Equity risk premium and Sovereign debt

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Abstract

Domestic economy of the emerging markets and international sovereign debt market, such as those of Latin America and Asia, are affected by changes in the U.S. monetary policy. We present a model, which uses a pricing kernel to evaluate risk and return in financial markets of a lending country while embedding it in a model of sovereign default of a borrowing country. Thus, we bring together two separate strands of literature and present a holistic model of international finance, which evaluates how pricing of risk affects the price of debt, spreads and behavior of the borrowing country, i.e., its ability to borrow and smooth consumption. This helps in matching high spreads with low default probabilities. We find that variation of sovereign spread is driven by business cycles of the lending country. Boom in lending country leads to cheaper debt contracts in international lending market. Moreover, higher debt is issued during booms in the borrowing or the lending country.

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1 Motivation

A change in the U.S. monetary policy or the world market influences the country risk and cost of funds of developing countries like Latin America, Asia and East Europe. It affects the debtor country’s ability to repay loans and to attract foreign investors. In 1994 when the U.S. monetary policy tightened, the country spreads widened in Latin America. In 1998, the reverse happened when the Fed eased its monetary policy; the sovereign spreads decreased (Figure 1, Arora and Cerisola (2001)). Data on Argentina show similar trend. The correlation between Fed fund rate and the sovereign spreads is around 60 percent for the period between 1983-2001 (Figure 2). In addition, some basic analysis illustrates that the U.S. Fed fund rate has a positive and significant influence on the magnitude of sovereign spread.

The key motivation of this paper is to study how choices and characteristics of a lending country influence the real business cycles and domestic economy of a borrowing country and the international sovereign debt market. Specifically, we focus our attention to the case of the U.S. and Argentina. As a source of funding, the prevailing world interest rate in the lending country influences not only the borrowing country’s ability to borrow but also the default probability, level of debt and mean spreads in the international sovereign debt market. For instance, when times are bad, the borrowing country’s government might find it difficult to roll over debt and seek new funding.

We introduce a model of international finance, which combines two separate economic frameworks into one. We follow closely and extend the work by Melino and Yang (2003), Eaton and Gersovitz (1981), Arellano (2008) and Lizarazo (2012). Melino and Yang (2003) incorporates properties, such as mildly procyclical inter-temporal elasticity of substitution and strongly counter-cyclical risk aversion in the framework with Epstein-Zin preferences for lenders. This specification is very useful in the evaluation of risk and generating moments of asset returns in the financial market. Eaton and Gersovitz (1981) incorporate endogenous default risk in an incomplete market setup. Recent models, such as Arellano (2008) and Lizarazo (2012), on endogenous sovereign debt have managed to explain real business cycles statistics for the borrower. While these two frameworks
have successfully generated the economic outcomes from the view of respective agents, i.e. borrower and lender, a model is still missing that can combine these two sides together and yield a holistic model of international finance.

We develop a stochastic general equilibrium model of a small open economy which borrows from the rest of the world in an international lending and borrowing market. This paper extends the analysis of model on sovereign debt by introducing a risk averse lending country, which faces uncertainty of future income. The lending country’s wealth is determined solely by its own income shocks and is not affected by the default decision of the borrowing country. The lending and borrowing in the international debt market takes place between governments of each country. They lend and borrow by issuance of bonds in the international market. As the international debt contracts are not enforceable, the borrowing government may choose to default. This results in an immediate output penalty and exclusion from the international financial market. The domestic agents have access to the domestic financial markets, which are not integrated across countries. Thus, the households or the private agents of a country do not have access to the foreign financial market. We assume that the decisions made by agents in the small open economy do not influence fundamental variables (quantities) and prevailing prices in the rest of the world.

The pricing kernel of the lending country is derived from state dependent preferences and it is crucial in evaluation of risk. We use this pricing kernel in our baseline economy and match data on real business cycle statistics of the borrowing country, financial outcomes in the lending country and the international sovereign debt market. We find that when lending country receives a higher (lower) income shock, the contract on international lending charges a lower (higher) interest rate. In addition, we observe higher issuance of debt when the borrowing or the lending country receives a higher income shock. The paper also highlights the importance of lending country’s characteristics by replacing a risk averse lending country with risk neutral lending country. This specification fails on all fronts. The default probability and mean spread are very high as compared to the data and equity risk premium puzzle remains unsolved. We also look at the standard model of sovereign debt, which is also unable to resolve equity risk premium puzzle, and thus the moments of asset return remain unaccounted for.
We find that the standard model with its pricing kernel has a property that cannot match high spreads without counter-factually high default probabilities. One explanation is that spread is completely tied to the default probability. In order to match the spread, we need a theory without generating high default probability. Since, the pricing of risk varies over cycles and the risk averse lending country demands a risk premium, the price of debt has to be very different from default probability. In standard models, variation of spread is explained by the default probability. However, in the baseline economy of this paper, the variation of spread is also driven by the pricing of risk, which are affected by the business cycles of the lending country. Thus, we obtain high spreads with low default probability. These findings are very striking given that we choose an i.i.d. instead of correlated income process for the lending country.

**Literature review.**

Arora and Cerisola (2001) studies how changes in the U.S. monetary policy – federal funds target rate – impact sovereign bond spreads in emerging economies like Latin America, Asia and East Europe. They present empirical evidences using secondary markets data\(^1\) to show that spreads widen in emerging economies when U.S. monetary policy is tightened. In 1994 and then in 1998, a similar but opposite instance happened. The U.S. monetary policy tightened in 1994, which resulted in widened spreads. A rise in U.S. rates tends to increase the debt-service burden of the borrowing country, which reduces the ability to repay. Bernie et.al. (IMF, 2008) present empirical results in support of linkages between the developed and emerging financial markets. In general, emerging economies are influenced by the disturbances in the developed economies. Fostel and Kaminsky (2007) show that fundamentals of the domestic economy, which could be attributed to good governance; better fundamentals and higher output growth, and the external factors influence the international capital market flows to emerging economies. Any changes in interest rate (or global liquidity) and risk aversion of mature economies tend to influence the supply of funds to these economies.

This paper marries two models together, wherein we will use more recent de-

\(^1\)Merrill Lynch Global emerging markets spread
velopment in the field of sovereign debt and equity risk premium puzzle. The aim of this paper is to see how business cycle statistics of an emerging economy vary as we incorporate explicitly lending country’s preferences. This paper enriches the standard business cycle model under endogenous default by assuming lenders to be risk averse and incorporating domestic and international financial markets. In doing so, we calculate the financial and the real business cycle outcomes of a borrowing country in a stationary recursive equilibrium with default risk. We will also look at the (domestic) financial market outcomes of the lending country.

The economic literature has recently seen a lot of research in the field of emerging economies and its debt. These papers study the choice of level of debt and default decision by these economies. They focus on real business cycle models of international lending and borrowing under endogenous default decision. Most of the literature explores the effects of the borrowing country’s characteristics on these decisions. The important factors could be output shocks, political stability, Hatchondo et al. (2009) etc. Some recent papers go beyond the borrower’s characteristics to explore how lenders might affect the business cycles of these emerging economies, which tend to behave quite differently from that in the developed economies, Lizarazo (2012). In all these research papers, it has been highlighted that the emerging economies have business cycles that are more volatile, counter-cyclical rate of interest, which lags the cycles and the volatility of consumption is higher than the volatility of income, Neumeyer and Perri (2005).

Another strand of literature has attempted at explaining the equity risk premium puzzle and match asset returns and other financial variable with data. Equity risk premium is one of the most important financial figures used by investors for making investment decisions. A change in global interest rate might lead to a change in the riskiness of government bonds. This can further affect the returns on riskier assets, such as stocks, which should be reflected in change in the equity risk premium. Researchers have been trying to explain it and match it with data. This line of research has brought to light something important, which is called as equity risk premium puzzle. The economic models with standard preferences have failed to explain the large equity risk premium observed in data when the investors are risk averse. While equity risk premium puzzle
does not exist when the lenders are highly risk averse, many explanations have come forth to illustrate shortcomings of these models with standard preferences. Many attempts have been made using models with habit preferences, Epstein-Zin preference etc. to account for mismatch between data and theory without much success.

Melino and Yang (2003) has highlighted that introducing state dependent preferences with Epstein-Zin formulation of recursive utility will help resolve the puzzle. They incorporate strongly counter-cyclical risk aversion and mildly pro-cyclical inter-temporal elasticity of substitution to the Epstein Zin preferences. The agents are sensitive to current state of economy. If a low state were realized today, agents would invest in assets which pays a higher return if low state is realized tomorrow and a low return if high state is realized tomorrow. However, if a high state were realized today then the agents would not be as sensitive as in the above case. They will invest in an asset, which pays similarly to either low or high state realization tomorrow.

Section 2 introduces the model and the environment with a discussion on agents and markets. It formally presents the agents' problems and equilibrium of this economy. Section 3 computes the model and discusses the quantitative
analysis. We report results from calibration in this section.

2 The Model

We study the world economy, which consists of a small open economy and a rest of the world. We follow the framework of Aguiar and Gopinath (2007) and Arellano (2008) to model default. We assume that the small open economy is small enough compared to the rest of the world so its policies do not affect international interest rates and consumption. It is the net borrower and rest of the world is the net lender in the international debt market.

Time is discrete and infinite, $t = \{0, 1, 2, \ldots\}$. The lending and the borrowing economies, $i \in \{l, b\}$, are inhabited by risk averse households of unit mass each and are ruled by benevolent government. The households are identical within each country and receive stochastic endowment stream every period. They have access to the domestic financial market through risky equity market and the risk-
free bond market. The governments have access to the international debt market and trade a one-period risky bond. This debt contract is not enforceable and the borrowing government may choose to default on its debt obligation. The defaulting government is penalized with a fraction of its output immediately and it remains in financial autarky for a certain period.

**Households.** The households in the borrowing country have CRRA preferences over consumption, $c_b$, while the households in the lending country have Epstein-Zin preferences over consumption, $c_l$. The households of the borrowing and the lending country have access to the domestic capital market\(^2\). They can buy risky equity, $s_i$, at a price, $q_i^e$ and risk free debt, $a_i$, at a price, $q_i^{rf}$. The equity pays stochastic dividend, $\theta_i(y_i)$, which is a function of the income realization $y_i$. Additionally, the households in the borrowing country receive a lump sum intra-period transfer of goods, $T$, from its government.

The households in the borrowing country receives endowments $y_b$. It follows the AR(1) process,

$$\log(y'_b) = \rho_b \log(y_b) + \epsilon'_b,$$  \hspace{1cm} (1)

where, the persistence parameter $\rho_b$ is within the unit circle and the income shocks $\epsilon'_b$ are i.i.d and normally distributed $N(0, \varsigma_b^2)$. $\varsigma_b$ is the standard deviation of the income shocks.

The households in the lending country receives income $y_l$ and its growth rate, $\tilde{g}'_l$, follows a Markov process such that

$$y'_l = \tilde{g}'_l y_l,$$  \hspace{1cm} (2)

where the Markov chain is ergodic and $\tilde{g}'_l$ is positive.

The household in the borrowing country takes transfers $T'$ as given. It is a stochastic process, which is endogenously determined from government’s problem discussed later. It follows a Markov process and depends on the state of the economy, i.e. level of debt in the international debt market and income of the borrowing and the lending countries. The Bellman equation for the household in

\(^2\)Refer to the appendix for more details on the domestic market.
the borrowing country is given by

\[ U_b(w_b, y_b, \mathcal{T}) = \max \{ u_b(c_b) + \beta_b E [U_b(w_b', y_b', \mathcal{T}')] \}, \]

(3)

where \( \beta_b \in (0, 1) \) is the borrower’s discount factor and \( w_b \) is its wealth and \( y_b \) is the income realization. The utility function is strictly increasing and concave, i.e. \( u'_b(.) > 0 \) and \( u''_b(.) < 0 \) and satisfies the standard Inada conditions. The maximization is subjected to the budget constraint

\[ c_b + q_e^b s'_b + q_f^b a'_b = w_b \]

(4)

and the law of motion

\[ w'_b = a'_b + s'_b [\theta'_b(y'_b) + q'_e^b] + \mathcal{T}' \]

(5)

The lending country is not affected by the borrowing country’s characteristics as we assume a small open economy framework. The Bellman equation for the household in the lending country is given by

\[ U_l(w_l, y_l) = \max \{ u_l(c_l) + \beta_l E [U_l(w'_l, y'_l, \mathcal{T})] \}, \]

(6)

where \( \beta_l \in (0, 1) \) is the lender’s discount factor, \( w_l \) is its wealth, \( y_l \) is its income realization, \( \gamma_l(y_l) \) is its risk aversion and \( \frac{1}{\varphi_l(y_l)} \) is its inter-temporal elasticity of substitution\(^3\). The utility function is strictly increasing and concave, i.e. \( u'_l(.) > 0 \) and \( u''_l(.) < 0 \) and satisfies the standard Inada conditions. The maximization is subjected to the budget constraint

\[ c_l + q_e^l s'_l + q_f^l a'_l = w_l \]

(7)

and the law of motion

\[ w'_l = a'_l + s'_l [\theta'_l(y'_l) + q'_e^l] \]

(8)

We assume that the lending country is very big such that the default decision

\(^3\)Hereon, we assume that \( \gamma_l \) and \( \frac{1}{\varphi_l} \) depend on \( y_l \)
of the borrowing country does not affect its consumption. We also assume that \( \beta_b < \beta_l \), so that small open economy \( b \) is the net borrower and rest of the world \( l \) is the net lender.

Given a transfers function, \( T \), a recursive competitive equilibrium for an individual country \( i \) is given by a value function \( U_i \), decision rules \( g^s_i \) and \( g^a_i \) and the pricing functions \( q^e_i \) and \( q^{rf}_i \) such that,

(i) Given the pricing functions \( q^e_i \) and \( q^{rf}_i \), the value function \( U_i \) and the decision rules \( g^s_i \) and \( g^a_i \) and solve the household’s problem.

(ii) Markets clear

\[
\begin{align*}
s'_i &= g^s_i = 1 \\
a'_i &= g^a_i = 0
\end{align*}
\]

In the domestic market of the borrowing country, the price of equity

\[
q^e_b(y'_b) = \sum_{y'_b} \Pi(y'_b, y_b)(q^e_b(y'_b) + y'_b)M_b(y'_b, y_b)
\]

where, \( M_b(y'_b, y_b) = \beta_b \left( \frac{c'_b}{c_b} \right)^{-\gamma_b} \), is the stochastic discount factor of the borrowing household. The price of risk free debt, \( q^{rf}_b \), in the domestic market is

\[
q^{rf}_b(y'_b) = \sum_{y'_b} \Pi(y'_b, y_b)M_b(y'_b, y_b)
\]

In the lending country, the price of equity

\[
q^e_l(y'_l) = \sum_{y'_l} \Pi(y'_l, y_l)(q^e_l(y'_l) + y'_l)M_l(y'_l, y_l)
\]

where, \( M_l(\cdot) \) is the marginal rate of substitution.

The price of risk free debt

\[
q^{rf}_l(y'_l) = \sum_{y'_l} \Pi(y'_l, y_l)M_l(y'_l, y_l)
\]
**Government in the borrowing country.** The government is benevolent and maximizes the utility of its representative households. It smooths consumption by trading a non-contingent bond, which pays a time- and state-invariant return. It receives $qd'$ units of goods today and delivers $d'$ units of goods tomorrow in the international debt market. For each state of the economy $(y_b, y_l, d)$, the relationship between transfers $T(y_b, y_l, d)$ and $d'$ is determined by

$$ T = qd' - d $$

(9)

The state variables for the borrowing government are income shocks, $y_b$ and $y_l$, and the debt holding of the risky bond, $d$; and the state variable for the lending government is $y_l$. The aggregate income space is $y = (y_b, y_l)$, the aggregate state space for the borrowing country is $(y, d) = (y_b, y_l, d) = S_b$ and the aggregate state space for the lending country is $y_l = S_l$. Let $\Pi(y', y)$ represent the income transition matrix.

We denote the value function of the borrowing government as $V_b(y, d)$. Let $V_b^d(y)$ denote the value function after default and $V_b^{nd}(y, d)$ be the value function after repayment. The recursive formulation of the borrowing government’s problem is expressed as

$$ V_b(y, d) = \max \{ V_b^{nd}(y, d), V_b^d(y) \} $$

When it chooses to default, it temporarily exits the international credit market and redeems access with probability $\lambda$. The decision to default leads to loss of output, $y_b^d = (1 - \delta) y_b$.

The value function is,

$$ V_b^d(y) = u_b(y_b^d) + \beta_b \sum_{y'_b} \sum_{y'_l} [(1 - \lambda) V_b^d(y') + \lambda V_b^{nd}(y', d' = 0)] \Pi(y', y) $$

If it chooses to repay the debt then the value function is,

$$ V_b^{nd}(y, d) = \max_{d'} \left\{ u(y_b + T(y, d)) + \beta_b \sum_{y'_b} \sum_{y'_l} V_b(y', d') \Pi(y', y) \right\} $$

4The output loss, $\delta$ is expressed in percentage.
At the start of the period, the borrowing government takes the default decision after observing the income shocks. If it decides to repay its debt, then the borrowing government chooses $d'$ in order to maximize the utility subject to the resource constraint. It takes the bond price schedule $q(y, d')$ as given. The households in two countries choose the quantity of equity and risk-free assets taking prices as given. All agents consume.

The default set of the borrowing government is characterized by the aggregate income space $y$ for which it finds optimal to default for a given level of debt $d$. More precisely, $D(y', d')$ is given by:

$$D(y', d') = \{ y' \in Y : V^{nd}(y', d') < V^d(y) \}$$  \hspace{1cm} (10)

The repayment set is defined as the $A(y', d')$

$$A(y', d') = \{ y' \in Y : V^{nd}(y', d') \geq V^d(y) \}$$  \hspace{1cm} (11)

The value function of the lending government is $V_l(s)$ and is given by

$$V_l(y_l) = \max_{c_l} \left\{ u(c_l) + \beta l \sum_{y'_b} \sum_{y'_l} V_l(y'_l) \Pi(y', y) \right\}$$

The lending government faces resource constraint given by

$$c_l = y_l$$

**Stationary Recursive Competitive Equilibrium.** A stationary recursive competitive equilibrium for a world economy, which faces default risk in international debt market, is a set of value functions $V^*_b: S_b \mapsto R$ and $V^*_l: S_l \mapsto R$; policy functions for the borrowing household $g^*_b: S_b \mapsto R$, $g^a_b: S_b \mapsto R$ and $g^d_b: S_b \mapsto R$; policy function for the borrowing government $g^d_b: S_b \mapsto R$ and transfers function $g^T: S_b \mapsto R$; policies for the lending households $g^*_l: S_l \mapsto R$, $g^a_l: S_l \mapsto R$ and $g^d_l: S_l \mapsto R$; pricing functions $q^e_l, q^f_l, q^e_b, q^f_b$ and $q$; a stationary measure $\pi^*$ such that
(i) Taking as given the borrowing government policy, $g^b_d$, households consumption $g^c_c$ satisfies the resource constraint.

(ii) Taking as given the risky bond price function $q(y, d')$, the governments policy functions $g^b_d$, the default set $D(y', d')$ and the repayment set $A(y', d')$ satisfy the borrowing government’s optimization problem.

(iii) Bonds prices $q(y, d')$ incorporate the borrowing government’s default probabilities and are consistent with lending government’s optimization.

(iv) The domestic goods, equity and risk free bond market and the international risky bond markets clear

$$s'_b = g^a_b = 1$$
$$s'_i = g^a_i = 1$$
$$a'_b = g^a_b = 0$$
$$a'_i = g^a_i = 0$$

The equilibrium in the international debt market implies that the price of the risky bond for state $(y, d)$ is,

$$q(y, d') = \sum_{(y'_b, y'_l) \in A'} \Pi(y', y) M_t(y'_l, y_l)$$

where, $M_t(y'_l, y_l) = \beta_t \left( \frac{c'_l}{c_l} \right)^{-\phi_l} \left( \frac{\mathcal{V}_l(y'_l)}{\mathcal{R}(\mathcal{V}_l(y'_l))} \right)^{\phi_l - \gamma_l}$, is the stochastic discount factor of the lending household and $\mathcal{R}(\cdot)$ is a function defining the certainty equivalence of tomorrow’s utility.

**Equity Premium and Spread.** We follow the standard approach of Mehra and Prescott (1985) to quantify the equity premium in the lending country. The equity premium, $EQ_t$, is given as the difference between un-conditional return on equity, $R^e_t$, and un-conditional return on risk-free debt, $R^{rf}_t$.

$$EQ_t = R^e_t - R^{rf}_t$$

Similarly, we quantify the financial variables for the borrowing country.

$$EQ_b = R^e_b - R^{rf}_b$$
The equity risk premium in the borrowing country, $EQ_b$, corresponds to the difference between un-conditional return on equity, $R^e_b$, and un-conditional return on risk-free debt of the borrowing country.

The spread, $S$, is the difference between un-conditional return on sovereign bond, $R^d$ and un-conditional return on risk-free debt, $R^{rf}_l$. All details on un-conditional returns are provided in the Appendix.

3 Quantitative Analysis

3.1 Calibration and functional forms

We solve the model numerically and use quantitative results to discuss default, financial features (for e.g. equity risk premium) and business cycle properties (for e.g. interest rates spreads, output, and consumption) of this economy. We use parameters derived from the literature, Arellano (2008), for Argentina and from combined sources, data and Mehra and Prescott (1985), for the U.S. economy. They represent the borrowing and the lending countries in the model of this paper. A period in the model refers to a quarter in data, which represents time-period from 1983 : Q3 until 2001 : Q3.

*Functional forms and parameters* We use parameters from Arellano (2008), Mehra and Prescott (1985) and Melino and Yang (2003) for the borrowing and the lending country. We vary these parameters in order to answer how characteristics of the borrowing and the lending countries change the model outcomes with respect to data. To begin with, we adopt a specific baseline for the analysis. In this setting, the lending country has state dependent recursive preferences while the borrowing country has CES preferences, as considered in standard literature. The lending country is risk averse when low-income state is realized and it is risk neutral when high-income state is realized. The borrowing country is very impatient and risk averse.

Table 1 lists all those exogenous benchmark parameters, which are taken from Arellano (2008), Mehra and Prescott (1985), Melino and Yang (2003) and derived from data. The risk free world interest rate is 0.8 percent, which matches the
quarterly yield on the five-year US Treasury bond. The probability of re-entry is 0.282, which matches 1.75 trade balance volatility\(^5\). Moreover, the output cost after default is set at 0.969 of the average output, which captures 5.53 percent debt service to GDP. Similar to Arellano (2008), we assume that default results in direct output cost, \(\tilde{y}_b\), of the following form

\[
f(y_b) = \begin{cases} 
\tilde{y}_b & \text{if } y_b > \tilde{y}_b \\
y_b & \text{otherwise.}
\end{cases}
\]

For the borrowing country, we set the CRRA, \(\gamma_b = 2\), and the discount factor at \(\beta_b = 0.882\). The output of the borrowing country follows an AR(1) process with persistence and standard deviation set at \(\rho_b = 0.945\) and \(\varsigma_b = 0.025\), respectively. In case of lending country, we use the environment and parameters for the endowment process as presented in Mehra and Prescott (1985). The consumption growth is calculated so that the average growth rate of per capita consumption is 0.018, its standard deviation is 0.036. The first order serial correlation is -0.14, which translates into probability 0.43 of remaining in same state. We incorporate this parameter space and the stochastic discount factor from Melino and Yang (2003) in order to account for financial variables, as observed for the lending country's actual data. Melino and Yang (2003) target the estimates of historical average return on equity and risk free rate, which are at 7 percent and 0.8 percent, respectively. The standard deviation of equity and risk free rate is 0.165 and 0.056, respectively. With these targets in mind Melino and Yang (2003) estimate the rates on equity and risk free processes, i.e. values attained for different income realizations. These processes are then linked to the stochastic discount factor and the first order conditions with respect to equity and risk free bond. The first order conditions are derived from maximization problem of the agents subject to wealth accumulation constraint. They find specific parameter spaces \(^5\)As in Aguiar and Gopinath (2007), probability of redemption implies that the economy cannot access international credit market for 2.5 years on average. Gelos et. al. (2003) reports it to be around three years in data.
of discount factors, risk aversion parameters and inter-temporal elasticity of substitution parameters, which satisfies the first order conditions\(^6\) that match the equity and risk free rate processes exactly. One such parameter space is used for this paper. We set the lending country’s discount factor at 0.98, the risk aversion parameter during recession is 23.25 and during boom it is 0.21, the intertemporal elasticity of substitution during recession is 2.1 and during boom, it is 2.98.

**Discretization of shocks.** We discretize the state space for the variables as follows. There are 21 grids for borrower’s income \(y_b\), 2 grids for lender’s consumption (income) growth rate \(g'_l\) and 400 grids for debt. This particular choice of grids are considered as we want to compare, later, the results using the benchmark parameters and the parameters from Arellano (2008), Mehra and Prescott (1985) and Melino and Yang (2003). Accordingly, the income space is approximated using the procedure from Tauchen and Hussey (1991).

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\(^6\)Refer to the Appendix
3.2 Calibration results

In this section, we report actual and simulated quarterly business cycles for Argentina from 1983 until 2001. The quantitative predictions are obtained by simulating the model over time to obtain short run and long run statistics. For short (or medium) run statistics, we consider 100 defaults events and 74 observations before the default. We report interest rate spread, output and consumption around default episodes. We also look at the volatility and correlation between these variables for the borrowing country. Then, we calculate the long-run average for interest rate spread, debt to output share and default probability. In addition, we measure financial variables such as return on equity, return on risk free bond, return on sovereign bond and equity risk premium.

Table 2 reports results on business cycle statistics and financial variables for actual data in column 1 and the baseline economy in column 2. In order to isolate the role of risk aversion, we also simulate the baseline under the assumption of a risk neutral lending country while keeping rest of the parameters fixed. The results from this exercise are reported in column 3. In the last experiment, results in column 4, we recalibrate the risk neutral lenders and match the default probability of the baseline economy.

In the baseline economy, we build on Melino and Yang (2003) using a pricing kernel, which helps resolve the equity risk premium puzzle and risk free rate puzzle, for the lending country. The pricing kernel provides a good theory about evaluation of risk in the lending country and international debt market as we embed it in the model of sovereign debt and default. A resolution of risk free rate puzzle ensures that the risk free rate in the lending country is low, as observed in data. Low risk free rate enables lenders to access funds at low cost of borrowing and invest in assets with higher return. The obvious choices are going to either domestic equity market or international debt market, which pay a higher risk premium.

The baseline economy focuses on specific structure - the borrowing and the lending countries are risk averse and the borrowing country is very impatient. The lending country is highly risk averse when the state of the economy is low while it is risk neutral when the state of the economy is high. The risk aversion, which
is strongly counter cyclical, is set at 23.25 in low state and 0.21 in high state. The inter temporal elasticity of substitution is mildly pro-cyclical but highly responsive. It is set at 2.1 in low state and 2.98 in high state. The discount factor for the lending country is set at 0.98 and for the borrowing country at 0.882.

The current state of the economy in lending country affects the investment decision. As future is also important, a low-income state in the lending country today would encourage investors to invest in an asset, which promises a higher payment when a low-income state realizes tomorrow. This asset could pay relatively lower amount in higher income realization tomorrow. The lending country demands compensation during recession, as they are more risk averse. A high-income state today, which punctuates risk neutrality, does not highlight differences of such a magnitude in future particularly.

These features - strongly counter cyclical risk aversion and mildly pro-cyclical inter temporal elasticity of substitution - generate a pricing kernel, which is more accurate in evaluation of risk and return. The return on equity, 7.01 percent, is comparatively much higher than the risk free return, 0.80 percent, in the lending country. At these levels, the model matches the data very well with respect to equity risk premium and risk free rate (puzzles). The equity risk premium is around 6.21 percent and the risk free rate is low. The return on risky sovereign bond, 15.23 percent, issued by the borrowing country is higher than the return from equity market in the lending country. Here, we observe a low default rate, 4.43 percent, and a moderately high debt issuance as a share of output, 7.67 percent. These two effects, together, result in a high long-run mean spread, 14.43 percent, which is closer to the data. In medium (or short) run, around default episodes, the model over estimates the mean spread, 53.31 percent, as compared to 28.6 percent in data.

When the lender is risk neutral with CES preferences and the borrower is very impatient, with the discount factor set at 0.882, we are in a very different economy. The return on borrowing country’s sovereign bond, 2.74 percent, is higher than the return on equity, 2.04 percent, in the lending country. Moreover, return on equity is same as return on risk free bond in lending country. Thus, the equity risk premium remains unresolved in this economy. These returns in
various markets, international and domestic, leads to high debt issuance as a share of output, 16.49 percent, high default rate, 16.94 percent and thus a very high long run mean spread, 16.43. The medium run mean spread, 37.22 percent, is an overestimate of the data. This variation of the economy misses the data on several points for both, the borrowing and the lending country.

We re-calibrate the above economy, which has a risk neutral lending country and a patient borrowing country. The borrowing country has a discount factor set at 0.953. In this economy, the asset markets have very similar returns. The return on equity and risk free return in the lending country is 2.04 percent and the return on borrowing country’s sovereign bond is 2.06 percent. The differences in these returns highlight moderate debt issuance as a share of output, 5.96 percent, and default rate, 2.99 percent. These two opposing effects lead to a low long run mean spread, 3.52 percent, which does not match data. Moreover, the returns on different assets do not resolve the equity risk premium puzzle. This model economy underestimates the medium (short) run mean spread, 23.63 percent, which is result contrary to the one in baseline model.

The model presented in this paper highlights the role of risk averse lending country. In three experiments, we simulate the economy to see how risk aversion alone changes results. Under the baseline economy, a low default probability at 4.43 is associated with high spread at 14.43 over the business cycle. However, in column 3 and column 4, the default probability and spread move one-to-one. When the preferences are changed from the baseline to a risk neutral lending country with everything else same, we get a high spread at 16.43 because default probability is high at 16.94. We observe this behaviour as the pricing of risk is incorrect. The pricing kernel does not react much to the level of debt. The borrowing country borrows a lot and tend to default a lot. When we recalibrate the model with risk neutral lending country to match the default probability we find that the spread is low at 3.52. This shows that the standard theory is missing an important link between spread and default probability. The relationship between spread and default probability is a necessary ingredient in determining the price of sovereign bonds.

All three model economies do well with respect to other business cycle statistics of the borrowing country. They are in line with empirical findings on emerging
economies. The volatility of consumption is higher than the volatility of income, consumption is pro-cyclical and spread is counter cyclical. However, the equity risk premium, in the borrowing country’s domestic financial market, is low. It stands at around 0.20 percent. It is worth reminding readers that the low equity risk premium could be attributed to the preference of the agent in the borrowing country. Our best guess is that if CES preference was replaced by the state dependent Epstein-Zin preference, we will be able to get a higher equity risk premium. The equity risk premium for emerging economies is quite difficult to estimate empirically as the actual data is available for short periods and it is highly volatile. Damodaran (2016) reports the annual equity risk premium of Argentina at 9.16 percent. He estimates Argentina’s annual equity risk premium for 2010-12 using the method of relative equity market standard deviations under the assumption that the equity risk premium for the U.S. is 6 percent. This method uses annualized standard deviations of the S&P 500 and the MERVAL index in the U.S and Argentina. The key underlying assumption of this method is a linear relationship between equity risk premiums and equity market standard deviations. Salomons and Grootveld (2003) reports the average monthly equity risk premium for Argentina from 1976 until 2001 at 3.16 percent. Other studies, which estimate equity risk premium for emerging economies, and specifically for Argentina, find it to be higher than the developed countries.

Figure 3 plots the bond price schedules faced by the borrowing country with different preferences of the lending country. The bond price schedules in panel (a) and panel (b) corresponds to the borrowing country’s income shocks 10 percent below and above the trend. The bond price schedules display standard properties across lending country’s preferences, i.e. it is an increasing function of assets or higher level of debt is associated with lower bond price (or, higher interest rates). The figure displays the baseline economy under two scenarios: high and low-income realizations for the lending country (risk averse (high) and risk averse (low)). When the borrowing country receives a high-income shock, it can choose from a set of contract, which offers lower interest rates, as compared to when it receives a low-income shock. If the lending country also receives a high-income shock, and thus almost risk neutral, contracts with lower interest rates and higher-level debts are available in the international debt market. If, however, it receives
a lower-income shock, higher interest rate is charged for the same level of debt. In case of a risk neutral lending country, a lower interest rate is offered, when compared with the state dependent (with high-income realization today) lending country, at lower levels of debt. This finding is in line with the description of the state dependent preferences of the lending country, which chooses different assets depending on current income realization.

Figure 4 presents the saving function of the borrowing country under different characteristics of the economic agents. The level of debt, \( d' \), is higher when the borrowing country is in boom\(^7\). In panel (a), high-income state of the borrowing country, under persistent income, ensures repayment of debt. On the contrary, recessions in the borrowing country limits the capacity to borrow in the international debt market. A high level of debt, \( d' \), is issued for high level of initial debt, \( d \), when the lending country is risk neutral and the borrowing country is very impatient. The debt level, \( d' \), is only slightly lower when the lending country has counter-cyclical risk aversion and pro-cyclical inter temporal elasticity of substitution. Risk aversion has one of the direct dampening impact on debt issuance. It is noteworthy that debt issuance, \( d' \), is lower during recession than during boom in the lending country. Moreover, the re-calibrated economy with risk neutral lending country allows lower debt issuance, \( d' \), when the initial debt level, \( d \), is already very high, as compared to the other preference specifications. In panel (b), the borrowing country’s income is below the trend. Here, the borrowing and lending does not take place at higher levels of initial debt. However, when the initial debt level is not so high, the debt issuance depends on the lending country’s preferences. Higher level of debt is issued with state dependent preferences during boom in the lending country. Similar level of debt is observed in case of a risk neutral lending country. However, in case of re-calibrated economy with risk neutral lending country and the economy with state dependent preferences during recession in the lending country, the debt issuance is much lower.

The next step involves further analysis by using the model in this paper as the starting point. An extension would involve looking at the returns on equity, returns on bond and equity risk premium in the borrowing country’s domestic financial markets when default episodes occur. These are expected to be much

\(^7\)Higher debt corresponds to a high negative value on the x-axis and the y-axis of Figure 4.
different from the long run average as default affects the output of the borrowing country, though that of the lender remains unchanged in this model. Currently, it is difficult to analyse complete (full) influence of lender’s characteristics on borrowing country’s decisions due to huge size of the modelling data (matrices) and limited states for lending country’s income. One possibility is to extend the analysis from an i.i.d. income process to a Markov income process and allow for some correlation between the incomes of the borrowing and the lending countries.

4 Conclusion

The real and financial markets of small open economies are vulnerable to changes in interest rates of the large open economies. Periods like in 1994 and 1998 had been a very clear indication that the Latin American, as well as Asian, economies were affected by changes in the monetary policy of the U.S. and hence its interest rates. We present a model, which uses a pricing kernel to evaluate risk (and return) in financial markets while embedding it in a model of sovereign default. In doing do, we bring together, two separate strands of literature, and present a holistic model of international finance.

The first strand sets its attention on the financial markets of the developed countries, such as the U.S. The pricing kernel, which we derive from state dependent preferences of the lending country, enables us to resolve the puzzle associated with equity risk premium. Melino and Yang (2003) emphasizes the role of (moderately) pro-cyclical inter temporal elasticity of substitution and (strongly) counter-cyclical risk aversion in resolving it. These preferences affect the choices that lending country makes while investing in assets, such as domestic equity, international sovereign bond etc. Even though, portfolio allocation is not the focal point of this paper, we show that the investment decisions of investors depend on various aspects, such as income shocks in two countries, characteristics of the borrowing and the lending country etc.

In order to align with the aim of this paper we look at how lending country might influence the sovereign debt market and the domestic economy of the borrowing country. This brings us to the second strand of literature, which focuses on the endogenous default model for emerging economies, such as Argentina.
These models have been successful, largely, in explaining business cycle statistics of emerging economies when contracts in the international lending and borrowing market are unenforceable.

We find that the model presented in this paper, after incorporating some key features, not only matches data concerning the real business cycle statistics of the borrowing country but also assesses the risk very well for the lending country. We observe that state dependent preferences are key in evaluating risk more accurately. We also find that when lending country receives a higher (lower) income shock, the contract on international lending charges a lower (higher) interest rate. In addition, we observe higher issuance of debt when the borrowing or the lending country receive a higher income shock.

Risk neutral preferences, under similar conditions, fail to explain the observed statistics for both the borrowing and the lending country. The default probability, mean debt to output ratio and mean spread are very high as compared to the data. Moreover, the puzzle on equity risk premium remains unsolved. Re-calibrating the model with risk neutral lending country by incorporating a patient borrower, as in the standard literature, explain the business cycle statistics for the borrowing country. However, re-calibrating the parameters does not allow the model to explain the asset markets in the lending country.

The three economies in this paper illustrate that the baseline economy’s spread is very different from the default probability. The standard model can generate huge spreads only with a high default probability. We have established evaluation of risk, which changes with the business cycles of the lending country. The level of spread is different from the default probability due to existence of risk premium. Thus, an accurate pricing kernel is very important in accounting for risk in the international debt market.

The model presented in this paper is the first attempt in bringing features of lending and the borrowing country explicitly, while accounting for statistics of each of the economies together. This paper has successfully highlighted the importance of lending country’s characteristics in defining not only the international sovereign debt market but also the domestic economy of the borrowing country. Thus, the model in this paper is very useful as the starting point for any future analysis, which has an intention to investigate a research question pertaining to
international finance, specifically to international debt market.

<table>
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<tr>
<th>Parameters</th>
<th>Data</th>
<th>Risk averse Lender</th>
<th>Risk neutral Lender</th>
<th>Risk neutral Re calibrated</th>
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<td>$\beta_b$</td>
<td>-</td>
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<td>0.88</td>
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<tr>
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**Table 2: Data and Model**

Note: Simulation results are reported for averages over 500000 periods. For short (or medium) run statistics, we consider 100 defaults events and 74 observations before the default. Standard deviations are reported in percentages.
Figure 3: Bond Price Schedule

(a) High income realization for borrower

(b) Low income realization for borrower
(a) High income realization for borrower

(b) Low income realization for borrower

Figure 4: Saving function
5 Appendix

Household’s problem in lending country with Epstein-Zin preferences.

In the section we use time based subscripts in order to illustrate the importance of state dependent preferences for the lender (also, subscript $l$ is removed for simplicity). The household’s problem with Epstein-Zin preferences in the lending country can be written as follows:

$$U_t(w_t, y_t) = \max_{\{s_{t+1} \geq 0, a_{t+1} \geq 0\}} \left[ c_{t}^{1-\phi_t} + \beta_t \left( E \left[ U_t(w_{t+1})^{1-\gamma_t} \right] \right) \right]^{\frac{1}{1-\gamma_t}}$$

subjected to following constraints

$$c_t + q^e_t s_{t+1} + q^r_t a_{t+1} = w_{t+1}$$

the law of motion of wealth, where $\theta(y_{t+1}) = y_{t+1}$

$$w_{t+1} = a_{t+1} + s_{t+1} \left[ \theta(y_{t+1}) + q^e_{t+1} \right]$$

and the law of motion of income with $\tilde{g}_{t+1}$ as growth rate

$$y_{t+1} = \tilde{g}_{t+1} y_t$$

The first order conditions w.r.t $s_{t+1}$ gives

$$q^e_t = E_t \left[ \beta_t \frac{c_{t+1}^{-\phi_{t+1}}}{c_t^{-\phi_t}} \left( R_t(U_{t+1})^{\gamma_t-\phi_t} U_t^{-\gamma_t+\phi_{t+1}} \right) (q^e_{t+1} + y_{t+1}) \right] \tag{15}$$

where, $R_t(U_{t+1}) = \left[ E_t \left( U_{t+1}^{1-\gamma_t} \right) \right]^{\frac{1}{1-\gamma_t}}$. Assume that the price is homogenous of degree one then $q^e_t = q^e y_t$. Also, a recursive competitive equilibrium of goods market implies $\tilde{g}_{t+1} = \frac{w_{t+1}}{y_t} = \frac{c_{t+1}}{c_t}$. Thus,

$$q^e_t = E_t \left[ \beta_t \tilde{g}_{t+1}^{-\phi_{t+1}} y_t^{\phi_t-\phi_{t+1}} \left( R_t(U_{t+1})^{\gamma_t-\phi_t} U_t^{-\gamma_t+\phi_{t+1}} \right) (q^e_{t+1} + 1) \right] \tag{16}$$

In a special case of this general specification, if $\phi_t = \phi_{t+1}$ and $\gamma_t = \gamma_{t+1}$ then the above equation reduces to the standard price equation.
\[ q_t^e = E_t \left[ \beta g_{t+1}^{\gamma} \left( \frac{U_{t+1}}{R(U_{t+1})} \right)^{\gamma + \varphi} (q_{t+1}^e + 1) \right] \] (17)

where, \( \beta g_{t+1}^{\gamma} \left( \frac{U_{t+1}}{R(U_{t+1})} \right)^{\gamma + \varphi} \) is the 'stochastic' marginal rate of substitution.

Similarly, the first order conditions w.r.t \( a_{t+1} \) gives

\[ q_t^{rf} = E_t \left[ \beta_t \left( \frac{c_{t+1}}{c_t} \right)^{\varphi_t} (R(U_{t+1})^{\gamma_t} - \varphi_t U_t - \gamma_t + \varphi_t + 1) \right] \] (18)

In a special case, eq(17) is given by

\[ q_t^{rf} = E_t \left[ \beta g_{t+1}^{\gamma} \left( \frac{U_{t+1}}{R(U_{t+1})} \right)^{\gamma + \varphi} \right] \] (19)

Here on, we follow the notations used in the paper, i.e. drop time subscripts \( t \).

**Domestic Capital Market of the lending country.** The domestic capital markets comprises of equity and risk free debt. We have used a variation of the Mehra and Prescott (1985) to calculate the returns for each of these investments. In discreet case, the price of equity, \( q_t(y_l) \), is rewritten as

\[ q_t^e(y_l) = \sum y'_l \Pi(y'_l, y_l)(q_t^e(y'_l) + y'_l)M_l(y'_l, y_l) \]

where, \( M_l(.) \) is the marginal rate of substitution. The realized return on equity is given by

\[ r_t^e(y'_l, y_l) = \left[ \frac{q_t^e(y'_l) + y'_l}{q_t^e(y_l)} - 1 \right] \]

The equilibrium in the domestic market implies that the conditional return on the equity is \( \tilde{R}_t^e(y_l) \) and the un-conditional return is \( R_t^e \)

\[ \tilde{R}_t^e(y_l) = \sum y'_l \Pi(y'_l, y_l)r_t^e(y'_l, y_l) \] (20)
The price of risk free debt, \( q_{rf}^f(y_t) \), is
\[
q_{rf}^f(y_t) = \sum_{y_t'} \Pi(y_t', y_t) M_t(y_t', y_t)
\]

The equilibrium in the domestic risk-free bond market implies that the unconditional return on the risk-free bond \( R_{rf}^c \) is given as
\[
R_{rf}^c = \sum_{y_t} \tilde{\Pi}(y_t) \tilde{R}_{rf}^c(y_t)
\] (21)

**Domestic Capital Market of the borrowing country.** The domestic capital markets comprises of equity and risk free debt. We use a variation of the Mehra and Prescott (1985) to calculate the returns for each of these investments. The price of equity, \( q_e^b(y) \), is
\[
q_e^b(y_b) = \sum_{y_b'} \Pi(y_b', y_b)(q_e^b(y_b') + y_b') M_b(y_b', y_b)
\]

where, \( M_b(.) \) is the marginal rate of substitution of the borrowing country’s households. The return on equity for an income realization is given by
\[
r_e^b(y_b', y_b) = \left[ \frac{q_e^b(y') + y'}{q_e^b(y)} - 1 \right]
\]

The equilibrium in the domestic market implies that the conditional return on the equity is \( \tilde{R}_b^e \) and the un-conditional return is \( R_b^e \)
\[
\tilde{R}_b^e(y_b) = \sum_{y_b'} \Pi(y_b', y_b)r_b^e(y_b', y_b)
\] (23)
\[
R_b^e = \sum_{y_b} \tilde{\Pi}(y_b) \tilde{R}_b^e(y_b)
\] (24)

The price of risk free debt, \( q_{rf}^f(y) \), is
\[ q_{b}^{rf} (y_{b}) = \sum_{y_{b}'} \Pi(y_{b}', y_{b}) M_{b}(y_{b}', y_{b}) \]

The equilibrium in the domestic risk-free bond market implies that the expected return on the risk-free bond \( R_{b}^{rf} \) is given as

\[ R_{b}^{rf} = \sum_{y_{b}} \Pi(y_{b}) \left[ \frac{1}{q_{b}^{rf} (y_{b})} - 1 \right] \] (25)

The price of government debt, \( q(y, d) \), is

\[ q(y, d) = \sum_{(y_{b}', y_{l}') \in D} \sum_{y_{b}' \in D} \Pi(y_{b}', y) M_{l}(y_{l}', y_{l}) \] (26)

The equilibrium in the domestic risk-free bond market implies that the expected return on the risk-free bond \( R_{d}^{d} \) is given as

\[ R_{d}^{d} = \sum_{y_{b}} \sum_{y_{l}} \Pi(y) \left[ \frac{1}{q(y, d)} - 1 \right] \] (27)
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