Externalities, Entry Bias and Optimal Subsidy Policy in Oligopoly

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Abstract

This article analyses alternative subsidy schemes and long-run entry bias in a new industry that creates positive environmental externalities. It demonstrates that per unit subsidy scheme, despite attracting fewer firms, results in higher industry output and economic surplus in the equilibrium compared to the expenditure equivalent lump-sum subsidy scheme. However, the later leads to higher total surplus, unless spill-over externalities is sufficiently small. Further, free entry equilibrium number of firms may be excessive or insufficient. The first best equilibrium outcome can be implemented through a unique combination of per unit subsidy and lump sum subsidy/tax, which involves positive government expenditure.

Keywords: Positive Externalities; Environmental Benefit; Free Entry; Cournot Oligopoly; Expenditure Equivalent Subsidy Schemes; Social Optimum

JEL Classification: D43; L52; H23; Q48; Q58

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1. Introduction

Several countries across the globe have active (direct and indirect) subsidy policies in place to promote industries with positive environmental externalities. Renewable energy, electric cars and other electric motor vehicles industries are prime examples of such subsidized industries. It is well argued that firms that create positive externalities, either through its production process or by producing environmentally friendly goods or both, tend to produce at a less than socially optimal level for any given intensity of product market competition. The reason is privately optimal decisions of profit maximizing firms do not take into account social benefits of positive environmental externalities generated by them. Further, firms in these industries often need to incur high setup costs and face oligopolistic market structure – two characteristics that lead to socially inefficient number of firms in the industry in the long run under free entry in absence of externalities (Mankiw and Whinston, 1986). Therefore, existence of positive externalities, high setup costs and oligopolistic market structure seem to justify government interventions in these industries. However, the question arises on the efficiency of alternative subsidy policies in the long run. Is it better for a budget constrained benevolent social planner to direct subsidies to reduce setup cost compared to incentivizing firms to produce more in the long run? Can the first best equilibrium outcome be achieved in the long run through subsidization? If yes, what is the socially optimal subsidy scheme? Empirical evidences of idle capacity creation through subsidized investments further emphasize the importance of answering these questions.[1]

The objective of this article is twofold. First, it attempts to compare and contrast welfare implications of two alternative *expenditure equivalent* subsidy schemes, lump sum subsidy versus per unit subsidy, when government subsidization is necessary to promote investments

[1] See, for example, Flora et al (2014), Wu et al (2014) and Zang et al (2016).

and production of the socially desirable goods. Second, it aims to characterize the socially optimal subsidy scheme in the long run. For these purposes, this article develops a partial equilibrium model of entry in a new industry considering a fairly general framework, in which firms are profit maximizing agents and upon entry each firm incurs a fixed setup cost, produces a homogeneous good using non-increasing returns to scale technology, generates externalities that has net environmental benefit and engages in Cournot competition in the product market. The social planner is considered to be benevolent and interested in maximizing the total surplus, which is the sum of economic surplus and net environmental benefit of externalities, creation by the industry in the long run. The set up of the model allows to distinguish between 'generation externalities', which refers to externalities created by production process and/or consumption of the goods produced, and 'spill over externalities', which arises due to spill over of technology and/or pro-environmental practices from the new industry to other industries.

It demonstrates that the lump sum subsidy scheme, which offers a fixed amount of subsidy to each firm that enters the industry, attracts more number of firms in the industry in the equilibrium compared to that under the expenditure equivalent per unit subsidy scheme, which offers a fixed amount of subsidy per unit of production such that total government expenditure on subsidy remains the same as in the case of lump sum subsidy. However, the per unit subsidy scheme, not only induces each firm to produce more, but also results in higher aggregate output of the industry despite attracting less number of firms in the industry in the equilibrium compared to those under the revenue equivalent lump sum subsidy scheme. The reason for this apparently striking result is as follows. A lump sum subsidy scheme enhances industry output by inducing more entry in the industry due to its direct positive effect on firms' profits without altering their effective marginal costs. In contrast, a per unit subsidy scheme has both a direct positive effect on industry output due to its effective marginal cost reducing effect and an indirect positive effect on industry output via its entry inducing effect. When these two subsidy schemes are expenditure equivalent, under the per unit subsidy scheme the increase in industry output due to effective marginal cost reduction is larger than the loss in industry output due to relatively less entry than that under the lump sum subsidy scheme and, thus, industry output is higher under the per unit subsidy scheme. It follows that economic surplus is always higher under the per unit subsidy scheme. But, the lump sum subsidy scheme leads to higher spill over externalities by attracting more entry and, thus, unless the marginal effect of entry on net environmental benefit of externalities is sufficiently small, it helps creating greater amount of total surplus than the expenditure equivalent per unit subsidy scheme.

This article also shows that, in absence of subsidy schemes, free entry equilibrium number of firms in the industry and a firm's privately optimal level of output may be greater than, or less than, or equal to the socially optimal number of firms and per firm output, respectively, whereas the equality cannot hold for both at the same time. This is due to the presence of environmental externalities along with the presence of business stealing effect and sunk cost of entry. Interestingly, the first best equilibrium outcome can be implemented through a unique combination of per unit subsidy and lump sum subsidy. To achieve the social optimum, (i) output of each firm needs to be subsidized at a rate equal to the marginal net environmental benefit of generation externalities plus the amount of money necessary to compensate for revenue loss due to marginal increase in output, which is always positive, and (ii) each entering firm needs to be offered a lump sum subsidy equal to marginal net environmental benefit of spill over externalities less marginal revenue loss of each firm due to entry, which may be positive or negative or zero. Further, the socially optimal subsidy scheme involves positive government expenditure, unlike as in absence of externalities. In the latter case, a balanced budget tax-subsidy policy involving an entry tax and a per unit output subsidy leads to the first best equilibrium outcome in the long run.

Two strands of literature are particularly relevant for the present analysis. The first strand of literature is concerned with free entry bias. Starting with Weizsacker (1980), the issue of inefficiency of free entry equilibrium has received considerable attention in the literature. Notably, by considering a fairly general framework in which the social planner can control only the number of firms in the industry and not firm's post entry output choices, Mankiw and Winston (1986) demonstrate that free entry leads to excessive number of firms in the equilibrium under Cournot competition compared to the second best level, due to the existence of business stealing effect. Note that the first best equilibrium outcome is never achievable in the Mankiw and Winston (1986)'s framework, as in the present model in case only the lump sum subsidy is allowed for. Amir et al (2014) extends this analysis to allow for limited increasing returns to scale in production and argue that under Cournot competition free entry equilibrium number of firms is excessive even compared to the first best solution - a result that emerges in a special case of the present analysis. Contribution of this article in this stream of literature is twofold. First, it extends the analysis of inefficiency of free entry equilibrium outcome by allowing for the possibility of externalities generated by firms. Second, it analyzes relative efficacy of alternative expenditure equivalent subsidy schemes, per unit versus lump sum, and characterizes the socially optimum subsidy policy to address the problem of inefficiency of free entry equilibrium in the long run.[2]

[2] Perry (1984), Suzumura and Kiyono (1987), Nachbar et al (1998) and Amir and Lambson (2003), to name a few, also demonstrate inefficiency of free entry equilibrium in alternative scenarios. However, to the best of our knowledge, the issue of externalities has not received much attention in this stream of literature.

A large number of studies in the second strand of literature on environmental regulation under imperfect competition in the product market have attempted to analyze implications of alternative policies, including tax-subsidy policies, to protect the environment considering alternative scenarios. Although most of these studies have focused on controlling negative environmental externalities created by firms, a few recent studies have analyzed implications of alternative policy instruments on performances of pro-environment industries as well. The broad result of this stream of literature conforms with the general intuitive argument of Tinbergen that the number of policy instruments must be no less than the number of policy targets, except in special cases (Tinbergen, 1952; Arrow, 1958). For example, Katsoulacos and Xepadadeas (1995) argue that social welfare may be enhanced by imposing a license fee on entry in addition to emission tax in the case of endogenous market structure under polluting Cournot oligopoly. Extending this analysis Cato (2010) shows that the first best solution can be implemented by imposing a balanced budget combination of entry tax, emission tax and tax refund based on market share. Considering a polluting mixed duopoly Pal and Saha (2014) show that the social optimum can be achieved by taxing emission at a lower rate than the rate of abatement subsidy coupled with full nationalization of the public firm. On the other hand, Reichenbach and Requate (2012), by considering a model of vertical integration with free entry of upstream equipment manufacturers for the competitive fringe renewable energy sector and an oligopolistic traditional energy sector, demonstrate that an appropriate combination of tax in traditional energy sector and output subsidy to upstream manufacturers can result in the first best solution. Andor and Voss (2016), by distinguishing capacity externalities from generation externalities, analyze optimal subsidy policy to promote renewable energy generation in a model of peak-load pricing under uncertainty. Matsumura and Yamangishi (2017) argue that energy conservation regulation over and above a tax on energy consumption improves welfare under imperfect competition in the long run.

The present analysis characterizes socially optimal policy in oligopoly with positive externalities under endogenous market structure and further reinforces the argument that two policy instruments are necessary to implement the first best solution when there are two distinct reasons for market imperfections. It demonstrates that in the present scenario the optimal policy consists of a per unit output subsidy to induce internalization of positive externalities by firms and a lump sum tax/subsidy to correct for entry bias, except that only per unit subsidy is sufficient in a special case. Further, it shows that the optimal policy intervention to attain the first best solution calls for positive government expenditure. In reality, many governments, particularly of developing countries, face strict budget constraint and, thus, are compelled to settle for a second best solution. This article offers new insights to understand the implications of alternative tax-subsidy policies under budget constraint on performance of oligopolistic industries in the long run, an aspect which has been largely ignored in the existing literature.

The rest of the article is organized as follows. Section 2 presents the framework of the model. Implications of two alternative expenditure equivalent subsidy schemes, per unit vs. lump sum, on the long run equilibrium outcomes are analyzed in Section 3. Section 4 characterizes the optimal policy to implement the first best equilibrium outcome. Section 5 concludes. All proofs are relegated to the Appendix.

2. The Model

Consider that there is a large (infinite) number of identical profit maximizing potential entrants in a new industry. To enter the industry each firm must incur a sunk fixed cost $K(> 0)$. Upon entry firms produce homogeneous goods using identical technologies of production, face the inverse market demand function $p(Q): \mathbb{R}^+ \rightarrow \mathbb{R}^+$ where Q denotes aggregate output in the market, and engage in simultaneous move quantity competition in the product market. Production cost of firm i is given by $C(q_i)$, where q_i denotes its output.

Assumption 1: $p(Q)$ is twice continuously differentiable, $p' < 0$ and $p''Q + p' \leq 0$.

Assumption 2: $C(q)$ is twice continuously differentiable, $C(0) = 0$, $C' > 0$ and $C'' \geq 0$.

Assumptions 1-2 are standard regulatory assumptions, implying that (a) the market demand function is not too convex and (b) variable cost function is not concave.

The industry creates environmental externalities through its production process and/or consumption of the good produced. For example, although electric vehicle manufacturing process emits pollutants, greater usage of electric vehicles reduces air pollution and perhaps induces pro-environmental behaviour as well. Similar is the case for solar power generating and storage systems and wind turbines. Use of ecosystem strengthening production technologies in an industry may have both direct and indirect positive environmental externalities, where indirect externalities arise due to inter-industry technology spill over. We refer to externalities created by production process of the industry and consumption of the good produced by the industry as '*generation externalities'* and externalities created due to inter-industry technology spill over as '*spill over externalities*'.

Following the arguments of Andor and Voss (2016), inter-industry technology spill over and, thus, spill over externalities is considered to be higher in case the parent industry is less concentrated. On the other hand, direct externalities due to production process of the industry and consumption of the good produced by the industry, i.e. generation externalities, is assumed to depend on the quantity of the good produced. Let $h(Q, N)$ be the net environmental benefit due to externalities created by the industry, if N (≥ 0) firms have entered. Although the net environmental benefit due to externalities created by the industry may be positive, zero or negative, depending on relative strengths of positive and negative externalities, in this article we consider that the industry is *pro-environment* in the sense that the net environmental benefit due to externalities created by it is positive: $h(Q, N) > 0$.

Assumption 3: $h(Q,\ N)$ is twice continuously differentiable in Q (≥ 0) and N (≥ 0) and satisfies the following.

(i) $h(0,\ 0) = h(0,\ N) = 0$.

(ii) $\frac{\partial h(Q,N)}{\partial Q} = h_Q(Q,\ N) > 0 \ and \ \frac{\partial h(Q,N)}{\partial N} = h_N(Q,\ N) \geq 0 \ \forall \ Q \geq 0 \ and \ N \geq 0$

(iii) $\frac{\partial^2 h(Q,N)}{\partial Q^2} = h_{QQ}(Q,\ N) \leq 0,\ \frac{\partial^2 h(Q,N)}{\partial N^2} = h_{NN}(Q,\ N) \leq 0 \ and \ \frac{\partial^2 h(Q,N)}{\partial Q\, \partial N} = h_{QN}(Q,\ N) = 0, \forall \ Q \geq 0 \ and \ N \geq 0$.

Assumption 3 states that the net environmental benefit of the society due to externalities generated by the industry is positive and concave in Q and N. It also implies that environmental benefits due to 'generation externalities' and 'spill over externalities' created by the industry are additively separable, which we assume for simplicity. In other words, marginal net environmental benefit of generation externalities is independent of spill-over externalities and vice-versa.

The social planner designs subsidy policy to regulate the industry. Existence of positive net environmental benefit of externalities generated by firms and imperfect competition in the product market may justify such interventions in the market. It is often argued that appropriately designed subsidy policy may induce firms to internalize externalities (at least partially) and reduce distortions due to imperfect competition in the product market.

There are two alternative subsidy schemes: (a) lump-sum subsidy scheme (denoted by L), under which the government offers a lump-sum subsidy S (> 0) to each firm, and (b) per-unit subsidy scheme (denoted by U), under which each firm receives a subsidy of amount $s > 0$) per unit of production. The social planner chooses one out of these two subsidy schemes or a combination of both. Expenditure for subsidization is financed through alternative sources, which is assumed to have no implication to market demand for the good produced.[3] However, the social planner operates under a strict budget constraint, which allows for only $\bar{G}$ ($0 < \bar{G} < \infty$) amount of money for the purpose of subsidization.

Firm i's ($i = 1, 2, 3, \dots N$) profit under lump-sum and per-unit subsidy schemes, respectively, are as follows.

Profit under lump-sum subsidy: $\Pi_i^L(q_i, q_{-i}, S) = p(Q)q_i - C(q_i) - K + S$ (1)

Profit under per-unit subsidy: $\Pi_i^U(q_i, q_{-i}, s) = p(Q)q_i - C(q_i) - K + sq_i$ (2)

The government's objective is to maximize total surplus, which is given by the sum of consumer surplus (CS), producer surplus (PS) and net environmental benefit due to externalities (h), minus the government's expenditure on subsidy payments ($G \leq \bar{G}$). Let, W

[3] Note that we can ignore income effects of tax-subsidy policy in a partial equilibrium approach, as is the case in the present analysis.

denotes the total surplus. Considering that all firms are domestic and each firm produces the same level of output (q), the expression for social surplus can be written as follows.

$$W(N, q) = \left[\int_0^Q p(z)dz - p(Q)Q\right] + [N\Pi(N, q)] + h(Q,\ N) - G, \tag{3}$$

where $Q = Nq$.

Alternatively, we can also write

$$W(N, q) = \int_0^Q p(z)dz + h(Q,\ N) - NC(q) - NK \tag{4}$$

3. Expenditure Equivalent Subsidy Schemes: Lump Sum vs. Per Unit

In this section we analyse relative performance of two alternative expenditure equivalent subsidy schemes, lump sum vis-à-vis per unit. A per unit subsidy scheme and a lump sum subsidy scheme are *expenditure equivalent*, if total subsidy amount remains the same under each of these two subsidy schemes, i.e., if $N^L S = s\sum_{i=1}^{N^U} q_i^U$, where N^L denotes the number of firms in case of lump sum subsidy and N^U and q_i^U, respectively, denote the number of firms and firm i's output in case of per unit subsidy.

Subsidy per unit of production reduces effective marginal cost of each firm, which has a direct positive effect on industry output and an indirect positive effect on entry via enhancing industry profitability, which in turn has a negative effect on firms' quantity choice. On the other hand, a lump sum subsidy reduces the effective sunk entry cost and, thus, has a direct positive effect on entry, which in turn results in higher industry output. However, it is not straightforward to understand relative effectiveness of these two subsidy schemes. For an equivalent amount of expenditure on subsidy, does lump sum subsidy scheme induces more entry than per unit subsidy scheme? Which subsidy scheme results in higher industry output?

Is per unit subsidy scheme preferred over lump sum subsidy scheme from the social planner's point of view? This section attempts to answer these questions.

The dilemma of the social planner is whether to offer a subsidy of amount s (> 0) per unit of production or to offer a lump-sum subsidy of amount S (> 0) to each firm that enters the industry, such that in each case total subsidy remains the same.

Stage 1: The social planner announces the type of the subsidy scheme, per unit or lump sum. The social planner sets the rate of subsidy s per unit of output (S per firm) in case per unit (lump sum) subsidy is offered, such that total surplus is maximized subject to the budget constraint.

Stage 2: Each of the many potential entrants in the market decides whether to enter or stay out, given a certain amount of entry cost K (> 0) and the subsidy scheme offered in stage 1.

Stage 3: Firms that enter the market in the second stage engage in Cournot competition given consumers' demand and the market clears.

Within the above framework, we derive the equilibrium number of firms in the industry and the equilibrium output per firm under lump sum subsidy scheme and per unit subsidy scheme, separately, by using the backward induction method.

3.1. Lump-sum Subsidy Scheme

Let us first consider the scenario in which the government offers lump sum subsidy $S(> 0)$ to each firm. In stage 3, given N^L number of firms have entered the market, these firms engage in Cournot competition and, thus, make production decisions simultaneously and independently. The problem of firm i $(= 1, 2, \ldots N^L)$ can be written as follows.

$$\max_{q_i} \Pi_i^L(q_i, q_{-i}, S, N^L) = p(Q)q_i - C(q_i) - K + S.$$

The first order condition of the above maximization problem is,

$$p(Q) + p'(Q)q_i - C'(q_i) = 0, i = 1, 2, \ldots\ N^L; \qquad (5)$$

where $Q = \sum_{i=1}^{N^L} q_i$. Second order condition is satisfied by Assumptions 1-2. From condition (5) it is evident that, given the number of firms, lump sum subsidy S does not have any effect on equilibrium output. Now, as firms are identical, under Assumptions 1-2 each firm chooses the same level of output in the equilibrium in stage 3.

Let $q^L = q^L(N^L)$ denote the equilibrium output per firm. Then, from condition (5), we have $q^L = q^L(N^L)$. Denoting the equilibrium profits per firm under lump sum subsidy by Π^L, we can write $\Pi^L = \Pi^L(q^L, N^L, S) = p(Q^L)q^L - C(q^L) - K + S$, where $q^L = q^L(N^L)$ and $Q^L = N^L q^L(N^L)$ is the equilibrium industry output given N^L.

Lemma 1: $\frac{dq^L}{dN^L} < 0$ and $\frac{\partial q^L}{\partial S} = 0$.

Proof: See Appendix.

Lemma 1 states that output per firm under Cournot competition is decreasing in the number of firms in the market and is independent of the lump sum subsidy. As the number of firms increases, product market competition becomes more intense and each firm's residual demand curve shifts inward. This is the business-stealing effect of new entry in the industry.

Next, we consider the free-entry stage, i.e., stage 2. In this stage, given the lump sum subsidy S, the equilibrium number of firms satisfies the following zero-profit condition.

$$p(Q^L)q^L - C(q^L) - K + S = 0, \text{ where } Q^L = N^L q^L .$$

Note that, in the stage 2 equilibrium, price $p(Q^L)$ must be equal to effective average cost, which is given by the average cost $(C(q^L) - K)/q^L$ minus average subsidy S/q^L.

Let $N^L = N^L(S)$ denote the equilibrium number of firms, for any given lump sum subsidy S. The corresponding equilibrium output per firm is $q^L(S) = q^L(N^L(S))$, By denoting the equilibrium profits per firm in stage 2 by $\Pi^L(S)$, we have

$$\Pi^L(S) = \Pi^L(q^L(S),\ N^L(S),\ S) = 0$$

Finally, let $Q^L(S)$ denote the total output in stage 2 equilibrium, then

$$Q^L(S) = \sum_{i=1}^{N^L(S)} q^L(S) = N^L(S) \cdot q^L(S).$$

From the first order condition of stage 3 and zero-profit condition of stage 2 we get the following.

$$p(Q^L(S)) + p'(Q^L(S))q^L(S) - C'(q^L(S)) = 0 \quad (6)$$

$$p(Q^L(S))q^L(S) - C(q^L(S)) - K + S = 0. \quad (7)$$

Conditions (6) - (7) together imply the following.

Lemma 2: $\frac{dN^L(S)}{dS} > 0$; $\frac{dq^L(S)}{dS} < 0$; $\frac{dQ^L(S)}{dS} > 0$.

Proof: See Appendix.

Lemma 2 states that higher rate of lump sum subsidy leads to more entry, less per firm output and more industry output. These are well known comparative statics results under Cournot competition.

Finally, in stage 1 the social planner sets the rate of lump sum subsidy S by solving the following problem.

$$\left.\begin{array}{c}\max\limits_{S>0} W^L(S) = \int_0^{Q^L(S)} p(z)dz - N^L(S)C\big(q^L(S)\big) - N^L(S)K + h\big(Q^L(S), N^L(S)\big) \\ \textit{Subject to the constraint} \\ N^L(S)S \leq \bar{G}\ , \end{array}\right\} \quad (8)$$

Let $S = S^L$ solves problem (8).

3.2. Per-unit Subsidy Scheme

We now consider the per unit subsidy scheme and solve the game along the same lines as in the case of lump sum subsidy scheme. First, given N^U firms have entered the market, the profit maximization problem of firm i $(= 1, 2, 3, \dots N^U)$ in stage 3 can be written as follows.

$$\max_{q_i} \Pi_i^U(q_i, q_{-i}, S, N^U) = p(Q)q_i - C(q_i) - K + sq_i.$$

The first order condition of the above maximization problem is given by

$$P(Q) + P'(Q)q_i - C'(q_i) + s = 0; i = 1, 2, \dots N^U. \quad (9)$$

The second order condition is satisfied by Assumptions 1 and 2. Condition (9) implies that in equilibrium each firm’s marginal revenue must be equal to its marginal cost $C'(q_i)$ minus the per unit subsidy s received. Solving the system of first order conditions given by (9), we get the equilibrium output of firm $i\ (= 1, 2, \dots, N^U)$ $q_i^U = q^U = q^U(N^U, s)$. Corresponding industry output and profit of each firm are, respectively, $Q^U = N^U q^U = Q^U(N^U, s)$ and $\Pi^U = P(Q^U)q^U - C(q^U) - K + sq^U = \Pi(q^U, N^U, s)$.

Lemma 3: *(i)* $\frac{\partial q^U(N^U,s)}{\partial N^U} < 0$ *and* $\frac{\partial q^U(N^U,s)}{\partial s} > 0$, *for all* $N^U \geq 0$ *and* $s \geq 0$ *and (ii)* $\frac{dq^U(N^U,s)}{dN^U} < 0$ *for all* $(N^U, s) \in \{(N^U, s) |\ N^U q^U s = G > 0,\ N^U \geq 0 \text{ and } s \geq 0\}$, *where* G *is a constant.*

Proof: See Appendix.

It is easy to observe from Lemma 3 that (a) for any given subsidy rate s, each firm chooses a lower quantity in the equilibrium in case there are more firms in the Industry, which is due to business stealing effect, and (b) for any given number of firms in the industry, higher per unit subsidy encourages firms to produce more, because it reduces effective marginal costs.

Next, for any given rate of per unit subsidy s, the following zero-profit condition should hold in equilibrium in stage 2.

$$p(Q^U)q^U - C(q^U) - K + sq^U = 0, \text{ where } Q^U = N^U q^U \text{ and } q^U = q^U(N^U, s).$$

That is, for any given rate of per unit subsidy s, the equilibrium number of firms $N^U(s)$ is such that price $p(Q^U)$ is be equal to the average total cost $(C(q^U) - K)/q^U$ minus the per unit subsidy s. Let, $q^U(s)$, $Q^U(s)$ and $\Pi^U(s)$, respectively, denote the corresponding equilibrium output of each firm, industry output and profit of each firm. Then $q^U(s) \equiv$

$q(N^U(s), s)$, $Q^U(s) = N^U(s)q^U(s)$ and $\Pi^U(s) = 0$. Therefore, from the first order condition in stage 3 and zero-profit condition in stage 2, we have the following.

$$p(Q^U(s)) + p'\big(Q^U(s)\big)q^U(s) - C'\big(q^U(s)\big) + s = 0 \tag{10}$$

$$p(Q^U(s)\ )q^U(s) - C(q^U(s)) - K + sq^U(s) = 0. \tag{11}$$

From (10) and (11), comparative static exercise with respect to the rate of per unit subsidy s reveals the following.

Lemma 4: $\frac{dN^U(s)}{ds} > 0$; $\frac{dq^U(s)}{ds} < (=)0$, iff $p'' < (=)0$; $\frac{dQ^U(s)}{ds} > 0$.

Proof: See Appendix.

Lemma 4 implies that in the presence of imperfect competition, the output per firm under per unit subsidy scheme is either less than or equal to that in absence of any subsidy in the long run. The equality occurs only in the case of linear market demand function. Now, Combining equation (10) and (11) yields the following.

$$p'\big(Q^U(s)\big)q^U(s) - C'\big(q^U(s)\big) + \frac{C\big(q^U(s)\big)+K}{q^U(s)} = 0.$$

Thus, in the long run, the equilibrium output per firm is achieved at the level where marginal cost $C'\big(q^U(s)\big)$ equals average cost $\frac{C\big(q^U(s)\big)+K}{q^U(s)}$ plus the cost of marginal price contraction with respect to output $p'\big(Q^U(s)\big)q^U(s)$, so per unit subsidy can only affect firms' production decision through the price-contraction effect. Thus, if and only if marginal price is constant and independent of the number of firms, the price contraction effect will be the same and irrelevant to the change of market scale, then firms' production decision will be made while taking it as constant. When marginal price is decreasing on the total output, price-contraction effect becomes stronger and the cost of marginal price contraction gets larger as market scale increases. Therefore, as subsidy increases, to set off the stronger price-contraction effect, firms' output will decrease.

Notice that when individual firm's production is small relative to the market scale, there will be no price-contraction effect, because all firms are price-taker and set output at the level that equates marginal cost to average cost. Then the output per firm under per unit subsidy will be the same as in free entry, so is the socially optimal output as discussed before.

Finally, in stage 1, under per unit subsidy scheme the social planner sets the rate of subsidy s by solving the following problem.

$$\left.\begin{array}{c} \max\limits_{s>0} W^U(s) = \int_0^{Q^U(s)} p(z)dz - N^U(s)C\big(q^U(s)\big) - N^U(s)K + h\big(Q^U(s), N^U(s)\big) \\ \text{Subject to the constraint} \\ N^U(s)q^U(s)s \leq \overline{G}\,, \end{array}\right\} \qquad (12)$$

Let $s = s^U$ solves problem (12).

3.3. Comparison of Equilibrium Outcomes

In this section, we compare the efficiency of the two alternative subsidy schemes – per unit vis-à-vis lump sum. In order to guarantee the existence and uniqueness of the optimal subsidy level in each case, we assume that $W^L(S)$ is strictly concave in S and $W^U(s)$ is strictly concave in s.

Assumption 4: $\frac{d^2}{dS^2}W^L(S) < 0$ and $\frac{d^2}{ds^2}W^U(s) < 0$.

It is very straightforward to observe, from Lemma 2 and Lemma 4, that total subsidy expenditure increases with the rate of subsidy, regardless of the subsidy scheme.

Lemma 5: $\frac{d}{dS}[N^L(S)S] > 0$ and $\frac{d}{ds}[N^U(s)q^U(s)s] > 0$.

Define S^* as the unique optimal subsidy level under lump sum subsidy scheme and s^* as the unique optimal per unit subsidy rate in absence of the budget constraint, i.e., $S^* := \arg\max W^L(S)$ and $s^* := \arg\max W^U(s)$.

As we are interested in comparing the efficiency of the two alternative subsidy schemes for any given level of the government's budget, without any loss off generality we assume that the government's budget constraint is binding in the case of at least one of the two subsidy schemes.

Assumption 5: $0 < \bar{G} < \min(N^L(S^*)S^*, Q^U(s^*)s^*)$.

Note that S^L and s^U are unique solutions of constrained optimization problems (8) and (11), respectively. Now, by Assumption 4, (a) $W^L(S)$ is increasing in S for all $S < S^*$ and (b) W^U(s) is increasing in s for all $s < s^*$. Further, by Lemma 5 we have $N^L(S)S$ and $N^U(s)q^U(s)s$ are increasing is S and s, respectively, for all S and s. Therefore, it follows from Assumption 5 that S^L and s^U satisfy respective budget constraints with equality.

$$N^L S^L = \bar{G} \text{ and } Q^U s^U = \bar{G},$$

where $N^L = N^L(S^L)$, $Q^U = Q^U(s^U)$. Therefore, the resulting optimal social welfare under lump sum subsidy scheme and per unit subsidy scheme are, respectively, as follows.

$$\textit{Lump sum}: \quad W^L = W^L(S^L) = \int_0^{Q^L} P(z)dz - P(Q^L)Q^L - \bar{G} + h(Q^L, N^L)$$

$$\textit{Per unit}: \quad W^U = W^U(s) = \int_0^{Q^U} P(z)dz - P(Q^U)Q^U - \bar{G} + h(Q^U, N^U)$$

Now, comparing the equilibrium number of firms that enter the industry under two alternative subsidy schemes we obtain the following.

Proposition 1: *Suppose that Assumptions 1-5 hold. Then, the lump sum subsidy scheme induces more entry in the market in the equilibrium compared to that under the expenditure equivalent per unit subsidy scheme:* $N^L > N^U$ *for all* S *and* s *such that* $SN^L = sq^U N^U > 0$.

Proof: See Appendix.

The intuition behind Proposition 1 is as follows. A lump sum subsidy intensifies product market competition by attracting more firms in the market. In contrast, a subsidy per unit of output intensifies product market competition by inducing firms to produce more output as well as by attracting new firms in the market. It implies that for any given number of firms, although each firm receives the same amount of total subsidy under alternative expenditure equivalent subsidy schemes, per unit subsidy increases firms' profit by a lesser amount than that under lump sum subsidy. Therefore, in order to attract the same number of firms in the market, a higher amount total subsidy needs to be paid under per unit subsidy scheme than that under lump sum subsidy scheme. It follows that, to keep the total subsidy expenditure equivalent, the rate of per unit subsidy needs to be less than the required rate for attracting the same number of firms as that under lump sum subsidy scheme. As a lower rate per unit subsidy attracts fewer firms in the market, the equilibrium number of firms is less under per unit subsidy scheme compared to that under an expenditure equivalent lump sum subsidy scheme.

Proposition 2: *Suppose that Assumptions 1-5 hold. Then in the equilibrium the following is true.*

(a) *Each firm produces more output under per unit subsidy scheme than that under an expenditure equivalent lump sum subsidy scheme:* $q^L < q^U$ *for all* S *and* s *such that* $SN^L = sq^U N^U > 0$.

(b) *The total output of the product under lump sum scheme is less than the total output under an expenditure equivalent per unit subsidy scheme:* $Q^L < Q^U$ *for all* S *and* s *such that* $SN^L = sq^U N^U > 0$.

Proof: See Appendix.

Proposition 2 implies that a per unit subsidy scheme leads to, not only higher output choice by each firm, but also higher aggregate output of the industry in the equilibrium compared to those under the expenditure equivalent lump sum subsidy scheme. The mechanism driving this result is as follows. Unlike lump sum subsidy, per unit subsidy has a direct positive effect on each firm's quantity choice. It implies that, for any given number of firms in the industry, each firm produces more under per unit subsidy scheme than that under lump sum subsidy scheme. Moreover, a per unit subsidy scheme attracts less number of firms in the industry compared to the expenditure equivalent lump sum subsidy scheme (Proposition 1). A lower number of firms in the industry results in less intense product market competition, which induces each firm to produce more (Lemma 1 and Lemma 3). It follows that a per unit subsidy scheme leads to higher per firm output than the revenue equivalent lump sum subsidy scheme. Next, note that for any given subsidy scheme, although more number of firm induces each firm to produce less due to business stealing effect, contribution of an additional firm to industry output is larger than the sum of corresponding reductions in each incumbent firm's output and, thus, the industry output is higher in case there is more entry. Lump sum subsidy leads to higher industry output by inducing more entry. In contrast, per unit subsidy enhances industry output by reducing each firm's effective marginal cost as well as by attracting more firms in the industry. Interestingly, it turns out that, although under a per unit subsidy scheme fewer firms enter the industry than that under the expenditure equivalent lump sum subsidy scheme, in the former case the increase in industry output due to effective marginal cost reduction is larger than the loss in industry output due to less entry.

Finally, we turn to examine relative efficiency of alternative expenditure equivalent subsidy schemes – per unit vis-à-vis lump sum. For this purpose, we compare economic surplus ES (which is given by the sum of consumers' surplus and producers' surplus minus the expenditure on subsidy, i.e. $\int_0^Q P(z)dz - P(Q)Q + N\,\Pi - \mathrm{G}$) and total surplus W (which is the sum of economic surplus and net benefit due to externalities created by the industry) under a per unit subsidy scheme with those under the revenue equivalent lump sum subsidy scheme.

Proposition 3: *Suppose that Assumptions 1-5 hold. Then, the following is true.*

(i) *The sum of consumers' surplus and producers' surplus minus the government's expenditure on subsidy, i.e. economic surplus, is higher under a per unit subsidy scheme compared to that under the expenditure equivalent lump sum subsidy scheme.*

(ii) *If spill over externalities do not exist or if the marginal effect of the number of firms on net benefit of externalities created by the industry is sufficiently small, a per unit subsidy scheme results in higher total surplus than the expenditure equivalent lump sum subsidy scheme.*

Proof: See Appendix.

Note that (a) consumers' surplus CS $(= \int_0^Q P(z)dz - P(Q)Q)$ is strictly increasing in industry output Q and it does not directly depend on the number of firms N in the industry; (b) the equilibrium producers' surplus PS $(= N\,\Pi)$ does not vary with the type of expenditure equivalent subsidy scheme - per unit or lump sum, because in the free entry equilibrium each firm earns zero profit $(\Pi = 0)$; (c) subsidy expenditure $G(\geq 0)$ remains unchanged regardless of the subsidy scheme in place, because subsidy schemes are expenditure equivalent; and (d) net benefit of generation (spill over) externalities depends only on industry output Q (number of firms in the industry N).[4] Now, as a per unit subsidy scheme results in higher Q, but lower N, compared to the expenditure equivalent lump sum subsidy scheme, the former results in higher consumers' surplus, higher economic surplus, higher generation externalities and lower spill over externalities than the later. Clearly, if there is no spill over externalities or if net benefit from spill over externalities is sufficiently insensitive to the number of firms (i.e. if $h_N(Q,\ N)$ is sufficiently small), total surplus is also higher under a per unit subsidy scheme than that under the expenditure equivalent lump sum subsidy scheme.

4. The First Best

[4] As $h_{QN}(Q,\ N) = 0, \forall\ Q \geq 0\ and\ N \geq 0$ (by Assumption 3), $h(Q, N)$ is additively separable in Q and N.

Consider that the social planner intervenes in the market by choosing a combination of per unit subsidy s ($\gtreqless 0$) and lump sum subsidy S ($\gtreqless 0$). A negative value of a subsidy rate implies a tax. It is of importance to examine whether there exists a pair of per unit and lump sum subsidies (s, S) such that the market equilibrium outcome attains the first best.

Let (N^{**}, q^{**}) be the pair of number of firms and the output of each firm which is socially optimal, i.e., the first best. Then, (N^{**}, q^{**}) is given by the solution of the unconstrained maximization problem (13).

$$\max_{q\geq 0, N\geq 0} W(N,q) = \int_0^{Nq} p(z)dz - NC(q) - NK + h(Nq, N) \tag{13}$$

Therefore, the socially optimal pair of number of firms and per firm output (N^{**}, q^{**}) solves the following first order conditions of problem (13).[5] Because firms are identical and Assumptions 1-3 hold, socially optimal output of each firm is considered to be the same.

$$p(Nq) - C'(q) + h_Q(Q,N) = 0 \tag{14}$$

$$[p(Nq)q - C(q) - K] + h_Q(Q,N)q + h_N(Q,N) = 0 \tag{15}$$

In the social optimum, (a) given the number of firms, price is equal to marginal cost of production minus marginal net environmental benefit of generation externalities and (b), given the output per firm, cost of having an additional firm $(C(q) - K)$ is equal to the associated social gain.

[5] Second order conditions for maximization are satisfied, because $W(N,q)$ is concave in $N \geq 0$ and $q \geq 0$ by Assumptions 1-3. As firms are identical, under Assumptions 1-3 it is optimal for the social planner to set the same level of output for each firm.

Next, given the pair of subsidies (s, S), in the case of free entry the problem of firm i $(= 1, 2, \ldots N)$ can be written as follows.

$$\max_{q_i} \; \Pi_i(q_i, q_{-i}, s, S, N) = p(Q)q_i - C(q_i) + sq_i - K + S$$

The first order condition of the above maximization problem is,

$$p(Q) + p'(Q)q_i - C'(q_i) + s = 0, i = 1, 2, \ldots N;$$

where $Q = \sum_{i=1}^{N} q_i$. Second order condition is satisfied by Assumptions 1-2. As firms are identical, under Assumptions 1-2 each firm chooses the same level of output in the equilibrium in stage 3. Therefore, the free entry equilibrium pair of number of firms and output per firm (N^f, q^f) solves the following two equations.

$$p(Nq) + p'(Nq)q - C'(q) + s = 0 \tag{16}$$

$$p(Nq)q - C(q) + sq - K + S = 0. \tag{17}$$

Equation (17) is the free entry equilibrium condition $(\Pi(s, S) = 0)$. Equations (16)-(17) imply that at the free entry equilibrium under the subsidy scheme (a) given the number of firms, price is greater than the effective marginal cost $(C'(q) - s)$ and (b) given the per firm output, total cost of the marginal entrant is equal to its revenue plus subsidy.

To illustrate the possibility of divergence of the free entry equilibrium from the social optimum, let us first consider the scenario in which there is no government intervention, i.e. $S = s = 0$. Then equations (14) and (16) together imply that, for any given number of firms in the industry, privately optimal output per firm is less than the socially optimal output per firm, because firms ignore the benefit of generation externalities. On the other hand, equations (15) and (17) together imply that, for any given level of per firm output, the free entry equilibrium number of firms is less than the social optimal number, because firms do not take in to account the benefit of spill over externalities. So, there is a tension between fixing the number of firms and setting the per-firm output level.

Note that in absence of externalities and subsidy schemes, if the social planner sets the number of firms and firms are free to choose their respective quantities of output (i.e., if the social planner can only ensure a second best equilibrium outcome by directly regulating entry), free entry equilibrium number of firms will be more than the second best social optimum, because entry of a new firm reduces each incumbent's output (business stealing effect) (Mankiw and Whinston, 1986). However, in the presence of externalities, this result of excess entry may be reversed, if marginal effect of entry on net environmental benefit of spillover externalities is sufficiently large.

Now, by comparing free entry equilibrium output per firm and the number of firms in absence of subsidy schemes ($S = s = 0$) with their respective first best levels, we can show the following.

Corollary 1: *Suppose that Assumptions 1-3 hold true and* $S = s = 0$. *Then the following is true.*

a) $q^{f0} > (\leq)\, q^{**}$, *if* $\frac{h_N(Q^{**},N^{**})}{q^{**}} > (\leq) - p'(N^{f0}q^{f0})q^{f0}$, *and*

b) $N^{f0} < (\geq) N^{**}$, *if* $h_Q(q^{**}N^{f0}, N^{f0})q^{**} + h_N(q^{**}N^{f0}, N^{f0}) > (\leq) - \Pi(N^{f0}, q^{**})$;

where superscript '$f0$*' denotes the equilibrium under free entry in absence of subsidies*

Proof: See Appendix.

Corollary 1 state that the free entry equilibrium output per firm (number of firms) may be greater than, equal to, or less than the socially optimal level, depending on relative magnitudes of marginal net environmental benefits and marginal price contraction effects of entry and output. The underlying reason is as follows. First, when there is imperfect competition in the product market, output expansion by firms causes the market price to fall. This price contracting effect of output expansion discourages firms to choose higher output. On the other hand, at the socially optimal output per firm, marginal cost must equal to effective average cost, which is given by average cost minus marginal net environmental benefit of spill over externalities per unit of output (from (14)-(15)). As the total effective average cost is lower in the presence of spill over externalities compared to that in absence of spill over externalities and marginal cost is non-decreasing in output per firm, it is optimal for the social planner to choose a lower level of output per firm compared to that in absence of spill over externalities. On the other hand, the presence of spill over externalities increases society's need for more entrants, which will cause the market price to contract, so marginal cost has to be lower at the chosen level of output per firm. The stronger the spillover effect, the more output per firm has to be given up to achieve social optimum. Next, we explain the divergence of free entry equilibrium number of firms from the social optimum. Note that the marginal entrant affects total surplus via two channels: directly through profits and indirectly by creating externalities. Although firms' private incentive for entry depends only on profitability, social planners incentive depends on both profitability of firms and net environmental benefit of externalities created. For any given level of per firm output, the marginal entrant's contribution to surplus creation through externalities is positive, which induces the social planner to have more entry than that under free entry equilibrium. On the other hand, given the per firm output at the social optimal level ($q = q^{**}$), the marginal entrant contributes in terms of negative profits, which induces the social planner to have less number of firms than the free entry equilibrium level. The relative magnitudes of these two opposing effects determine whether free entry equilibrium number of firms will be excessive or insufficient or socially optimal.

Further, note that both $N^{f0} = N^{**}$ and $q^{f0} = q^{**}$ cannot be true at the same time, as there does not exist any (N, q) pair of number of firms and per unit output that satisfies equations (14)-(15) as well as equations (16)-(17) when $S = s = 0$, for any $h_Q \geq 0$ and $h_N \geq 0$. . It implies that, if $S = s = 0$, the free entry equilibrium outcome is not socially optimal, regardless of whether the industry creates externalities or not. The extent of divergence of free entry equilibrium outcome from the social optimum crucially depends on the strength of network externalities created by the industry and characteristics of market demand and cost functions.

Corollary 1 also implies that, if $h_Q = h_N = 0$, $q^{f0} < q^{**}$ and $N^{f0} > N^{**}$ (because, $\Pi(N^{f0}, q^{**}) < \Pi(N^{f0}, q^{f0}) = 0)$. That is, in absence of externalities, the free entry equilibrium output of each firm is insufficient and the free entry equilibrium number of firms is excessive compared to the first best solution. Clearly, the result of Amir et al (2014) emerges in a special case of the present analysis.

Finally, we turn to examine whether the social planner can implement the first best equilibrium outcome by intervening in the market through a tax-subsidy scheme. Comparing equations (14)-(15) with equations (16)-(17), we can say that there exists a pair of per unit and lump sum subsidies (s, S), as in Proposition 4, such that the market equilibrium outcome attains the first best.

Proposition 4: *Suppose that Assumptions 1-3 hold. Then, the social planner will set* $s^{**} = h_Q(Q^{**}, N^{**}) - p'(N^{**}q^{**})q^{**} > 0$ *and* $S^{**} = p'(N^{**}q^{**})(q^{**})^2 + h_N(Q^{**}, N^{**}) \gtreqless 0$ *in order to implement the social optimum.*

Proof: See Appendix.

Proposition 4 says that, in the presence of positive net environmental benefit of externalities created by the industry, the social planner will offer a combination of per unit subsidy and lump sum subsidy/tax to achieve the social optimum. The per unit subsidy rate needs to be set equal to the amount that compensates for revenue loss due to marginal increase in output plus the marginal net environmental benefit of generation externalities. The reason is, at the free entry equilibrium number of firms in the industry, socially optimal output per firm is more than the privately optimal output per firm. On the other hand, if the social planner does not have control over firms' output choice, in absence of spill over externalities there will be excess entry due to business stealing effect (Mankiw and Whinston, 1986), which calls for an entry tax equal to marginal loss in revenue of each firm due to entry. However, in the presence of generation externalities there is a social gain of entry, which is equal to marginal net environmental benefit of spill over externalities. Therefore, in the presence of generation externalities, free entry equilibrium number of firms will be insufficient (excessive) from the social planner's point of view, if marginal loss in revenue of each firm due to entry is less (more) than marginal net environmental benefit of spill over externalities. It implies that the social planner needs to offer a lump sum subsidy equal to marginal net environmental benefit of spill over externalities less marginal revenue loss of each firm due to entry, which may be positive or negative depending on whether the former dominates the later or not. In sum, the social planner always needs to directly influence both entry decisions (by offering the lump sum subsidy S^{**} to each firm) and post entry behavior of firms (by offering the subsidy s^{**}

per unit of production) to achieve the first best equilibrium outcome, except when marginal net environmental benefit of spill over externalities exactly compensates the marginal revenue loss of each firm due to entry. In the latter case only per unit subsidy at rate s^{**} is necessary to achieve the first best. Intervention through a lump sum subsidy alone can never lead to the first best.

From Proposition 4 it follows that to achieve the first best equilibrium outcome the social planner needs to spend $G^{**} = N^{**}\, [s^{**}q^{**} +\ S^{**}] = N^{**}\big[h_Q(Q^{**}, N^{**}) + h_N(Q^{**}, N^{**})\big]$ (> 0) amount of money to subsidize the industry. It is easy to observe that, in absence of externalities (i.e. when $h(Q, N) = 0\ \forall\ Q,\ N \geq 0$), to implement the first best equilibrium outcome it is necessary to subsidize production at the rate $s^{**}|_{h=0} = -\,p'(N^{**}q^{**})q^{**}(> 0)$ and tax entry at the rate $T^{**}|_{h=0} = -S^{**}|_{h=0} = -p'(N^{**}q^{**})(q^{**})^2(> 0)$, which balances the budget ($G^{**}|_{h=0} = 0$).

5. Conclusion

In this article we have developed a partial equilibrium model of entry in a Cournot oligopolistic industry with externalities, which create positive net environmental benefit. The model allows us to distinguish between generation externalities and spill over externalities. First, by comparing and contrasting the long run equilibrium outcomes under two most commonly used subsidy schemes, subsidy per unit of production versus lump sum subsidy, we have demonstrated the following. (i) A per unit subsidy scheme results in production of more output by each entering firm as well as higher aggregate output of the industry, despite attracting fewer firms in the industry, in the long run equilibrium compared to those under the expenditure equivalent lump sum subsidy scheme. (ii) Although the equilibrium economic surplus is always higher in the case of per unit subsidy scheme, the expenditure equivalent lump sum subsidy scheme generates higher total surplus in the equilibrium unless the strength of spill over externalities is sufficiently weak. These results suggest that budget constrained governments' optimal choice of subsidy policy for industries with positive externalities crucially depends on marginal net environmental benefit of spill over externalities. Next, by comparing the free entry equilibrium outcome with the first best solution, we show that the free entry equilibrium is socially inefficient even in the presence of positive externalities and, unlike as in absence of externalities, the free entry equilibrium number of firms and per firm output in the presence of positive externalities may be less than, equal to, or greater than the socially optimal level depending on strengths of generation and spill over externalities. It also shows that the first best solution is implementable by offering each entering firm a unique combination of per unit subsidy and lump sum subsidy/tax, which requires positive government expenditure.

We have analyzed free entry bias and implications of subsidy policies on industry structure and market efficiency in the presence of positive environmental externalities created by firms by considering a new industry in isolation. The present analysis, therefore, ignores possible demand and cost interdependencies faced by firms in the industry under consideration with their counterparts in other industries. For example, electric cars and cars using fossil fuels are considered to be imperfect substitutes. Intuitively, we can say that in the presence of such demand interdependencies per unit subsidy will assume greater importance than that in absence of any demand interdependencies, as in the former case per unit subsidy will also have a demand shifting effect from the polluting industry to the pro-environment industry. Nonetheless, it seems to be important to extend the present analysis to understand the implications of interdependencies between sectors on optimal policy choice in the presence of externalities. We leave this for future research.

APPENDIX

Proof of Lemma 1

By differentiating equation (5) with respect to N^L, we have

$$p' q^L + p' N^L \frac{dq^L}{dN^L} + p'' {q^L}^2 + p'' Q^L \frac{dq^L}{dN^L} + p' \frac{dq^L}{dN^L} - C'' \frac{dq^L}{dN^L} = 0,$$

where $Q^L = N^L q^L$, $q^L = q^L(N^L)$, $p' = p'(Q^L)$, $p'' = p''(Q^L)$, $C'' = C''(q^L)$. Solving the equation for $\frac{dq^L}{dN^L}$, we have

$$\frac{dq^L}{dN^L} = -\frac{p' q^L + p'' {q^L}^2}{p' N^L + p'' Q^L + p' - C''}.$$

By Assumptions 1 and 2, $\frac{dq^L}{dN^L} < 0$. Further, because q^L is independent of S for any given N^L, we have $\frac{\partial q^L}{\partial S} = 0$.

[QED]

Proof of Lemma 2

Differentiating equations (6) and (7) with respect to S, we have

$$H \begin{pmatrix} \frac{dq^L(S)}{dS} \\ \frac{dN^L(S)}{dS} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \qquad (A.1)$$

where

$$H = \begin{pmatrix} p' N^L + p'' Q^L + p' - C'' & p' q^L + p'' {q^L}^2 \\ p' Q^L + p - C' & p' {q^L}^2 \end{pmatrix}.$$

$$|H| = p' {q^L}^2 (p' - C'') - (p - C') \left(p' q^L + p'' {q^L}^2 \right).$$

Note that equation (6) implies $p - C' = -p' q^L > 0$. Therefore, by Assumptions 1 and 2, we have

$$|H| > 0.$$

Now, as $|H| > 0$ and Assumptions 1 and 2 hold true, we have

$$\frac{dq^L(S)}{dS} = \frac{P' q^L + P'' {q^L}^2}{|H|} < 0,$$

$$\frac{dN^L(S)}{dS} = -\frac{P' N^L + P'' Q^L + P' - C''}{|H|} > 0,$$

$$\frac{dQ^L(S)}{dS} = \frac{\partial Q^L}{\partial q^L} \frac{dq^L}{dS} + \frac{\partial Q^L}{\partial N^L} \frac{dN^L}{dS} = \frac{-P' q^L + C''}{|H|} > 0.$$

[QED]

Proof of Lemma 3

First we define function

$$M(q^U, N^U, s) \equiv p(Q^U) + p^{'}(Q^U)q^U - C^{'}(q^U) + s,$$

where $Q^U = N^U q^U$, $q^U = q^U(N^U, s)$. Because $q_i^U = q^U$ satisfies equation (9), $M(q^U, N^U, s) = 0$. Therefore, we have

$$\frac{\partial q^U}{\partial N^U} = -\frac{\frac{dM}{dN^U}}{\frac{dM}{dq^U}} = -\frac{p^{'}q^U + p^{''}q^{U^2}}{p^{'}N^U + p^{''}Q^U - C^{''}} < 0, \text{ by Assumptions 1 and 2.}$$

Similarly,

$$\frac{\partial q^U}{\partial s} = -\frac{\frac{dM}{ds}}{\frac{dM}{dq^U}} = -\frac{1}{p^{'}N^U + p^{''}Q^U - C^{''}} > 0, \text{ by Assumptions 1 and 2.}$$

Next, we have proved that $\frac{\partial q^U(N,s)}{\partial N} < 0$, as $q^U = q^U(N^U, s)$.

Note that $\frac{dq^U(N^U,s)}{dN^U} = \frac{\partial q^U(N^U,s)}{\partial N^U} + \frac{\partial q^U(N^U,s)}{\partial s}\frac{\partial s}{\partial N^U}$.

Consider that N^U and s such that $N^U s q^U(N^U, s) = Constant$. Then, by differentiating both sides of this condition by N^U, we get

$sq^U(N^U, s) + N^U q^U(N^U, s)\frac{\partial s}{\partial N} + N^U s \frac{\partial q^U(N^U,s)}{\partial s} = 0$. It implies that

$$\frac{\partial s}{\partial N^U} = -\frac{s}{N^U q^U}\left(N^U \frac{\partial q^U(N^U, s)}{\partial N^U} + q^U\right)$$

$$= -\frac{s}{N^U q^U}\left(-\frac{p^{'}q^U + p^{''}q^{U^2}}{p^{'}N^U + p^{''}Q^U - C^{''}} N^U + q^U\right)$$

$$= -\frac{s}{N^U q^U}\left(\frac{-C^{''}q^U}{p^{'}N^U + p^{''}Q^U - C^{''}}\right)$$

$$< 0, \text{by Assumptions } 1-2.$$

Therefore, we have the following.

$$\frac{dq^U(N^U, s)}{dN^U} < 0$$

[QED]

Proof of Lemma 4

Differentiating equation (10) and (11) with respect to s, we have

$$I\begin{pmatrix}\frac{dq^U(s)}{ds} \\ \frac{dN^U(s)}{ds}\end{pmatrix} = \begin{pmatrix}-1 \\ -q^U\end{pmatrix}, \qquad (A.2)$$

where

$$I = \begin{pmatrix} p'N^U + p''Q^U + p' - C'' & p'q^U + p''{q^U}^2 \\ p'Q^U + p - C' + s & p'{q^U}^2 \end{pmatrix}.$$

$|I| = p'{q^U}^2(p' - C'') - (p - C' + s)\left(p'q^U + p''{q^U}^2\right).$

By equation (10), $p - C' + s = -P'q^U > 0$. Therefore, by Assumptions (1) and (2), we have $|I| > 0$. Thus,

$$\frac{dq^U(s)}{ds} = \frac{-p'{q^U}^2 + q^U\left(p'q^U + p''{q^U}^2\right)}{|I|} = \frac{p''{q^U}^3}{|I|} \leq 0,$$

with equality being satisfied if and only if $P'' = 0$.

$$\frac{dN^U(s)}{ds} = \frac{-q^U(p'N^U + p''Q^U + p' - C'') + (p'Q^U + p - C' + s)}{|I|} = -\frac{q^U(p''Q^L + 2p' - C'')}{|I|} > 0,$$

$$\frac{dQ^U(s)}{ds} = \frac{\partial Q^U}{\partial q^U}\frac{dq^U}{ds} + \frac{\partial Q^U}{\partial N^U}\frac{dN^U}{ds} = -\frac{{q^U}^2(2p' - C'')}{|I|} > 0.$$

[QED]

Proof of Proposition 1

To prove: $N^L > N^U$ *for all* S *and* s *such that* $SN^L = sq^UN^U = \bar{G}$.

First, we prove that, if $N^L = N^U = \bar{N}$, $N^LS^L < N^Uq^U(N^U)s^0$, where s^0 is the rate of subsidy needed to induce $N^U(= \bar{N})$ firms to enter under the per unit subsidy scheme. That is, if the government wants to induce entry of the same number of firms in the market regardless the type of subsidy scheme, it needs to pay more amount of total subsidy to the industry under per unit subsidy scheme comparing to that under lump sum subsidy scheme.

Let $N^L = N^U = \bar{N}$, then zero-profit conditions must hold in free entry equilibrium under both subsidy schemes:

$$p(\bar{N}\,q^L)q^L - C(q^L) - K + S^L = 0, \qquad \text{(A.3)}$$

$$p(\bar{N}\,q^U)q^U - C(q^U) - K + s^0q^U = 0, \qquad \text{(A.4)}$$

$$(\text{A}.4) \Rightarrow p(\bar{N}\,q^U)q^U - C(q^U) - K + S^L = S^L - s^0q^U. \qquad \text{(A.5)}$$

We have proved that, given the number of firms, the Cournot-Nash equilibrium output satisfies $\frac{\partial q^U(N^U,s)}{\partial s} > 0$ (Lemma 3). Therefore, if $N^L = N^U = \bar{N}$, $q^L < q^U$. (A.6)

(A.3) & (A.6) $\Rightarrow$ $P(\bar{N}\, q^U)q^U - C(q^U) - K + S^L < 0$. (A.7)

(A.5) & (A.7) $\Rightarrow$ $S^L - s^0 q^U < 0$,

$\Rightarrow$ $S^L < s^0 q^U$,

$\Rightarrow$ $N^L S^L < N^L s^0 q^U(s^0, N^L)$. (A.8)

Now, from Lemma 5, we know

$\frac{d}{ds} N^U(s) q^U(s) s > 0$. (A.9)

[(A.8) and (A.9)] $\Rightarrow s^U < s^0$, for $N^L S^L = N^L s^U q^U$ to be satisfied.

But, if $s^U < s^0$, we must have $N^U < N^L$, because by Lemma 4, $\frac{dN^U}{ds} > 0$.

Therefore, if under alternative subsidy schemes – per unit and lump sum – the total amount of subsidy payment to the industry remains the same, the free-entry equilibrium number of firms will be less under per unit subsidy scheme compared to that under lump sum subsidy scheme:

$$N^U < N^L, \text{ if } N^L S^L = N^U s^U q^U(N^U, s^U).$$

[QED]

Proof of Proposition 2

(a) From Proposition 1, we have

$$N^U < N^L$$

(A.10)

From Lemma 1 and Lemma 3 we have, for any given N,

$$q^L(N) < q^U(N)$$

(A.11)

By Lemma 3, we have

$$\frac{dq^U(N,s)}{dN} < 0. \quad (A.12)$$

Therefore, we have the following.

$$(i)\ \ q^U(N^L) < q^U(N^U), \qquad by\ (A.12)\ \ and\ (A.10).$$

$$(ii)\ \ q^L(N^L) < q^U(N^L), \qquad by\ (A.11).$$

From (*i*) and (*ii*), we get

$$q^L(N^L) < q^U(N^L) < q^U(N^U).$$

(A.13)

Alternatively,

$$q^L(N^U) < q^U(N^U), \qquad by\ (A.11). \qquad (A.11a)$$

$$\frac{dq^L}{dN^L} < 0, \qquad by\ Lemma\ 1 \qquad (A.11b)$$

Thus, (A.10), (A.11), (A.11a) and (A.11b) together imply that $q^L(N^L) < q^U(N^U)$.

(b)

We have the following.

$$p(\,Q^U)q^U - C(q^U) - K + s^U q^U = 0\ \ \big(by\ equation\ (11)\big), \qquad (A.14)$$

$$p(\,Q^L)q^L - C(q^L) - K + S^L = 0\ (by\ equation\ (7)), \qquad (A.15)$$

$$N^L > N^U\ \ (by\ Proposition\ 1), \qquad (A.16)$$

$$q^L < q^U\ \big(by\ Proposition\ 2(a)\big), \qquad (A.17)$$

and

$$N^L S^L = N^U s^U q^U\ (by\ the\ expenditure\ equivalant\ criterion). \qquad (A.18)$$

Now,

$$(A.18) \quad \Rightarrow \quad \frac{N^L q^L S^L}{q^L} = Q^U s^U \quad \Rightarrow \quad \frac{Q^L S^L}{q^L} = Q^U s^U \ \Rightarrow \frac{Q^L}{Q^U} = \frac{q^L s^U}{S^L} \qquad (A.19)$$

For any given p, let us define q_0 as follows.

$$q_0 = \underset{q}{Argmax}[pq - C(q) + s^U\, q] \qquad (A.20)$$

Therefore, for any given p and for any q_1 and q_2 such that $q_1 < q_2 \le\ q_0$, we have the following, because $(pq - C(q) + s^U\, q)$ is concave in q (by Assumption 2).

$$pq_1 - C(q_1) + s^U q_1 < pq_2 - C(q_2) + s^U q_2 \qquad (A.21)$$

Note that, for any given p, $\frac{\partial}{\partial q}[pq - C(q) + s^U q] = p - C^{'}(q) + s^U$ and $\frac{\partial^2}{\partial q^2}[pq - C(q) + s^U q] = -C^{''}(q) \leq 0$. It follows that, when $C^{''}(q) = 0$, $q_0 = \begin{cases} \infty, & if\ p > C^{'}(q) - s^U \\ 0, & if\ p < C^{'}(q) - s^U \end{cases}$.

Let us first prove that, if $p = p(Q^U)$, $q^U < q_0$.

By equation (10), $p'(Q^U)q^U + p(Q^U) - C'(q^U) + s^U = 0$. It implies that $p(Q^U) = C^{'}(q^U) - s^U - p^{'}(Q^U)q^U > C^{'}(q^U) - s^U$ (by Assumption 1). However, if $p = p(Q^U)$ and $C^{''}(q^U) = 0$, $p > C^{'}(q^U) - s^U = C^{'}(q^0) - s^U \Rightarrow q_0 = \infty$. It implies that $q^U < q_0$, whenever $p = p(Q^U)$ and $C^{''}(q^U) = 0$.

Further, when $C^{''} > 0$ and $p = p(Q^U)$,
$(A.21) \Rightarrow p(Q^U) - C^{'}(q^0) + s^U = 0$ and
$(10) \Rightarrow P(Q^U) - C^{'}(q^U) + s^U = -P^{'}(Q^U)q^U > 0$. Therefore, we must have $q^U < q_0$ whenever $p = p(Q^U)$ and $C^{''}(q^U) > 0$. Thus, if $p = p(Q^U)$, $q^U < q_0$ must hold true regardless of whether $C^{''}(q^U) = 0$ or $C^{''}(q^U) > 0$.

Now, as by *(A.17)* $q^L < q^U$ and we have proved that $q^U < q_0$, we have
$p(Q^U)q^L - C(q^L) + s^U q^L < p(Q^U)q^U - C(q^U) + s^U q^U = K$, by inequality *(A.21)* and equation (1). It implies the following.

$$P(Q^U)q^L - C(q^L) + s^U q^L - K < 0. \qquad (A.22)$$

Now, suppose that $Q^L = Q^U$. Then $p(Q^L) = p(Q^U)$ and, thus,

$$\begin{aligned} (A.22) &\Rightarrow p(Q^U)q^L - C(q^L) + s^U q^L - K < 0 \\ &\Rightarrow p(Q^L)q^L - C(q^L) - K + S^L < S^L - s^U q^L \\ &\Rightarrow 0 < S^L - s^U q^L \ \big(\text{by equation } (A.15)\big) \\ &\Rightarrow \frac{s^U q^L}{S^L} < 1 \qquad (A.22b) \end{aligned}$$

By (*A.19*) and (*A.22b*), $\frac{Q^L}{Q^U} < 1$. Hence the contradiction. So, $Q^L \neq Q^U$.

Next, suppose that $Q^L > Q^U$. Then $p(Q^L) < p(Q^U)$. Therefore, $p(Q^L)q^L - C(q^L) + s^U\, q^L < p(Q^U)q^L - C(q^L) + s^U\, q^L < K$, by (*A.22*). It implies that

$$p(Q^L)q^L - C(q^L) - K + S^L < S^L - s^U\, q^L \;\Rightarrow 0 < S^L - s^U\, q^L$$

$$\Rightarrow \frac{s^U\, q^L}{S^L} < 1 \;\Rightarrow \frac{Q^L}{Q^U} < 1 \text{ (by } (A.19)) .$$

Hence the contradiction. So, $Q^L \ngtr Q^U$.

We have proved that $Q^L \neq Q^U$ and $Q^L \ngtr Q^U$. Therefore, we must have $Q^L < Q^U$.

[QED]

Proof of Proposition 3

Let $ES = Economic\ Surplus$

$$= (Consumers'\ Surplus) + (Producers'Surplus) - Total\ Subsidy$$

$$= \left[\int_0^Q P(z)dz - P(Q)Q\right] + N\,\Pi - \text{G}.$$

We have the following.

(a) Under free entry, profit of each firm $\Pi = 0$, regardless of the subsidy scheme.

(b) $\frac{d}{dQ}\left[\int_0^Q P(z)dz - P(Q)Q\right] = P(Q) - P(Q) - P'(Q)Q = -P'(Q) > 0$, by Assumption 1.

(c) $Q^L < Q^U$, by Proposition 2.

(d) Because the two subsidy schemes are equivalent in terms of total expenditure on subsidy, total subsidy (G) does not depend on the subsidy scheme.

From (a)-(d) it follows that $ES(Q^L) < ES(Q^U)$, i.e, economic surplus (the sum of consumers surplus and producers surplus minus the expenditure on subsidy) is more under a per unit subsidy scheme compared to that under the revenue equivalent lump sum subsidy scheme.

Next, total surplus is given by

$$W = ES + Net\ Benefit\ of\ Externalities = ES + h(Q, N),$$

As $ES(Q^L) < ES(Q^U)$, for $W^L < W^U$ to hold true it is sufficient to have the following.

$$h(Q^L, N^L) < h(Q^U, N^U) \qquad (A.23)$$

$(A.23) \Rightarrow \frac{h(Q^U,N^U)-h(Q^L,N^U)}{Q^U-Q^L}(Q^U - Q^L) - \frac{h(Q^L,N^L)-h(Q^L,N^U)}{N^L-N^U}(N^L - N^U) > 0$. Therefore,

it is straightforward to observe that the condition (A.23) is satisfied,

(a) if $h_N(Q, N) = 0$, because $h_Q(Q, N) > 0$ (by Assumption 3) and $Q^L < Q^U$ (by Proposition 2); or

(b) if $h_N(Q, N) > 0$ but sufficiently small.

Now,

$W^L < W^U$

$\Leftrightarrow ES(Q^L) + h(Q^L, N^L) < ES(Q^U) + h(Q^U, N^U)$

$\Leftrightarrow \left[\frac{h(Q^L,N^L)-h(Q^L,N^U)}{N^L-N^U}\right](N^L - N^U) < [ES(Q^U) - ES(Q^L)] + \left[\frac{h(Q^U,N^U)-h(Q^L,N^U)}{Q^U-Q^L}\right](Q^U - Q^L)$

As $h_{QN}(Q, N) = 0, \forall\, Q \geq 0$ and $N \geq 0$ (by Assumption 3), $h(Q, N)$ is additively separable in Q and N. So, we can write $h(Q, N) = h^{gen}(Q) + h^{spill}(N)$, where $h^{gen}(Q)$ and $h^{spill}(Q)$ denote, respectively, net benefit due to general externalities and net benefit due to spill over externalities. Therefore,

$W^L < W^U$

$\Leftrightarrow \left[\frac{h^{spill}(N^L)-h^{spill}(N^U)}{N^L-N^U}\right](N^L - N^U) < [ES(Q^U) - ES(Q^L)] + \left[\frac{h^{gen}(Q^U)-h^{gen}(Q^L)}{Q^U-Q^L}\right](Q^U - Q^L)$

$\Leftrightarrow \frac{\Delta h^{spill}}{\Delta \mathrm{N}}\Delta \mathrm{N} < [ES(Q^U) - ES(Q^L)] + \frac{\Delta h^{gen}}{\Delta \mathrm{Q}}\Delta \mathrm{Q}$,

where $\Delta \mathrm{N} = N^L - N^U > 0$, $\Delta \mathrm{Q} = Q^U - Q^L > 0$, $\Delta h^{spill} = h^{spill}(N^L) - h^{spill}(N^U) > 0$ and $\Delta h^{gen} = h^{gen}(Q^U) - h^{gen}(Q^L) > 0$.

[QED]

Proof of Corollary 1

(N^{f0}, q^{f0}) is the solution of the system of equations (16)-(17), if $s = S = 0$. Therefore, we have the following.

$$p(N^{f0}q^{f0}) + p'(N^{f0}q^{f0})q^{f0} - C'(q^{f0}) = 0 \qquad (A.24)$$

$$p(N^{f0}q^{f0})q^{f0} - C(q^{f0}) - K = 0. \qquad (A.25)$$

(N^{**}, q^{**}) is the solution of the system of equations (14)-(15). Therefore, we have the following.

$$p(N^{**}q^{**}) - C'(q^{**}) + h_Q(Q^{**}, N^{**}) = 0 \qquad (A.26)$$

$$[p(N^{**}q^{**})q^{**} - C(q^{**}) - K] + h_Q(Q^{**}, N^{**})q^{**} + h_N(Q^{**}, N^{**}) = 0 \qquad (A.27)$$

a) First, we prove that $q^{f0} > (\leq)\ q^{**}$, $if\ \frac{h_N(Q^{**},N^{**})}{q^{**}} > (\leq) - p'(N^{f0}q^{f0})q^{f0}$.

By dividing equation (A.25) with q^{f0}, we get

$$p(N^{f0}q^{f0}) - \frac{C(q^{f0})+K}{q^{f0}} = 0. \qquad \text{(A.28)}$$

Equation (A.24) and equation (A.28) together implies

$$C'(q^{f0}) - \frac{C(q^{f0})+K}{q^{f0}} = p'(N^{f0}q^{f0})q^{f0} < 0 \text{ (by Assumption 1)} \qquad \text{(A.29)}$$

Similarly, from equations (A.26) and (A.27), we get

$$C'(q^{**}) - \frac{C(q^{**})+K}{q^{**}} = -\frac{h_N(Q^{**},N^{**})}{q^{**}} < 0 \text{ (by Assumption 3)} \qquad \text{(A.30)}$$

Further note that $\forall\ q < (>) q_{min},\ C'(q) < (>) \frac{C(q)+K}{q}$, where q_{min} is such that $C'(q_{min}) = \frac{C(q_{min})+K}{q}$. Therefore, both $q^{f0} < q_{min}$ and $q^{**} < q_{min}$ hold true.

Let $F(q) = C'(q) - \frac{C(q)+K}{q}$. It is evident that $F(q)$ is continuous and differentiable in q (by Assumption 2). Differentiating $F(q)$ with respect to q, we get

$$F'(q) = C''(q) - \frac{C'(q)}{q} + \frac{C(q)+K}{q^2} = C''(q) - \frac{1}{q}[C'(q) - \frac{C(q)+K}{q}].$$

Clearly, for all $q < q_{min}$, $F'(q) > 0$ (by Assumption 2). It follows that

(a) $F(q^{**}) > F(q^{f0}) \Leftrightarrow q^{**} > q^{f0}$,

(b) $F(q^{**}) = F(q^{f0}) \Leftrightarrow q^{**} = q^{f0}$ and

(c) $F(q^{**}) < F(q^{f0}) \Leftrightarrow q^{**} < q^{f0}$.

Therefore, we have the following.

(a) $q^{*} > q^{f0} \Leftrightarrow \frac{h_N(Q^{**},N^{**})}{q^{**}} < -p'(N^{f0}q^{f0})q^{f0}$,

(b) $q^{*} = q^{f0} \Leftrightarrow \frac{h_N(Q^{**},N^{**})}{q^{**}} = -p'(N^{f0}q^{f0})q^{f0}$ and

$$\text{(c)}\ \ q^* < q^{f0} \iff \frac{h_N(Q^{**},N^{**})}{q^{**}} > -p^{'}(N^{f0}q^{f0})q^{f0}.$$

b) Now, we prove that $N^{f0} < (\geq) N^{**}$, *if*

$$h_Q(q^{**}N^{f0},N^{f0})q^{**} + h_N(q^{**}N^{f0},N^{f0}) > (\leq) - \Pi(N^{f0},q^{**}).$$

Differentiating $W(N,q)$ with respect to N, we get the following.

$$\frac{\partial W(N,q)}{\partial N} = [p(Nq)q - C(q) - K] + h_Q(Q,N)q + h_N(Q,N)$$

$$= \Pi(N,q) + h_Q(Q,N)q + h_N(Q,N)$$

Therefore, $\left.\frac{\partial W(N,q)}{\partial N}\right|_{\substack{N=N^{f0}\\ q=q^{**}}} = \Pi(N^{f0},q^{**}) + h_Q(q^{**}N^{f0},N^{f0})q^{**} + h_N(q^{**}N^{f0},N^{f0})$.

On the other hand, we have $\left.\frac{\partial W(N,q)}{\partial N}\right|_{\substack{N=N^{**}\\ q=q^{**}}} = 0$.

Now, note that $\frac{\partial^2 W(N,q)}{\partial N^2} = p^{'}(Nq)q^2 + h_{QQ}(Q,N)q^2 + h_{QN}(Q,N)q + h_{NQ}(Q,N)q + h_{NN}(Q,N) < 0$, by Assumption 1 and Assumption 3. That is, the function $W(N,q)$ is strictly concave in N. Therefore, $N^{f0} < (\geq) N^{**}$, if and only if $\left.\frac{\partial W(N,q)}{\partial N}\right|_{\substack{N=N^{f0}\\ q=q^{**}}} > (\leq) \left.\frac{\partial W(N,q)}{\partial N}\right|_{\substack{N=N^{**}\\ q=q^{**}}}$.

It follows that

$$N^{f0} < (\geq) N^{**} \iff \Pi(N^{f0},q^{**}) + h_Q(q^{**}N^{f0},N^{f0})q^{**} + h_N(q^{**}N^{f0},N^{f0}) > (\leq) 0$$

$$\iff h_Q(q^{**}N^{f0},N^{f0})q^{**} + h_N(q^{**}N^{f0},N^{f0}) > (\leq) - \Pi(N^{f0},q^{**}).$$

[QED]

Proof of Proposition 4

Solving equation (14) and equation (16), we obtain

$$s = h_Q(Q,N) - p^{'}(Nq)q. \qquad (A.31)$$

Solving equation (15) and equation (17), we obtain

$$S = h_Q(Q,N)q + h_N(Q,N) - sq. \qquad (A.32a)$$

By substituting the value of s from equation (A.31) to equation (A.32a) and then solving for S, we get the following

$$S = p^{'}(Nq)(q)^2 + h_N(Q,N) \qquad (A.32b)$$

Note that, under the Assumptions 1-5, (N^{**}, q^{**}) and (N^f, q^f) are unique solutions of the system of equations (14)-(15) and the system of equations (16)-(17), the necessary and sufficient condition for $(N^{**}, q^{**}) \equiv (N^f, q^f)$ to be true is $(s, S) = (s^{**}, S^{**})$, where s^{**} and S^{**} are given by (A.33).

$$\begin{cases} s^{**} = h_Q(Q^{**}, N^{**}) - p'(N^{**}q^{**})q^{**} \\ S^{**} = p'(N^{**}q^{**})(q^{**})^2 + h_N(Q^{**}, N^{**}) \end{cases} \quad (A.33)$$

Clearly, by Assumption 1 and Assumption 3, $s^{**} > 0$ and $S^{**} \gtreqless 0$. The sign of the later inequality depends on relative magnitudes of $p'(N^{**}q^{**})(q^{**})^2$ and $h_N(Q^{**}, N^{**})$.

[QED]

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