truthful equilibrium similar to the optimal separating network, with any possible difference emerging from agents of some type θ_k being connected to some, but not all, agents of the marginal type $\mathcal{M}(\theta_k)$.

Suppose there is such an equilibrium with agent i of type θ_k connected to $n^i_{\mathcal{M}(k)} < n_{\mathcal{M}(k)}$ agents of the marginal type $\mathcal{M}(\theta_k)$. Now, for each marginal link formed, i receives a transfer $\max[0, -MG_k]$. It follows that forming more marginal links must be (at least weakly) beneficial. Now, a truthful equilibrium must be pairwise-stable, where the latter is defined so that an agent forms all links that are at least weakly beneficial. It follows that each agent would form all possible links with marginal types. This, in turn, implies that the optimal separating structure is the only truthful equilibrium.||

7 Credit rating benefits as transfers

The previous section specified a general transfer design that implements the optimal separating network as the unique PWS network. We now show that so long as credit ratings generate well-defined benefits that are increasing in ratings and high enough ratings can generate high enough benefits, we can match the transfers that we designed above with rating benefits. In other words, if we have well-defined benefits from ratings, instead of paying a certain transfer to an agent, we can provide a similar payoff by assigning an appropriate credit rating to the agent. In this section we clarify the conditions for this to be possible.

As noted in Section 2, a credit rating is a real number belonging to a finite interval $[r_{\min}, r_{\max}]$ where $r_{\max} > r_{\min} > 0$, with higher numbers conveying greater creditworthiness. Receiving a rating confers a benefit to an agent. For instance, those with high ratings may have better access to formal sector credit, allowing them access to capital for profitable investment. For credit rating benefits to be able to mimic the transfer function, we need to impose some constraints at the outset.

First, for an individual without a credit rating and without access to formal credit, credit rating can only provide positive benefits (even the worst rating can only bar access to credit, which is the initial situation). Therefore, a credit rating system can only achieve positive transfers (i.e. $t_k \geqslant t_{k+1}$ for all $k \in \{1, \ldots, m-1\}$). This is already true of our transfer design mechanism.

Second, for any two types with different ratings, the payoff of the higher-rated type must strictly exceed that of the other. From the previous section, we know that for $c > c^*$, marginal links generate losses across type classes, i.e. $\mathrm{MG}_k < 0$ for all k. Therefore from Lemma 2, for $c > c^*$, $A_k > 0$ for all k, and therefore transfers are given by

$$t_k = A_k + t_{k+1} > t_{k+1}$$

In other words, transfers are positive, and, for $c > c^*$, strictly monotonic, where c^* is given by equation (6.1). Figure 6 below shows a case of positive monotonic transfers. We assume $c > c^*$ for the rest of the analysis.

7.1 Mimicking transfers through rating benefits

To see how rating benefits can mimic transfers, let us start with a simpler case where the benefit is purely a function of the rating received by an agent. Let r_k be the rating given to an agent assigned type θ_k. Then the benefit function can be written as $B(r_k)$ which maps ratings to positive real numbers, and is strictly increasing in r_k. So long as the benefits can be tuned through ranking, they can play the same role as the positive transfers. Suppose there is a low rating $r_{\min}$ such that $B(r_{\min}) \leqslant t_m$ and a high rating $r_{\max}$ such that $B(r_{\max}) \geqslant t_1$. Then we can find $r_1, \ldots, r_m$ such that

$$B(r_k) = t_k.$$

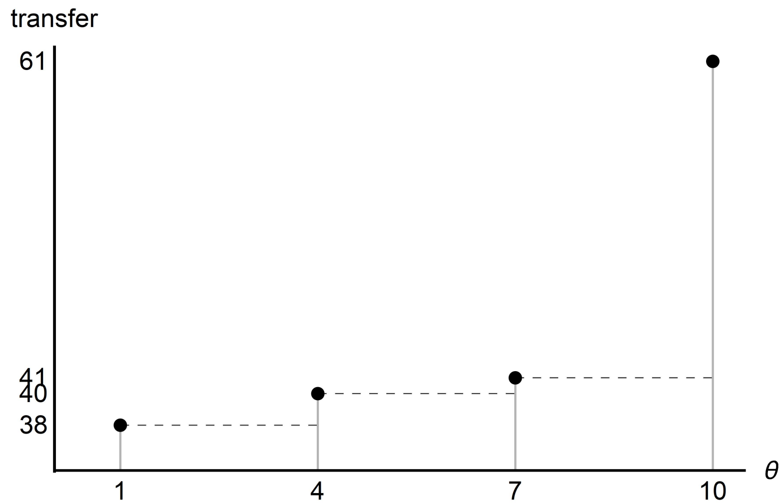


Figure 6: Positive monotonic transfers with types $\theta_1 = 10, \theta_2 = 7, \theta_3 = 4, \theta_4 = 1$ and two agents per type. The figure is drawn assuming $g(\theta_k^i, \theta_q^j) = \theta_k\theta_q$, and $c = 29$, where the cost threshold for monotonic transfers is $c^* = 28$.

However, in general, given the same credit terms, a more productive agent derives a greater benefit.[11] It is therefore natural for the benefit derived from ranking to depend directly on the agent's type. That is, the benefit that an agent of type θ_k derives from rating r_k depends not just on the rating, but also on the agent's type. A general benefit function therefore take the form $B(r_k, \theta)$, which is strictly increasing in both arguments. As above, assume $B(r_{\max}, \theta_1) \geqslant t_1$ and $B(r_{\min}, \theta_m) \leqslant t_m$. In this case, however, it may not be possible to match transfers exactly with ranking benefits - the latter might exceed the former. To see why, recall (from Lemma 2 and given the case $c > c^*$) that

$$t_1 = (n_{\mathcal{M}(1)} - 1)(-\mathrm{MG}_1) + (-\mathrm{MG}_{k+1}) + t_2 > t_2$$

Suppose $B(r_2, \theta_2) = t_2$. Now, suppose $B(r_2, \theta_1) - B(r_2, \theta_2) \geqslant t_1 - t_2$. This implies

$$B(r_2, \theta_1) \geqslant t_1 + B(r_2, \theta_2) \geqslant t_1 - t_2 = t_1$$

Then for any $r_1 > r_2$, $B(r_1, \theta_1) > t_1$. Note that the same conclusion would obtain if we assumed $B(r_2, \theta_2) \geqslant t_2$. So in general $B(r_k, \theta_k) \geqslant t_k$, with strict inequality possible in some cases.

However, this presents no incentive problem since the inequality is caused by the impact of the type θ_1 being higher than θ_2 - so that the fact that benefits exceed transfers does not

[11] See footnote 2 for an example.

cause any incentive problems (type θ_2 cannot get the extra benefit of type θ_1 by imitating type θ_1). To ensure that the higher rating r_1 does not cause any incentive problems it is sufficient to have r_1 such that for θ_2, forming additional links to increase rating is not beneficial:

$$B(r_1, \theta_2) - B(r_2, \theta_2) < n_m(c - g(\theta_2, \theta_m))$$

where the left hand side is the increase in benefit from a higher ranking for type θ_2, and the right hand side is the extra cost from additional links required to imitate type θ_1. Since B is continuous in r, given any r_2 it is always possible to find a r_1 that is higher than r_2 that satisfies the above inequality. Figure 7 illustrates the point.

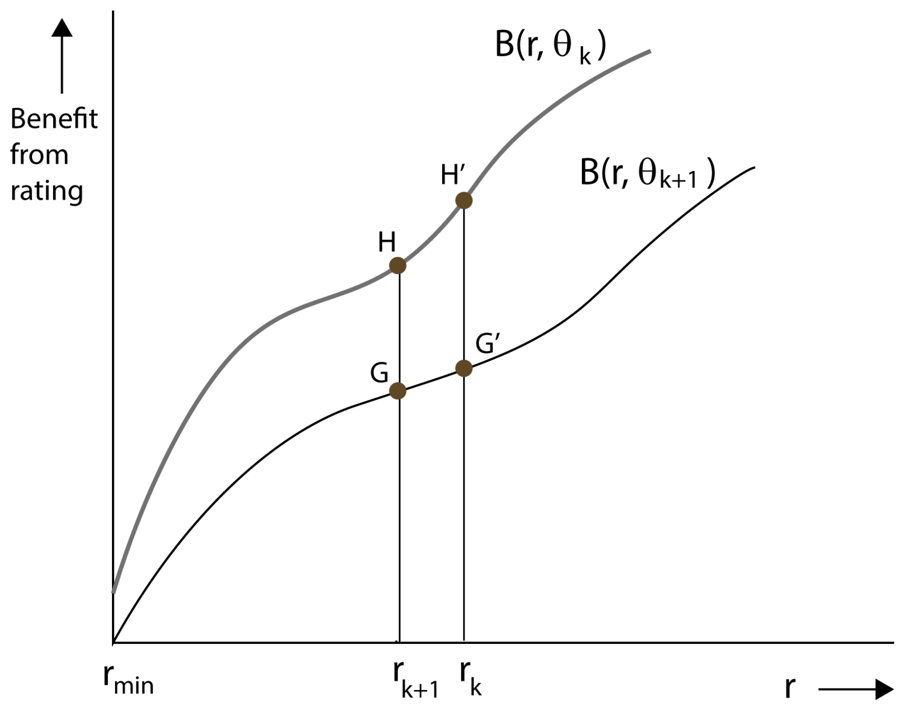


Figure 7: The benefit from rating for different types. If the distance GH is larger than $t_k - t_{k+1}$, then $B(r_{k+1}, \theta_k) \geqslant t_k$ implying that for any $r_k > r_{k+1}$, $B(r_k, \theta_k) > t_k$. This does not create incentive problems. Suppose r_k is close enough to r_{k+1} such that the difference between the heights G' and G (the extra benefit from higher rating for an agent of type θ_{k+1} if they were able to form the extra marginal links of type θ_k) is lower than the loss from forming these links (given by $n_{\mathcal{M}(k)}(c - g(\theta_{k+1}, \mathcal{M}(\theta_k))))$. This is sufficient to maintain incentives. It is clear that such a value of $r_k > r_{k+1}$ can always be found.

The discussion above shows that in general we can write $B(r_k, \theta_k) \geqslant t_k$, and if strict inequality holds, a sufficient condition for preserving incentives is that r_k is close enough to r_{k+1} so that type θ_{k+1} does not want to form extra connections to mimic type θ_k. Thus the benefit function can be written in a recursive form as follows.

Definition 8. ***(Mimicking transfers)*** *A ratings vector $r_1, \ldots, r_m$, where r_i denotes the rating given to agents assigned type θ_i, $i \in \{1, \ldots, m\}$ is said to mimic transfers if the following holds. Using values of transfers and A_k from Lemma 2, we can find $r_1, \ldots, r_m$ such that*

$$\begin{aligned} B(r_m, \theta_m) &= t_m \\ B(r_k, \theta_k) &\geqslant A_k + B(r_{k+1}, \theta_{k+1}) \quad \textit{for } k < m \end{aligned}$$

Further, if strict inequality holds for any $k < m$, then $r_k > r_{k+1}$ is such that

$$B(r_k, \theta_{k+1}) - B(r_{k+1}, \theta_{k+1}) < n_{\mathcal{M}(k)} \left(c - g(\theta_{k+1}, \mathcal{M}(\theta_k))\right)$$

Since the benefits function $B(r, \cdot)$ is continuous in r, it is always possible to find $r_1, \ldots, r_m$ that satisfy the properties above, implying that the same result as under transfers holds under ratings as well. This is noted in the result below.

Proposition 4. *For $c > c^*$, where c^* is given by equation (6.1), and given ratings that mimic transfers in the sense given by definition 8, the network that is optimal among those that lead to consistent rating (the optimal separating network) is obtained as the unique truthful equilibrium.*

Proof: Assume $c > c^*$. Since $B(r_k, \theta_k) \geqslant t_k$, the fact that no type has any incentive to drop any links under transfers still applies. Further, the construction of ratings rules out the incentive to form any extra links to mimic a higher type even when $B(r_k, \theta_k) > t_k$. Therefore credit-rating benefits satisfy the incentive compatibility constraints in the optimal separating network, implying that Proposition 2 holds under such benefits. Further, Proposition 3 applies as before, showing that the optimal separating network is the unique truthful equilibrium.||

8 Related Literature

König, Tessone, and Zenou (2014) study a decentralized dynamic link formation game where linking opportunities arrive at random. Links decay but links formed with more central agents decay more slowly. Here agents prefer to form link with an agent of high centrality and the resulting equilibrium structure is in the Nested Split Graph (NSG) class. Belhaj, Bervoets, and Deroïan (2016) study the problem of a planner who aims to design the network that maximizes the sum of Bonacich centralities of agents where the cost per link is also a function of the linking types of the agents. They show that the efficient networks belong to the NSG family, and they single out specific members of this family as function of the assumed cost structure. Our work complements these, but our focus is different: given different productivity types, we study how a mechanism designer can use network features to extract information embedded in the network by creating credit rating incentives that lead to endogenous reorganization of the network. Interestingly, the networks that allow the planner to both rank agents consistently with their types and maximize social benefit under the constraint of consistent ranking belong to the NSG class.

Bloch and Olckers (2018) study the question of ordinal ranking of agents who are connected in a network. Agents can rank others directly connected to them. In this setting they consider a mechanism design exercise to elicit the ranking information from agents who have local information: they can rank their neighbors ordinally. The mechanism asks agents to report this ranking and shows that an ex post efficient ranking can be achieved. Here the network itself is exogenous, and is simply a conduit for local information. Our exercise is very different. In our model, a designer uses the network structure itself to elicit local information to construct credit ratings. Since credit ratings confer benefits to agents, the structure of the network is endogenous: knowing that the network architecture will affect rating changes the link-formation incentives.

In an interesting paper Wei, Yildirim, Van den Bulte, and Dellarocas (2016) too study the problem of credit ratings that take into account the social network of the population. They propose a model of network formation where individuals with heterogeneous types have a preference for homophily. A planner imperfectly observes agents' types but can use the social network to refine his posterior. Since it is common knowledge that agents prefer to connect to similar types, agent i provides a signal about i's type. In their model the network matters, but only through the fact that linkages are informative in homophilous networks, so the equilibrium network becomes more assortative. In contrast, our model does not assume any preference for homophily, although homophilous structures might still arise in equilibrium through complementarities. Further, our focus is quite different: we are directly interested in the structure of the entire network and how that can be used in extracting information seeded in the network. In our setting the pairwise-stable equilibrium network does not shrink and become more assortative but expands to become a CNSG, which also maximizes social benefit among the class of separating networks.

A feature of our work is that we study a network with agents of heterogeneous types. Galeotti, Goyal, and Kamphorst (2006) study a model of network formation with heterogeneity in both costs and benefits of forming links. The benefits arise from flow of information: agent i can access j's information (including information obtained by j from nodes j alone connects to) and vice versa. Their paper concludes that high centrality and short average distances are robust features of equilibrium. Our setting is different: the benefit from a connection arises from synergy between types of the agents connected rather than from accessing information. This creates different link-formation incentives. Our focus is also different since we take a design approach on networks where the planner reformulates the network through rating incentives to uncover underlying information.

Finally, it is worth mentioning an empirical paper in a different context. Benson, Iyer, Kemper, and Zhao (2018) study company director networks with a large data set and show that companies where directors have greater degree centrality receive higher ratings. Further, companies with directors who score higher on other centrality measures such as eigenvector centrality or Bonacich centrality are also rated higher. To the extent that links here display complementarities, our results would imply that an NSG structure might emerge in such networks, explaining that the data would show precisely such correlations, although, as our results explain, centrality does not cause higher rating, rather the underlying NSG structure implies that in equilibrium, higher rated agents would also have higher centrality.

9 Conclusion

We present a model in which credit ratings are based on the network formed among productive agents. Individuals differ in their innate productivity or type and can observe the types of other individuals. Links take the form of bilateral collaborative projects. We assume that outside planners or agencies can observe the network but not the types of the individuals. We ask whether it is possible for the planner to infer the productivity of individuals from observing network characteristics alone. We consider networks that allow separation of types and show that the optimal separating structure is a connected nested split graph.

The intention of the credit rating system is to increase individuals' access to credit and, with higher ratings conferring greater benefits. Since ratings depends on network properties, they affect the incentives to form links. This implies that given any rating system and the scheme of associated benefits, the network structure and credit ratings are jointly determined in equilibrium. We call an equilibrium "truthful" if the network structure satisfies certain properties that allow separation of types from observing the network. We show that when the cost of link formation is not too low, it is possible to design a credit rating system that implements the optimal separating network as the unique truthful equilibrium.

The paper clarifies a crucial aspect of emerging 'fin-tech' ideas that intend to exploit network characteristics to sort individuals according to their creditworthiness. However the very realization that such metrics are credit relevant alter incentives and behavior, thereby altering the network and therefore the network characteristics on which the metrics are based. The key consideration in designing a credit rating scheme is to recognize this impact on incentives, to ensure that the scheme's design can sort individuals in a consistent manner.

A Appendix

A.1 Proof of Lemma 1

Consider a connected separating-dense network with $m > 1$ type classes. Label types according to the size of Θ^k, with $k \in \{1, \ldots, m\}$, thus $|\Theta^1| \geq |\Theta^2| \geq \ldots \geq |\Theta^m|$. The network is connected so the minimal size of a set Θ^k must be 1, while the maximal size is m since there are m type classes. This implies that, in order to have type separation with m distinct sizes for the sets Θ^k, we must have the following sets

$$\Theta^1 = \{\theta_1, \theta_2, \ldots, \theta_m\}, \Theta^2 = \{\theta_1, \theta_2, \ldots, \theta_{m-1}\}, \ldots, \Theta^m = \{\theta_1\}$$

This implies

$$\Theta^m \subset \Theta^{m-1} \subset \ldots \subset \Theta^2 \subset \Theta^1 \tag{A.1}$$

Finally, (A.1) together with the fact that, by Definition 3, in a separating-dense network each agent with type θ_k is connected to all the agents with types in Θ^k, imply that the neighborhood structure must be nested. This concludes the proof. $\|$

A.2 Proof of Proposition 1

Consider an optimal separating network G. Since G is separating-dense, by Lemma 1, G is also a CNSG. We then need to show that higher types are connected to more type-classes compared to lower types.

Suppose this is not true. Suppose $\theta_{k_1} > \theta_{k_2}$ and agents of type θ_{k_2} connect to agents of type θ_{k_3}, while agents of type θ_{k_1} do not. Let i be an agent of type θ_{k_i}, $i \in \{1, 2, 3\}$. If there were a link between 1 and 3, this would generate marginal externality E_{13} as given by equation (2.3). Now,

$$E_{13} - E_{23} = e(T_{G \setminus \{23\}}) - e(T_{G \setminus \{13\}})$$

Since $g(\theta_{k_1}, \theta_{k_3}) > g(\theta_{k_2}, \theta_{k_3})$, this implies taking away link 23 reduces externality less than taking away link 13, so that $e(T_{G \setminus \{23\}}) - e(T_{G \setminus \{13\}}) > 0$. Thus $E_{13} - E_{23} > 0$.

Now replace the existing link 23 with the non-existing link 13. For the original configuration to be an optimal separating network, we need, from Definition 4, that the replaced link has higher value, i.e.

$$g(\theta_{k_2}, \theta_{k_3}) + E_{23} > g(\theta_{k_1}, \theta_{k_3}) + E_{13}$$

But since $g(\theta_{k_1}, \theta_{k_3}) > g(\theta_{k_2}, \theta_{k_3})$ and $E_{13} > E_{23}$, this inequality does not hold (indeed the opposite is true). Therefore we have a contradiction.

It is also clear that by replacing the link 23 with the link 13 we can generate higher value, so that, in any optimal separating network, agents of θ_{k_1} rather than θ_{k_2} must be connected to the type class θ_{k_3}. It follows that higher types must be connected to more type classes, so that $\Theta^j \subset \Theta^i$ for all agents i, j where i has higher type than j.‖

A.3 Proof of Lemma 2

In this case, since all marginal and infra-marginal links are formed, we have $n_q^i(\theta_k) = n_q$ for all k and q. This implies that the transfer to all agents of the same type is the same - so the transfer does not depend on the agent's identity (the superscript i) any longer. We can therefore drop the superscript i in t_k^i. For $k > \ell$,

$$\begin{aligned} t_k &= \sum_{q=1}^{\mathcal{M}(k)} n_q \max\left[0, -\mathrm{MG}_q\right] \\ &= n_{\mathcal{M}(k)} \max\left[0, -\mathrm{MG}_k\right] + \sum_{q=1}^{\mathcal{M}(k+1)} n_q \max\left[0, -\mathrm{MG}_q\right] \\ &= n_{\mathcal{M}(k)} \max\left[0, -\mathrm{MG}_k\right] + t_{k+1} \end{aligned} \tag{A.2}$$

where $t_{m+1} \equiv 0$.

Next, for $k = \ell$,

$$t_\ell = (n_\ell - 1) \max\left[0, -\mathrm{MG}_\ell\right] + \sum_{q=1}^{\mathcal{M}(\ell+1)} n_q \max\left[0, -\mathrm{MG}_q\right] = (n_\ell - 1) \max\left[0, -\mathrm{MG}_\ell\right] + t_{\ell+1}$$

where the second step follows from equation (A.2) using $k = \ell + 1$.

Finally, consider a type θ_k where $k < \ell$. An agent of type θ_k forms connections with all other agents of types 1 to $\mathcal{M}(\theta_k)$. Therefore

$$t_k = (n_k - 1)\max\left[0, -\mathrm{MG}_k\right] + \sum_{q=1}^{k-1} n_q \max\left[0, -\mathrm{MG}_q\right] + \sum_{q=k+1}^{\mathcal{M}(k)} n_q \max\left[0, -\mathrm{MG}_q\right]$$

Similarly,

$$t_{k+1} = (n_{k+1} - 1)\max\left[0, -\mathrm{MG}_{k+1}\right] + \sum_{q=1}^{k} n_q \max\left[0, -\mathrm{MG}_q\right] + \sum_{q=k+2}^{\mathcal{M}(k+1)} n_q \max\left[0, -\mathrm{MG}_q\right]$$

Therefore

$$\begin{aligned} t_k - t_{k+1} &= -\max\left[0, -\mathrm{MG}_k\right] + \max\left[0, -\mathrm{MG}_{k+1}\right] + n_{\mathcal{M}(k)} \max\left[0, -\mathrm{MG}_k\right] \\ &= (n_{\mathcal{M}(k)} - 1)\max\left[0, -\mathrm{MG}_k\right] + \max\left[0, -\mathrm{MG}_{k+1}\right] \end{aligned}$$

This completes the proof.‖

A.4 Proof of Lemma 5

Lemma 4 shows that all infra-marginal connections must be formed in any truthful equilibrium. Next consider marginal connections. Consider first the case where an agent drops some (but not all) of the marginal connections in an optimal separating network. This does not alter type assignment. Further, by construction an agent cannot lose (and might strictly gain) by forming marginal connections. Therefore there can be no strict benefit from such a deviation that drops a marginal link.

Next, suppose an agent of type θ_k drops all marginal connections for that type. Such an agent would then be classified as θ_{k+1}. Let the new transfer to agents classified as θ_{k+1} be given by $\widetilde{t}_{k+1}$.

First, consider a type θ_k for $k \leqslant \ell$.

The deviation would change the agent's payoff as follows. First, each agent of type $\theta_{\mathcal{M}(k)}$ now has one fewer link, so that $t_{\mathcal{M}(k)}$ goes down by $\max[0, -\mathrm{MG}_k]$. Since transfers are recursive (Lemma 2), the transfer to θ_{k+1} also goes down by that amount. Further, there is now one fewer agent classified as θ_k (so that transfer goes down by $\max[0, -\mathrm{MG}_k]$) and one more classified as θ_{k+1} (raising transfer by $\max[0, -\mathrm{MG}_{k+1}]$), which implies that the new transfer to type θ_{k+1} (which is the transfer received by the deviating agent) is given by

$$\widetilde{t}_{k+1} = t_{k+1} - 2\max[0, -\mathrm{MG}_k] + \max[0, -\mathrm{MG}_k]$$

From Lemma 2, this implies

$$\widetilde{t}_{k+1} = t_k - (n_{\mathcal{M}(k)} + 1)\max[0, -\mathrm{MG}_k]$$

Clearly, deviation leads to a transfer that is weakly lower than the original transfer for the deviating agent.

Next, consider a type θ_k for $k > \ell$. In this case, dropping all marginal connections simply leads to the loss of transfer for these connections:

$$\widetilde{t}_{k+1} = t_k - n_{\mathcal{M}(k)}\max[0, -\mathrm{MG}_k]$$

which is again implies a weak loss from deviation.

Finally, if there are an odd number of degree-classes and there are exactly two agents in class θ_ℓ (lowest type in clique), dropping a single link between agents will change the number of degree classes. This is exactly the case covered by case 3 in the example in Section 6.1.1. Type θ_ℓ ($\ell = 3$ in the example) cannot benefit since infra-marginal types for ℓ will now have higher marginal types, weakly reducing transfers.

This completes the proof that under the transfers specified, starting from the optimal separating structure, no agent can benefit by cutting any links.$\|$

References

BELHAJ, M., S. BERVOETS, AND F. DEROÏAN (2016): "Efficient networks in games with local complementarities," *Theoretical Economics*, 11, 357–380. 33

BENSON, B. W., S. R. IYER, K. J. KEMPER, AND J. ZHAO (2018): "Director Networks and Credit Ratings," *Financial Review*, 53, 301–336. 34

BLOCH, F. AND M. OLCKERS (2018): "Friend-based Ranking," Working paper. 33

GALEOTTI, A., S. GOYAL, AND J. KAMPHORST (2006): "Network formation with heterogeneous players," *Games and Economic Behavior*, 54, 353–372. 34

JACKSON, M. O. AND A. WOLINSKY (1996): "A Strategic Model of Social and Economic Networks," *Journal of Economic Theory*, 71, 44–74. 7

KÖNIG, M. D., C. J. TESSONE, AND Y. ZENOU (2014): "Nestedness in networks: A theoretical model and some applications," *Theoretical Economics*, 9, 695–752. 33

WEI, Y., P. YILDIRIM, C. VAN DEN BULTE, AND C. DELLAROCAS (2016): "Credit Scoring with Social Network Data," *Marketing Science*, 35, 234–258. 33