On Green Growth with Sustainable Capital*

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Abstract

We develop an endogenous growth model to address a long standing question whether sustainable green growth is feasible by re-allocating resource use between green (natural) and man-made (carbon intensive) capital. In our model, final output is produced with two reproducible inputs, green and man-made capital. The growth of the man-made capital causes depreciation of green capital via carbon emissions which the private sector does not internalize. A benevolent government uses carbon taxes to encourage firms to substitute carbon intensive man-made capital with green capital that the production technology allows. Doing so, the damage to natural capital by emissions can be partly reversed through a lower socially optimal long run growth. This trade-off between environmental policy and long-run growth can be overcome by a combination of an investment in pollution abatement and higher total factor productivity.

Key words: Green growth, sustainability, carbon tax, clean growth, resource substitution.
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1. Introduction

In the UK, a major focus of the flagship clean growth industrial policy is to boost green growth through the promotion of cost effective low carbon technologies. While the industrial strategy lays out the goals of clean growth, it is less clear about the trade-offs facing the economy in meeting this clean growth target. The challenge emanates from a long standing theoretical and policy debate in resource and environment economics on whether natural capital can substitute man-made carbon intensive capital without sacrificing long run growth. The crux of the debate is whether a sustainable low carbon growth is achievable with a socially acceptable degree of substitution between man-made carbon intensive capital and natural capital.[1] If so, what policy instruments could accomplish this task?

This paper addresses the question using the lens of a simple endogenous growth model. Sustainable growth in our model implies a long-run low-carbon balanced growth. Man-made carbon intensive capital is augmented by private investment. The private sector, while determining its optimal accumulation of man-made capital, does not internalize the damage it inflicts on the green capital base due to carbon emissions. A benevolent government designs a Pigovian type tax-subsidy and public investment programme to correct for this externality. Doing so, the government seeks a Pareto optimal mix of man-made to green capital. Following the directives of European Commission, green capital is defined as environment friendly replenishable

[1] Proponents of substitutability are Solow (1974), Nordhaus and Tobin (1972). On the other hand, Daly (1997) and Ayres (2007) among others hold the view that these two forms of capital cannot be substituted for each other to maintain sustainable growth. For a recent survey on the sustainability issues of growth, see Cerkez (2018).

resources.[2] The underlying production technology is kept general in our model to allow for different degrees of substitution between green and man-made capital. Since the focus of this paper is on sustainable growth, we specialize to the steady state analysis and abstract from transitional dynamics.

There is a wave of literature on the effect environmental tax on economic growth. Forster (1973) analyzes optimal capital accumulation in the presence of pollution. His framework was subsequently extended by Gruver (1976), Luptacik and Schubert (1982), Siebert (1987). Gradus and Smulders (1993) do a comprehensive analysis of the environmental policy in terms of pollution abatement.[3] Using a learning by doing technology and pollution distaste in the utility function, Michel and Rotillon (1995) argue that capital should be mostly taxed to combat the pollution distaste. An unpleasant feature of their model is that a social optimum that internalizes pollution distaste might lead to a zero long run growth unless there is strong consumption compensation for pollution distaste.

The novelty of our model is that we have two kinds of capital, man-made and green capital in the production technology. Although the stock of green capital erodes due to carbon emissions from man-made capital, there is still complementarity between them in the production process. This complementarity could give rise to a socially optimal positive sustainable growth. We demonstrate this by setting up

[2] Examples of replenishable resources are reforestation, use of solar energy, improving air and water quality.

[3] Using a two good general equilibrium model, Hollady et al. (2018) examine the effect of environmental regulation on the emissions leakage in the presence trade frictions They analyze the effect of an emissions tax but abstract from capital accumulation, growth and production based externality from emission which is our primary focus in this paper.

a social planning problem which lays out the Pareto optimal ratio of man-made to green capital in an economy where man-made capital can damage the green capital base of the economy. We then describe a market economy where the private sector fails to internalize the adverse effect of its investment in man-made capital on green capital. A corrective tax-subsidy and green public investment programme are then designed which could replicate the socially optimal green growth rate.

We show various lines of extensions of our growth model. First, we present a scenario of strict complementarity between man-made and green capital that disallows any substitution between these two types of capital advocated by Daly (1997) and Ayres (2007). A carbon tax can thus only finance a public investment in green capital to replenish the damage caused by man-made capital. Such a tax cannot alter the ratio of green to man-made capital given by the technology. In this case, growth is unambiguously lower if the emissions is higher.

Second, we add an emissions disutility as in Michel and Rotillon (1995) in our model. Our results contrast sharply from Michel and Rotillon (1995) who argue that pollution distaste can lead to a gloomy stationary state with no long run growth if the social planner internalizes the pollution disutility. In our model due to the complementarity between green and man-made capital, the social optimum growth rate is positive even though emissions impose a negative externality on household's utility .

In the above scenario, higher carbon emissions always lowers growth due to the damage from investment in man-made capital to green capital. This adverse effect on growth is due to the absence of a pollution abatement technology. We then present a scenario where an emissions abatement technology is in place. In this scenario, a combination of carbon tax, public investments in abatement and green capital replenishment could restore the Pareto optimal proportion of man-made to green capital. Greater efficiency in pollution abatement boosts the long run growth and lowers the depreciation of green capital and also lowers the carbon tax. The positive growth effect could be further enhanced by augmenting TFP by a general R&D programme making the abatement technology more affordable.

Our results are consistent with the current environmental policy of net-zero carbon emissions which aims to lower emissions recognizing the constraint that a zero emissions is not possible.[4] In our model, green depreciation can be effectively eliminated by an optimal carbon tax and an efficient carbon abatement technology. The cost of such carbon tax is the distortion inflicted on the private sector which can be considerably lowered by making abatement technology more efficient. After netting out this cost, a net-zero carbon emissions is still possible.

The paper is organized as follows. Section 2 sets up a social planning problem which characterizes the socially optimal sustainable growth with optimal public and private investment in green and man-made capital. Section 3 develops a model of a decentralized economy with a benevolent government to determine the optimal carbon tax, subsidy and public investment which could replicate the allocation of the social planning optimum. In section 4, we extend our model to a scenario where carbon emissions cause negative utility externality. Section 5 extends the model

[4] The UK is the first major economy that has committed itself to a legally binding net-zero carbon emission target by 2050.

further to include public investment in pollution abatement. Section 6 analyzes the case of consumption based emissions. Section 7 concludes.

2. A social planning problem of sustainable growth

The economy produces the final output (Y_t) with broad based capital (K_t) and a unit raw labour with a linear technology as in Rebelo (1991):

$$Y_t = AK_t \tag{1}$$

where A is a constant total factor productivity (TFP) term. The aggregate capital is composed of man-made (K_t^p) and green capital (K_t^g) based on the following constant elasticity of substitution (CES) aggregator:

$$K_t = \left[(1-\nu)K_t^{p^{\varphi}} + \nu K_t^{g^{\varphi}}\right]^{1/\varphi} \tag{2}$$

with $0 < \nu < 1$, .and $\varphi = (\sigma - 1)/\sigma$ where σ is the elasticity of substitution. Note that since σ is positive by construction $-\infty < \varphi < 1$.

The man-made capital evolves according to the linear depreciation rule:

$$K_{t+1}^p = (1-\delta_p)K_t^p + I_t^p \tag{3}$$

where I_t^p is the level of private investment in man-made capital and δ_p is its rate of depreciation.

A benevolent social planner invests a fraction of final output, i_{yt}^g to replenish green capital by planting trees among other means.[5] The law of motion of the green capital stock is given by:

$$K^g_{t+1} = (1 - \delta_{gt})K^g_t + i^g_{yt} Y_t \tag{4}$$

In a similar spirit to Grudus and Smulders (1993), the depreciation rate of green capital (δ_{gt}) is proportional to the ratio of private to green capital. More man-made capital relative to green capital causes erosion of green capital (in the form of deforestation and climate change). In other words:

$$\delta_{gt} = \omega_t \frac{K^p_t}{K^g_t} \tag{5}$$

A few clarifications about the green depreciation rate, δ_{gt} are in order. The term ω_t represents erosion of green capital per unit of man-made capital due to the carbon emissions of the latter capital. This erosion is caused by the technology of investment, but it can be managed by pollution abatement technology to which we turn later. For our baseline model, we assume that ω_t is time invariant meaning $\omega_t = \overline{\omega}$ for all t and is exogenous. Hereafter we call $\overline{\omega}$ the rate of green erosion. The social planner takes the emission technology (5) and the erosion rate as given and designs a Pareto optimal ratio of man-made to green capital and a path of public investment in green capital.

[5] We represent the investment in man-made capital in level but green investment in rate. This distinction is crucial yo justify a carbon tax rate in a decentralized economy.

Plugging (5) into (4), the law of motion of green capital reduces to:

$$K_{t+1}^{g} = K_t^g - \overline{\omega} K_t^p + i_{yt}^g Y_t \tag{6}$$

The social planner determines a socially desirable sustainable green growth that maximizes the welfare of a representative infinitely lived agent. Noting that C_t is the consumption of the agent at date t and β is a constant discount factor, formally the optimization problem is written as:

$$Max \quad \sum_{t=0}^{\infty} \beta^t \ln C_t \tag{7}$$

s.t.

$$C_t + I_t^p \leq (1 - i_{yt}^g) Y_t \tag{8}$$

and (1), (2), (3), (6), (8) and subjec to the inequality constraint $i_{yt}^g \leq 1$. We do not impose any non-negativity constraint on either i_{yt}^g and I_t^p because we allow for disinvestment in both types of capital.

Assuming an interior solution, the planner chooses the time paths of man-made and green capital to equate the marginal product of man-made with the marginal product of green capital net of depreciation rates of both types of capital.[6] In other

[6]The derivation is available in the appendix. We assume an interior solution for the social planning problem assuming the green investment rate i_{yt}^g does not hit the upper bound. For plausible parameter values, we find that this is a reasonable assumption which keeps the growth self sustained.

words, the following static efficiency condition must hold:

$$\Theta\left(\frac{K_t^g}{K_t^p}\right) = \Psi\left(\frac{K_t^g}{K_t^p}\right) + \overline{\omega} + \delta_p \tag{9}$$

where

$$\Theta\left(\frac{K_t^g}{K_t^p}\right) = A\frac{\partial K_t}{\partial K_t^p} = A(1-\nu)\left[(1-\nu) + \nu\left(\frac{K_t^g}{K_t^p}\right)^{\varphi}\right]^{\frac{1-\varphi}{\varphi}} \tag{10}$$

and

$$\Psi\left(\frac{K_t^g}{K_t^p}\right) = A\frac{\partial K_t}{\partial K_t^g} = A\nu\left[\nu + (1-\nu)\left(\frac{K_t^g}{K_t^p}\right)^{-\varphi}\right]^{\frac{1-\varphi}{\varphi}} \tag{11}$$

We have the following proposition.

Proposition 1. *Based on the static efficiency condition (9), a unique ratio of green to man made capital,* $\frac{K_t^g}{K_t^p}$ *exists.*

Proof: It follows from the fact that $\Theta(0) = A(1-\nu)^{1/\varphi}$, $\Theta'\left(\frac{K_t^g}{K_t^p}\right) > 0$ and $\Psi(0) = \infty$, $\Psi'\left(\frac{K_t^g}{K_t^p}\right) < 0$. Thus there exists a unique crossing point in the positive quadrant between $\Theta\left(\frac{K_t^g}{K_t^p}\right)$ and $\Psi\left(\frac{K_t^g}{K_t^p}\right) + \overline{\omega} + \delta_p$ schedules. Figure 1 demonstrates graphically the existence of a unique K_t^g/K_t^p //

Next note that since there is no non-negativity restriction on both types of investment, regardless of the initial stocks of both types of capital, the following balanced growth path is attained immediately.

$$1 + \gamma = \beta\left[1 + \Psi\left(\frac{K_t^g}{K_t^p}\right)\right] \tag{12}$$

where γ is the balanced growth rate.

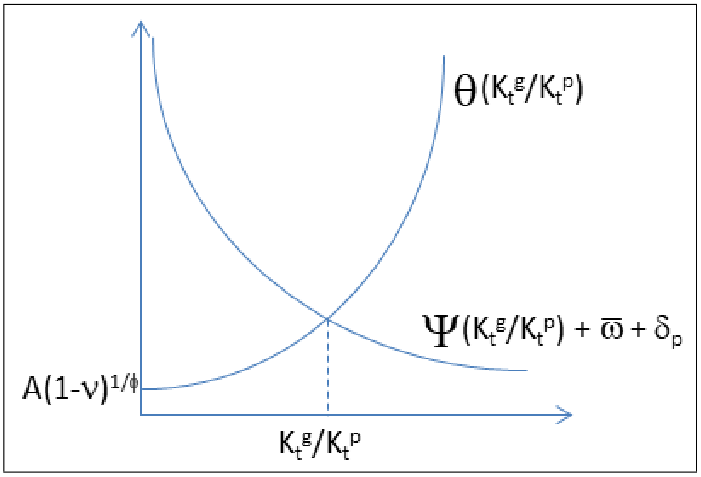

Fig 1: Existence of K_t^g/K_t^p

Using the implicit function theorem, and exploiting the fact that $\Theta'\left(\frac{K_t^g}{K_t^p}\right) > 0$ and $\Psi'\left(\frac{K_t^g}{K_t^p}\right) < 0$, it is straightforward to verify that

$$\frac{\partial(K_t^g/K_t^p)}{\partial\overline{\omega}} = \frac{1}{\left[\Theta'\left(\frac{K_t^g}{K_t^p}\right) - \Psi'\left(\frac{K_t^g}{K_t^p}\right)\right]} > 0$$

The efficiency condition dictates that a shift to a technology that causes greater erosion of green capital (higher $\overline{\omega}$) requires more stringent quantity control of man-made capital by either divesting in man-made capital or investing in green capital. Either of these two actions or a combination of them boosts the ratio K_t^g/K_t^p. The social planner mandates a higher ratio of green to man-made capital when the environmental damage is higher. This can also be easily checked from Fig 1. Higher $\overline{\omega}$ makes the $\Psi(.) + \overline{\omega} + \delta_p$ shift out resulting a higher equilibrium K_t^g/K_t^p.

The balanced growth rate (γ) must satisfy the following conditions:

$$1+\gamma=\beta\left[1+\Psi\left(\frac{K_t^g}{K_t^p}\right)\right] \tag{13}$$

Since $\Psi'(.)<0$, the implication is that a higher green erosion rate ($\overline{\omega}$) unambiguously lowers the balanced growth rate via a rise in K_t^g/K_t^p. Growth is, therefore, highest with zero erosion.

Using (6), the steady state investment ratio in green capital is given by:

$$i_y^g=\frac{\gamma+\overline{\omega}(K_t^p/K_t^g)}{A\left[(1-\nu)(K_t^p/K_t^g)^{-\varphi}+\nu\right]^{1/\varphi}} \tag{14}$$

Higher erosion ($\overline{\omega}$) lowers growth (γ) and the socially optimal ratio of man-made to green capital (K_t^p/K_t^g). The effect on the fraction of final output invested to replenish green capital, i_y^g is nonlinear. It depends on the erosion rate ($\overline{\omega}$) and the resulting substitution of man-made by green capital. If this substitution is strong, the efficient investment in green capital could fall due to a decline in $\overline{\omega}(K_t^p/K_t^g)$.

3. A Decentralized Economy with Carbon Tax

We now describe how a government can replicate the social planning allocation described in the preceding section by a corrective tax-subsidy scheme in a decentralized economy. The private sector consists of firms and households. Competitive firms produce final goods using the production function (2). Households own the man-made capital, accumulate it and rent it at a competitive price (r_t) every period to the firms for final goods production. Households supply one unit of labour for the production of final goods at a competitive wage (w_t). While producing final goods, the private sector does not internalize the damage caused to green capital based on (5). The government imposes a carbon tax (τ_t) on the rental income of firms in a Pigovian fashion to correct for the externality and uses the tax proceeds to finance green investments and transfers (T_t) to households. The government budget constraint is:

$$\tau_t r_t K_t^p = i_{yt}^g Y_t + T_t \tag{15}$$

where the public investment ratio $\{i_{yt}^g\}$ satisfies (6).

The household takes the stock of green capital $\{K_{gt}\}$ as well as the sequences $\{\tau_t\}$, $\{T_t\}$, $\{w_t\}$ and $\{r_t\}$ as parametrically given, and maximizes (7) subject to the following flow budget constraints and the private investment technology (3)[7]:

$$C_t + I_t^p = w_t + (1 - \tau_t) r_t K_t^p + T_t \tag{16}$$

The Euler equation facing the household is:

$$\frac{C_t}{C_t} = \beta \left[(1 - \tau_{t+1}) r_{t+1} + 1 - \delta_p \right] \tag{17}$$

The zero profit condition dictates that the competitive rental price of capital equals

[7]Since the household takes K_t^g as given, it faces a constant returns to scale technology involving K_t^p and inelastic labour which is normalized at unity. .

the marginal product of private capital which means

$$r_{t+1} = \Theta\left(\frac{K^g_{t+1}}{K^p_{t+1}}\right) \tag{18}$$

3.1. Optimal carbon tax

The government designs the time path of the carbon tax such that the private marginal benefit of investing in man-made capital exactly balances the social marginal benefit given by the social planner's Euler equation (13). The optimal carbon tax is:

$$\tau_t = \frac{\overline{\omega}}{\Theta\left(\frac{K^g_t}{K^p_t}\right)} \tag{19}$$

Plugging the efficient time path of K^g_t/K^p_t from the social planning problem, one can generate the time path of the carbon tax, τ_t.

3.2. Simulations

We perform model simulations based on the baseline model to assess the effects of green erosion the aggregate economy. Given that we are targeting low frequency annual data, the social discount factor β is fixed at the conventional level 0.96 and the depreciation rate of man-made capital, δ_p at 0.01. The TFP parameter, A is fixed at 0.145 to set a long run annual growth target of 2% for the UK economy.

Figure 2 plots the effect of green erosion ($\overline{\omega}$) on the optimal ratio of green to man-made capital, the long run balanced growth rate and the optimal carbon tax rate fixing the structural parameter, $\varphi = 0.5$, $\nu = 0.5$, $\beta = 0.96$, $\delta_p = 0.01$, $A = 0.14$.[8]

[8]These parameters are fixed at these values for illustrative purposes ensuring that the model

Starting from a zero erosion, as $\overline{\omega}$ rises, the long run growth rate falls while the carbon tax rate rises sharply which encourages the firms to substitute man-made capital with green capital. The decline in growth rate reflects the distortionary effects of carbon tax. Public investment in green capital required to replenish green capital first rises but as the depreciation of green capital is prevented by rapid substitution from man-made to green capital, it levels off.

A more carbon intensive economy (higher $\overline{\omega}$) necessitates a higher carbon tax which finances the public investment in green capital. The cost of such carbon tax is a lower growth due to the distortionary effect. This is the classic environmental policy and growth trade-off. In the absence of any pollution abatement programme, once can boost the growth by raising TFP. Figure 3 compares the effect of green erosion on the economy when the TFP parameter A is fixed at a 10% higher level. The long run growth is, however, higher for a given level of green erosion ($\overline{\omega}$). If the technology cannot be altered via an abatement technology, the way to boost growth is to raise TFP by R&D innovations.

solution exists. The direction of comparative statics is reasonably in the neighborhood of these parameter values.

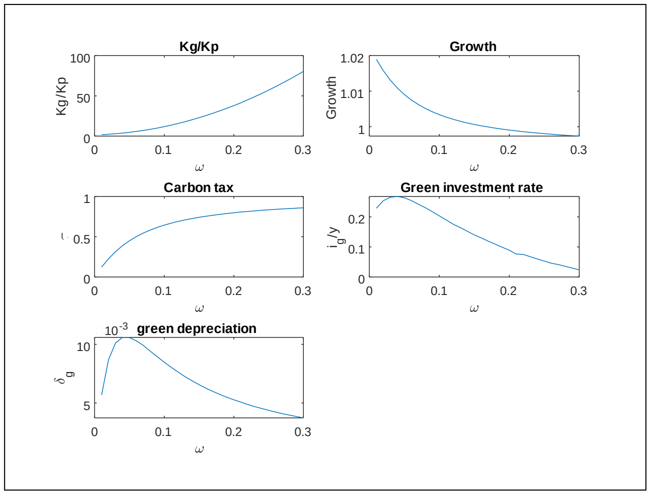

Fig 2: Effect of higher carbon emisson

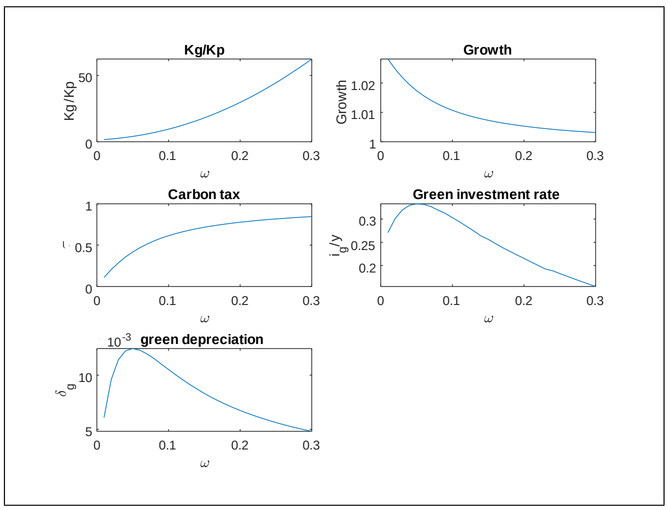

Fig 3: Effect of carbon emission on the economy when TFP is 10% higher

4. Extensions

4.1. Strict complementarity

Strict complementarity between man-made and natural capital arises as a special case when the production technology has zero elasticity of substitution ($\varphi- > -\infty$).

In this case the production function (1) takes the Leontief form:

$$Y_t = A \ \min\left[K_t^p, K_t^g\right] \tag{20}$$

The efficient ratio of green to man-made capital is unity. Based on (5), the green depreciation rate along a balanced growth path is given by:

$$\delta_{gt} = \overline{\omega} \tag{21}$$

Since strict complementarity disallows any substitution between two types of capital, the green depreciation rate cannot be altered by any carbon tax. The government has to engage in a public investment programme to replenish green capital damaged by man-made capital.

The balanced growth rate is given by:

$$1 + \gamma = \beta \left(1 + \frac{A - \overline{\omega} - \delta_p}{2}\right) \tag{22}$$

Higher erosion rate unambiguously lower the long run growth rate as in the previous scenario because of the destruction caused by man-made capital. The optimal investment rate in green capital (14) is:

$$i_y^g = \frac{\beta\{1 + 0.5(A - \delta_p)\} - 1 + \overline{\omega}(1 - 0.5\beta)}{A} \tag{23}$$

A higher emissions rate ($\overline{\omega}$) needs to be matched by higher public investment in green capital because no substitution is possible between green and man-made capital.[9]

The optimal carbon tax in the case of a fixed coefficient production function (20) is given by

$$\tau = 0.5 + A^{-1}(1 - 0.5\delta_p) + 0.5A^{-1}\overline{\omega} \tag{24}$$

Thus both the investment ratio i_y^g in (23) and carbon tax rate rise unambiguously with respect to $\overline{\omega}$. Since green capital cannot be substituted by man-made capital, destruction of green capital is compensated by greater investment in green capital by imposing higher carbon taxes on the private sector.

4.2. Pollution Disutility

Until now we have modelled green depreciation causing a pure production externality by lowering the green capital base of the economy. The model can be easily extended to a scenario where this erosion of green capital causes consumption externality. Following Gradus and Smulders (1993) and Michel and Rotillon (1995), we include green erosion affecting the direct utility function of the household. We specify the direct utility function as $\ln C_t - \chi \ln \delta_{gt}$ where $\chi > 0$. This basically means that citizens have a distaste for green depreciation (δ_{gt})which is the same as pollution distaste and the parameter χ represents the extent of distaste.[10] The private sector does not internalize this pollution distaste while undertaking optimal investment decisions. The pollution has both production and consumption based externalities. Thus the Euler equation (17) remains unaffected.

[9] See the appendix for a proof of (23).

[10] Luptacik and Schubert (1982) also model disutlity from pollution in a similar manner. .

It is straightforward to verify that along a balanced growth path the static efficiency condition (9) changes to

$$\Theta\left(\frac{K_t^g}{K_t^p}\right) = \Psi\left(\frac{K_t^g}{K_t^p}\right) + \overline{\omega} + \delta_p + \chi\frac{C_t}{K_t^p}\left(1 + \frac{K_t^p}{K_t^g}\right) \tag{25}$$

where

$$\frac{C_t}{K_t^p} = A\left[(1-\nu) + \nu\left(\frac{K_t^g}{K_t^p}\right)^{\varphi}\right]^{\frac{1}{\varphi}} - \gamma\left(1 + \frac{K_t^g}{K_t^p}\right) - \delta_p - \overline{\omega} \tag{26}$$

and the balanced growth rate is given by:

$$1 + \gamma = \beta\left[1 + \Psi\left(\frac{K_t^g}{K_t^p}\right) + \frac{\chi C_t}{K_t^p}.\frac{K_t^p}{K_t^g}\right] \tag{27}$$

Comparing (25) with (9) and noting that $\Theta'(.) > 0$ and $\Psi'\left(\frac{K_t^g}{K_t^p}\right) < 0$, it immediately follows that the socially optimal ratio of green to man-made capital (K_t^g/K_t^p) is unambiguously higher in this economy with distaste for pollution compared to the baseline model with no pollution distaste. By inspection of Fig 1, also note that the socially optimal ratio of green to man made capital is higher in this economy with pollution distaste $(\chi > 0)$. There is also no transitional dynamics because regardless of the initial condition, growth rate γ and the capital proportion K_t^g/K_t^p hold which satisfy (25), (26) and (27). Note also that in a decentralized economy, the private sector does not internalize the disutility from pollution. Thus the Euler equation for private investment stays the same as in (17) which after equating to the social planner's Euler equation (A.20) one obtains the optimal carbon tax formula as follows.

$$\tau_t = \frac{\overline{\omega} + \frac{\chi C_t}{K_t^p}}{\Theta\left(\frac{K_t^g}{K_t^p}\right)} \tag{28}$$

The expression for public investment ratio in green capital (14) remain the same. The appendix outlines the derivation of the key equations of this augmented model.[11]

In Michel and Rotillon (1995), a socially optimal long run growth rate is not sustainable if there is pollution distaste. In contrast, in our model, due to the presence of two reproducible inputs, namely man-made and green capital, a positive long run socially optimal growth rate given by (27) is sustainable even though citizens have distaste for pollution. This happens in our model due to the complementarity between man-made and green capital. A carbon tax and public investment package can also be designed to reproduce the Pareto optimal balanced growth rate. Figure 4 describes the effect of green erosion in our augmented model by bringing a small dose of emission distaste (setting χ at 0.05). All other parameters are fixed at the same level as in the baseline model of no emission disutility. The effect of carbon emissions is similar to our baseline model without any pollution distaste as in Fig 1. However, the ratio of green to man-made capital and the carbon tax rate are significantly higher than the baseline model. due to pollution distaste.

[11] In the special case of strict complementarity, it is straightforward to verify that the optimal carbon tax rate (24) remains unaffected because $K_t^p = K_t^g$ due to the Leontief production function. Thus the extra term in the direct utility function drops out.

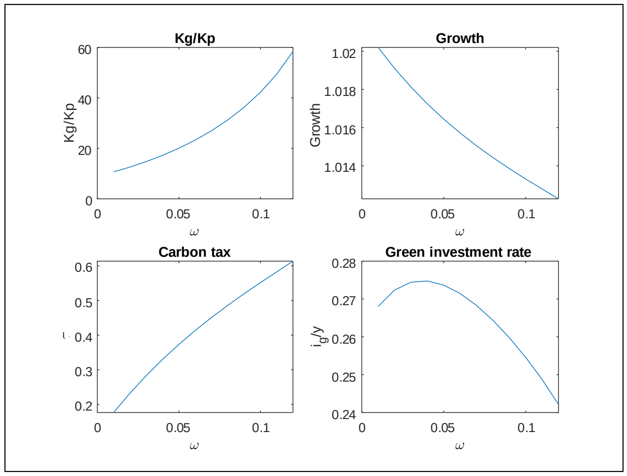

Fig 4: Effects of carbon emissions when there is emissions disutlity

5. Overcoming the adverse effect on growth: Decarbonisation

The punch-line of the previous models is that there is a painful trade-off between environmental policy and long-run growth unless there is an effort to abate the emissions by lowering ω_t. This requires public investment in emissions abatement. In this section, we extend our model to explore such possibility. Suppose in addition, to green investment (i^g_{yt}), a fraction of GDP (i^ω_{yt}) is spent on emission abatement. To put this formally, we introduce an emissions abatement technology as follows:

$$\omega_t = \varpi - \varkappa(\varpi) i^\omega_{yt} \tag{29}$$

If there is no public investment in emission abatement, emission is simply ϖ. The higher the investment in emissions abatement, the lower the emissions via the abatement technology (29). The effectiveness of the emissions abatement is captured by the parameter $\varkappa$ which is an increasing function of the exogenous emissions $/\varpi$.

We call $\varkappa(\varpi)$ an intervention function for combating climate shock. A higher green house effect (higher ϖ) can be combated by a more efficient abatement technology (e.g. efficient carbon capturing) which means a higher $\varkappa$. The exact functional form for $\varkappa(\varpi)$ depends on the time to intervene and proactiveness of the pollution agency in response to a climate shock. In the following section, we give an illustration of a specific intervention pattern.

The social planning problem (7) now changes to:

$$Max \quad \sum_{t=0}^{\infty} \beta^t \ln C_t \tag{30}$$

s.t.

$$C_t + I_t^p \leq (1 - i_{yt}^g - i_{yt}^\omega) Y_t \tag{31}$$

and (1), (2), (3), (6), (8) and $i_{yt}^g + i_{yt}^\omega < 1$.

The new first order condition for abatement investment (i_{yt}^ω) equates the marginal benefit of abatement investment to the marginal cost in terms of foregone national output. In other words,

$$\varkappa(\varpi) K_t^p = AK_t \tag{32}$$

which immediately pins down the Pareto optimal ratio of green to man-made capital as follows:

$$\frac{K_t^g}{K_t^p} = \left[\frac{(\varkappa(\varpi)/A)^\phi - 1 + \nu}{\nu} \right]^{1/\phi} \tag{33}$$

Notice that the ratio of green to man-made capital is constant and it holds in both short run and long run equilibrium. Higher abatement efficiency ($\varkappa$) unambiguously raises the ratio of green to man-made capital.

The static efficiency condition (9) is modified after including abatement investment as follows:

$$\Theta\left(\frac{K_t^g}{K_t^p}\right) = \Psi\left(\frac{K_t^g}{K_t^p}\right) + (\omega_t + \delta_p)/(1 - i_{yt}^{\omega}) \tag{34}$$

Plugging (33) into the modified static efficiency condition (34), the optimal abatement investment is given by:

$$i_{yt}^{\omega} = \frac{\Psi\left(\frac{K_t^g}{K_t^p}\right) - \Theta\left(\frac{K_t^g}{K_t^p}\right) + \delta_p + \bar{\omega}}{\varkappa + \Psi\left(\frac{K_t^g}{K_t^p}\right) - \Theta\left(\frac{K_t^g}{K_t^p}\right)} \tag{35}$$

The balanced growth equation (13) now nets out the abatement investment. It is given by:

$$1 + \gamma = \beta\left[1 + (1 - i_{yt}^{\omega})\Psi\left(\frac{K_t^g}{K_t^p}\right)\right] \tag{36}$$

The steady state green investment ratio (14) changes to

$$i_y^g = \frac{\gamma + (\varpi - \varkappa i_{yt}^{\omega})(K_t^p/K_t^g)}{A\left[(1-\nu)(K_t^p/K_t^g)^{-\varphi} + \nu\right]^{1/\varphi}}$$

and finally note that the private investors do not internalize the investment in green capital and emissions abatement. The Pigovian tax has to be adjusted to make them pay for the both types of investment. The optimal carbon tax is given by:

$$\tau_t = \frac{\bar{\omega}}{\Theta\left(\frac{K_t^g}{K_t^p}\right)} + i_{yt}^{\omega}[1 - \varkappa/\Theta(K_t^g/K_t^p)] \tag{37}$$

The appendix presents an outline of the key equations of this model. For a linear technology ($\varphi = 1$), the model admits the following closed form solutions for the optimal abatement investment rate, growth rate and the depreciation rate are given by:

$$i_{yt}^{\omega} = \frac{A(2\nu - 1) + \delta_p + \overline{\omega}}{A(2\nu - 1) + \delta_p + \varkappa} \tag{38}$$

$$1 + \gamma = \beta[1 + (1 - i_{yt}^{\omega})A\nu] \tag{39}$$

$$\delta_{gt} = (\bar{\omega} - \varkappa i_{yt}^{\omega})[\nu/\{\varkappa A^{-1} - 1 + \nu\}] \tag{40}$$

The optimal carbon tax rate (37) reduces to:

$$\tau_t = \frac{\overline{\omega}}{A(1 - \nu)} + i_{yt}^{\omega}.(1 - \varkappa/A(1 - \nu)) \tag{41}$$

5.1. Combating a climate shock with a technological intervention

Consider an abatement technology with a specific intervention function which combats emission with a four year time lag. A climate shock hits the economy in the form of a green house effect and this effect progressively rises. This is modelled by raising $\overline{\omega}$ from unity to 1.05 over a period of five years. An intervention takes place in the form an efficient abatement technology after five years from the onset of the green house effect which is modelled by an upward shift of $\varkappa$. In the next eleven years, another technological discovery takes places which means a further upward shift of $\varkappa$.After then $\varkappa$ progressively rises. We fix the other parameter values at $A = 0.4$, $\nu = 0.5$, $\delta_p = 0.01$. Figure 5 illustrates the effects of this intervention. Growth rate initially falls due to this climate shock but as soon as the technology is in place, it starts rising. Abatement investment initially rises at the expense of a lower green investment. As soon as a more efficient abatement technology is in place, abatement investment falls due to lower cost of such abatement which is offset by a rise in green investment. The green capital base expands reflected by a higher ratio of green to man made capital. The carbon tax initially rises and then it falls due to a lower cost of abatement. Green depreciation first rises and then falls and eventually turns negative.

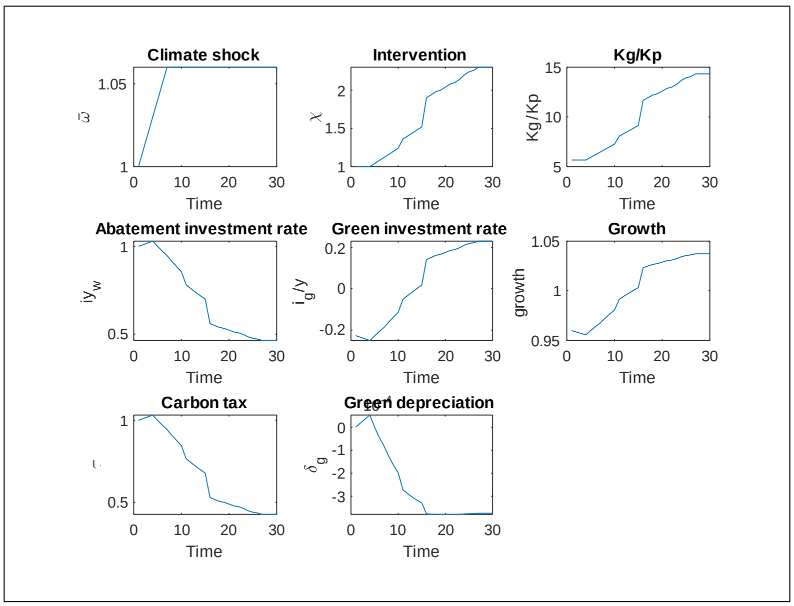

Figure 5: Effect of a climate change when a proactive abatement technology is in place

6. Conclusion

This paper extends conventional endogenous growth models to demonstrate the trade-offs facing the policy maker to balance sustainable growth with a clean environment policy. Since the private sector does not internalize the damage to the environment by carbon emissions, the policy maker imposes a corrective carbon tax on the private sector. Using alternative models, we show that higher carbon tax can nearly eliminate the depreciation of green capital caused by emissions if the production technology allows sufficient substitution of man-made capital by green capital. However, the distortionary effect of this tax lowers long run growth. To have a sustainable clean growth and to meet the UK Industrial Policy goal, efforts should be made to develop carbon free technologies.

To demonstrate the role of a carbon free technology, we extend our model environment to include public investment in emissions abatement. Our model shows that with a highly efficient pollution abatement technology, the adverse growth effect of environmental control can be mitigated and can be even reversed if the abatement technology is efficient and proactive to climate shock. The carbon tax could be also lowered. The policy lesson is that the adverse effect of carbon tax on growth can be reversed by new technology of emissions abatement in the form of carbon capture solutions such as forestation and carbon capture and storage. In addition, this alternative technology should be supplemented by a general R&D to boost the TFP. A carbon tax can help the transition to this new technology.

Currently in the UK, the public investment in natural capital is 0.1% of GDP. The environmental goods and services sector contribute to an estimated 1.6% of GDP. The ratio of natural capital to man-made capital is estimated at 0.0752.[12]

[12] From the Office of National Statistics, we get the monetray value of the natural capital in teh US economy which is £496.9 billion in 2014. The man made capital is the UK gross capital stock which is estimated at £6.6 trilion at the end of 2013 according to ONS. The ratio of natural and

According to the Department for Business, Energy and Industrial Strategy (2017) provisional figures, in 2017 there was a 3.2% decrease in carbon emission from 2016 figures and 21% decrease from 2009 estimate. Thus emissions are on a downward trend. However, decarbonisation efforts in some difficult sectors require development of new technologies which takes time. The annual growth rate of GDP is stabilizing around 1.8% according to the ONS estimates which is close to a 2% target. At the same time, the UK Committee for Climate Change (CCC) taxes amount to about 2.5% of the GDP three-quarters of which are from energy taxes (ONS, 2015). The committee of carbon tax has predicted that carbon tax will increase from 16 pounds per ton to 78 pounds per by 2030 and then by 220 pounds by 2050. These statistics suggest that the UK may not too far from a zero emissions target.

A future extension of our model is to consider adverse health effect of emissions as in Gradus and Smulders (1993). Such an extension would strengthen the case for a steeper Pigovian carbon tax. However, the effect on growth caused by the carbon tax is likely to be ambiguous. While the distortionary effects of carbon would lower the long run growth, a positive effect on health may promote growth via human capital.

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man made capital turns out to be 0.0752 for 2014.

A. Appendix

The present value Lagrangian is given by:

$$L^p = \sum_{t=0}^{\infty} \beta^t \ln C_t + \sum_{t=0}^{\infty} \lambda_t \left[(1 - i_{yt}^g) A K_t + (1 - \delta_p) K_t^p - C_t - K_{t+1}^p\right] \qquad \text{(A.1)}$$
$$+ \sum_{t=0}^{\infty} \mu_t \left[K_t^g + i_{yt}^g A K_t - \omega K_t^p - K_{t+1}^g\right]$$

where $\{\lambda_t\}$ and $\{\mu_t\}$ are the lagrange multipliers. The first order conditions are:

$$C_t : \ \beta^t / C_t \ - \lambda_t = 0 \qquad \text{(A.2)}$$

$$K_{t+1}^p : \ \ - \lambda_t + \lambda_{t+1} \left\{ (1 - i_{yt+1}^g) A \frac{\partial K_{t+1}}{\partial K_{t+1}^p} + 1 - \delta_p \right\} - \mu_{t+1} \omega + \mu_{t+1} A i_{yt+1}^g \frac{\partial K_{t+1}}{\partial K_{t+1}^p} = 0 \qquad \text{(A.3)}$$

$$K_{t+1}^g : \ \lambda_{t+1} (1 - i_{yt+1}^g) A \frac{\partial K_{t+1}}{\partial K_{t+1}^g} - \mu_t + \mu_{t+1} \left\{ 1 + A i_{yt+1}^g \frac{\partial K_{t+1}}{\partial K_{t+1}^g} \right\} = 0 \qquad \text{(A.4)}$$

$$i_{yt}^g : \ \ - \lambda_t + \mu_t = 0 \qquad \text{(A.5)}$$

Eq (A.5) is the foundation of the crucial static efficiency condition that equates the marginal distortion from the tax rate to the marginal benefit of the tax to finance green capital. Plugging (A.5) into (A.3) and using (A.2), we get:

$$\frac{C_t}{C_t} = \beta \left[A \frac{\partial K_{t+1}}{\partial K_{t+1}^p} + 1 - \delta_p - \bar{\omega} \right] \qquad \text{(A.6)}$$

Likewise, plugging (A.5) into (A.4) and using (A.2), we get:

$$\frac{C_t}{C_t} = \beta \left[A \frac{\partial K_{t+1}}{\partial K_{t+1}^g} + 1 \right] \qquad \text{(A.7)}$$

Equating (A.6) to (A.7), one obtains the static efficiency condition (9).

To get the optimal carbon tax formula (19), equate the right hand sides of (A.6) and (A.7).

A.1. Case of strict complementarity

Since the production function in eq (14) is Leontief type, the efficient ratio K_t^P/K_t^g is pinned down by the technology and is equal to unity. Eq (6) reduces to

$$1+\gamma = 1-\omega + i_y^g A \tag{A.8}$$

To get the optimal green investment ratio i_y^g, we need to recast the social planning problem and derive the balanced growth from the social planner's perspective. The social planner now no longer chooses the ratio of green to made made capital because it is pinned down by the technology at a fixed proportion ($K_t^P/K_t^g = 1$). Setting $K_t^g = K_t^p$, the economy wide resource constraint can be reduced to:

$$C_t + 2K_{t+1}^p - (2-\delta_p-\omega)K_t^p = AK_t^p$$

The present value lagrangian can be written as:

$$L^p = \sum_{t=0}^{\infty} \beta^t \ln C_t + \sum_{t=0}^{\infty} \lambda_t^{'} \left[(2+A-\omega-\delta_p) - C_t - 2K_{t+1}^p\right] \tag{A.9}$$

where $\{\lambda_t^{'}\}$ is the sequence of lagrange multipliers associated with the flow resource constraints.

The first order conditions are:

$$C_t:\ \beta^t/C_t\ - \lambda_t^{'} = 0 \tag{A.10}$$

$$K_{t+1}^p: \quad -2\lambda_t^{'} + \lambda_{t+1}^{'}(2 + A - \omega - \delta_p) = 0 \tag{A.11}$$

It is straightforward now using (A.10) and (A.11) that the balanced growth rate is given by:

$$1 + \gamma = \beta \left(1 + \frac{A - \omega - \delta_p}{2}\right) \tag{A.12}$$

Using (A.8) and (A.12), the optimal investment ratio in green capital given by:

$$i_y^g = \frac{\beta\{1 + 0.5(A - \delta_p)\} - 1 + \omega(1 - .5\beta)}{A} \tag{A.13}$$

To get the optimal carbon tax, we need to use the household's Euler equation (19) which reduces to

$$\frac{C_t}{C_t} = \beta\left[(1 - \tau_{t+1})A + 1 - \delta_p\right] \tag{A.14}$$

Along the balanced growth path (A.14) reduces to:

$$1 + \gamma = \beta\left[(1 - \tau)A + 1 - \delta_p\right] \tag{A.15}$$

Equating (A.12) with (A.15), we get, .

$$\tau = 0.5 + A^{-1}(1 - 0.5\delta_p) + 0.5A^{-1}\omega \tag{A.16}$$

A.2. Model with Distaste for Emissions

Plugging $\delta_{gt} = \omega(K_t^p/K_t^g)$ and suppressing the constant term ω , the present value Lagrangian changes to:

$$L^p = \sum_{t=0}^{\infty} \beta^t [\ln C_t - \chi \ln(K_t^p/K_t^g)] + \sum_{t=0}^{\infty} \lambda_t \left[(1 - i_{yt}^g)AK_t + (1 - \delta_p)K_t^p - C_t - K_{t+1}^p\right] \tag{A.17}$$

$$+ \sum_{t=0}^{\infty} \mu_t \left[K_t^g + i_{yt}^g AK_t - \omega K_t^p - K_{t+1}^g\right]$$

Only first order conditions (A.3)and (A.4) now change to:

$$K_{t+1}^p: \quad -\frac{\beta^{t+1}\chi}{K_{t+1}^p} - \lambda_t + \lambda_{t+1}\left\{(1 - i_{yt+1}^g)A\frac{\partial K_{t+1}}{\partial K_{t+1}^p} + 1 - \delta_p\right\}$$

$$-\mu_{t+1}\omega + \mu_{t+1}Ai_{yt+1}^g\frac{\partial K_{t+1}}{\partial K_{t+1}^p} = 0 \tag{A.18}$$

$$K_{t+1}^g: \quad \frac{\beta^{t+1}\chi}{K_{t+1}^g} + \lambda_{t+1}(1 - i_{yt+1}^g)A\frac{\partial K_{t+1}}{\partial K_{t+1}^g} -$$

$$\mu_t + \mu_{t+1}\left\{1 + Ai_{yt+1}^g\frac{\partial K_{t+1}}{\partial K_{t+1}^g}\right\} = 0 \tag{A.19}$$

Plugging (A.2) and (A.5) into (A.18), we get:

$$\frac{C_t}{C_t} = \beta\left[A\frac{\partial K_{t+1}}{\partial K_{t+1}^p} + 1 - \delta_p - \omega - \frac{\chi C_t}{K_{t+1}^p}\right] \tag{A.20}$$

Plugging (A.2) and (A.5) into (A.19), we get:

$$\frac{C_t}{C_t} = \beta\left[A\frac{\partial K_{t+1}}{\partial K_{t+1}^g} + 1 + \frac{\chi C_t}{K_{t+1}^g}\right] \tag{A.21}$$

Combining (A.20) and (A.21) and backshifting the time subscript we get (25). The balanced growth rate equation (27) follows directly from (A.21). To get the expression for C_t/K_p^p in equation (26), combine (6) and (8) to get:

$$C_t + K_{t+1}^p + K_{t+1}^g - (1-\delta_p-\omega)K_t^p - K_t^g = AK_t \tag{A.22}$$

Then divide through by K_t^p and impose the balanced growth restriction, $K_{t+1}^p/K_t^p = K_{t+1}^g/K_t^g = 1+\gamma$.To get the optimal carbon tax formula (37), equate the right hand sides of (A.20) and (17).

A.3. Model with pollution abatement

The present value Lagrangian is given by:

$$\begin{aligned} L^p = &\sum_{t=0}^{\infty}\beta^t \ln C_t + \sum_{t=0}^{\infty}\overline{\lambda}_t\left[(1-i_{yt}^g-i_{yt}^{\omega})AK_t + (1-\delta_p)K_t^p - C_t - K_{t+1}^p\right] \qquad \text{(A.23)} \\ &+\sum_{t=0}^{\infty}\overline{\mu}_t\left[K_t^g + i_{yt}^g AK_t - \omega(i_{yt}^{\omega})K_t^p - K_{t+1}^g\right] \end{aligned}$$

where $\{\overline{\lambda}_t\}$ and $\{\overline{\mu}_t\}$ are the lagrange multipliers. The first order conditions are:

$$C_t:\ \beta^t/C_t\ -\overline{\lambda}_t = 0 \tag{A.24}$$

$$K_{t+1}^p : -\overline{\lambda}_t + \overline{\lambda}_{t+1}\left\{(1-i_{yt+1}^g - i_{yt+1}^{\omega})A\frac{\partial K_{t+1}}{\partial K_{t+1}^p} + 1 - \delta_p\right\} - \overline{\mu}_{t+1}\left\{\omega(i_{yt+1}^{\omega}) + Ai_{yt+1}^g\frac{\partial K_{t+1}}{\partial K_{t+1}^p}\right\} = 0 \tag{A.25}$$

$$K_{t+1}^g : \overline{\lambda}_{t+1}(1-i_{yt+1}^g - i_{yt+1}^{\omega})A\frac{\partial K_{t+1}}{\partial K_{t+1}^g} - \overline{\mu}_t + \overline{\mu}_{t+1}\left\{1 + Ai_{yt+1}^g\frac{\partial K_{t+1}}{\partial K_{t+1}^g}\right\} = 0 \tag{A.26}$$

$$i_{yt}^g :\ \ -\overline{\lambda}_t + \overline{\mu}_t = 0 \tag{A.27}$$

$$i^{\omega}_{yt}: \quad -\overline{\mu}_t \omega'(i^{\omega}_{yt}) K^p_t - \overline{\lambda}_t A K_t = 0 \tag{A.28}$$

Use (29), (A.27) and (A.28) to verify (33). Use (A.25) and (A.27) to get the balanced growth equation (36).

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