Redistributive Policy Shocks and Optimal Monetary Policy with Heterogeneous Agents.

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Abstract

In the name of food security, many emerging market and developing economies (EMDEs) have enacted more generous food security laws that have led to an increase in the procurement and redistribution of agricultural output. We refer to such changes as a "redistributive policy shock." What is less understood in the literature is the impact of such shocks on monetary policy design. To address this, we build a two-sector (agriculture and manufacturing) two-agent (rich and poor) New Keynesian DSGE model with procurement and redistribution. We show that the economy has steeper AS and AD curves compared to the benchmark economy, leading to a more pronounced impact of supply side and demand side shocks on inflation. We calibrate the model to the Indian economy and discuss how the transmission of redistributive policy shocks affects sectoral inflation rates, the economy wide inflation rate and output gap, sectoral movements in labor, rich and poor agent consumption, and aggregate welfare. We show that heterogeneity matters for whether monetary policy responses to shocks raise aggregate welfare or not. Our paper contributes to a growing literature on understanding the role of heterogeneity in monetary policy.

Keywords  TANK models, Inflation Targeting, Emerging Market Economies, Indian Economy, DSGE. JEL codes: E31, E32, E44, E52, E63.

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1 Introduction

With the goal of ensuring food security, many emerging market and developing economies (EMDEs) have enacted food security laws that involve an increase in the redistribution of procured agricultural output to the poorest population in a country. In 2013, India enacted a new National Food Security Act (NFSA) under the umbrella of a new "rights-based" approach to food security. The Act legally entitles "up to 75% of the rural population and 50% of the urban population to receive subsidized food grains under a Targeted Public Distribution System." Under the new act, about two thirds of the population is covered to receive highly subsidized food grains. The stated rationale of the Act is "to ensure that all people, at all times, should get access to the basic food for their active and healthy life and is characterized by availability, access, utilization and stability of food." The ostensible goal is to smooth the purchasing power of poor populations that are food insecure.

The enactment of a new national food security act with wider coverage, or intervening when there are large price shocks in food commodities such as the world rice price crisis of 2008, implies higher procurement and redistribution of food commodities by the government. For instance, in the Philippines, the National Food Authority (NFA) is mandated to purchase and distribute rice and other commodities across the country. In response to the rise in world prices of grains in the last quarter of 2007, the Philippines government provided higher funding support to implement its Economic Resiliency Program part of which involved scaling up a rice production enhancement program called "Ginintuang Masaganang Ani". The total fiscal cost of the NFA rice subsidy jumped to .6% of GDP in 2008 compared to .08% per cent of GDP in 2007 (Balisacan et al, 2010). In Bangladesh, the government has intervened in food markets for several years in order to reduce price fluctuations and procure rice for safety net programs (Hossain and Deb, 2010). To ensure food security in Indonesia in 2008, the Indonesian government, through its BULOG operational strategy doubled the amount of rice distributed to cover all poor families under the RASKIN program through targeted market operations requested by local governments. Regular rice distribution for the poor was achieved by increasing domestic rice procurement. BULOG’s heavy procurement added to demand, helping farmers maintain prices at a profitable level (Saifullah, 2010). The Korean government also motivates its agricultural policy for food security reasons based on self-sufficiency (Beghin et al., 2003).

We refer to exogenous changes in the procurement and redistribution of the agriculture good motivated by the above examples as "redistributive policy shocks." What is less

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1 See https://dfpd.gov.in/nfsa-act.htm
2 See https://dfpd.gov.in/nfsa-act.htm
understood in the literature is the impact of such redistributive policy shocks on the sectoral and aggregate dynamics of inflation, sectoral and aggregate output gaps and monetary policy design in this environment. A recent focus in the monetary policy literature explores the impact of monetary policy when there is consumer heterogeneity. As in this research, we ask how does heterogeneity matters for whether monetary policy responses to shocks raise aggregate welfare or not? How does heterogeneity affects the transmission of monetary policy? Does monetary policy have both output effects and redistributive effects, and if so, what is the quantifiable impact on these variables when there is a redistributive and procurement policy shock.

Why is it important to take into account heterogeneity? In our model heterogeneity determines the differential response that rich and poor consumption, and therefore aggregate demand, has to shocks. Heterogeneity therefore matters for whether monetary policy responses to shocks raise aggregate welfare or not. In addition, heterogeneity affects the transmission of monetary policy, as higher is the fraction of agents that are Ricardian, the more real interest rates affect the output gap.

1.1 Description of Model

We build a two-sector (agriculture and manufacturing) two agent (rich and poor) New Keynesian DSGE model. Our model builds on earlier work by Debortoli and Gali (2018), Aoki (2001), and Ghate, Gupta and Mallick (2018). More specifically, the main methodological contribution of our framework is that we extend the two agent New Keynesian DSGE framework of Debortoli and Gali to two sectors (agriculture and manufacturing) in a tractable way. On the production side, the agriculture sector is perfectly competitive with flexible prices while the manufacturing sector is characterized by monopolistic competition and sticky prices. As in Debortoli and Gali, we assume that there are two types of agents, rich and poor. Rich agents are Ricardian and buy one period risk free bonds. Poor agents are assumed to be rule of thumb consumers. Both rich and poor households consume both the agriculture good and the manufacturing good. To provide the subsidized agriculture good to the poor, the government taxes the rich via lump sum taxes and uses the proceeds to procure agricultural output from the open market. It then re-distributes a fraction of the procured agriculture good to the poor. Further, we assume that rich agents have a higher inter-temporal elasticity of substitution of consumption compared to the poor.\(^3\)

We calibrate the model to the Indian economy (a proto-typical EME economy) and discuss how the transmission of productivity shocks, redistributive policy shocks affects sectoral

\(^3\)In Debortoli and Gali, all agents have the same inter-temporal elasticity of substitution.
inflation rates, the economy wide inflation rate and output gap, sectoral movements in labor, and consumption of the rich and poor agents.

1.2 Main Results

We derive the DIS (Dynamic IS Curve) and the NKPC (New Keynesian Phillips Curve) and show that procurement and re-distribution affects the DIS curve by affecting the terms of trade as well as the natural rate of interest. Because procurement creates a divergence between the steady state share of labor in agriculture and consumption of the agricultural good, procurement affects the slopes of the aggregate DIS and NKPC. In particular, compared to the benchmark case (no procurement and every agent is Ricardian), we show that the DIS is flatter while the NKPC is steeper. Hence, monetary transmission, or the impact from real interest rate changes to the output gap, is weaker. This happens for two reasons. First, because procurement reduces the amount of final good available for consumption in the economy monetary policy is operative over less of the final good. Second, having two types agents - Ricardian and rule of thumb - means that only a fraction of agents are able to adjust their consumption when the real interest rate changes. Thus, adding agent heterogeneity and procurement in the model hinders monetary transmission.

We show that monetary policy has both redistributive effects as well as output effects. A positive agricultural productivity shock leads to a rise in both poor and rich consumption, and therefore higher welfare. In contrast, with a procurement-redistribution shock, aggregate consumption falls, leading to lower welfare, even though monetary policy is successful in raising agricultural consumption by the poor. The implication is that if there is a "sufficiently" high level of redistribution procurement and redistribution shocks can lead a rise in the consumption of the poor that off-sets the decline in rich consumption, thereby raising aggregate consumption. A contractionary monetary shock (that tightens the interest rate) shows that the impact effect on aggregate output in the model with frictions is more muted (compared to the benchmark model) because of weaker transmission. In contrast, the impact effect on inflation is higher in the model with frictions, because of adjustments in the price level of the flexible price good. Hence, on impact, monetary policy, leads to a smaller negative output gap, but a larger negative inflation gap in the model with frictions compared to the benchmark model. We discuss the intuition behind these results.

1.3 Literature Review

Our model builds on the seminal work by Gali and Monacelli (2005) and Aoki (2001). The main difference with respect to papers is that Gali and Monacelli (2005) consider an open
economy framework, whereas we consider a closed economy framework. In Aoki (2001) there are two production sectors, a flexible agriculture sector that is perfectly competitive, and a sticky price manufacturing sector that is monopolistically competitive. The production side of our model is similar to this. However, Aoki’s model has a single representative agent. In our model, we allow for two types of agents, rich (Ricardian) and poor (rule of thumb) with different inter-temporal elasticities of substitution in consumption and different budget constraints. Another difference with respect to Aoki (2001) is that the government in our model taxes rich agents, procures grain from the agriculture sector, and provides lump sum transfers to poor agents. In Aoki’s framework there is no government intervention.4

Debortoli and Gali (2018) build a two agent New Keynesian (TANK) DSGE model in which agents are Richardian/rich and rule of thumb/poor. They show that a tractable TANK model is a good approximation to the study the impact of aggregate shocks to aggregate variables in a baseline HANK (Heterogenous agent New Keynesian) model. In Debortoli and Gali (2018), there is however only one production sector (manufacturing). The main methodological contribution of our paper is to extend the two agent-one sector framework of Debortoli and Gali to two sectors in a tractable way.

Our paper also builds on previous work in Ghate, Gupta, Mallick (2018), or GGM. In GGM, there are three production sectors (grain, vegetables, and manufacturing). In that framework, all three sectors are monopolistically competitive, with the agriculture sector having flexible prices. The manufacturing sector is the sticky price sector. In the current framework, there are two production sectors (agriculture, manufacturing). Unlike GGM, the agriculture sector is just characterized by a grain sector which is assumed to be perfectly competitive. Like GGM, the manufacturing sector is the sticky price sector. In GGM, there is a single representative agent, i.e., it is a RANK (Representative Agent New Keynesian) model. Our model has two types of agents. In the current framework, we do not model minimum support prices as we did in GGM. Here, the government procures the agriculture sector good from the open market and then redistributes it back to poor agents at a subsidized price. Our focus is on the impact of redistributive policy shocks on rich-poor consumption, sectoral inflation dynamics, and monetary policy design in this context. Like GGM however, our model illustrates how the terms of trade between agriculture and manufacturing plays a crucial role in the transmission of monetary policy changes to aggregate outcomes.

Our paper builds on a growing literature on heterogenous agent New Keynesian (HANK) models (McKay, Nakamura, and Steinsson, 2016; Kaplan, Moll, and Violante, 2018; Auclert, 2019, and Broer et al., 2019). Like Auclert (2019), we isolate, between the rich and poor, 4Gali, Lopez-Salido, and Valles (2007) use a two agent framework (rule of thumb and Ricardian) to account for evidence on government spending shocks, but their focus is on fiscal policy, not monetary policy.
who gains and who loses from monetary policy changes, in response to redistributive policy shocks. Our paper merges a two sector production structure along the lines of Aoki with a TANK framework along the lines of Debortoli and Gali to understand the transmission of monetary policy using a tractable New Keynesian DSGE framework. Like Broer et al. (2019), we are interested in assessing whether monetary policy has redistributive and/or output effects when there is consumer heterogeneity.

2 The Model

The model has two sectors: agriculture $A$ and manufacturing $M$. The $A$-sector is characterized by perfect competition and flexible prices, and produces a single homogenous good. The $M$-sector is characterized by monopolistic competition and staggered price setting. We assume that there are two types of households: poor ($P$) and rich ($R$). The fraction of households which are rich is exogenously given and denoted by $\mu_R$. The rest ($1 - \mu_R$) are poor. The poor and rich can either work in the $A$ sector or the $M$ sector. Poor households are assumed to be rule of thumb (or hand to mouth consumers) and do not have bond holdings. Rich households are forward-looking Ricardian consumers and hold bonds. The rich households own the firms and also supply labor to their own firms, and so they have both dividend and labor income. The poor households only supply labor to the firms owned by the rich, and so their only income is labor income. This implies that the total number of firms equals the sum of rich and poor households.

Like GGM, the government procures grain in the open market. It does this by taxing (lump-sum) the rich and uses the proceeds to procure/buy $A$-sector output from the market at the market price.\(^5\) It then redistributes a fraction of the procured $A$ good to poor households. Hence redistribution goes to the poor households, rather than any particular sector. The rich households also have higher incomes than the poor since the poor households only have labor income, whereas rich households have labor and dividend income (which is lump-sum).\(^6\)

We also assume that poor and rich households have different inter-temporal elasticities of substitution. In particular, we assume that the poor have a lower inter-temporal elasticity of substitution than the rich, which means that they are less willing to substitute consumption across time periods.

\(^5\)It is important to note that the the seller of the $A$ good can be either poor or rich.
\(^6\)Given that the tax is lump-sum, there is no welfare loss to the rich for this tax-redistribution scheme.
2.1 Households

All households are assumed to have identical preferences. At time \( t = 0 \), a household of type \( K \) (= \( R \) and \( P \)) maximizes its expected lifetime utility given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t [U(C_{K,t}) - V(N_{K,t})]
\]

where \( C_{K,t} \) is a consumption index. The subscript \( K \in \{R, P\} \) specifies the household type. A household of type \( K \in \{R, P\} \) derives utility from consumption, \( C_{K,t} \), and disutility from labor supply, \( N_{K,t} \). \( \beta \in (0, 1) \) is the discount factor. The period utility function is specified as

\[
U(C_{K,t}) = \frac{C_{K,t}^{1-\sigma_K}}{1-\sigma_K}
\]

\[
V(N_{K,t}) = \frac{N_{K,t}^{1+\varphi}}{1+\varphi}
\]

where \( \sigma_K \) and \( \varphi \), respectively, are the inverse of the inter-temporal elasticity of substitution for consumer type \( K \), and the inverse of the Frisch labor supply elasticity, which is assumed to be the same for both types of households. Consumption of both rich and poor households follow Cobb-Douglas indices of agriculture (\( A \)) and manufacturing (\( M \)) consumption and is given by

\[
C_{K,t} = \frac{C_{K,A,t}^{\delta_K} \cdot C_{K,M,t}^{1-\delta_K}}{\delta_K^{\delta_K} \cdot (1-\delta_K)^{1-\delta_K}}, \quad \text{for } K = R \text{ and } P.
\]

where \( \delta_R \in [0, 1] \) is the share of agricultural goods in consumption for the rich while \( \delta_P \in [0, 1] \) is the share of out of pocket expenditure on agricultural goods.

Rich households maximize their current utility (1) subject to the following inter-temporal budget constraint

\[
\int_0^1 [P_{M,t}(j)C_{R,M,t}(j)] dj + P_{A,t}C_{R,A,t} + E_t \{ Q_{t+1}B_{t+1} \} \leq B_t + W_t N_{R,t} - T_{R,t} + Div_t
\]
is \( P_{A,t} \). Consumption in the manufacturing sector is a CES aggregate of a continuum of differentiated goods indexed by \( j \in [0, 1] \), where \( P_{M,t}(j) \) is the price level of the \( j \)th variety of the \( M \)-sector good, i.e.,\(^7\)

\[
C_{M,t} = \left( \int_0^1 C_{M,t}(j) \frac{\varepsilon - 1}{\varepsilon} \, dj \right)^{\frac{\varepsilon}{\varepsilon - 1}}, \quad \varepsilon > 1.
\]

To model the procurement-redistribution set-up in an emerging market economy, the government in every period procures the agriculture good at the open market price, \( P_{A,t} \). Part of the procured agriculture good is rebated back to poor to each household as a subsidy, \( C^S_{P,A,t} \), while the remaining portion is put into a buffer stock.\(^8\) Of the total consumption of the agriculture good by the poor household, \( C_{P,A,t} \), a fraction, \( \lambda_t \), is subsidized (it is given for free). The remaining fraction, \( (1 - \lambda_t) \) of \( C_{P,A,t} \) is purchased from the open market \( (C^O_{P,A,t}) \). That is, \( C^S_{P,A,t} = \lambda_tC_{P,A,t} \) and

\[
C^S_{P,A,t} + C^O_{P,A,t} = C_{P,A,t}.
\]

implies \( C^O_{P,A,t} = (1 - \lambda_t)C_{P,A,t} \).

Poor households maximize their current utility (1) subject to the following (static) budget constraint

\[
\int_0^1 \left[ P_{M,t}(j)C_{P,M,t}(j) \right] dj + P_{A,t}C^O_{P,A,t} \leq W_t N_{P,t} + P_{A,t}C^S_{P,A,t}
\]

where \( P_{A,t}C^S_{P,A,t} \) denotes the nominal value of the subsidy given to the poor, evaluated at the market price, \( P_{A,t} \), and \( P_{A,t}C^O_{P,A,t} \) denotes the nominal value of open market purchases of the agriculture done by the poor. Inherent in equation (7) is a procurement-redistribution policy which has two components. First, the government provides \( C^S_{P,A,t} \) to the poor for free, which augments their income by \( P_{A,t}C^S_{P,A,t} \). Second, poor households use this augmented income to off-set their open market purchases of the agriculture good, \( C^O_{P,A,t} \).

\(^7\)The demand functions for goods within manufacturing varieties are

\[
C_{K,M,t}(j) = \left( \frac{P_{M,t}(j)}{P_{M,t}} \right)^{-\varepsilon} C_{K,M,t}
\]

for \( K = R \) and \( P \).

\(^8\)An equivalent interpretation is that non-redistributed procured output is wasted, or "thrown into the ocean." We do not endogenize buffer stock dynamics in this paper.
2.1.1 Optimal allocations

Optimal consumption allocations by the rich for $A$ and $M$ goods are given, respectively, by

$$C_{R,A,t} = \delta_R \left( \frac{P_{A,t}}{P_t} \right)^{-1} C_{R,t}$$  \hspace{1cm} (8)$$

$$C_{R,M,t} = (1 - \delta_R) \left( \frac{P_{M,t}}{P_t} \right)^{-1} C_{R,t}$$  \hspace{1cm} (9)$$

where the aggregate price level is given by $P_t = P_{A,t}^{\delta_R} P_{M,t}^{1-\delta_R}$.

For poor households, consumption allocations for the $A$ and $M$ goods are given respectively by

$$C_{P,A,t} = \frac{\delta_P}{1 - \lambda_t} \left( \frac{P_{A,t}}{P_t} \right)^{-1} C_{P,t}$$  \hspace{1cm} (10)$$

$$C_{P,M,t} = (1 - \delta_P) \left( \frac{P_{M,t}}{P_t} \right)^{-1} C_{P,t}$$  \hspace{1cm} (11)$$

Using the fact that $C_{R,M,t}(j) = \left( \frac{P_{M,t}(j)}{P_{M,t}} \right)^{-\varepsilon} C_{R,M,t}$ and the demand functions in (8)-(9) implies that the budget constraint for the rich can be rewritten as

$$P_tC_{R,t} + E_t\{Q_{t+1}B_{t+1}\} \leq B_t + W_tN_{R,t} - T_{R,t} + Div_t$$  \hspace{1cm} (12)$$

For the poor, using equations (10)-(11) implies

$$P_tC_{P,t} (1 - \delta_P\lambda_{s,t}) \leq W_tN_{P,t}$$  \hspace{1cm} (13)$$

where $C_{P,t}$ denotes consumption (of both the agriculture good and manufacturing good), $C_{R,t}$ denotes consumption of the rich household. As seen in equation (13), the impact of subsidizing the agriculture good for poor households reduces the effective price on their consumption to $P_t (1 - \delta_P\lambda_{s,t})$ where $\lambda_{s,t} = \frac{\lambda}{1 - \lambda_t}$.

The solutions to maximizing equation (1) subject to equation (12) for the rich and equation (13) for the poor yield the following optimality conditions:

$$1 = \beta E_t \left[ \left( \frac{C_{R,t+1}}{C_{R,t}} \right)^{-\sigma_R} \frac{P_t}{P_{t+1}} R_t \right]$$  \hspace{1cm} (14)$$

$$\frac{W_t}{P_t} = \frac{N_{R,t}^\phi}{C_{R,t}^{-\sigma_R}} \text{ for the rich}$$  \hspace{1cm} (15)$$

See the Technical Appendix for all derivations.
\[
\frac{W_t}{P_t (1 - \delta_P \lambda_{s,t})} = \frac{N_{P_t}^p}{C_{P_t}^{\sigma_p}} \text{ for the poor} \quad (16)
\]

where \( R_t = \frac{1}{E_t(Q_{t+1})} \) is the gross nominal return on the riskless one-period bond.

### 2.1.2 Sectoral aggregates

We define aggregate agriculture consumption as a weighted average of rich and poor agriculture consumption:

\[
C_{A,t} = \mu_R C_{R,A,t} + (1 - \mu_R) C_{P,A,t} \quad (17)
\]

The total amount of redistributed grain and the consumption subsidy to the poor is given by:

\[
(1 - \mu_R) C_{P,A,t}^S = \phi_t Y_{A,t}^P \quad (18)
\]

Substituting out for \( C_{P,A,t} \) from (10) and noting that \( C_{P,A,t}^O = (1 - \lambda_t) C_{P,A,t} \) yields

\[
C_{A,t} = \mu_R \delta_R \left( \frac{P_{A,t}}{P_t} \right)^{-1} C_{R,t} + (1 - \mu_R) \delta_P \left( \frac{P_{A,t}}{P_t} \right)^{-1} C_{P,t} + \phi_t Y_{A,t}^P \quad (19)
\]

where total agriculture consumption by the rich and poor is broken up into market purchases of agriculture by the rich, market purchases of agriculture by the poor, and redistributed agriculture output to the poor. Define \( C_t = \mu_R C_{R,t} + (1 - \mu_R) C_{P,t} \). This implies

\[
C_{A,t} = \left( \frac{P_{A,t}}{P_t} \right)^{-1} (\mu_R \delta_R C_{R,t} + (1 - \mu_R) \delta_P C_{P,t}) + \phi_t Y_{A,t}^P
\]

Likewise, \( C_{M,t} = \mu_R C_{R,M,t} + (1 - \mu_R) C_{P,M,t} \) which implies

\[
C_{M,t} = \left( \frac{P_{M,t}}{P_t} \right)^{-1} (\mu_R (1 - \delta_R) C_{R,M,t} + (1 - \mu_R) (1 - \delta_P) C_{P,M,t}) \quad (20)
\]

### 2.2 Terms of trade

Terms of trade (TOT) between the agriculture and the manufacturing sectors is defined as \( T_t = \frac{P_{A,t}}{P_{M,t}} \). CPI inflation is then given by \( \pi_t = \ln P_t - \ln P_{t-1} \), and the sectoral inflation rates are given by as \( \pi_{A,t} = \ln P_{A,t} - \ln P_{A,t-1} \) and \( \pi_{M,t} = \ln P_{M,t} - \ln P_{M,t-1} \), respectively, for the agriculture and the manufacturing sectors. From the aggregate price index, CPI inflation can also be written in terms of TOT as
\[ \pi_t = \delta_R \pi_{A,t} + (1 - \delta_R) \pi_{M,t} = \delta_R \Delta T_t + \pi_{M,t}. \]  

(21)

### 2.3 Firms

In the manufacturing sector, there is a continuum of firms indexed by \( j \). Each firm produces a differentiated good with a linear technology given by the production function \( Y_{M,t}(j) = A_{M,t} N_{M,t}(j) \). We assume that productivity shocks are the same across firms follows a AR(1) process,

\[ \log A_{M,t} - \log A_M = \rho_M (\log A_{M,t-1} - \log A_M) + \varepsilon_{M,t} \]

\( \varepsilon_{M,t} \sim i.i.d(0, \sigma_M) \). The nominal marginal costs are common across firms and are given by \( MC_{M,t} = (1 + \tau_M) \frac{W_t}{A_{M,t}} \) where \( \tau_M \) is the employment subsidy given to manufacturing production. Real marginal costs is written as

\[ mc_{M,t} = \frac{MC_{M,t}}{P_{M,t}} = (1 + \tau_M) \frac{W_t T^\delta R}{P_t} \frac{1}{A_{M,t}}. \]  

(22)

Let \( Y_{M,t} = \left( \int_0^1 Y_{M,t}(j)^{\varepsilon-1} d\xi \right)^{\frac{1}{\varepsilon}} \), where \( \varepsilon > 1 \). Output demand is given by \( Y_{M,t}(j) = \left( \frac{P_{M,t}(j)}{P_{M,t}} \right)^{-\varepsilon} Y_{M,t} \). The labor supply allocation in manufacturing sector is obtained as

\[ N_{M,t} = \int_0^1 N_{M,t}(j) d\xi = \frac{Y_{M,t}}{A_{M,t}} Z_{M,t} \]  

(23)

where \( Z_{M,t} = \int_0^1 \left( \frac{P_{M,t}(j)}{P_{M,t}} \right)^{-\varepsilon} d\xi \) represents the price dispersion term. Equilibrium variations in \( \ln \int_0^1 \left( \frac{P_{M,t}(j)}{P_{M,t}} \right)^{-\varepsilon} d\xi \) around perfect foresight steady state are of second order. Given that the agriculture sector is characterized by flexible price and perfect competition, we can write the sectoral aggregate production as

\[ Y_{A,t} = A_{A,t} N_{A,t} \]  

(24)

where the productivity shock follows an AR(1) process,

\[ \log A_{A,t} - \log A_A = \rho_A (\log A_{A,t-1} - \log A_A) + \varepsilon_{A,t}. \]  

(25)

where \( \varepsilon_{A,t} \sim i.i.d(0, \sigma_A) \). Nominal marginal costs are given by \( MC_{A,t} = \frac{W_t}{A_{A,t}} \)
2.3.1 Price setting in the manufacturing sector

Price setting follows Calvo (1983), and is standard in the literature. Firms adjust prices with probabilities \((1 - \theta)\) independent of the time elapsed since the previous adjustment. The inflation dynamics under such price setting is

\[ \pi_{M,t} = \beta E_t\{\pi_{M,t+1}\} + \kappa \hat{m}_{M,t} \]  

where \(\kappa = \frac{(1-\beta)(1-\theta)}{\theta}\), and \(\hat{m}_{M,t}\) is the deviation of the real marginal cost in the manufacturing sector from its natural rate (to be defined later).

2.4 Government procurement

In each period, the government procures \(Y_{A,t}^P\) amount of agricultural output at the market price \(P_{A,t}\), using the tax receipts from the rich and redistributes a fraction \((\phi_t \in [0,1])\) of procured goods to the poor.\(^{10}\) The redistributed amount is given by \(\phi_t Y_{A,t}^P\). The agricultural sector output is the sum of consumption and the amount accumulated by the buffer stock

\[ Y_{A,t} = C_{A,t} + (1 - \phi_t) Y_{A,t}^P \]  

where the total consumption of the agricultural good \(C_{A,t}\) consists of the total amount purchased (by both the rich and poor) from the market and the amount redistributed to the poor, \(\phi_t Y_{A,t}^P\). A procurement shock is given by an AR(1) process,

\[ \ln Y_{A,t}^P - \ln Y_{A,t}^P = \rho_{Y_A} (\ln Y_{A,t-1}^P - \ln Y_{A,t}^P) + \varepsilon_{Y_A,t} \]  

where \(\rho_{Y_A} \in (0,1)\) and \(\varepsilon_{Y_A,t} \sim i.i.d(0,\sigma_{Y_A})\). Re-distributive policy shocks, captured by changes in \(\phi_t\), capture sudden increases in the fraction of procured grain re-distributed to the poor,

\[ \ln \phi_t - \ln \phi = \rho_{\phi} (\ln \phi_{t-1} - \ln \phi) + \varepsilon_{\phi} \]  

where \(\rho_{\phi} \in (0,1)\) and \(\varepsilon_{\phi} \sim i.i.d(0,\sigma_{\phi})\).

\(^{10}\)Please note that when \(P\) is super-script, it refers to 'procurement'. When it is sub-script, it refers to 'poor'.

3 Equilibrium Dynamics

3.1 Market Clearing

Market clearing is given by the following equations:

\[ C_t = \mu_R C_{R,t} + (1 - \mu_R) C_{P,t} \quad (30) \]

\[ N_t = N_{A,t} + N_{M,t} \quad (31) \]

\[ Y_{M,t} = C_{M,t} \quad (32) \]

\[ Y_t = C_t + T_t^{1-\delta_R} Y_{A,t}^P \quad (33) \]

\[ Y_t = T_t^{1-\delta_R} Y_{A,t} + T_t^{-\delta_R} Y_{M,t} \quad (34) \]

\[ \mu_R T_{R,t} = [(1 - \phi_t) Y_{A,t}^P + C_{P,A,t}^S (1 - \mu_R)] P_{A,t} \quad (35) \]

and equation (27). Equation (30) corresponds to aggregate consumption by both rich and poor households, weighted by their respective masses, \( \mu_R \) and \( 1 - \mu_R \) in the population. The labor market clearing condition is given by equation (31). The manufacturing goods market clearing condition is given by equation (32). The aggregate goods market clearing condition is given by equation (33) which can be written in terms of \( T_t \) as in equation (34). Equation (35) is the government budget constraint, which equates lump sum taxes collected from the rich to the amount of redistribution \( (2C_{P,A,t}^S (1 - \mu_R)) \) and the fraction of procured output that goes towards buffer stock accumulation \( ((1 - \phi_t) Y_{A,t}^P) \).

3.2 Steady state

Define \( X \) (without \( t \) subscript) as the steady state value of the variable, \( X_t \). Assuming no trend growth in productivity, \( A_s = 1 \) for \( s = A, M \). Define the steady state consumption share of the rich, \( s_R \), as

\[ s_R = \frac{\mu_R C_R}{C} \quad (36) \]
and that of the poor as
\[ 1 - s_R = \frac{(1 - \mu_R)C_P}{C}. \] (37)

We can also define the steady state employment share of the rich, \( N_R \)
\[ N_R = \mu_R N \] (38)

and the employment share of the poor as \( N_P \)
\[ N_P = (1 - \mu_R)N. \] (39)

The steady state level of aggregate consumption is\(^{11} \)
\[ C = \Gamma \frac{1}{\sigma_R - \sigma_P} \] (40)

where \( \Gamma \) is a constant. Once we know the expression for \( C \), equations (36) and (37) yield \( C_R \) and \( C_P \), respectively. This implies that Manufacturing employment, \( N_M \), output, \( Y_M \), and consumption, \( C_M \), is given by
\[ N_M = Y_M = C_M = (1 - \bar{\delta})C = (1 - \bar{\delta})\Gamma \frac{1}{\sigma_R - \sigma_P}. \] (41)

where \( \bar{\delta} = s_R \delta_R + (1 - s_R) \delta_P \).

The steady state inter-sectoral terms of trade, \( T = 1 \). Aggregate employment in the steady state is given by,
\[ N = \frac{N_M}{1 - \mu_A} = \frac{1 - \bar{\delta}}{1 - \mu_A} C. \]

The steady state level of agricultural output procured, \( Y_A^P \), is given by
\[ Y_A^P = C \frac{\mu_A - \bar{\delta}}{1 - \mu_A}. \] (42)

For \( Y_A^P > 0 \), we require that \( \mu_A > \bar{\delta} \), which implies that the steady state labor share in agriculture is greater than its consumption share since a fraction of agricultural output is not consumed. Note that in the absence of procurement (\( Y_A^P = 0 \)), and these two steady state shares are equal as \( C \frac{\mu_A - \bar{\delta}}{1 - \mu_A} = 0 \implies \mu_A = \bar{\delta} \). The steady state relation in the agricultural sector then becomes
\[ N_A = Y_A = C_A + (1 - \phi)Y_A^P = \bar{\delta}C + Y_A^P = C \frac{\mu_A}{1 - \mu_A}(1 - \bar{\delta}) \] (43)

\(^{11}\)Please refer to the Technical Appendix for derivations.
Solving for $\lambda$, the subsidized proportion of the poor’s agriculture consumption, we obtain\(^{12}\)

$$\lambda = \frac{\phi(\mu_A - \bar{\delta})}{\phi(\mu_A - \bar{\delta}) + \bar{\delta}(1 - \mu_A)(1 - s_R)}. \quad (44)$$

which is increasing in the fraction of procured agricultural output redistributed to the poor, $\phi$.

Since $A_s = 1$ for $s = A, M, P_A = W$, and $P_M = W$ (with the employment subsidy), in the steady state

$$\frac{W}{P_A} = \frac{W}{P_M} = \frac{W}{P} = 1.$$  

We will require two parameters later: $c = \frac{C}{Y}$ and $c_A = \frac{C_A}{Y_A}$. These are given by

$$c = \frac{1 - \mu_A}{1 - \delta} \quad (45)$$

and

$$c_A = \frac{\bar{\delta}(1 - \mu_A) + \phi(\mu_A - \bar{\delta})}{\mu_A(1 - \delta)} \quad (46)$$

### 3.3 Log-linearized model

Given the steady state, we log-linearize the key relationships. Define $\hat{X} = \ln X_t - \ln X$, as the log deviation of $X_t$ from its steady state value. Log linearization of the aggregate market clearing condition (equation (33)) gives

$$\hat{Y}_t = c\hat{C}_t + (1 - c) \left[ (1 - \delta_R)\hat{Y}_t + \hat{Y}_{A,t} \right]$$

\(= \left( \frac{1 - \mu_A}{1 - \delta} \right) \hat{C}_t + \left( \frac{\mu_A - \bar{\delta}}{1 - \delta} \right) (1 - \delta_R)\hat{Y}_t + \left( \frac{\mu_A - \bar{\delta}}{1 - \delta} \right) \hat{Y}_{A,t} \quad (47)$$

where $c$ is the steady state consumption share in output and is defined in equation (45). Log linearization of aggregate consumption, $C$, in equation (30) gives

$$\hat{C}_t = s_R\hat{C}_{R,t} + (1 - s_R)\hat{C}_{P,t}$$

\(= \hat{C}_t = s_R\hat{C}_{R,t} + (1 - s_R)\hat{C}_{P,t} \quad (48)$$

where $s_R$ is the steady consumption share of the rich households. Log linearization of the first order conditions (equations (15) and (16)) for the rich and poor households give

\(^{12}\)Note that $\lambda = \frac{\phi Y_P^t}{(1 - \mu_R)C_P^t}$. Please see the appendix.
\[
\hat{W}_t - \hat{P}_t = \varphi \hat{N}_{R,t} + \sigma_R \hat{C}_{R,t} \tag{49}
\]
and
\[
\hat{W}_t - \hat{P}_t = \varphi \hat{N}_{P,t} + \sigma_P \hat{C}_{P,t} - \lambda_p \hat{x}_{s,t}. \tag{50}
\]
where \( \lambda_p = \frac{\delta_p}{1 - \delta_p} = \frac{\delta_P}{1 - \lambda(1 - \delta_p)} \), given that \( \lambda_s = \lambda/(1 - \lambda) \).

The log-linearized consumption of the poor, \( \hat{C}_{P,t} \), is given by
\[
\hat{C}_{P,t} = \frac{\sigma_R}{\sigma_P + \lambda_p} \hat{C}_{R,t} + \frac{\lambda_p}{\sigma_P + \lambda_p} \left[ \hat{\phi}_t + \hat{Y}_{A,t}^P + (1 - \delta_R) \hat{T}_t \right]. \tag{51}
\]
which is increasing in the redistribution shock, \( \hat{\phi}_t \), the steady state deviation of procurement, \( \hat{Y}_{A,t}^P \), and the steady state deviation of the terms of trade, \( \hat{T}_t \). Log linearization of the Euler equation (14) for the rich households around zero inflation in the steady state gives
\[
\hat{C}_{R,t} = E_t \{ \hat{C}_{R,t+1} \} - \frac{1}{\sigma_R} \left[ \hat{R}_t - E_t \{ \Pi_{t+1} \} \right] \tag{52}
\]
Note that if both the rich and poor households have the same inter-temporal elasticity of substitution, (i.e., \( \sigma_R = \sigma_P \)) and there is no redistribution, i.e., \( \lambda = 0 \), then \( \hat{C} = \hat{C}_{R,t} = \hat{C}_{P,t} \).

Substituting \( \hat{C}_{P,t} \) in equation (51).into (48), solving for \( \hat{C}_{R,t} \), and substituting the resulting expression for \( \hat{C}_{R,t} \) in equation (52), gives us the Euler equation in terms of aggregate consumption, \( \hat{C}_t \), as
\[
\hat{C}_t = E_t \{ \hat{C}_{t+1} \} - \frac{1}{\Phi} \left[ \hat{R}_t - E_t \{ \Pi_{t+1} \} \right] - \Psi E_t \left\{ \Delta \hat{\phi}_{t+1} + \Delta \hat{Y}_{A,t+1}^P + (1 - \delta_R) \hat{T}_{t+1} \right\} \tag{53}
\]
where
\[
\Phi = \frac{\sigma_R(\sigma_P + \lambda_p)}{s_R(\sigma_P + \lambda_p) + (1 - s_R)\sigma_R} \tag{54}
\]
is the weighted average of the (inverse) inter-temporal substitution elasticity of the rich and poor households with the weights being the share of rich and poor in total population.

and
\[
\Psi = \frac{\lambda_p(1 - s_R)}{\sigma_P + \lambda_p}
\]
With \( \sigma_R = \sigma_P \) and \( \lambda = 0 \), equation (53) becomes the standard Euler equation for homogenous households.

Combining equations (53) and (47), we obtain the Euler equation in terms of aggregate consumer
\[ \hat{Y}_t = E_t \{ \hat{Y}_{t+1} \} - c \Phi^{-1} \left[ \hat{R}_t - E_t \{ \Pi_{t+1} \} \right] 
- (1 - \delta_R) \left[ (1 - c) + c \Psi \right] E_t \left\{ \Delta \hat{T}_{t+1} \right\} 
- [(1 - c) + c \Psi] E_t \{ \Delta \hat{Y}_{A,t+1} \} - [c \Psi] E_t \{ \Delta \hat{\phi}_{t+1} \} \] (55)

Log-linearization of the market clearing condition in the agricultural sector (equation (27)) gives (show steps in the Appendix)

\[ \hat{Y}_{A,t} = \frac{c (s_R \delta_R \hat{C}_{R,t} + (1 - s_R) \delta_P \hat{C}_{P,t})}{\mu_A} - \frac{c \delta (1 - \delta_R) \hat{T}_t}{\mu_A} + \frac{(1 - c) \hat{Y}_{A,t}}{\mu_A} \] (56)

Log-linearization of the optimal demand for manufacturing output (equation (20)) gives

\[ \hat{Y}_{M,t} = \delta_R \hat{T}_t + \frac{(s_R (1 - \delta_R) \hat{C}_{R,t} + (1 - s_R) (1 - \delta_P) \hat{C}_{P,t})}{1 - \delta} \] (57)

Log-linearization of the labor market clearing condition (31) gives

\[ \hat{N}_t = \mu_A \hat{N}_{A,t} + (1 - \mu_A) \hat{N}_{M,t} = \mu_A \hat{Y}_{A,t} + (1 - \mu_A) \hat{Y}_{M,t} - \hat{A}_t \] (58)

where \( \hat{A}_t = \mu_A \hat{A}_{A,t} + (1 - \mu_A) \hat{A}_{M,t} \), and \( \mu_A = \frac{N_A}{N} \) is the steady state employment share in agriculture. The last line uses log linearization of the sectoral production functions.

From equations (49) and (81) and noting that \( \hat{N}_{R,t} = \hat{N}_t \), we can write equation (15) as

\[ \hat{W}_t - \hat{P}_t = \varphi \hat{N}_t + \Phi \hat{C}_t - \Psi \Phi \left[ \hat{\phi}_t + \hat{Y}_{A,t} + (1 - \delta_R) \hat{T}_t \right] \] (59)

Substituting equations (56) and (57) into (58), and the resulting equation into (59), we get

\[ \hat{W}_t - \hat{P}_t = \Lambda \hat{C}_t - \left[ \varphi c (\delta_P - \delta_R)(1 - s_R) + \Psi \Phi (1 - \delta_R) \right] \hat{T}_t \]
\[ + \left[ \varphi (1 - c) - \Psi \Phi \right] \hat{Y}_{A,t} - \left[ \Psi \Phi \right] \hat{\phi}_t - \hat{A}_t \] (60)

where \( \Lambda = \{ \varphi c + \Phi \} \).

Finally, the log linearized real marginal cost in the manufacturing sector is given by

\[ \hat{m}_{C,M,t} = \hat{W}_t - \hat{P}_t + \delta_R \hat{T}_t - \hat{A}_{M,t} \] (61)
3.4 Gap Variables

Define, $\hat{X}_t^N$ as the deviation of $\ln X_t$ under flexible prices from the steady state, $\hat{X}_t^N = \ln X_t^N - \ln X$. Also, define a gap of a variable as $\hat{X}_t = X_t - X_t^N$. Then, the dynamic IS equation (DIS) is given by

$$\tilde{Y}_t = E_t \left\{ \tilde{Y}_{t+1} - c \Phi^{-1} \left[ \tilde{R}_t - E_t \{ \Pi_{t+1} \} - \tilde{R}_t^N \right] - (1 - \delta_R) [(1 - c) + c \Psi] E_t \{ \Delta \tilde{T}_{t+1} \} \right\} \quad (62)$$

where $\tilde{R}_t^N$ is the real natural interest rate and is given by

$$\tilde{R}_t^N = - \left\{ \Psi \Phi (1 - \Lambda^{-1} \Phi) + \varphi (1 - c) \Lambda^{-1} \Phi \right\} E_t \left\{ \Delta \tilde{Y}_{PA,t+1} \right\} \quad (63)$$

$$- \left\{ \Psi \Phi (1 - \delta_R) (1 - \Lambda^{-1} \Phi) + \Lambda^{-1} \Phi (\delta_R - \varphi c (\delta_P - \delta_R) (1 - s_R)) \right\} E_t \left\{ \Delta \tilde{T}_{t+1} \right\}$$

$$- \left[ \Psi \Phi (1 - \Lambda^{-1} \Phi) \right] E_t \{ \Delta \tilde{\phi}_{t+1} \}$$

$$+ \Phi \Lambda^{-1} E_t \left\{ \varphi \Delta \hat{A}_{t+1} + \Delta \hat{A}_{M,t+1} \right\}$$

The NKPC (New Keynesian Phillips Curve) in terms of manufacturing sector inflation, the output gap, and the terms of trade gap (shown in appendix),

$$\pi_{M,t} = \beta E_t \{ \pi_{M,t+1} \} + \kappa \Lambda (1 - \bar{\delta}) \tilde{Y}_{M,t} - \kappa [\varphi c (\delta_P - \delta_R) (1 - s_R) + \Psi \Phi (1 - \delta_R) - \delta_R - \Lambda (\bar{\delta} - \delta_R)] \tilde{T}_t \quad (64)$$

We can also express the NKPC in terms of aggregate inflation and the output gap,

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa \frac{\Lambda}{c} \tilde{Y}_t - \kappa \left[ \frac{\Lambda (1 - c) (1 - \delta_R)}{c} - \delta_R + \Psi \Phi (1 - \delta_R) + \varphi c (\delta_P - \delta_R) (1 - s_R) \right] \tilde{T}_t \quad (65)$$

$$+ \delta_R \Delta \tilde{T}_t - \beta \delta_R E_t \{ \Delta \tilde{T}_{t+1} \}. \quad (66)$$

3.5 Monetary Policy Rule

Monetary policy, as in GGM, follows a simple Taylor rule with the nominal interest rate as a function of aggregate inflation and the economy-wide output gap. We use a simple generalization of Taylor (1993):

$$R_t = (R_{t-1})^{\phi_y} (\pi_t)^{\phi_y} \left( \frac{Y_t}{Y_n} \right)^{\phi_y} \quad (67)$$
The log-linearized version of the Taylor rule shows that
\[ \hat{R}_t = \phi_r \hat{R}_{t-1} + \phi_\pi \pi_t + \phi_y \bar{Y}_t, \]  
(68)
i.e., the nominal interest rate, \( \hat{R}_t \), depends on its lagged value, \( \hat{R}_{t-1} \), aggregate inflation’s deviation from its target, \( \pi_t \), and the aggregate output gap, \( \bar{Y}_t \). This closes the model.

3.6 Difference in NKPC and DIS with two agents and two sectors

Equations (62), the Dynamic IS curve, and (65), the New Keynesian Phillips curve, summarize the non-policy block of the economy. How do these equations differ compared to a benchmark model, i.e., one where there is a single agent and a single sticky price sector (see Gali (2015, Chapter 3))? There are three key differences between the current framework and the benchmark model. The first difference is that there are two sectors which implies that the terms of trade, \( T_t \), appears in the NKPC and the DIS.

The second difference is that we have two types of agents, rich and poor, where \( \sigma_R \neq \sigma_P \) and \( s_R \neq 1 \).

The third difference is that there is (steady state) procurement and redistribution in the current framework, i.e., \( \mu_A - \hat{\delta} > 0 \), and \( \lambda > 0 \). When \( \mu_A - \hat{\delta} > 0 \), this implies that the employment share and consumption share in agriculture diverge i.e., \( c = \frac{C}{Y} = \frac{1-\mu_A}{1-\hat{\delta}} < 1 \). Hence, \( \mu_A - \hat{\delta} > 0 \) drives a wedge between consumption and production in the aggregate economy. We refer to \( \mu_A - \hat{\delta} > 0 \) as the procurement wedge. If \( \mu_A = \hat{\delta} = 0 \) and \( \phi = 0 \) (which implies \( \lambda = 0 \)), there is no procurement wedge.

Remark 1 Suppose \( s_R = 1, \mu_A = \hat{\delta} = 0, \) and \( \phi = 0 \) Then equation (62) is given by
\[ \bar{Y}_t = E_t \left\{ \bar{Y}_{t+1} \right\} - \frac{1}{\sigma_R} \left[ \hat{R}_t - E_t \{ \Pi_{t+1} \} - \hat{R}_t^N \right] \]  
(69)
where \( \hat{R}_t^N = \frac{\sigma_R(1+\varphi)}{\varphi+\sigma_R} E_t \left[ \triangle \hat{A}_{M,t+1} \right] \), which is the DIS equation in the benchmark model. Further, the New Keynesian Phillips Curve in equation (65) is given by
\[ \pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa(\varphi + \sigma_R) \bar{Y}_t \]  
(70)
which is the NKPC in the benchmark model where \( \pi_t = \pi_{M,t} \) and \( \bar{Y}_t = \bar{Y}_{M,t} \).

Remark 2 Suppose \( \mu_A - \hat{\delta} > 0, \sigma_R \neq \sigma_P, s_R \neq 1, \) and \( \lambda > 0 \). Then, (i) \( \frac{\varphi}{\sigma} < \frac{1}{\sigma_R} \) and (ii) \( \kappa \frac{\lambda}{c} > \kappa(\varphi + \sigma_R) \) if and only if \( s_R < \frac{1}{c} \left[ \frac{\sigma_p+\lambda_p-\sigma_R}{\sigma_p+\lambda_p-\sigma_R} \right] \).
Statement (i) in the above remark shows that monetary transmission is weaker compared to the benchmark model. Statement (i) can be checked by comparing the slope of the DIS curve in equation (62) with respect to the deviation of the real interest rate from its natural level ($\frac{\sigma_R}{c}$) and noticing that it is strictly less than $\frac{1}{\sigma_R}$ (the corresponding term in (69)) provided that $s_R < \frac{1}{c} \left[ \frac{\sigma_P + \lambda \rho - \sigma_R}{\sigma_P + \lambda \rho - \sigma_R} \right]$. Given the above restrictions, this will always be satisfied. Even if household had the same IES ($\sigma_R = \sigma_P$), the presence of the procurement wedge ($\mu_A - \delta > 0$), and redistribution ($\lambda > 0$) is sufficient for transmission to be weaker in our model. The procurement wedge weakens monetary transmission because there is less consumption (relative to production) and so monetary policy influences a lower level of final good consumption. With $\sigma_R \neq \sigma_P$ and $s_R \neq 1$, the consumption decisions of rule of thumb agents are not directly influenced by monetary policy. Hence, both features contribute towards weakening monetary transmission compared to the benchmark model. It can also be checked that heterogeneity affects the transmission of monetary policy, as higher is the fraction of agents that are Ricardian ($s_R$), the greater is the sensitivity of the output gap to the real interest rate.

Statement (ii) shows that the slope of the NKPC in equation (65) with respect to $\ddot{Y}_t$ is steeper compared to the benchmark model given by (70). This can be easily checked by comparing the slope magnitudes $\kappa \frac{\sigma_R}{c}$ with $\kappa (\varphi + \sigma_R)$ in these equations. This means that compared to the benchmark model, the aggregate supply (AS) curve for the economy is steeper. The aggregate demand (AD) curve is obtained by substituting the monetary policy rule, equation (68) into the DIS equation, (62). This yields (assuming $\phi_r = 0$)

$$\pi_t = \left[ \frac{1}{c \Phi - 1 \phi_\pi} \right] E_t \left\{ \ddot{Y}_{t+1} \right\} - \left[ \frac{1 + c \Phi - 1 \phi_y}{c \Phi - 1 \phi_\pi} \right] \ddot{Y}_t + \frac{1}{\phi_\pi} \left[ E_t \{ \pi_{t+1} \} + \ddot{R}_{N,t} \right] \tag{71}$$

It can be seen that the slope of the AD curve (with respect to $\ddot{Y}_t$) given in equation (71) is steeper compared to the benchmark model. This can easily be checked analytically, as $\Phi > \sigma_R$ and $c < 1$ ensures that

$$\frac{\phi_c + \phi_y}{\phi_\pi} > \frac{\sigma_R + \phi_y}{\phi_\pi}$$

This leads to our third Remark.

**Remark 3** The AD and AS curve are steeper compared to the benchmark model.

Given that we are interested in comparing the impact of an agricultural productivity (supply) shock and a procurement-redistribution (demand) shock on the economy, Remark 3 suggests that a positive productivity shock will lead to a larger fall in inflation and a smaller increase in the output gap compared to the benchmark model. Also, a positive procurement-
redistribution shock leads to a greater increase in inflation and a smaller increase in the output gap compared to the benchmark model. The implication of this is that with steeper AS and AD curves, in response to shocks that raise inflation, monetary policy needs to be tighter. Heterogeneity impacts monetary policy since the "composite" inter-temporal elasticity substitution, $\Phi^{-1}$, given by (54) is a function of $\sigma_R$, $\sigma_P$, $\lambda_p$, and $s_R$. First, since $\Phi > \sigma_R$ this implies that $\Phi^{-1} < \frac{1}{\sigma_R}$. In response to a change in the real interest rate, the economy inter-temporally substitutes less, making monetary policy less effective in closing gaps in response to shocks. Second, since $c < 1$ monetary policy is less effective since consumption diverges from output because of the procurement wedge. Third, since $\lambda_p$ augments $\sigma_P$ in this expression, the presence of redistribution raises the "effective" coefficient of relative risk aversion of poor agents ($\sigma_P + \lambda_p$) and reducing $\Phi^{-1}$.

4 Quantitative Analysis

4.1 Calibration

In this section, we calibrate the model to Indian data. Our goal is to understand the quantitative implications of a positive procurement and redistributive (P-R) shock (a demand side shock) to the economy and compare it with a positive agricultural productivity shock (a supply side shock). This is done to determine the differential impacts of a positive demand side and positive supply side shock on the economy. We also allow for all shocks (orthogonalized) to hit the economy together. We use the impulse response functions to assess implications for monetary policy design, highlighting implications for emerging market central banks who face terms of trade shocks.

4.1.1 Description of parameters

We use Levine et al. (2012) to set the discount factor for India at $\beta = 0.9823$. Following Anand and Prasad (2010), we choose the value of the inverse of the Frisch elasticity of substitution, $\varphi = 3$. Using Atkeson and Ogaki (1996), we fix the value of the inter-temporal elasticity of substitution (IES) for the rich and poor to be 0.8 and 0.5, respectively. We use the 2011-2012 Indian National Sample Survey (NSS) 68th round to set the share of workers in agriculture to 0.48 (this figure excludes allied activities) and the share of rich in population, $s_R$, to be 0.3279. We take a weighted average of urban and rural population with 0.5 and

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13 It can easily be checked that $\Phi$ is increasing in $\lambda_p$.
14 We use Dynare Version 4.5.7 to calibrate the model.
15 This share of the rich in the population is the weighted average of the share of the urban and rural populations who do not have access to grain under the NFSA with weights being 0.5 and 0.25, respectively.
0.25 being the weights in urban and rural areas.\textsuperscript{16} The share of agriculture in consumption for the rich is determined by the share of cereals and cereal substitutes in total expenditures net of expenditures on services, durables, vegetables, fuels and is equal to 0.4. The share of poor agent’s market purchase of agricultural good is found to be 0.3.***\textbf{Needs to be updated}***

We set the measure of price stickiness for the manufacturing sector, $\theta = 0.75$, as estimated in Levine et al. (2012) for the formal sector in India. We set the value of the persistence parameters and standard errors for agricultural and manufacturing productivity equal to those given in Anand and Prasad (2010). Thus, for productivity shocks in the agriculture sector, the AR(1) coefficient is calibrated to be, $\rho_A = 0.25$ and for the manufacturing sector, $\rho_M = 0.95$. The standard error of the regressions are given by $\sigma_A = 0.03$ and $\sigma_M = 0.02$, respectively. Following Levine et. al. (2012), the elasticity of substitution (EIS) between varieties of manufacturing goods is set to $\varepsilon = 7.02$ for the Indian case.

We estimate an AR (1) processes on procurement and redistribution as described in equation (28) and (29) using the procurement and off-take data from Table 27: Public Distribution System – Procurement, off-take and stocks.\textsuperscript{17} In our paper, we confine our analysis to procurement of grain and rice, two of the major grains procured under the NFSA and distributed under the PDS (the Public Distribution System). Using data from 1980-2018, we first make the procurement series stationary by subtracting the natural log of the average (value of the series) from the natural log of total procurement (wheat and rice) series and regress it on a constant, trend and AR(1) term.\textsuperscript{18} This yields the persistence coefficient and the standard error of the regression. The estimated persistence parameters for procurement ($\rho_{Y_A}$) and redistribution ($\rho_\phi$) processes are 0.44 and 0.54, respectively, while the standard errors are $\sigma_{Y_A} = 0.13$ and $\sigma_\phi = 0.10$.

We estimate the steady state share of the rich in consumption as 0.3636. This is calculated by computing the share of consumption by the rich in total consumption. This is done using the NFSA definition of the rich and the MPCE (monthly per-capita consumption expenditure) of agents from the 68\textsuperscript{th} round of the NSS.\textsuperscript{19} We calculate the parameter $\lambda$ to be a weighted average of the PDS (public distribution system) share in rice and wheat consumption in rural and urban areas with population shares as weights.\textsuperscript{20} This implies $\lambda = 0.209$. We calculate the steady state share of redistribution, $\phi$, by solving for $\phi$ from

\begin{align*}
\text{RBI’s Handbook of Statistics on the Indian Economy, 2017-2018.} \\
\text{Since } \phi_t \in [0,1], \ln (\phi_t) < 0. \text{ Hence, we use the logs of total (rice and wheat) off-take (instead of fractions) to estimate } \rho_\phi \text{ and } \sigma_\phi. \\
\text{See the Data Appendix.} \\
\text{See the Data Appendix.}
\end{align*}
Following Levine et al. (2012), we fix the interest rate smoothing parameter to be \( \phi_r = 0.66 \), with weights on inflation to be \( \phi_\pi = 1.2 \), and the weight on the output gap, \( \phi_y = 0.5 \). Table 1 below summarizes the structural parameters used in the calibration exercise in our model and their values.

\[
\phi = \frac{\lambda \delta_P (1-s_R)(1-\mu_A)}{(\mu_A - \delta)}.
\]

Using the parameter values in Table 1, this yields, \( \phi = 0.0656 \).

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### Structural Parameters

<table>
<thead>
<tr>
<th>Structural Parameters</th>
<th>Notation</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
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<td>Discount factor</td>
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<td>IES - Rich</td>
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<td>IES - Poor</td>
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<td>Population share of rich</td>
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<td>Employment share in agriculture</td>
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<td>Expenditure share of agriculture - Rich</td>
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### Shock Parameters

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<td>Productivity shock in A-sector</td>
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<tr>
<td>Redistribution shock</td>
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### Standard Errors

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<td>Redistribution shock</td>
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### Monetary Policy Parameters

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<td>Weight on inflation gap</td>
<td>( \phi_y )</td>
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<td>Levine et al. (2012)</td>
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\[^{21}\text{This yields} \phi = \frac{\lambda \delta_P (1-s_R)(1-\mu_A)}{(\mu_A - \delta)}.\]
4.2 Impulse response analysis

In this section, we study the impulse response functions (IRFs) of the relevant macroeconomic variables with respect to shocks to agriculture productivity (a supply shock) and procurement and redistribution (a demand side shock). We allow for the procurement wedge to be positive, i.e. $\mu_A - \delta > 0$, and $\lambda > 0$.

4.2.1 Transmission of a single period positive productivity shock in the $A$-sector

A positive agricultural productivity shock raises the supply of agricultural output, which in turn leads to a reduction in price of the agricultural good, $P_A$. This leads to a fall in agriculture inflation, $\pi_A$, overall inflation, $\pi$, and a worsening in the terms of trade. Nominal wages rise on impact since the value of the marginal product in agriculture ($=P_A A_A$) rises despite $P_A$ falling. Real wages also rise since $P_M$ is sticky, and $P_A$ falls. The income effect from a rise in real wages would suggest that $N$ falls, and $C_A$ and $C_M$ rise. The substitution effect suggest that $N$ would rise. The inter-good substitution effect also implies that $C_M$ falls since the manufacturing good has become relatively more expensive compared to the agricultural good. Given the calibrated parameters, the income effect on labor supply dominates the substitution effect and $N$ falls. However, $C_M$ rises, as the income effect also leads to an increase in the demand for the manufacturing good, dominating the inter-good substitution effect, which raises $Y_M$ and $N_M$ on impact. Hence, $N_A$, or labor supplied to the agriculture sector falls on impact. Inflation in the manufacturing sector falls because real marginal costs in the manufacturing sector, given by equation (61) fall. This is because while there is an increase in real wages, the decline in the terms of trade reduces real marginal costs. This underlines the importance of the terms of trade expression in assessing real marginal costs in two sector NK DSGE models.

Aggregate output rises because from equation (34), this depends on both $Y_A$ and $Y_M$ and the terms of trade. Overall, $Y$ increases, and the output gap increases. However, the decline in inflation induces the central bank from the Taylor rule, equation (68), to cut nominal interest rates. Real rates also fall, which induces a rise in the consumption of rich households, $C_R$. From equation (51), it is apparent that there are two terms that impact poor household consumption ($C_P$): consumption of the rich household’s and the terms of trade. Overall, $C_P$ rises leading to aggregate consumption, $C$, to rise.
In sum, a positive agriculture productivity shock leads to a rise in both poor and rich consumption, and therefore higher welfare.
4.2.2 Transmission of a single period procurement and redistribution shock

A procurement and re-distribution (which are orthognoalized) shock acts like a demand shock to the economy.\footnote{The reason why we consider them simultaneously is because the government’s desire to increase procurement is driven by its desire for higher re-distribution.} On impact, a procurement and redistribution shock, i.e., a positive demand shock, leads to higher agricultural output, $Y_A$, higher $P_A$ and a higher $\pi_A$. This leads to an increase in the terms of trade. Since $Y_A$ has increase, but productivity in the agriculture sector has not, this implies $N_A$ increases. Note that the value of the marginal product of labor also rises, which leads to a rise in the nominal wage, $W$, and a rise in $W/P$. A rise in the real wage has competing income and substitution effects, and higher real wages also induce higher household demand for $C_A$ and $C_M$. Since the terms of trade has risen, $P_M$ is lower relative to $P_A$ which from the inter-good substitution effect, also induces a rise in $C_M$, higher $N_M$ and higher $Y_M$. Overall, $N$, rises along with a rise in $N_A$ and $N_M$. Real marginal costs in the manufacturing sector rise because real wages have gone up, and there is also an increase in the terms of trade. Hence, manufacturing inflation, $\pi_M$ rises, which leads to aggregate inflation, $\pi$, rising. Aggregate output also rises from equation (34)), as does the output gap, which induces the central bank to raise the nominal interest rate. This makes real rates rise This makes consumption of the rich, $C_R$, fall. Given the calibrated parameters, it turns out that $C_P$ rises, as the second term in equation (51) dominates the first term. Given the parametrization of the model, aggregate consumption falls, leading to lower welfare, even though monetary policy is successful in raising agricultural consumption by the poor. Thus, monetary policy has both redistributive effects as well as output effects.
4.2.3 Transmission of a single period monetary policy shock

We now consider a single period contractionary monetary policy shock, which increases the nominal interest rate. This exercise is included to emphasize how the two sector TANK DSGE model leads to muted impact (less monetary transmission) on impact compared to the benchmark model (one sector, one agent). The blue dashed line depicts the benchmark model where we verify that the benchmark model has IRFs that are consistent with Gali
The red line depicts the current model.

In response to a rise in the nominal interest rate, real rates rise, and $C_R$ and $C_P$ fall. Hence, aggregate consumption, $C$, falls. The decline in rich and poor consumption induces a decline in the demand for the agriculture and manufacturing goods. However, since the agriculture sector has flexible prices, $P_A$ falls by more than $P_M$. The terms of trade falls. On impact, monetary policy therefore has a stronger effect on aggregate inflation, as this falls more than the amount of inflation in the benchmark model where there is just a single sticky price sector. The large swing in inflation compared to the benchmark model is brought about large changes in the price of the flexible price good.

The decline in the demand for the manufacturing good induces $N_M$ and $Y_M$ to fall. The value of the marginal product of labor also falls because $A_A$ is unchanged and $P_A$ falls. Hence, nominal wages fall, which induces real wages to fall in both models. The real wage however falls more in the benchmark model due to the absence of the flexible price sector. Aggregate employment rises because the income effect dominates the substitution effect although the effects are more muted in the model with frictions. Hence the impact of monetary policy on aggregate employment is weaker compared to the benchmark model. Aggregate output and the output gap fall because of competing sectoral output and terms of trade effects although the impact effect on output in the benchmark model is larger. Real marginal costs fall because of falling real wage (off-setting the rising terms of trade effect), which makes manufacturing inflation, $\pi_M$, also fall.

Because aggregate inflation and the output gap have fallen, the central banks in the next period reduces the nominal interest rate. The real interest rate falls, and both the benchmark model and the model with frictions stabilize and converge back to their steady state values.

This exercise underscores two aspects. First, the impact effect on aggregate output in the model with frictions is more muted (compared to the benchmark model) because of weaker transmission. In contrast, the impact effect on inflation is higher in the model with frictions, because of adjustments in the price level of the flexible price good. Hence, on impact, monetary policy, leads to a smaller negative output gap, but a larger negative inflation gap in the model with frictions compared to the benchmark model.
Figure 4: Impact of single period contractionary monetary policy shock

5 Optimal Monetary Policy

To be done.
6 Conclusion

We build a two sector two agent NK DSGE model with procurement and redistribution. Our paper contributes to a growing literature on understanding the role of heterogeneity in monetary policy. The novel aspect of our framework is that we extend the framework of Debortoli and Gali to two sectors. We are interested in understanding how a variety of shocks (redistributive policy shocks, agricultural productivity shocks, and monetary policy shocks) affect aggregate demand, and what role heterogeneity has on the design of monetary policy when there are two production sectors.

Why is it important to take into account heterogeneity? We show that heterogeneity matters for whether monetary policy responses to shocks raise aggregate welfare or not. In addition, heterogeneity affects the transmission of monetary policy, as higher is the fraction of agents that are Ricardian, the more real interest rates affect the output gap. We also show that the structural features in our model involving procurement and redistribution lead to steeper AD and AS curves.
References


7 Technical Appendix

7.1 The Model

**Derivation of Equation (14):** In the first stage, rich agents maximize equation (4) for a given level of expenditure, $X_t$ subject to the period budget constraint given by: $P_{A,t}C_{R,A,t} + P_{M,t}C_{R,M,t} = X_t$. This yields equations (8) and (9). In the second stage, rich household maximize (1) subject to the inter-temporal budget constraint (5) choosing $C_{R,t}$, $N_{R,t}$, and $B_{t+1}$ optimally. This yields the following first order conditions:

$$C_{R,t}^{-\sigma_R} = \mu_t P_t$$

$$N_{R,t}^\varphi = \mu_t W_t$$

and

$$-E_t\{Q_{t+1}\} \beta^t \mu_t + \beta^{t+1} E_t\{\mu_{t+1}\} = 0$$

where $\mu_t$ is the Lagrangian multiplier. Using $\frac{1}{\varphi_t\{Q_{t+1}\}} = R_t$, this yields equation (14).

**Derivation of Equation (16):** Poor agents maximize (4) subject to: $P_{A,t}C_{P,A,t}^\varphi + P_{M,t}C_{P,M,t} = M_t$, where $M_t$ corresponds to the income of the poor, by choosing $C_{P,A,t}$ and $C_{P,M,t}$ optimally. Note that $C_{P,A,t}^\varphi = (1 - \lambda_t)C_{P,A,t}$ given equation (6). This yields equation (10) and (11). Substituting equations (10) and (11) into equation (7) implies

$$P_{A,t}(1 - \lambda_t)C_{P,A,t} + P_{M,t}C_{P,M,t} \leq W_tN_{P,t} + P_{A,t}C_{P,A,t}^\varphi$$

which can be simplified to

$$P_tC_{P,t} - \delta_p \lambda_{s,t} P_tC_{P,t} = W_tN_{P,t}.$$ 

where $\lambda_{s,t} = \frac{\lambda_t}{1 - \lambda_t}$. In the second stage, poor households maximize (1) subject to the above equation which yields equation (16). Note that we require the regularity condition: $\lambda_t < \frac{1}{1 + \delta_p}$.

7.2 Steady State

From the FOCs for the rich and poor (equations (15) and (16)) the steady state condition is $\frac{N_{R}^{\varphi}}{C_{R}^{\sigma_R}} = \frac{N_{P}^{\varphi}(1 - \delta_p \lambda_s)}{C_{P}^{\sigma_P}}$. Since $N_{R} = \mu_R N$ and $N_{P} = (1 - \mu_R)N$, we have $\mu_R^{\varphi}C_{R}^{\sigma_R} = (1 - \mu_R)\varphi C_{P}^{\sigma_P}(1 - \delta_p \lambda_s) \implies \mu_R^{\varphi} \left( \frac{\lambda_R}{\mu_R} C \right)^{\sigma_R} = (1 - \mu_R)^{\varphi} \left( \frac{1 - \lambda_R}{1 - \mu_R} C \right)^{\sigma_P}(1 - \delta_p \lambda_s) \implies C^{\sigma_R - \sigma_P} = \ldots$
\[\left(\frac{1-\mu_R}{\mu_R}\right)^{\phi} \left(\frac{1-s_R}{1-\mu_R}\right)^{\sigma_R} (1-\delta_P \lambda_s) = (1-\mu_R)^{(\phi-\sigma_P)} \mu_R^{-\sigma_R} (1-s_R)^{\sigma_R} (1-\delta_P \lambda_s) = \Gamma.\] Here \(\lambda_s = \lambda/(1-\lambda)\). The steady state aggregate consumption is therefore,

\[C = \Gamma^{\frac{1}{\sigma_R-\sigma_P}}.\] (72)

Since \(A_M = A_A = 1\), nominal marginal costs are given by: \(MC_M = MC_A = W\). Given that the agricultural sector is characterized by perfect competition and flexible prices, price equals nominal marginal cost, so \(P_A = W\), while in the manufacturing sector the price is a markup over nominal marginal cost \(P_M = \frac{\varepsilon}{\varepsilon-1} W\). Therefore, the steady state term of trade is \(T = \frac{P_A}{P_M} = \frac{\varepsilon-1}{\varepsilon}\). With the employment subsidy in the manufacturing sector in place,

\[T = 1.\]

From the market clearing condition (equation (32)), the production function for manufacturing, and the optimal demand allocation (equation (20)) for manufacturing goods, we have

\[N_M = Y_M = C_M = (1-\delta)C = (1-\bar{\delta})\Gamma^{\frac{1}{\sigma_R-\sigma_P}}.\]

Denoting \(\mu_A\) as the steady state employment share in agricultural sector, we can write aggregate employment, \(N\), as

\[N = \frac{N_M}{1-\mu_A} = \frac{1-\bar{\delta}}{1-\mu_A} C\] (73)

and also

\[N = \frac{N_A}{\mu_A} = \frac{Y_A}{\mu_A} = \frac{1}{\mu_A} \left[\bar{\delta}C + Y_A^P\right].\] (74)

The last line uses the market clearing condition for the agriculture sector (equation (27)), and the optimal demand allocation for agricultural goods (equation (27)). Equating (73) and (74), we obtain

\[Y_A^P = C \left[\frac{\mu_A}{1-\mu_A} (1-\bar{\delta}) - \bar{\delta}\right] = C \frac{\mu_A - \bar{\delta}}{1-\mu_A}.\]

This is the steady state level of agricultural output procured. For \(Y_A^P > 0\), it needs to be that \(\mu_A > \bar{\delta}\), which implies that the steady state labor share in agriculture is greater than its consumption share since a fraction of agricultural output is not consumed. Note that in the absence of procurement \((Y_A^P = 0)\), and these two steady state shares are equal as
\( C^{\frac{\mu_A - \bar{\delta}}{1 - \mu_A}} = 0 \implies \mu_A = \bar{\delta}. \) The steady state relation in the agricultural sector then becomes

\[
N_A = Y_A = C_A + (1 - \phi)Y_P^A = \bar{\delta}C + Y_P^A = C\frac{\mu_A}{1 - \mu_A}(1 - \bar{\delta})
\]

Combining the expressions for \( Y_A \) and \( Y \), we also have that \( \mu_A = Y_P^A \) is the steady state output share in agriculture. From the aggregate market clearing condition (equation (33)), \( Y = C + Y_P^A = C\frac{1 - \bar{\delta}}{1 - \mu_A} \). The steady state share of consumption in output \( c = \frac{C}{Y} \) equals

\[
c = \frac{1 - \mu_A}{1 - \bar{\delta}}
\]

(75)

Note that \( c < 1 \) given \( \mu_A > \bar{\delta} \), as a fraction of agricultural good is not consumed.

We now relate \( c \) with the steady state share of consumption in output in the agricultural sector \( c_A = \frac{C_A}{Y_A} \), which will be found useful later. We already have \( Y_A = C\left(\frac{\mu_A}{1 - \mu_A}\right)(1 - \bar{\delta}) \), and \( C_A = \bar{\delta}C + \phi Y_P^A \). Therefore,

\[
c_A = \frac{\bar{\delta}(1 - \mu_A) + \phi(\mu_A - \bar{\delta})}{\mu_A(1 - \bar{\delta})}.
\]

(76)

Combining the above expression with equation (75), we can write

\[
c_A = \left[\frac{\bar{\delta}}{\mu_A} + \frac{\phi(\mu_A - \bar{\delta})}{\mu_A(1 - \mu_A)}\right] c
\]

(77)

Note that \( c_A < c \) given that \( \mu_A > \bar{\delta} \). We next derive the steady state value of \( \lambda \). Note that \( \lambda = \frac{\phi Y_P^A}{(1 - \mu_R)C_{PA}} \). From (10), \( C_{PA} = \frac{\delta_P C_P}{1 - \lambda} \) (as \( T = 1 \)) and using the relation between \( C_P \) and \( C \) from (36). Therefore,

\[
\lambda = \frac{\phi Y_P^A(1 - \lambda)}{(1 - \mu_R)\delta_P C_P} = \frac{\phi Y_P^A(1 - \lambda)}{\delta_P(1 - s_R)C}.
\]

Using \( Y_P^A = \frac{(\mu_A - \bar{\delta})}{(1 - \mu_A)}C \). This implies

\[
\lambda = \frac{(\mu_A - \bar{\delta})\phi(1 - \lambda)}{\delta_P(1 - \mu_A)(1 - s_R)}
\]

(78)

Solving for \( \lambda \), we obtain

\[
\lambda = \frac{\phi(\mu_A - \bar{\delta})}{\phi(\mu_A - \bar{\delta}) + \delta_P(1 - \mu_A)(1 - s_R)}.
\]

(79)

It is easy to check from equation (76) that \( c_A < 1 \) if and only if \( \phi < 1 \), which is true by
assumption. Likewise, from (78), define \( \lambda_s = \frac{\lambda}{1 - \lambda} \). Then we can re-write \( \phi \) as

\[
\phi = \frac{\lambda \delta_P (1 - \mu_A) (1 - s_R)}{\mu_A - \delta}.
\] (80)

Note that since \( \lambda_s > 0 \) by assumption, this implies that \( \phi > 0 \), given the other parameter restrictions in the model \( (\mu_A - \bar{\delta} > 0, \mu_A < 1, s_R < 1, \delta_P > 0) \). Hence \( 0 < \phi < 1 \).

Since \( \phi < 1 \), this is equivalent to

\[
\lambda = \frac{1}{1 + \frac{\delta_P (1 - \mu_A) (1 - s_R)}{\mu_A - \delta}} < 1
\]

which is satisfied given the above parameter restrictions. Hence, \( \lambda \in (0, 1) \) implies that \( \lambda_s \in (0, \infty) \).

7.3 The Log-Linearized Model

**Derivation of Equation (51):** To derive an expression for the log-linearized consumption for the poor, using the definition of \( \lambda_t = \frac{\phi_t Y_P^P}{C_{P,A,t} (1 - \mu_R)} \), and using equation (10), we have

\[
\lambda_t = \frac{\phi_t Y_P^P (1 - \lambda_t)}{\delta_P T - \delta C_{P,t} (1 - \mu_R)} \implies \lambda_t = \frac{\lambda_s}{1 - \lambda_t} = \frac{\phi_t Y_P^P}{\delta_P T - \delta C_{P,t} (1 - \mu_R)}. \]

Log linearization of this equation gives \( \tilde{\lambda}_{s,t} = \hat{\phi}_t + \hat{Y}_A^P + (1 - \delta_R) \hat{T}_t - \hat{C}_{P,t} \). The log-linearized first order condition (equation (16)) for the poor is given by

\[
\tilde{W}_t - \tilde{P}_t = \varphi \tilde{N}_{P,t} + (\sigma_p + \lambda_p) \tilde{C}_{P,t} - \lambda_p \left[ \hat{\phi}_t + \hat{Y}_A^P + (1 - \delta_R) \hat{T}_t \right].
\]

Given that \( N_{R,t} = \mu_R N_t \) and \( N_{P,t} = (1 - \mu_R) N_t \), we have \( \tilde{N}_{R,t} = \tilde{N}_{P,t} = \tilde{N}_t \). Combining this with equations (49) we get equation (51).

**Derivation of Equation (81):** To derive an expression for \( \tilde{C}_{R,t} \), substituting equation (51) for \( \tilde{C}_{P,t} \) into equation (48), the log-linearized consumption of the rich is given by,

\[
\tilde{C}_{R,t} = \left[ s_R + \frac{(1 - s_R) \sigma_R}{\sigma_p + \lambda_p} \right]^{-1} \tilde{C}_t - \Psi \left[ s_R + \frac{(1 - s_R) \sigma_R}{\sigma_p + \lambda_p} \right]^{-1} \left[ \hat{\phi}_t + \hat{Y}_A^P + (1 - \delta_R) \hat{T}_t \right], \tag{81}
\]

where \( \Psi = \frac{\lambda_p (1 - s_R)}{\sigma_p + \lambda_p} \).

7.4 Flexible price equilibrium and the natural rate

**Derivation of DIS in Equation (62):** Given that under flexible prices, real marginal cost is a constant, so that \( \tilde{m}_{C,M,t}^N = 0 \), equation (61) becomes

\[
0 = \tilde{W}_t^N - \tilde{P}_t^N + \delta_R \hat{T}_t^N - \tilde{A}_{M,t}^N.
\]
Combining this with the flexible price counterpart of equation (60), we get

\[
\hat{C}_t^N = \Lambda^{-1} \{ \varphi c(\delta_P - \delta_R)(1 - s_R) + \Psi \Phi(1 - \delta_R) - \delta_R \} \hat{T}_t^N - \Lambda^{-1} [\varphi(1 - c) - \Psi \Phi] \hat{Y}_{A,t} + \Lambda^{-1} \left[ \varphi \hat{A}_t \right] (\varphi \hat{A}_t) 
\]

Note that procurement is the same under both sticky and flexible prices. Substituting out for \( c \) and \( 1 - c \) in the above expression, the flexible price counterpart of equation (47) is

\[
\hat{Y}_t^N = \hat{c} \hat{C}_t^N + (1 - c) \left[ (1 - \delta_R) \hat{T}_t^N + \hat{Y}_{A,t} \right] = \left( \frac{1 - \mu_A}{1 - \delta} \right) \hat{C}_t^N + \left( \frac{\mu_A - \delta}{1 - \delta} \right) (1 - \delta_R) \hat{T}_t^N + \left( \frac{\mu_A - \delta}{1 - \delta} \right) \hat{Y}_{A,t} 
\]

Substituting equation (82) into equation (83), forwarding one period and then subtracting from each other, we obtain

\[
\hat{Y}_t^N = E_t \left\{ \hat{Y}_{t+1}^N \right\} - (1 - \delta_R) \{ 1 - c + c \Psi \} E_t \left\{ \Delta \hat{T}_{t+1}^N \right\} 
- [c\Lambda^{-1} \{ \varphi c(\delta_P - \delta_R)(1 - s_R) - \delta_R \} + c\Psi(1 - \delta_R)(\Lambda^{-1} \Phi - 1)] E_t \left\{ \Delta \hat{T}_{t+1}^N \right\} 
+ [c\Lambda^{-1} \{ \varphi(1 - c) - \Psi \Phi \} - (1 - c)] E_t \left\{ \Delta \hat{Y}_{A,t+1} \right\} 
- \{ c\Lambda^{-1} [\Psi \Phi] \} E_t \left\{ \Delta \hat{A}_{t+1} \right\} - c\Lambda^{-1} E_t \left\{ \varphi \Delta \hat{A}_{t+1} + \Delta \hat{A}_{M,t+1} \right\} 
\]

Finally, substituting (53) into (47) and then subtracting equation (85) we obtain the dynamic IS (DIS) curve given by equation (62).

**Derivation of NKPC in Equation (65):** From equation (47) and (83), the consumption gap is written as

\[
\hat{C}_t = \frac{1}{c} \left[ \hat{Y}_t - (1 - c)(1 - \delta_R) \hat{T}_t \right] 
\]

From equation (61) and given that \( \hat{m}_c_{M,t} = 0 \),

\[
\hat{m}_c_{M,t} = \hat{W}_t - \hat{P}_t + \delta_R \hat{T}_t. 
\]

And from equation (60),

\[
\hat{W}_t - \hat{P}_t = \Lambda \hat{C}_t - \{ \varphi c(\delta_P - \delta_R)(1 - s_R) + \Psi \Phi(1 - \delta_R) \} \hat{T}_t 
\]

Substituting equation (88) in equation (87) yields the manufacturing sector real marginal cost gap in terms of the aggregate consumption gap and the terms of trade gap.
\[ \tilde{m}c_{M;\lambda} = \Lambda \tilde{C}_t - [\varphi c(\delta_P - \delta_R)(1 - s_R) + \Psi \Phi (1 - \delta_R) - \delta_R] \tilde{T}_t \]  \hspace{1cm} (89)

We use equation (86) and the gap version of equation (34) i.e., \( \tilde{Y}_t = (1 - \mu_A) \tilde{Y}_{M;\lambda} + (\mu_A - \delta_R) \tilde{T}_t \) to get consumption gap in terms of manufacturing sector output gap and terms of trade gap \( \tilde{C}_t = (1 - \delta) \tilde{Y}_{M;\lambda} + (\delta - \delta_R) \tilde{T}_t \). Substituting this expression in equation (89) and using the resultant expression in equation (26) to get the manufacturing sector NKPC.

\[ \pi_{M;\lambda} = \beta E_t\{\pi_{M;\lambda+1}\} + \kappa \Lambda (1 - \delta) \tilde{Y}_{M;\lambda} - \kappa [\Psi \Phi (1 - \delta_R) + \varphi c(\delta_P - \delta_R)(1 - s_R) - \delta_R - \Lambda (\delta - \delta_R)] - \tilde{T}_t \]  \hspace{1cm} (90)

We also have the relationship that connects CPI inflation with sectoral inflation and TOT as

\[ \pi_t = \pi_{M;\lambda} + \delta \Delta \tilde{T}_t \]  \hspace{1cm} (91)

Substituting equations (86) and (91) into equation (26) yields equation (65).
8 Data Appendix

Description of $\lambda$: We calibrate $\lambda$ using the following formula

$$\lambda = \left[ \frac{\text{Wheat}_{\text{PDS}}^R + \text{Rice}_{\text{PDS}}^R}{\text{Wheat}_{\text{Total}}^R + \text{Rice}_{\text{Total}}^R} \right] \cdot \frac{\text{Pop}^R}{\text{Pop}_{\text{Total}}} + \left[ \frac{\text{Wheat}_{\text{PDS}}^U + \text{Rice}_{\text{PDS}}^U}{\text{Wheat}_{\text{Total}}^U + \text{Rice}_{\text{Total}}^U} \right] \cdot \frac{\text{Pop}^U}{\text{Pop}_{\text{Total}}},$$

where $\text{Wheat}_{\text{PDS}}^R = \text{consumption of PDS wheat in rural areas}$, $\text{Wheat}_{\text{Total}}^R = \text{Total consumption (PDS + market) consumption of wheat in rural areas}$, $\text{Rice}_{\text{PDS}}^R = \text{consumption of PDS rice in rural areas}$, $\text{Rice}_{\text{Total}}^R = \text{Total consumption (PDS + market) consumption of rice}$, $\text{Pop}^R = \text{Rural Population}$, $\text{Pop}_{\text{Total}} = \text{Total (Rural + Urban) Population}$, $\text{Wheat}_{\text{PDS}}^U = \text{consumption of PDS wheat in urban areas}$, $\text{Wheat}_{\text{Total}}^U = \text{Total consumption (PDS + market) consumption of wheat in urban areas}$, $\text{Rice}_{\text{PDS}}^U = \text{consumption of PDS rice in urban areas}$, $\text{Rice}_{\text{Total}}^U = \text{Total consumption (PDS + market) consumption of rice in urban areas}$, $\text{Pop}^U = \text{Urban population}$, and $\text{Pop}_{\text{Total}} = \text{Total (Rural + Urban) Population}$.

Calculation of $s_R$: We calibrate $\lambda$ using the following formula

$$s_R = \frac{\text{MPCE}_{\text{Total}}^R \cdot 0.25 \cdot \text{Pop}^R + \text{MPCE}_{\text{Total}}^U \cdot 0.50 \cdot \text{Pop}^U}{\text{MPCE}_{\text{Total}}^R \cdot \text{Pop}^R + \text{MPCE}_{\text{Total}}^U \cdot \text{Pop}^U},$$

where $\text{MPCE}^R = \text{monthly per-capita consumption expenditure in rural areas}$, $\text{Pop}^R = \text{Rural Population}$, $\text{MPCE}^U = \text{monthly per-capita consumption expenditure in urban areas}$, $\text{Pop}^U = \text{Urban Population}$. 0.5 and 0.25 are the fraction of urban and rural population not receiving grain under NFSA.

Calculation of $\mu_R$: We calibrate $\mu_R$ using the following formula

$$\mu_R = 0.5 \cdot \frac{\text{Pop}^U}{\text{Pop}_{\text{Total}}} + 0.25 \cdot \frac{\text{Pop}^R}{\text{Pop}_{\text{Total}}},$$

where $\text{Pop}^U = \text{Urban population}$, $\text{Pop}^R = \text{Rural population}$ and $\text{Pop}_{\text{Total}} = \text{Total (Rural + Urban) Population}$.