The Weakness of Weak Ties in Referrals

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Abstract

I build a model of employee referrals with two main features: unemployed workers choose which employed workers to ask for referrals based on the type of ties (weak or strong) they have with them, and firms try to infer some information about the abilities of the unemployed workers through the recommendations of its employees. The model predicts that the returns to using a tie vary with the unemployed worker's ability, the tie strength, and the proportion of workers who have access to different types of ties. I then develop two applications of this model. (1) There is significant evidence suggesting that the black-white wage gap widens as one moves up the wage hierarchies of the private sector in the US. The model shows that the lack of access to strong ties for blacks can be behind this empirical finding. (2) In the second application, I explore some implications of the employee referrals for job search. The model can explain (i) the mixed evidence about the use of different types of ties in job search, and (ii) the mixed evidence about the wage differentials between workers who found jobs through referrals and workers who found jobs by formally applying to firms.

Keywords: Employee Referrals, Strength of Ties, Wage Differentials, and Wage Hierarchies.

JEL Codes: D82, D85, J31, J70.

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1 Introduction

It is well known that on average whites earn higher wages than blacks in the US.[1] A little less known fact is that the wage gap between whites and their black counterparts (same individual characteristics) widens as one moves up the wage hierarchies. Kaufman (1983) was the first to demonstrate that black men face the greatest disadvantage in labor market divisions (based on occupation and industry) at the high end of the wage hierarchy (based on mean wages). Using more contemporary data, Grodsky and Pager (2001) also explore the relationship between mean wages of occupations and black-white wage gaps. Although they do not find any relationship in the public sector, they do find a positive relationship in the private sector (on average, occupations with higher mean wages face wider black-white wage gaps). Huffman (2004) also confirms this finding for the wage hierarchy of local labor markets (labor market divisions based on occupation, industry, and metropolitan area). Despite the mounting evidence, it remains unclear which mechanisms are behind the increase in wage discrimination and inequality as one moves up the wage hierarchies.[2] Gaining a better understanding of these mechanisms is of paramount importance in forming public policy and the subject of this paper.

At this point, there is an empirical consensus, both in economics and in sociology, on the widespread use of employee referrals in the labor market.[3] About 50% of U.S. jobs are found through social networks and about 70% of firms have programs encouraging referral-based hiring.[4] In this paper, I show that the employee referrals can be behind the increase in wage penalty for blacks as one moves up the wage hierarchies. I build a model of employee referrals with three sorts of agents: firms, employed workers, and unemployed workers. The firm has no relationships/ties with the unemployed workers, but the employed workers have ties with them. As a result, the firm is indirectly connected to the unemployed workers through its employees. A tie between an employed worker and an unemployed worker can be either weak (acquaintance) or strong (close friends).[5] An unemployed worker is characterized by

[1] For an excellent review of the black-white wage gap literature in economics, see Altonji and Blank (1999).

[2] This increase in black-white wage gaps is in both a relative and an absolute sense. Grodsky & Pager (2001) also demonstrate that their finding cannot be explained by the association between patterns of wage dispersion and average pay levels across occupations. In other words, they find that wages are not more dispersed in the highest-paying occupations. Similar to Grodsky & Pager (2001), Huffman (2004) also shows that his result is not an artifact of relatively high levels of wage variability at the upper end of the wage hierarchy. They show that the wage variability is largely independent of the average overall pay in a job.

[3] See Topa (2011), for a survey of the economics literature, and Marsden and Gorman (2001), for a survey of the sociology literature.

[4] Granovetter (1974) showed that roughly 50% of workers are referred to their jobs by social contacts, a finding that has been confirmed in more recent data (Topa 2011). A leading online job site estimates according to their internal data that 69% of firms have a formal employee referral program (CareerBuilder 2012).

[5] The issues of strong versus weak ties have been considered before. For instance, see Granovetter (1974),

his ability level and the type of ties that he has with the employed workers. Each unemployed worker's ability level is his private information, but his ties (employed workers) observe a signal about his ability level. The type of signal that an employed worker receives depends on his relationship with the unemployed worker (weak ties have more noise in their signals). This model involves two main features: unemployed workers choose which employed workers to ask for referrals based on the type of ties (weak or strong) they have with them, and firms try to infer some information about the abilities of the unemployed workers through the recommendations of its employees. An employed worker receives some gratitude from the unemployed worker for providing him with a recommendation, and he faces reputation costs of providing an inaccurate recommendation. The value of this gratitude is strictly increasing in the wage offered to the unemployed worker, and the strength (weak or strong) of his relationship with the unemployment worker. The reputation costs depend on the distance between his recommendation and his expected value of the unemployed worker's ability. A firm can influence the reputation costs of its employees by choosing the magnitude of their penalty for providing an inaccurate recommendation. After presenting the main features of this model, I extend it to consider the races (blacks or whites) of unemployed workers and how their races can determine the type of ties they have access to. This extension allows me to explain the observed pattern of increasing black-white wage gap as one moves up the wage hierarchies.

The starting point of my analysis is to consider how the wage offered by a firm depends on the recommendations of the employed worker, and the type of tie between the employed worker and the unemployed worker. A strong tie is more informative than a weak tie, in that the interval of ability levels that a weak tie can infer through his signals $[0, \alpha_{weak})$ is included in the interval of ability levels that a strong tie can infer through his signals $[0, \alpha_{strong})$, i.e. $\alpha_{strong} > \alpha_{weak}$. This follows from the fact that an employed worker with a weak tie has more noise in his signals.[6] An employee's recommendations are strictly increasing in ability values until either the recommendations hit the upper bound of recommendation values (the highest ability value) or the ability value reaches the informativeness level of the tie (α_{weak} for

Montgomery (1992, 1994), Karlan et al (2009) and Zenou (2013, 2015). Similar to my model, most works have focused on the ties between the unemployed and employed workers. However, Karlan et al (2009) consider the ties between the employed workers and firms.

[6]The underlying idea behind this is that the employed worker can learn about the unemployed worker's ability θ, but there is an upper-bound α (interpreted as the ability level of the employed worker) to how much he can learn. An employed worker with a strong tie receives a precise signal if the unemployed worker's ability is below his threshold ability level (for $\theta \leq \alpha$), and signals are noisy otherwise. An employed worker with a weak tie can learn less about the unemployed worker's ability. Therefore, he only receives a precise signal if the unemployed worker's ability is sufficiently below his threshold ability level (for $\theta \leq \alpha - \varepsilon$ where $\alpha \geq \varepsilon > 0$), and signals are noisy otherwise. Thus, an employed worker with a weak tie can infer smaller interval of ability levels, because he has more noise in his signals.

weak ties and α_{strong} for strong ties). For any ability levels above this point, the employee's recommendations are the same and firms will not be able to infer ability levels. Whether the firm can infer a bigger interval of ability levels from strong ties or weak ties depends on two opposing forces in the model. On the one hand, strong ties get higher gratitude value from providing referral, which leads them to send higher recommendation values. If a strong tie runs out of recommendation values that he can send before the signal reaches the informativeness level of the weak tie, then firms can infer smaller interval of ability levels from a strong tie. On the other hand, a weak tie is less informative than a strong tie, and therefore he will reach his informativeness level before a strong tie. At the equilibrium, firms maximize the number of signals they can infer by setting the highest penalty value for receiving an inaccurate recommendation from its employees. As a result, an employee's recommendation values will be influenced by his reputation costs more than his gratitude benefits, and firms will be able to infer a bigger interval of ability levels from a strong tie than from a weak tie. In particular, firms will be able to infer sufficiently high ability levels (above some threshold) from only strong ties. If some high ability workers don't have strong ties, then low ability workers (whose abilities the firm cannot infer using a weak tie) can get higher wage offers from using weak ties. This is because their use of weak ties leads to firm pooling them with these high ability workers who don't have strong ties. On the other hand, high ability workers can get better wage offers from using strong ties because this leads to firm infering that they are high ability workers.

I develop two applications of the employee referrals model. In the first application, I consider an extension of this model with two races (blacks and whites) and two occupations. I assume that fewer blacks have strong ties with employed workers than whites. This assumption follows from two empirical facts: employment differentials between blacks and whites in the US and the homophily feature of social networks (tendency to interact with others that have similar characteristics).[7] Since blacks have lower employment rates than whites, they have to rely on their cross-race ties more than whites do.[8] Such cross-race ties tend to be weaker because individuals tend to interact more often with others that belong to their own race.[9] There are two types of occupations, manual labor intensive occupation

[7]See McPherson, Smith-Lovin and Cook (2001) for a survey on homophily.

[8]See Lang & Lehmann (2012) for a brief review of employment differentials between blacks and whites in the US.

[9]See Lin et al (1981) for a discussion on the extension of homophily principle which considers the strength of ties. See Homans (1950), Laumann (1966), Laumann and Senter (1976), Verbrugge (1979) for some references. See Tsui and O'Reilly (1989), and Thomas (1990) for some further evidence. Based on a national survey, Marsden (1987, 1988) reports that only 8 percent of people have any people of another race with whom they discuss important matters, which suggests the weakness of most cross-race ties. Jackson (2007) shows that in a network of the friendships in a high school from the Ad Health Data Set, most cross-race ties are weak.

and human capital intensive occupation. In the manual labor intensive occupation, output is only increasing in ability up to some level, and having human capital beyond that doesn't add to output. In the human capital intensive occupation, output is always increasing in ability. In this extension of the model, I show that the lack of access to strong ties for blacks can explain the observed pattern of increasing black-white wage gap as one moves up the wage hierarchies. In human capital intensive occupation, firms will offer higher wages to high ability workers with strong ties because firms can infer that they are high ability workers. If only few blacks have strong ties, then both below and above average ability black workers enter higher earning occupations through weak ties, firm pools them together, and on average high ability black workers get relatively lower wage than their white counterparts (same ability levels).

In the second application, I explore some implications of the employee referrals for job search. The model provides new insights about the employee referrals mechanism, which can help explain the empirical findings about the use of different types of ties in job search and the returns to ties. In the pioneering work of Granovetter (1974), he documents that a large proportion of jobs are found through weak ties. He argues that the weak ties to individuals with whom one has few common friends are most useful for job search, because they provide access to otherwise unobtainable information about job openings. This finding of the frequent use of weak ties led to the coining of the well-known phrase, the "strength of weak ties". However, the evidence about the use of weak ties is mixed. Many studies have found the frequent use of strong ties (Murray, Rankin, and Magill (1981), Bridges and Villemez (1986), Marsden and Hurlbert (1988)). Similarly, the empirical evidence about the returns to ties is mixed. Some studies show that workers who found their jobs through family, friends, and acquaintances earned more than those using formal and other informal job-search methods (Rosenbaum et al. (1999), Marmaros and Sacerdote (2002)). Others found no significant effect (Bridges and Villemez (1986), Holzer (1987), Marsden and Gorman (2001)) or even negative effect (Elliott (1999), Green, Tigges, and Diaz (1999)).

The employee referrals model provides a novel finding: returns to using a tie varies with the unemployed worker's ability, the tie strength, and the proportion of workers who have access to different type of ties. This finding can then simultaneously explain the mixed evidence about the use of different types of ties in job search, and the mixed evidence about the wage differentials between workers who found jobs through referrals and workers who found jobs by formally applying to firms. (1) The higher is the number of high ability workers who don't have strong ties, the higher are the returns from using weak ties, and the more workers use weak ties to pool with high ability workers. Contrary to the existing explanation, the frequent use of weak ties may not be due to its efficiency in matching workers and firms. When the access to strong ties is really scarce for high ability workers, many workers (even those with access to strong ties), use weak ties to pool with high ability workers. (2) The lower is the informativeness of the weak tie, the lower are the returns from pooling with high ability workers. This follows from the fact that really low ability workers are also included in the pool. Applying directly to the firm is equivalent to applying through a tie with the minimal informativeness level. If some high ability workers have access to ties, then the wage offer from pooling through direct application is below average ability. As a result, for above average ability workers, the returns to using a tie is always positive. However, for below average ability workers, the returns to using a tie can be small, insignificant, and even negative. Although these predictions do not emerge in the existing model of employee referrals, they are consistent with existing empirical evidence, suggesting that the tie selection and the strategic recommendation are important aspects of employee referrals and are useful for understanding job search.

Before presenting the model formally, let me mention some closely related literature. Firstly, there is a large literature on the employee referrals. The existing models in this literature either do not consider the incentives of the referrers (employed workers providing referrals) or they do not consider the type of ties selected by the unemployed workers.[10] As a result, these models are unable to explain the mixed evidence about the use of different types of ties in job search, and the mixed evidence about the wage differentials between workers who found jobs through referrals and workers who found jobs by formally applying to firms.[11] Secondly, there is a growing literature which considers social networks to explain racial inequality in labor market outcomes.[12] I contribute to this literature by using the employee referrals (a social networks based mechanism) to explain the widening of the black-white wage gap as one moves up the wage hierarchies. It is hard to explain this effect outside of my employee referrals model. In particular, there is a large literature on statistical

[10] There are two main classes of referral models: (i) models in which ties transmit information about job opportunities, and (ii) models in which ties transmit information about the productivity of workers. Calvo-Armengol (2004), Calvo-Armengol and Jackson (2004, 2007), Calvo-Armengol and Zenou (2005), Ioannides and Soetevent (2006), Fontaine (2008), Cahuc and Fontaine (2009), Bramoulle and Saint Paul (2009), and Gaelotti and Merlino (2014) belong to the first class of models. Montgomery (1991), Simon and Warner (1992), Arrow and Borzekowski (2004), Dustman et al (2011), and Galenianos (2012) belong to the second class of models. Both of these classes of referral models suggest that the returns to ties are positive, which is inconsistent with the mixed evidence about the returns to ties. See Saloner (1985) for a model which consider the incentives of the referrers. However, his model only focuses on the referrers and is therefore unable to provide any predictions about the unemployed workers (such as their returns to finding jobs through ties and their use of different type of ties).

[11] Loury (2006) has an explanation for the mixed evidence about the returns to ties but not the use of ties. Loury argues that workers with limited access to wage offers through other channels may rely on employee referrals as a last resort. Many of these workers would have lower rather than higher wages compared with those using other means to find jobs.

[12] See Ioannides & Loury (2004) for a survey.

discrimination.[13] In this literature, firms have uncertainty about the ability of the unemployed workers and they use the race of a worker as a signal of his ability level, which results in similarly skilled workers of different races to have different wages. Most mechanisms that can reduce this uncertainty would suggest shrinking of the wage discrimination as one moves up the wage hierarchies. For instance, the informational asymmetries between the firm and unemployed workers likely decrease for highly educated workers because education acts as a "signal" about their abilities (Spence (1973)).[14] At the same time, firms have a stronger incentive to invest in technologies (formal screening mechanisms such as aptitude tests and other attribute measurements) that accurately reveal the cognitive ability of workers in human capital intensive occupations. Since high earning occupations require higher education and they are more human capital intensive, there is arguably lower statistical discrimination in high earning occupations relative to low earning occupations.

Thus, the contribution of this paper is to provide an employee referrals model which is consistent with the mixed findings about the use of ties and the returns to ties, as well as with the widening of the black-white wage gap as one moves up the wage hierarchies. The rest of the paper is organized as follows. Section 2 presents the employee referrals model. I will first present a motivating example and then provide the general results. In section 3, I extend the base model by considering multiple races and occupations to show the widening of the black-white wage gap as one moves up the wage hierarchies. In section 4, I explore some implications of the employee referrals for job search. I extend the base model by allowing workers to apply directly to the firm, and through ties. This extension helps explain the mixed findings about the returns to ties. Section 5 concludes with some policy implications of the model. All proofs are available in the appendix.

2 A Model of Employee Referrals

The model involves an environment with three sorts of agents: unemployed workers, employed workers, and a single firm. The firm has no relationships/ties with the unemployed workers, but the employed workers have ties with the unemployed workers. As a result, the firm is indirectly connected to the unemployed workers through its employees. A tie $t \in (t_{weak}, t_{strong})$ between an employed worker and an unemployed worker can differ by its strength (weak or strong). To keep the presentation neat, I assume that each employed worker has a tie (either weak or strong) with only one unemployed worker, and each unemployed worker has at most one tie of each type. Thus, an unemployed worker's type is a

[13]See Lang & Lehmann (2012) for a review of theories about race discrimination.

[14]See Lang & Manove (2011) for a detailed discussion and justification of this argument.

pair (θ, T), where θ is his ability level and the set T indicates the type of ties that he has with the employed workers. There is a continuum of ability levels $\theta \in [0,1]$ and they are uniformly distributed $\theta \sim u[0,1]$. I assume that each unemployed worker has access to a weak tie $Prob(t_{weak} \in T) = 1$, but only β of the unemployed workers have access to a strong tie $Prob(t_{strong} \in T) = \beta$. An employed worker's type is a pair (θ, t) where θ is the ability level of the unemployed worker he has a tie with, and t is the strength of his tie with this unemployed worker. I take the network of relationships/ties between the agents as given, and examine its influence on the wage determination process.

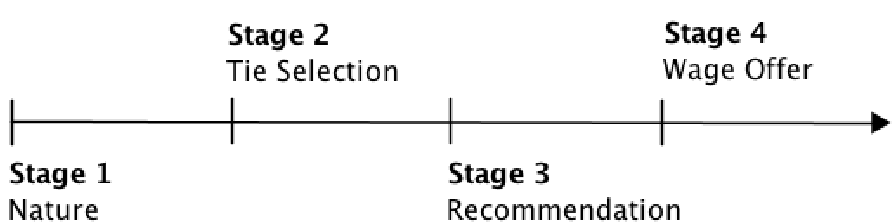
The role of network in this paper is to transmit information. An unemployed worker's ability level is his private information, but his ties receive a signal $s \in [0,1]$ about his ability level, and the firm tries to infer some information about the ability level through the recommendations of the employed workers. The firm can only infer the lowest ability level, weak ties can infer ability levels up till $\alpha_{weak} \geq 0$, and strong ties can infer ability levels up till $\alpha_{strong} > \alpha_{weak}$. In this sense, both type of ties are more informative than firms about the ability levels of the unemployed workers, and strong ties are more informative than weak ties. Formally, employed ties and the firm receive the following signals about the ability levels of the unemployed workers.

$$s(\theta, \alpha_t) = \begin{cases} \theta & \text{if } \theta \in [0, \alpha_t) \\ x & \text{if } \theta \in [\alpha_t, 1] \end{cases} \tag{1}$$

where $x \sim u[\alpha_t, 1)$, $\alpha_t \in \{0, \alpha_{weak}, \alpha_{strong}\}$, $\alpha_{strong} > \alpha_{weak} \geq 0$, and $\alpha_t = 0$ for the firm.

My model consists of four stages, which are depicted in figure 1. I begin by describing the model and then discuss the economic content of my modeling assumptions.

Fig 1. Model Timeline



Stage 1: Nature. At the beginning of this stage, nature determines the ability levels of unemployed workers and the type of ties they have access to. Then, firms and employed workers receive signals about the ability levels of unemployed workers.

Stage 2: Tie Selection. At this stage, each unemployed worker (θ, T) chooses tie type $t \in T$ that maximizes his wage,

$$\pi_U(\theta, T) = w(\rho(s,t), t) \tag{2}$$

where wage depends on the tie's recommendation $\rho(s,t)$, and the tie's type t. Observe that the tie's recommendation $\rho(s,t)$ is also a function of the tie's type t and the signal s that he received. At equilibrium, the proportion of unemployed workers with ability level θ who chose tie type t is denoted by $f(t|\theta)$. I assume rational expectations in that employed ties and firm know $f(t|\theta)$ for each ability level θ and tie type t, and they use it to form their expectations.

Stage 3: Recommendation. An employed worker faces both gratitude benefits and reputation costs of providing a recommendation $\rho \in [0,1]$. The value of gratitude is strictly increasing in the wage $w(\rho,t)$ offered to the unemployed worker. The reputation cost of providing an inaccurate recommendation is measured by the square of the difference between the recommendation and the employed worker's expected value of ability $(\rho - E[\theta|s,t])^2$. Given the signal s, and tie type t, each employed worker provides recommendation ρ that maximizes his payoff,

$$\pi_E(s,t) = w(\rho,t) - r(\rho - E[\theta|s,t])^2 \tag{3}$$

where $r > 0$ is the reputation costs parameter. The bigger is the reputation costs parameter, the lower are the incentives to provide inaccurate recommendations. Note that the employed worker's expected value of ability is equal to the signal if $s \in [0, \alpha_t)$ and is equal to the weighted average of ability values $E[\theta|s,t] = \int_{\alpha_t}^1 \theta \frac{f(t|\theta)}{\int_{\alpha_t}^1 f(t|\theta)d\theta} d\theta$ if $s \in [\alpha_t, 1]$.

Stage 4: Wage Offer. Given the recommendation ρ, and the tie type t, the firm forms its valuation $v(\rho,t) = E[\theta|\rho,t]$ of the unemployed worker's ability, and offers wage equal to its valuation $w(\rho,t) = E[\theta|\rho,t]$.

My model is a multistage sequential game, so I will derive its equilibrium through backward induction. In particular, stage 3 involves a signaling subgame. I will now define some key concepts of signaling games in the context of my employee referrals model. The basic problem in a signaling game is to analyze whether the receiver (firm) can infer the information (expected ability of unemployed worker) of the sender (employed worker) through his messages (recommendations).

Definition 1. An employed worker's recommendation strategy ρ *reveals his information* $E[\theta|s,t]$ if and only if:

(1) for $s \in [0, \alpha_t)$, $\{s' : \rho(s', t) = \rho(s, t)\} = \{s\}$, or

(2) for $s \in [\alpha_t, 1]$, $\{s' : \rho(s', t) = \rho(s, t)\} = [\alpha_t, 1]$.

For signals $s \in [0, \alpha_t)$, the employed worker knows the ability level and it is equal to the signal value. The first part of this definition says that if an employed worker received such a signal, then he is revealing his information if the firm can also infer the signal/ability value. For any signal in the interval $s \in [\alpha_t, 1]$, the employed worker expects the same ability value and it is equal to the weighted average of ability values in this interval. The second part of this definition says that if an employed worker received such a signal, then he is revealing his information if the firm can also infer that the signal belongs to this interval $[\alpha_t, 1]$.

An unemployed worker with ability level θ is *separating through an employed tie* t if the messages sent by the employed tie reveals his ability value to the firm. If the firm can't differentiate between a set of ability levels, then the firm *pools* unemployed workers with such ability values together. For this signaling game, I will focus on an appealing class of pure strategy Perfect Bayesian Equilibria (PBE), where employed workers reveal maximum information in the cheapest way possible. See appendix for a formal justification. I will now formally define this class of PBE and provide some intuition for its attractiveness.

Definition 2. An employed worker's recommendation strategy $\rho(s, t)$ is *cost efficient* if and only if:

(1) The initial value condition is $\rho(0, t) = 0$.

(2) If $\rho(s^*, t) = 1$, then for $s < min\{s^*, \alpha_t\}$, recommendations are increasing in signals $\rho_1(s, t) > 0$.

(3) For $s \geq min\{s^*, \alpha_t\}$, recommendations value minimizes reputation costs $\rho(s, t) \in argmin_{\rho \in [0,1]}(\rho - E[\theta|s, t])^2$ subject to $\rho(s, t) \neq \rho(s', t)$ for all $s' < min\{s^*, \alpha_t\}$.

A cost efficient recommendation strategy satisfies three conditions. The first condition is the "Riley Condition" of least costly separation in signaling games. For the lowest signal value, the employed worker can reveal the signal to the firm even by minimizing reputation costs. The second condition requires that the employed worker reveals information to the firm until he either runs out of recommendation values or the signal reaches his informativeness level. Since reputation costs depend on the distance between the recommendation value and the signal value, it is cheaper to reveal information by increasing recommendations as signal increases. Similar to the lowest signal value, for the set of signals at the top, the employed worker can reveal his information to the firm even by minimizing reputation costs. However, it may not be feasible to choose the recommendation value that minimizes reputation costs if it is already chosen for some lower signal value. In such case, the employed worker chooses the constrained minimum.

A cost efficient recommendation strategy is appealing in that (a) it maximizes the set of ability levels that a firm can infer, and (b) among all recommendation strategies that also maximize the set of ability levels that a firm can infer, the employed worker incurs lowest reputation costs by revealing information through a cost efficient recommendation strategy. I will focus on equilibria of the employee referrals model where recommendation strategies of the employed workers are cost efficient. I will refer to this class of equilibria as Cost Efficient Equilibria (CEE) hereafter.

2.1 Discussion of Modeling Assumptions

I now discuss some of the assumptions underlying my model.

Access to Ties. I made several assumptions regarding access to ties. Firstly, I assumed that each unemployed worker has access to weak ties. This assumption is quite natural in that everyone knows many acquaintances (weak ties). However, knowing acquaintances does not necessarily mean that they will provide referrals. Thus, I relax this assumption in section 4. I consider an extension of my model where some proportion of unemployed workers do not have access to weak ties. Secondly, I assumed that if an unemployed worker has access to a certain type of tie, then he has only one such tie. In section 2.4, I consider an extension of the model which relaxes this assumption. The main results of this paper are robust to both of these extensions (sections 4, and 2.4). Finally, I assume that each employed worker has a tie (either weak or strong) with only one unemployed worker. This assumption is based on the idea that an employed worker knows his most preferred unemployed tie. Thus, I define "an unemployed worker to have access to a tie" to mean that the employed worker will provide referral to this unemployed worker among all the unemployed workers that he has ties with.

Information Structure. The underlying idea behind the information structure is that the employed worker can learn about the unemployed worker's ability, but there is an upper-bound α (interpreted as the ability level of the employed worker) to how much he can learn. An employed worker with a strong tie receives a precise signal if the unemployed worker's ability is below his threshold ability level (for $\theta \leq \alpha$), and signals are noisy otherwise. An employed worker with a weak tie can learn less about the unemployed worker's ability. Therefore, he only receives a precise signal if the unemployed worker's ability is sufficiently below his threshold ability level (for $\theta \leq \alpha - \varepsilon$ where $\alpha \geq \varepsilon > 0$), and signals are noisy otherwise. Thus, an employed worker with a weak tie can infer smaller interval of ability levels, because he has more noise in his signals.

Firm's Role. So far I have assumed a passive role for the firm. The firm simply offers a wage equal to its valuation of the unemployed worker's ability. In the employee referral mechanism, a firm can not only choose the wage it offers to the unemployed worker, but it can also influence the reputation costs of its employees. In section 2.4, I consider an extension of the model which takes into account these two roles of the firm.

The fact that weak ties are less informative than strong ties does not necessarily imply that strong ties will provide more information to a firm through referrals. It depends on how much the employed worker values gratitude over reputation costs. The firm plays an important role here in that it can influence the reputation costs of its employees.[15]

Recommendation. I assumed that recommendation values are bounded $\rho \in [0, 1]$. The recommendation value is the ability value that an employed worker conveys to the firm. Thus, it is bounded by the values that ability variable can take $\theta \in [0, 1]$. Note that the firm does not blindly believe what the employed worker conveys to it. The firm tries to infer the signal that its employee received through his recommendation value, and forms its own beliefs.

The underlying idea behind the recommendation variable is as follows. An employed worker providing a referral puts some effort $e \in [e_{min}, e_{max}] \supseteq [0, 1]$ into recommending an unemployed worker. The more effort he puts into his recommendation, the higher the ability value he conveys to the firm $\rho'(e) \geq 0$. The recommendation function maps the employed worker's effort into recommendation value $\rho : [e_{min}, e_{max}] \rightarrow [0, 1]$. Instead of having an additional effort variable, I simply assume that the employed worker can choose the recommendation value. Note that this model belongs to the costly signaling class of model where recommendation (or effort value) is the costly message.

Reputation Costs. The recommendation value is the ability value that an employed worker conveys to the firm. So reputation cost is the difference between the ability value than an employed worker conveys to the firm and the ability value that the employed worker actually believes $(\rho - E[\theta|s, t])^2$. Note that the reputation cost comes from recommendations that are lower and higher than the employed worker's expected value of ability. The results

[15] Firms can influence the reputation costs of its employees in several ways. Firms can prolong the "probation period" of workers hired through employee referrals. This will allow firms to better learn about the abilities of workers hired through employee referrals, increases the likelihood that an employee will be caught if he provided an inaccurate recommendation, and therefore increases the expected reputation costs of employee for providing an inaccurate recommendation. Firms could worsen punishment for employees who provide inaccurate recommendations (see Heath (2013) and Beaman and Magruder's (2012)). The main idea here is that the firm can choose the reputation cost levels in the employee referral method of hiring, which is a key distinction from non-employee referrals. See Montgomery (1991) for some early references on reputation costs. See Fernandez & Mateo (2015) for some references and a discussion.

of this paper are robust to a variation of the model in which reputation costs only come from recommendations that are higher than the expected ability value (see remark at the end of appendix B).

It is clear that there are reputation costs to going over the expected ability value. By recommending higher than the ability value, the employed worker is putting his reputation as an employee at stake. The employed worker doesn't want to be responsible for recommending a bad worker. However, there are several reasons to believe that there are reputation costs of going under the expected ability value as well. By recommending lower than the expected ability value, the employed worker is putting his reputation as a "friend" (or whatever his relationship is with the unemployed worker) at stake. The employed worker doesn't want to be responsible for under-selling his "friend". Alternatively even as an employee, the employed worker will have a better reputation if he recommends accurately as oppose to always giving really low recommendations. The base model takes such considerations into account, and therefore the reputation costs come from both going under and over the expected ability value.

2.2 Motivating Example

I will first consider a simple example where weak ties have minimal information $\alpha_{weak} = 0$, strong ties have full information $\alpha_{strong} = 1$, and only half of the unemployed workers have access to strong ties $\beta = 0.5$. This example will provide intuition on how to characterize the equilibrium for the general case.

It is a multistage sequential game, so I will derive the equilibrium through backward induction. In stage 4, the firm sets wage equal to its valuation $w(\rho, t) = E[\theta|\rho, t]$. If the tie is weak, then the employee's signals do not provide any new information and therefore the firm's valuation does not depend on such employee's signals $w(\rho, t_{weak}) = E[\theta|t_{weak}] = \int_0^1 \theta \frac{f(t_{weak}|\theta)}{\int_0^1 f(t_{weak}|\theta)d\theta} d\theta$. If the tie is strong, then the firm's wage offer depends on whether it can infer its employees signals. In a cost efficient recommendation strategy, recommendations are (weakly) increasing in signal $\rho_1(s, t_{strong}) \geq 0$. For the interval of signals that the firm can infer, the firm sets wage equal to the signal value $w(\rho(s, t_{strong}), t_{strong}) = s$. Since recommendations are strictly increasing in signals $\rho_1(s, t_{strong}) > 0$ for such interval of signals, the firm's wage is strictly increasing in recommendation. Plugging this wage in the first order condition of the employed worker's maximization problem gives the following differential equation (DE).

$$\rho_1(s, t_{strong}) = \frac{1}{2r(\rho(s, t_{strong}) - s)} \tag{DE}$$

This differential equation then implies that employee recommends higher than his signal $\rho(s, t_{strong}) > s$. This follows from the fact that the firm's wage is strictly increasing in recommendation. As a result, the employed worker has an incentive to inflate his recommendations (recommend higher than his signal) in order to get more gratitude (which is increasing in wage). The reader can verify that the family of solutions to this differential equation is given by

$$\rho(s, t_{strong}) + c = -\frac{1}{2r} ln\left[\frac{1}{2r} + s - \rho(s, t_{strong})\right]$$

where c is a constant which can be determined by the initial value condition $\rho(0, t_{strong}) = 0$. So I get $c = -\frac{1}{2r} ln\left[\frac{1}{2r}\right]$, and

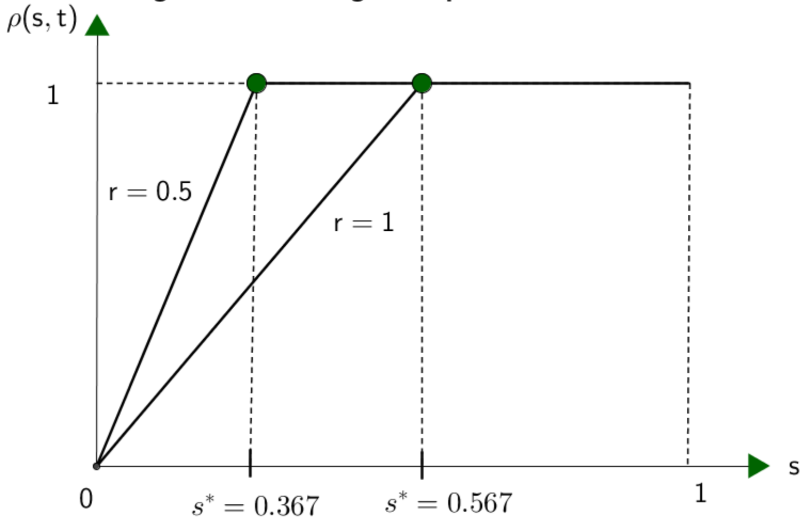
$$\begin{aligned}
\rho(s, t_{strong}) &= \frac{1}{2r} ln\left[\frac{1}{2r}\right] - \frac{1}{2r} ln\left[\frac{1}{2r} + s - \rho(s, t_{strong})\right] \\
\implies \rho(s, t_{strong}) &= \frac{1}{2r} ln\left[\frac{\frac{1}{2r}}{\frac{1}{2r} + s - \rho(s, t_{strong})}\right] \qquad (4)
\end{aligned}$$

For a given reputation parameter r, there exists a unique signal s^* such that recommendation is equal to one.

$$\begin{aligned}
s^* &= \frac{1}{2r}\left[\frac{1 - e^{2r}}{e^{2r}}\right] + 1 \\
&= \frac{1}{2re^{2r}} - \frac{1}{2r} + 1 \qquad (5)
\end{aligned}$$

The reader can verify that as $r \to 0$, $s^* \to 0$ (using L'Hopital Rule), as $r \to \infty$, $s^* \to 1$, and the threshold signal is increasing in reputation cost parameter $s^{*'}(r) > 0$. Thus, for any finite reputation cost parameter, $s^* \in (0, 1)$. Figure 2a below depicts this.

Fig 2a. Motivating Example



So recommendations are strictly increasing in signal and inflated until they hit the upper bound of one. The employed worker's recommendations are equal to one $\rho(s, t_{strong}) = 1$ for any signal higher than this threshold signal $s \geq s^*$, which makes it impossible for firm to infer these signals from the employee's recommendations (firm can only infer signals below this threshold).

Similarly, wage offers from strong tie is strictly increasing in signal below threshold signal $s < s^*$, and equal to the average of signals $s \geq s^*$ otherwise.

$$w(\rho, t_{strong}) = \begin{cases} s & \text{if } s < s^* \\ E[\theta|s \geq s^*, t_{strong}] & \text{if } s \geq s^* \end{cases} \tag{6}$$

Observe that wage offers from weak ties is fixed, and wage offers from strong ties is increasing in signals. As a result, if $\widetilde{\theta}$ prefers strong tie, then all $\theta > \widetilde{\theta}$ prefer strong tie as well. Thus, the equilibrium involves a threshold ability θ^* such that $\theta < \theta^*$ prefer weak tie, and $\theta > \theta^*$ prefer strong tie. The value of threshold ability θ^* depends on threshold signal s^*. Let $\widehat{\theta} = \{\theta : \theta = E[\theta|t_{weak}]\}$ be the unemployed worker who is indifferent between choosing wage offer from a strong tie and revealing his ability or choosing wage offer from a weak tie. The reader can easily verify that $\widehat{\theta} = 0.41$. If threshold signal is high enough $s^* > \widehat{\theta}$, then threshold ability is $\theta^* = \widehat{\theta}$. If threshold signal is not high enough $s^* \leq 0.41$, then unemployed workers whose abilities are revealed with strong ties $\theta < s^*$ prefer weak tie. For unemployed workers with abilities $\theta \geq s^*$, they can get wage offer $E[\theta|s \geq s^*, t_{strong}] = \frac{1}{1-s^*}\int_{s^*}^{1} s ds$ by pooling with a strong tie or $E[\theta|t_{weak}] = \int_0^1 \theta \frac{f(t_{weak}|\theta)}{\int_0^1 f(t_{weak}|\theta)d\theta} d\theta$. The reader can easily verify that the wage offer from strong tie is strictly higher then any wage offer from weak ties

$E[\theta|s \geq s^*, t_{strong}] > 0.5 \geq E[\theta|t_{weak}]$ since the threshold signal is positive $s^* > 0$.

$$\theta^* = \begin{cases} s^* & \text{if } 0 < s^* \leq 0.41 \\ 0.41 & \text{if } s^* > 0.41 \end{cases} \tag{7}$$

In this example, strong ties are more informative to the firm since the threshold signal is positive $s^* > 0$. As a result, low ability workers use weak ties to pool with high ability workers without strong ties, and high ability workers use strong ties to separate themselves. This example suggests that the cost efficient equilibrium will involve a threshold ability where strong ties are more informative to the firm. Is there always a cost efficient equilibrium with a threshold ability? If so, are strong ties always more informative to the firm?

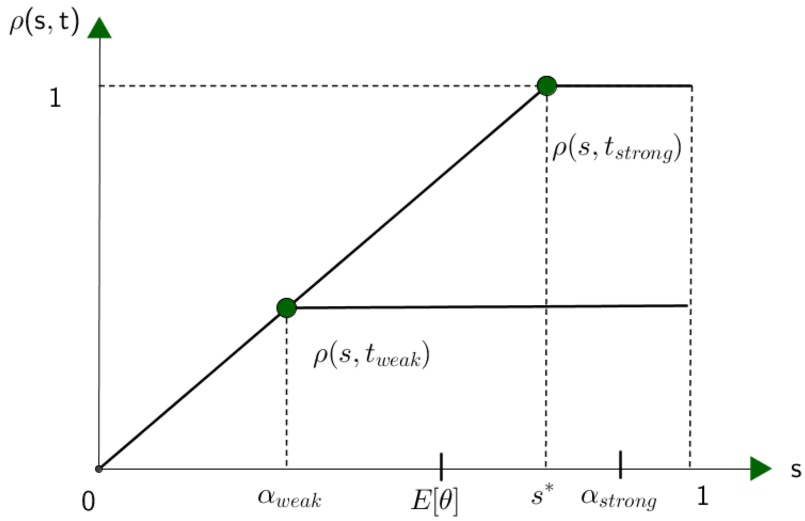
2.3 Cost Efficient Equilibrium (CEE)

I will first characterize the cost efficient recommendation strategy for each type of employed worker. Similar to the motivating example, recommendations are strictly increasing in signal and inflated $\rho(s,t) > s$ until either the recommendations hit the upper bound of one or the signal reaches informativeness level of the tie, i.e. for $s < min\{s^*, \alpha_t\}$. If recommendations hit the upper bound of one, then recommendation value is equal to one for any higher signal, i.e. $\rho(s,t) = 1$ for $s \geq s^*$. If the signal reaches the informativeness level of the tie, then the employed worker chooses recommendation value that minimizes his reputation costs $\rho(s,t) = E[\theta|s,t]$ as long as it was not used for some lower signal. If it was used for some lower signal, then the employed worker chooses the lowest value that was not used for some lower signal. See figure 2b below for a case where $\alpha_{weak} < s^* < \alpha_{strong}$, and $\rho(s, t_{weak}) = lim_{s' \to \alpha_t} \rho(s', t)$ for $s \geq \alpha_{weak}$.

> **Proposition 1 -** An employed worker's cost efficient recommendation strategy $\rho(s,t)$ satisfies:
>
> (1) The initial value condition $\rho(0,t) = 0$.
>
> (2) For $0 < s < min\{s^*, \alpha_t\}$, $\rho(s,t) > s$, and
>
> $$\rho(s,t) = \frac{1}{2r} ln \left[\frac{\frac{1}{2r}}{\frac{1}{2r} + s - \rho(s,t)} \right].$$
>
> (3) For $s \geq min\{s^*, \alpha_t\}$,

$$\rho(s,t) = \begin{cases} 1 & \text{if } min\{s^*, \alpha_t\} = s^* \\ E[\theta|s,t] & \begin{array}{c}\text{if } min\{s^*, \alpha_t\} = \alpha_t, \\ \text{and } E[\theta|s,t] \neq \rho(s',t) \text{ for all } s' < \alpha_t\end{array} \\ lim_{s' \to \alpha_t} \rho(s',t) & \begin{array}{c}\text{if } min\{s^*, \alpha_t\} = \alpha_t, \\ \text{and } E[\theta|s,t] = \rho(s',t) \text{ for some } s' < \alpha_t\end{array} \end{cases}.$$

Fig 2b. Cost Efficient Referral Strategy



For the interval of signals that the firm can infer $s < min\{s^*, \alpha_t\}$, the firm sets wage equal to the signal value $w(\rho(s,t),t) = s$. Since recommendations are strictly increasing in signals $\rho_1(s,t) > 0$ for such interval of signals, the firm's wage is strictly increasing in recommendation. For the interval of signals that the firm can not infer $s \geq min\{s^*, \alpha_t\}$, the firm sets wage equal to the weighted average of these signal/ability values i.e. $w(\rho(s,t),t) = E[\theta|s \geq min\{s^*, \alpha_t\}, t] = \int_{min\{s^*,\alpha_t\}}^{1} \theta \frac{f(t|\theta)}{\int_{min\{s^*,\alpha_t\}}^{1} f(t|\theta) d\theta} d\theta$, where weights are determined by the the proportion of unemployed workers $f(t|\theta)$ with ability level θ who chose tie type t.

Proposition 2 - Given a recommendation ρ, and the tie type t, the firm offers wage

$$w(\rho,t) = \begin{cases} s & \text{if } s < min\{s^*, \alpha_t\} \\ E[\theta|s \geq min\{s^*, \alpha_t\}, t] & \text{if } s \geq min\{s^*, \alpha_t\} \end{cases}.$$

If the threshold signal is greater than the informativeness level of the weak tie $s^* > \alpha_{weak}$, then there is an interval of signals $s \in (\alpha_{weak}, min\{s^*, \alpha_{strong}\})$ where wage offers from weak ties is fixed, and wage offers from strong ties is increasing in signals. As a result, if $\widetilde{\theta} \in (\alpha_{weak}, 1)$ prefers strong tie, then all $\theta > \widetilde{\theta}$ prefer strong tie as well. Thus, the cost efficient equilibrium involves a threshold ability θ^* such that ability levels below the threshold ability $\theta < \theta^*$ (weakly) prefer weak ties ($\theta \in [0, \alpha_{weak})$ are indifferent between the two type of ties, and $\theta \in [\alpha_{weak}, \theta^*)$ prefer weak ties), and unemployed workers with ability levels above the threshold ability $\theta \geq \theta^*$ prefer strong ties.[16] The value of threshold ability θ^* depends on threshold signal s^* and the informativeness level of both type of ties α_{weak}, α_{strong}. Let $\widehat{\theta} = \{\theta : \theta = E[\theta | s \geq \alpha_{weak}, t_{weak}]\}$ be the unemployed worker who is indifferent between choosing wage offer from a strong tie and revealing his ability or choosing wage offer from a weak tie.

> **Theorem 1 -** If $s^* > \alpha_{weak}$, then the Cost Efficient Equilibrium exists and is characterized by a threshold ability $\theta^* = min\{\widehat{\theta}, s^*, \alpha_{strong}\}$ such that:
>
> Unemployed workers with ability levels below the threshold ability $\theta < \theta^*$ (weakly) prefer weak ties, and unemployed workers with ability levels above the threshold ability $\theta > \theta^*$ prefer strong ties.

Recall that the threshold signal is increasing in reputation cost parameter $s^{*'}(r) > 0$ (see equation 5). If the reputation cost parameter is low enough, then $s^* \leq \alpha_{weak}$ and both type of ties are equally informative to the firm. In such case, unemployed workers are indifferent between the two type of ties, and the cost efficient equilibrium does not exist. Moreover, if strong ties get more gratitude from providing recommendations (gratitude benefits parameter varies by tie strength $\gamma(t_{strong}) > \gamma(t_{weak})$), then weak ties can be even more informative to the firm.

To ensure that the equilibrium involves a threshold ability where strong ties are more informative to the firm, I need the reputation costs parameter to be high enough. In the next section, I show that if a firm can choose the reputation costs parameter for its employees, it will indeed set reputation costs parameter to be high enough. I consider an extension of the model in which there are two firms, strong ties get more gratitude from providing recommendations (gratitude benefits parameter varies by tie strength $\gamma(t_{strong}) > \gamma(t_{weak})$) and firms can influence the reputation costs of its employees (each firm $j \in \{1, 2\}$ chooses reputation costs parameter for its employees $r_j \in [0, r_{max}]$). After nature moves and before tie selection, firms compete by setting wages and reputation costs parameters. In this extension, each firm sets wage equal to its valuation and the reputation cost parameter to the maximum level. The maximum reputation cost level is such that strong ties are more informative

[16] If the highest ability level ($\theta = 1$) is also common knowledge, then it only changes the threshold equilibrium in that the highest ability workers will be indifferent between the two type of ties as well, i.e. $\theta \in [0, \alpha_{weak}) \cup \{1\}$ are indifferent between the two type of ties, $\theta \in [\alpha_{weak}, \theta^*)$ prefer weak ties, and $\theta \in [\theta^*, 1)$ prefer strong ties.

to the firm $s^* > \widehat{\theta} \geq \alpha_{weak}$. Thus, I will focus on the case where the threshold signal $(\alpha_{strong} \geq s^* > \widehat{\theta})$ is high enough hereafter.

2.4 Firm's Role

I extend the base model by having two identical firms $j \in \{1,2\}$. I assume that if an unemployed worker has access to a certain type of tie, then he has one such type of tie at each firm. As a result, these firms are competitive. Each firm can set reputation cost level for its employees $0 < r_j \leq r_{max}$ where the maximum reputation cost level is such that strong ties are more informative to the firm $s^*_{strong} > \widehat{\theta} \geq \alpha_{weak}$. Stage 1 (nature) and stage 2 (tie selection) are the same as before, but a new stage (Stage 1.5) is introduced. After nature moves and before tie selection, firms compete by setting wages and reputation costs parameters.

Stage 1.5: Wage And Reputation Cost Setting. If an unemployed worker with ability level θ prefers wage offer from firm $j \in \{1,2\}$ using tie $t \in T$, then $1_j(\theta,t) = 1$. If he is indifferent between the two firms, then $1_j(\theta,t) = 0.5$ and $1_j(\theta,t) = 0$ otherwise. In stage 2, I am assuming that if an unemployed worker is indifferent between the two firms, then he is equally likely to select any one of them. The set of unemployed workers who prefer weak ties is denoted by Θ_{weak}, and the set of unemployed workers who prefer strong ties is denoted by Θ_{strong}. Since only β of the unemployed workers have access to strong ties, $(1-\beta)$ of the unemployed workers who prefer strong ties have to choose weak ties. Firm's profit from hiring an unemployed worker with recommendation $\rho(s,t,r_j)$ and tie type t is $\theta - w(\rho,t)$. Observe that the wage $w(\rho,t)$ depends on the recommendation $\rho(s,t,r_j)$, and recommendation is a function of the reputation costs level r_j set by the firm. Each firm $j \in \{1,2\}$ chooses wage $w(\rho,t)$ and reputation costs level $0 < r_j \leq r_{max}$ that maximizes its profit

$$\begin{aligned}
\pi_j &= \int_{\theta\in\Theta_{weak}} 1_j(\theta,t_{weak})\left[\theta - w(\rho,t_{weak})\right] d\theta \\
&+ \beta \int_{\theta\in\Theta_{strong}} 1_j(\theta,t_{strong})\left[\theta - w(\rho,t_{strong})\right] d\theta \\
&+ (1-\beta) \int_{\theta\in\Theta_{strong}} 1_j(\theta,t_{weak})\left[\theta - w(\rho,t_{weak})\right] d\theta.
\end{aligned} \tag{8}$$

Whether the firm can infer a bigger interval of ability levels from strong ties or weak ties depends on two opposing forces in the model. On the one hand, strong ties get higher gratitude value from providing referral, which leads them to send higher recommendation values. If a strong tie runs out of recommendation values that he can send before the signal reaches the informativeness level of the weak tie, then firms can infer less ability levels from a strong tie. On the other hand, a weak tie is less informative than a strong tie, and therefore he will reach his informativeness level before a strong tie.

Stage 3 is the same as before but now an employed worker's gratitude benefits are strictly increasing in the tie strength between the employed worker and the unemployed worker $\gamma(t)$. A strong tie gets more gratitude from providing recommendations than a weak tie, i.e. $\gamma(t_{strong}) > \gamma(t_{weak}) > 0$.

$$\pi_E = \gamma(t)w(\rho, t) - r_j(\rho - E[\theta|s, t])^2 \tag{9}$$

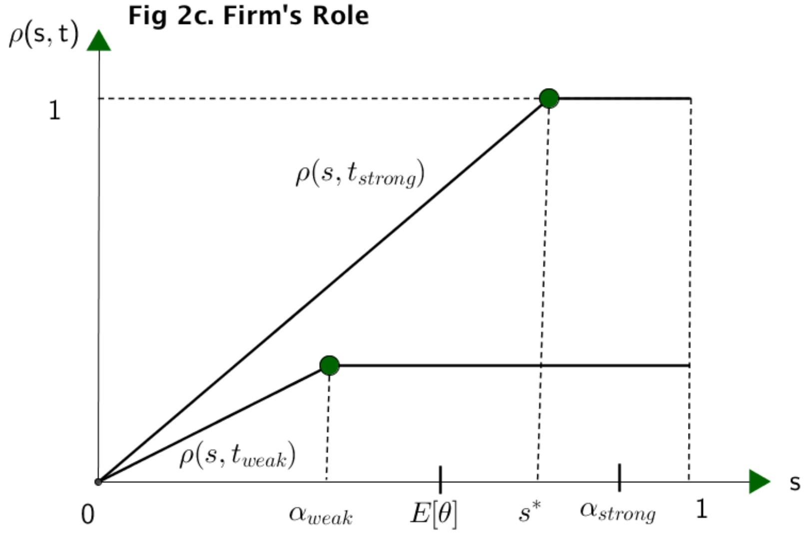
At stage 4, each firm $j \in \{1, 2\}$ forms its valuation $v(\rho, t)$ of the unemployed worker's ability, and offers the wage according to rule set in stage 1.5. At the equilibrium, firms maximize the number of signals they can infer by setting the highest penalty value for receiving an inaccurate recommendation from its employees. As a result, an employee's recommendation values will be influenced by his reputation costs more than his gratitude benefits, and firms will be able to infer a bigger interval of ability levels from a strong tie than from a weak tie. In particular, firms will be able to infer sufficiently high ability levels (above some threshold) from only strong ties. Figure 2c depicts this.

Proposition 3 - Each firm $j \in \{1, 2\}$ sets:

(1) reputation costs to the maximum level $r_j = r_{max}$, and

(2) wage equal to its valuation of the unemployed worker's ability $w(\rho, t) = v(\rho, t)$.

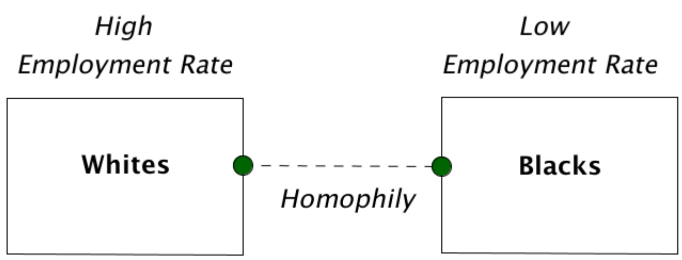
Fig 2c. Firm's Role



3 Wage Gap and Wage Hierarchies

In this section, I extend the employee referrals model by having two races (blacks and whites), and two types of occupations. I assume that fewer blacks have strong ties with employed workers than whites. This assumption follows from two empirical facts: employment differentials between blacks and whites in the US and the homophily feature of social networks (tendency to interact with others that have similar characteristics). Since blacks have lower employment rates than whites, they have to rely on their cross-race ties more than whites do. Such cross-race ties tend to be weaker because individuals tend to interact more often with others that belong to their own race (by homophily). Figure 3 below depicts this. Moreover, unemployed workers can choose the type of tie and the type of occupation they prefer. I then examine how the lower access to strong ties for blacks influences the black-white wage gap up and down wage hierarchies.

Fig 3. Access to Strong Ties



3.1 Races and Occupations

Now consider an environment with two groups $g \in \{black, white\}$ with blacks having less access to strong ties than whites, i.e. the proportion of unemployed blacks with access to a strong tie is less than the proportion of unemployed whites with access to a strong tie $\beta_{black} < \beta_{white}$. There is a single firm offering two type of occupations $occ \in \{occ_m, occ_h\}$, manual labor intensive occ_m and human capital intensive occ_h . In the manual labor intensive occupation, output is only increasing in ability up to some level. This ability level is below average $\theta_{low} < E[\theta]$, and having "human capital" (ability) beyond that doesn't add to output.

$$y_m = \begin{cases} \theta & \text{if } \theta \leq \theta_{low} \\ \theta_{low} & \text{if } \theta > \theta_{low} \end{cases} \tag{10}$$

In the human capital intensive occupation, output is always increasing in ability

$$y_h = \theta. \tag{11}$$

To work in human capital intensive occupation, an unemployed worker needs to pay a cost (getting education) k. However, it is "efficient" for above average ability workers to work in human capital intensive occupation in that their net value of working is higher $\theta - k > \theta_{low}$ for $\theta > E[\theta]$, and inefficient for below average ability workers to work in human capital intensive occupation in that their net value of working is lower $\theta - k < \theta_{low}$ for $\theta < E[\theta]$. I assume that strong ties can distinguish between below and above average ability workers and weak ties can't, i.e. $\alpha_{strong} \geq E[\theta] > \alpha_{weak}$.[17]

In this extension of the model, stages 1, 3, and 4 are same as described in section 2. However, stage 2 is modified in that each unemployed worker chooses the tie type and also the occupation type.

Stage 2': Tie Selection. At this stage, each unemployed worker (θ, T, g) chooses tie type $t \in T$ and the occupation type $occ \in \{occ_m, occ_h\}$ that maximizes his payoff,

$$\pi_u(\theta, T, g) = w(\rho(s,t), t, \beta_g, occ) - 1_{occ_h} k \tag{12}$$

where 1_{occ_h} is an indicator function which takes value of one if occupation is human capital intensive $occ = occ_m$ and zero otherwise. All the other aspects of the model are the same as before.

At stage 2', if an unemployed workers can get wage offer at least as high as the average ability $E[\theta]$, then he will choose the human capital intensive occupation. This follows from the fact that it is efficient for above average ability workers to work in human capital intensive occupation $\theta - k > \theta_{low}$ for $\theta > E[\theta]$, and inefficient for below average ability workers to work in human capital intensive occupation $\theta - k < \theta_{low}$ for $\theta < E[\theta]$. As a result, it is individually beneficial for an unemployed worker to get costly education if he can make firm believe that he is above average ability worker, even if he is actually not above average ability.

> **Proposition 4 -** An unemployed worker chooses the human capital intensive occupation if and only if $w(\rho(s,t), t, \beta_g, occ_h) \geq E[\theta]$ for some tie type $t \in T$.

Note that this result also implies that the mean wage of workers in the human capital

[17] This assumption is quite natural. To keep the presentation neat, I have assumed that there are only two strength levels, weak and strong. If there were many different strength levels, then sufficiently strong ties will be able to distinguish below and above average ability workers. I can then partition the set of strength levels into two sets, the weak set includes the set of tie strengths which can't distinguish between below and above average ability workers, and the strong set includes the set of tie strengths which can distinguish between below and above average ability workers.

intensive occupation is at least as high as average ability $\overline{w}_{occ_h} \geq E[\theta]$, and the mean wage of workers in the manual labor intensive occupation is below average ability $\overline{w}_{occ_l} \leq \theta_{low} < E[\theta]$. Thus, I can now show the stylized fact, i.e. the wage gap in occupation with higher mean wage (human capital intensive) is higher than the wage gap in occupation with lower mean wage (manual labor intensive). Wage gap in a given occupation is the average wage gap between whites and their black counterparts (same ability level). The stylized fact is satisfied whenever the wage gap in human capital intensive occupation is higher than the wage gap in manual labor intensive occupation, i.e. $\triangle WG = WG_h - WG_m > 0$.

3.2 Motivating Example

I will first consider a simple example, which is an extension of the motivating example considered in section 2.1. Recall, the weak ties have minimal information $\alpha_{weak} = 0$, strong ties have full information $\alpha_{strong} = 1$. Suppose only half of the unemployed white workers have access to strong ties $\beta_{white} = 0.5$, and no black workers have access to strong ties $\beta_{black} = 0$.

Since the firm can infer the lowest ability level. All unemployed workers with the lowest ability level choose manual labor intensive occupation, and get wage equal to their ability values. For all the other ability levels $\theta \in (0, 1]$, an unemployed black worker will be offered wage equal to the average ability by choosing the human capital intensive occupation, i.e. $w(t_{weak}, \beta_{black} = 0, occ_h) = E[\theta]$. Since this wage is at least as high as the average ability $E[\theta]$, all unemployed black workers with ability levels above the lowest $\theta \in (0, 1]$ will choose the human capital intensive occupation. From section 2.1, I know that all unemployed white workers with ability levels $\theta \in (0, 0.41)$ will prefer weak ties. They will get wage lower than the average ability by choosing the human capital intensive occupation, i.e. $w(t_{weak}, \beta_{white} = 0.5, occ_h) = 0.41 < E[\theta]$. Thus, they will choose the manual labor intensive occupations. On the other hand, unemployed white workers with ability $\theta \in [0.41, 1]$ will prefer strong ties. Half of these workers without access to strong ties will choose manual labor intensive occupation for the same reason as $\theta \in (0, 0.41)$. For $\theta \in [0.41, 0.5)$ with access to strong ties, they will get wage lower than the average ability by choosing the human capital intensive occupation, i.e. $w(t_{weak}, \beta_{white} = 0.5, occ_h) = \theta < E[\theta]$. Thus, they will choose the manual labor intensive occupations. For $\theta \in [0.5, 1]$ with access to strong ties, they will get wage higher than the average ability by choosing the human capital intensive occupation, i.e. $w(t_{weak}, \beta_{white} = 0.5, occ_h) = \theta > E[\theta]$. Thus, they will choose the human capital intensive occupations.

Manual Labor Intensive Occupation	Whites	wage	Blacks	wage
$\theta = 0$	all	θ	all	θ
$\theta \in (0, 0.41)$	all	$min\{\theta_{low}, 0.41\}$	none	-
$\theta \in [0.41, 0.5)$	half	$min\{\theta_{low}, \theta\}$	none	-
$\theta \in [0.41, 1]$	half	$min\{\theta_{low}, 0.41\}$	none	-
$WG_m = 0$				

The table above indicates for each ability level and for each group, the proportions and the wages of unemployed workers that chose manual labor intensive occupation. In this occupation, only the lowest ability blacks and whites can be compared. The firm can infer this ability level and therefore black and white workers get the same wage. In this occupation, the wage gap between whites and their black counterparts is zero.

Human Capital Intensive Occupation	Whites	wage	Blacks	wage
$\theta \in (0, 0.5)$	none	-	all	$E[\theta]$
$\theta \in [0.5, s^*)$	half	θ	all	$E[\theta]$
$\theta \in [s^*, 1)$	half	$\frac{1+s^*}{2}$	all	$E[\theta]$
$WG_h = \frac{1}{0.5}\left[E[\theta \vert \theta \geq 0.5] - E[\theta \vert \theta \geq 0]\right] > 0$				

Similarly, the table above indicates for each ability level and for each group, the proportions and the wages of unemployed workers that chose human capital intensive occupation. Since the informativeness level of weak tie is minimal, and above average ability white workers with access to strong ties use their strong ties, firm expects a white worker with a weak tie to be below average ability. As a result, an above average ability white worker with access to a strong tie can separate himself from below average ability white workers, and only above average ability white workers enter the human capital intensive occupation through strong ties. On the other hand, both below and above average ability black workers enter the human capital intensive occupation through weak ties, and firm pools them together. On average, high ability black workers get relatively lower wage than their white counterparts because the ability distribution of whites with a strong tie first order stochastically dominates the ability distribution of blacks with a weak tie.

3.3 Main Results for Wage Gap

In this section, I will provide conditions under which the wage gap in occupation with higher mean wage (human capital intensive) is higher than the wage gap in occupation with lower mean wage (manual labor intensive). The results in this section are derived from the assumption that only strong ties can distinguish between below and above average ability workers $\alpha_{strong} \geq E[\theta] > \alpha_{weak}$.[18] If no black workers have access to strong ties, then some below average ability workers enter the human capital intensive occupation through weak ties because firm cannot infer their abilities. On the other hand, an above average ability white worker with access to a strong tie can separate himself from below average ability white workers if the proportion of whites with access to strong ties β_{white} is high enough (above some threshold β^*). If the proportion of whites with access to strong ties is high enough, then many above average ability workers will use strong ties. As a result, if the firm cannot infer the ability of a white worker with a weak tie, then the firm will give less "benefit of the doubt" to this worker, and will expect him to be below average ability. Thus, only above average ability white workers with strong ties will enter the human capital intensive occupation. Since blacks don't have access to strong ties, both below and above average ability workers enter human capital intensive occupation through weak ties, and the rest of the arguments follow from the example (see last four lines of section 3.1).

The higher is the informativeness level of a weak tie, the more of the lower ability levels the firm can infer through a weak tie. As a result, if the firm cannot infer the ability of a white worker with a weak tie, then the firm knows that his ability is sufficiently high. Thus, the higher is the informativeness level of a weak tie, the more benefit of the doubt the firm gives, and the higher the proportion of whites with access to strong ties needs to be. As long as the weak tie cannot distinguish between below and above average ability workers $\alpha_{weak} < 0.5$, there exists a threshold value $\beta^* < 1$ such that if the proportion of whites with access to strong ties β_{white} is above this threshold, then the Black-White wage gap is higher in the occupation with higher mean wage.

> **Proposition 5 -** If no black workers have access to strong ties $\beta_{black} = 0$, and the informativeness level of weak tie is α_{weak}, then there exists a threshold value $\beta^* < 1$ such that:
>
> (1) If the proportion of whites with access to strong ties is higher than this threshold value $\beta_{white} > \beta^*$, then the Black-White wage gap is higher in the occupation with higher mean wage $\triangle WG = WG_h - WG_m > 0$.

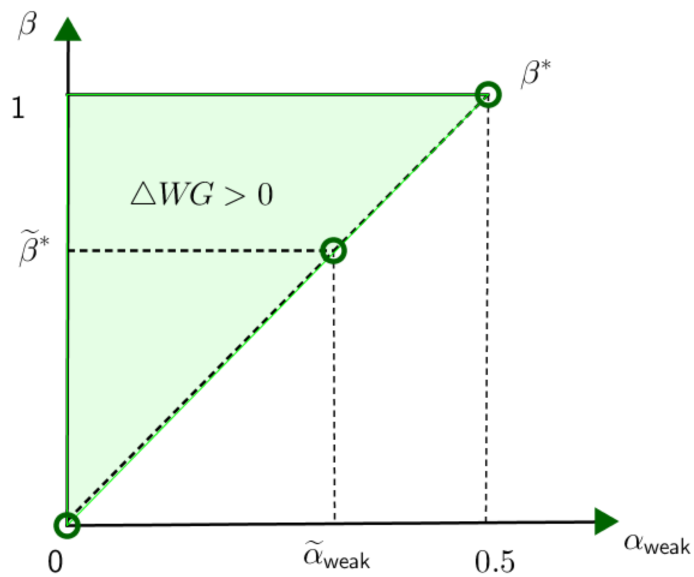
[18] I have assumed that the set of ability levels that an unemployed worker can infer is downwardly biased $[0, \alpha_t)$, but the results will also hold if this interval was upwardly biased $(\alpha_t, 1]$. In either case, blacks don't have access to strong ties and only strong ties can distinguish between below and above average ability workers. As a result, some below average ability black workers $[\alpha_{weak}, E[\theta])$ will use weak ties to pool with higher ability black workers and enter human capital intensive occupation. This result follows from the assumption that only strong ties can distinguish between below and above average ability workers. Observe that this assumption is $\alpha_{strong} \geq E[\theta] > \alpha_{weak}$ if the interval is downwardly biased and it is reversed $\alpha_{weak} \geq E[\theta] > \alpha_{strong}$ if the interval is upwardly biased.

(2) If $\alpha_{weak} = 0$, then $\beta^* = 0$.

(3) If $\alpha'_{weak} > \alpha_{weak}$, then $\beta^{*'} > \beta^*$.

In the figure 3 below, the informativeness level of weak tie α_{weak} is indicated in the horizontal axis, and the proportion of white workers with access to strong ties is indicated in the vertical axis. The diagonal dotted line indicates the threshold value β^* for a given α_{weak}. Since the values strictly above the threshold value β^* leads to the Black-White wage gap to be higher in the occupation with higher mean wage, the diagonal line itself is not included. For convenience, a straight diagonal line has been drawn in the figure but it may not be a straight line. The shaded region indicates the values of α_{weak} and β_{white} for which the Black-White wage gap is higher in the occupation with higher mean wage $\triangle WG = WG_h - WG_m > 0$.

Fig 4. Wage Gap



If some blacks have access to strong ties, then some high ability blacks enter human capital intensive occupation with strong ties. As long as the proportion of blacks with strong ties is small enough ($\beta_{black} \leq \beta^*$), some below average ability black workers enter the human capital intensive occupation through weak ties because firm cannot infer their abilities. On the other hand, as long as the proportion of whites with strong ties is high enough ($\beta_{white} > \beta^*$), an above average ability white worker with access to a strong tie can separate himself from below average ability white workers. As a result, only above average ability white workers enter human capital intensive occupation. It can then be shown (with some alpha that the Black-White wage gap is higher in the occupation with higher mean wage.

Theorem 2 - If the informativeness level of weak tie is α_{weak}, then there exists a threshold value β^* such that:

For $\beta_{black} \leq \beta^* < \beta_{white}$, Black-White wage gap is higher in the occupation with higher mean wage $\triangle WG = WG_h - WG_m > 0$.

In the figure 3 above, if the informativeness level of weak tie is $\widetilde{\alpha}_{weak}$, the proportion of whites with access to strong ties is high enough (above the diagonal line $\beta_{white} > \widetilde{\beta}^*$) and the proportion of blacks with access to strong ties is high enough (above the diagonal line $\beta_{white} > \widetilde{\beta}^*$) and the proportion of blacks with access to strong ties is low enough (below the diagonal line $\beta_{black} \leq \widetilde{\beta}^*$), then the Black-White wage gap is higher in the occupation with higher mean wage $\triangle WG = WG_h - WG_m > 0$.

4 Use of Ties and Returns to Ties

In this section, I will explore some implications of the employee referrals model for job search. For instance, when should one expect weak ties to be used more than strong ties for job search? When should one expect higher returns from finding a job through a tie (weak or strong) than finding the same job through direct application to the firm? The employee referrals model has new insights about the use of different type of ties and the returns to ties, which can help answer these questions.

4.1 Use of Ties

It is well known that many jobs are found through social networks (see Ioannides and Loury (2004), and Topa (2011) for two surveys). Granovetter (1974) documents that a large proportion of jobs are found through weak ties. Granovetter (1973) argues that the weak ties to individuals with whom one has few common friends are most useful for job search, because they provide access to otherwise unobtainable information about job openings. This finding of the frequent use of weak ties led to the coining of the well-known phrase, the "strength of weak ties". However, the evidence about the use of ties is mixed. Studies in U.S. cities (Murray, Rankin, and Magill (1981), Bridges and Villemez (1986), Marsden and Hurlbert (1988)) find that both weak and strong ties are important for job search. In Japan, Watanabe (1987) documents that small business employers screen applicants using strong ties. In China, Bian (1997, 1999) argues that the guanxi system of personal relationships allocates jobs using strong ties and paths.

The employee referrals model can help explain the mixed evidences about the use of different type to ties. I will now provide conditions under which one type of tie will be used more often than the other type of tie to find jobs. To show this, I relax the assumption that each unemployed worker has access to a weak tie. Only β_{weak} of the unemployed workers have access to a weak tie, β_{strong} of the unemployed workers have access to a strong tie, and β_{both} of the unemployed workers have access to both weak and strong ties. As before, there is a threshold ability level, workers with abilities lower than this threshold prefer weak ties, and workers with abilities higher than this threshold prefer strong ties. Low (below threshold) ability workers use weak ties to pool with workers who don't have access to strong ties and have higher abilities than theirs. On the other hand, high ability workers use strong ties to reveal their abilities to the firm. The lower is the access to strong ties, the higher is the threshold ability, and the higher is the abilities of workers who prefer to pool with even higher ability workers through weak ties. If the access to strong ties is sufficiently low (β_{strong} and β_{both} are low or β_{weak} is high), then even above average ability workers will prefer to use weak ties to pool with even higher ability workers. In such case, the threshold ability is above average ability, and most jobs are found through weak ties. In the proposition 6 below, I state this result formally.

> **Proposition 6 -** If the access to strong ties is sufficiently low $\beta_{weak} > max\{\beta^*, \beta_{strong}\}$, then the threshold ability level is above average ability $\theta^* > E[\theta]$, and most jobs are found through weak ties $f(t_{weak}) > f(t_{strong})$.

This result provides an alternative explanation for the frequent use of weak ties (or the "strength of weak ties"). A worker will use the type of tie which gives him higher returns, if he has access to such type of tie. If many high ability workers don't have access to strong ties, then weak ties will be used more often in job searches because it allows below average ability workers and some above average ability workers to pool with even higher ability workers. Contrary to the existing explanations, the frequent use of weak ties may not be due to its efficiency in matching workers and the firm. Instead, when access to strong ties are really scarce, many workers use weak ties to pool with really high ability workers.

4.2 Returns to Ties

The empirical evidence about the returns to ties is mixed. Some studies show that workers who found their jobs through family, friends, and acquaintances earned more than those using formal and other informal job-search methods (Rosenbaum et al. (1999), Marmaros and Sacerdote (2002)). Others indicate that the initial wage advantage declined over time (Corcoran, Datcher, and Duncan (1980), Simon and Warner (1992)). Some analysts found no general initial or persistent wage effects (Bridges and Villemez (1986), Holzer (1987), Marsden and Gorman (2001)). In fact, some studies (Elliott (1999), Green, Tigges, and Diaz (1999)) show that those using contacts earned less than those using formal methods.

The employee referrals model can help explain the mixed evidences about the returns to ties. I will now consider a variant of the model where workers can directly apply to the firm "d" and through a tie "t". Applying directly to the firm is equivalent to applying through a tie with the lowest informativeness level $\alpha_d = 0$. Suppose the informativeness level of the tie is positive $\alpha_t > 0$. Thus, it can be seen as a special case of the model where weak ties have the lowest informativeness level. I will assume that everyone can apply directly to the firm, but only δ of these workers get the job through direct application, and only β of the unemployed workers have access to ties. Let $w_{t-d}(\theta < 0.5)$ denote the average returns to using ties than direct applications for below average ability workers.

> **Proposition 7 -** (1) If the proportion of workers with access to ties is positive $\beta > 0$, then the threshold ability is below average $\theta^* < E[\theta]$. (2) In addition, if the informativeness level of the tie is above average ability $\alpha_t \geq E[\theta]$, the probability of finding jobs through direct application is $0 \leq \delta < 1$, then there exists a threshold value β^{**} such that:
>
> (i) If $\beta \approx \beta^{**}$, then $w_{t-d}(\theta < 0.5) \approx 0$.
>
> (ii) If $\beta < \beta^{**}$, then $w_{t-d}(\theta < 0.5) < 0$.
>
> (iii) If $\delta' < \delta$, then $\beta^{**'} > \beta^{**}$.

As before, there is a threshold ability level. Workers with abilities lower than this threshold prefer to apply directly to the firm, and workers with abilities higher than this threshold prefer to use their ties. The wage offer from directly applying to the firm equals this threshold ability. Since the informativeness level of the firm is minimal $\alpha_d = 0$, workers can at most get a wage equal to the average ability by directly applying to the firm. This proposition states that (1) if the proportion of workers with access to ties is positive $\beta > 0$, then the threshold ability value is below average ($\theta^* < E[\theta]$). As a result, above average ability workers always get higher returns from ties than directly applying to the firm. This result is consistent with the empirical finding of Cappellari and Tatsiramos (2015). (2 (i), (ii)) The lower is the proportion of workers with access to ties, the higher is the threshold ability, and the lower are the returns to ties. If the proportion of workers with access to ties is sufficiently small ($\beta \leq \beta^{**}$), then for below average ability workers, the returns to tie can be small ($\beta \approx \beta^{**}$) and even negative ($\beta < \beta^{**}$). (2 (iii)) For workers who prefer to find jobs through direct applications, their returns to ties are negative. The lower is the probability of finding a job through a direct application, the more such workers will use ties to find jobs, and the lower are the average returns to ties. This result is consistent with the empirical finding of Loury (2004).

5 Conclusion

For many years, social scientists and policy makers have tried to understand mechanisms that determine the black-white wage gap in the US. There is significant evidence suggesting that the wage gap between blacks and their white counterparts (same individual characteristics) widens as one moves up the wage hierarchies of the private sector. This paper shows that the widespread use of employee referrals in the labor market, and the lack of access to strong ties for blacks (via black-white employment differentials and homophily) can be behind this empirical finding. The model predicts that low (below average) ability workers can get higher wages from finding jobs through weak ties. This is because their use of weak ties leads to firm pooling them with these high ability workers who don't have strong ties. On the other hand, high ability workers can get better wage offers from using strong ties because this leads to firm infering that they are high ability workers. Only high ability white workers enter higher earning occupations because they can separate themselves from low ability white workers through strong ties. As long as sufficiently low number of blacks have access to strong ties, both low and high ability black workers enter higher earning occupations through weak ties, firm pools them together, and on average high ability black workers get relatively lower wage than their white counterparts.

Similar to aptitude tests and other attribute measurements used by firms in formal hiring, the employee referrals mechanism is a useful device for screening job applicants because employees can convey some information to the firm about the abilities of their unemployed ties. However, the employee referrals mechanism is by its nature discriminatory in that not all unemployed workers have the same access to employed ties. Government can help level the playing field for the two races by subsidizing formal screening mechanisms in the higher earning occupations. Such policy suggestion seems counter-intuitive in that the firm already has strong incentives to use formal screening mechanisms in the higher earning (human capital intensive) occupations, and the informational asymmetries between the firm and unemployed workers are arguably lower in the such occupations. However, the lack of access to strong ties for black workers gives white workers an informational advantage in the higher earning occupations. The employee referrals mechanism reduces more asymmetric information between the firm and white workers than between the firm and black workers in the higher earning occupations. This informational advantage of whites at the high end of wage hierarchies is an obstacle for the upwardly mobile blacks towards obtaining equal wages to their white counterparts.

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Appendix A. Proofs

This appendix contains the formal arguments for the results in the text.

Proof of proposition 1

(1) The initial value condition $\rho(0,t) = 0$ follows from the definition of the cost efficient recommendation strategy.

(2) For $0 < s < min\{s^*, \alpha_t\}$, the employed worker's expected value of ability is equal to the signal for such interval of signals. Thus, an employed worker solves the following maximization problem.

$$Max_{\rho\in[0,1]}\left\{w(\rho,t) - r(\rho - s)^2\right\}$$

FOC:

$$[\rho] \Longrightarrow w_1(\rho,t) - 2r(\rho - s) = 0$$

Since recommendations are increasing in signals $\rho_1(s,t) > 0$, the employed worker's recommendations reveal his signal value to the firm $\{s' : \rho(s',t) = \rho(s,t)\} = \{s\}$ for all such signals. As a result, the firm offers wage equal to the signal value for such signals $w(\rho(s,t),t) = s \Longrightarrow w_1(\rho,t)\rho_1(s.t) = 1$. Plugging this expression in the first order condition (FOC) gives the following differential equation (DE).

$$\rho_1(s,t) = \frac{1}{2r(\rho(s,t) - s)} \qquad (DE)$$

Since recommendations are increasing in signals $\rho_1(s,t) > 0$, then the differential equation implies that the employee recommends higher than his signal $\rho(s,t) > s$. The reader can easily verify that the family of solutions to this differential equation is given by

$$\rho(s,t) + c = -\frac{1}{2r}ln\left[\frac{1}{2r} + s - \rho(s,t)\right]$$

where c is a constant which can be determined by the initial value condition $\rho(0,t) = 0$. So I get $c = -\frac{1}{2r}ln\left[\frac{1}{2r}\right]$, and

$$\begin{aligned}\rho(s,t) &= \frac{1}{2r}ln\left[\frac{1}{2r}\right] - \frac{1}{2r}ln\left[\frac{1}{2r} + s - \rho(s,t)\right] \\ \Longrightarrow \rho(s,t) &= \frac{1}{2r}ln\left[\frac{\frac{1}{2r}}{\frac{1}{2r} + s - \rho(s,t)}\right].\end{aligned}$$

(3) If $min\{s^*, \alpha_t\} = s^*$, then recommendations hit the upper bound value of one for some signal. For any signal higher than this threshold signal, employee's payoff maximizing recommendation values are not feasible (greater than one). Thus, the employed worker chooses recommendation values as close to the payoff maximizing recommendation values as possible. The closest value to the payoff maximizing recommendation values for all such signals is one.

If $min\{s^*, \alpha_t\} = \alpha_t$, then the signal reaches the informativeness level of the employed worker. For any signal higher than the informativeness level, the employed worker chooses recommendation value that minimizes his reputation costs $E[\theta|s, t]$ as long as it was not used for some signal lower than the informativeness level. If it was used for some lower signal, then the employed worker chooses the lowest value that was not used for some lower signal, i.e. $lim_{s' \to \alpha_t} \rho(s', t)$. ■

Proof of proposition 2

(1) Since recommendations are increasing in signals for $s < min\{s^*, \alpha_t\}$, the employed worker's recommendations reveal his signal value to the firm $\{s' : \rho(s', t) = \rho(s, t)\} = \{s\}$ for all such signals. As a result, the firm offers wage equal to the signal value for such signals $w(\rho(s, t), t) = s$.

(2) For the interval of signals that the firm can not infer $s \geq min\{s^*, \alpha_t\}$, the firm sets wage equal to the weighted average of these signals $w(\rho(s, t), t) = E[\theta|s \geq min\{s^*, \alpha_t\}, t] = \int_{min\{s^*, \alpha_t\}}^{1} \theta \frac{f(t|\theta)}{\int_{min\{s^*, \alpha_t\}}^{1} f(t|\theta) d\theta} d\theta$, where weights are determined by the the proportion of unemployed workers $f(t|\theta)$ with ability level θ who chose tie type t. ■

Proof of theorem 1

(1) If the threshold signal is greater than the informativeness level of the weak tie $s^* > \alpha_{weak}$, then there is an interval of signals $s \in (\alpha_{weak}, min\{s^*, \alpha_{strong}\})$ where wage offers from weak ties is fixed, and wage offers from strong ties is increasing in signals. As a result, if $\widetilde{\theta} \in (\alpha_{weak}, 1)$ prefers strong tie, then all $\theta > \widetilde{\theta}$ prefer strong tie as well. Thus, the cost efficient equilibrium involves a threshold ability θ^* such that ability levels below the threshold ability $\theta < \theta^*$ weakly prefer weak ties ($\theta \in [0, \alpha_{weak})$ are indifferent between the two type of ties, and $\theta \in [\alpha_{weak}, \theta^*)$ prefer weak ties), and unemployed workers with ability levels above the threshold ability $\theta > \theta^*$ prefer strong ties. At equilibrium, wage offer from a weak tie is

$$
\begin{aligned}
E[\theta|s &\geq \alpha_{weak}, t_{weak}] = \int_{\alpha_{weak}}^{1} \theta \frac{f(t_{weak}|\theta)}{\int_{\alpha_{weak}}^{1} f(t_{weak}|\theta) d\theta} d\theta \\
&= \frac{1}{\int_{\alpha_{weak}}^{\theta^*} d\theta + \int_{\theta^*}^{1} (1-\beta) d\theta} \left[\int_{\alpha_{weak}}^{\theta^*} \theta d\theta + \int_{\theta^*}^{1} \theta (1-\beta) d\theta \right] \\
&= \frac{1}{2\left[\beta\theta^* + (1-\beta) - \alpha_{weak}\right]} \left[\beta(\theta^*)^2 + (1-\beta) - \alpha_{weak}^2\right]
\end{aligned}
$$

The value of threshold ability θ^* depends on threshold signal s^* and the informativeness levels of both type of ties α_{weak}, α_{strong}. Let $\widehat{\theta} = \{\theta : \theta = E[\theta|s \geq \alpha_{weak}, t_{weak}]\}$ be the unemployed worker who is indifferent between choosing wage offer from a strong tie and revealing his ability or choosing wage offer from a weak tie. If the threshold signal and the informativeness level of strong ties are high enough $min\{s^*, \alpha_{strong}\} > \widehat{\theta}$, then threshold ability is $\theta^* = \widehat{\theta}$.

If $min\{s^*, \alpha_{strong}\} < \widehat{\theta}$, then the threshold ability is $\theta^* = min\{s^*, \alpha_{strong}\}$. This follows from the following two observations. Firstly, since $E[\theta|s \geq \alpha_{weak}, t_{weak}] > min\{s^*, \alpha_{strong}\}$ for all $\alpha_{weak} \leq \theta < min\{s^*, \alpha_{strong}\}$, unemployed workers whose abilities are revealed with strong ties will prefer weak ties. Secondly, unemployed workers who can pool with strong ties, they get higher returns from pooling with strong ties than pooling with weak ties.

$$
\begin{aligned}
E[\theta|s &\geq min\{s^*, \alpha_{strong}\}, t_{strong}] = \int_{min\{s^*, \alpha_{strong}\}}^{1} \theta \frac{f(t_{strong}|\theta)}{\int_{min\{s^*, \alpha_{strong}\}}^{1} f(t_{strong}|\theta) d\theta} d\theta \\
&= \int_{min\{s^*, \alpha_{strong}\}}^{1} \theta \frac{\beta}{\int_{min\{s^*, \alpha_{strong}\}}^{1} \beta d\theta} d\theta \\
&= \frac{1}{1 - min\{s^*, \alpha_{strong}\}} \int_{min\{s^*, \alpha_{strong}\}}^{1} \theta d\theta \\
&> \frac{1}{1-\alpha_{weak}} \int_{\alpha_{weak}}^{1} \theta d\theta \\
&\geq \frac{1}{\int_{\alpha_{weak}}^{\theta^*} d\theta + \int_{\theta^*}^{1} (1-\beta) d\theta} \left[\int_{\alpha_{weak}}^{\theta^*} \theta d\theta + \int_{\theta^*}^{1} \theta(1-\beta) d\theta \right] \\
&= E[\theta|s \geq \alpha_{weak}, t_{weak}]
\end{aligned}
$$

The second equality follows from the fact that the workers with abilities $\theta \geq min\{s^*, \alpha_{strong}\}$ get the same wage offers. Thus, any equilibrium involves all such workers preferring same

type of tie. Suppose, all such workers prefer and use weak ties. They will get wage equal to the average of abilities $\theta \geq min\{s^*, \alpha_{strong}\}$,
i.e. $\frac{1}{1-min\{s^*,\alpha_{strong}\}} \int_{min\{s^*,\alpha_{strong}\}}^{1} \theta d\theta$. Then, $\theta > \frac{1}{1-min\{s^*,\alpha_{strong}\}} \int_{min\{s^*,\alpha_{strong}\}}^{1} \theta d\theta$ ability workers can profitably deviate by using a strong tie to get a higher wage. Thus, I consider equilibrium where all workers with ability levels $\theta \geq min\{s^*, \alpha_{strong}\}$ prefer strong ties. In such case, only those with access to strong ties will be able to use it. Then, the unemployed workers who can pool with strong ties $\theta \geq min\{s^*, \alpha_{strong}\}$ get wage equal to average of abilities $\theta \geq min\{s^*, \alpha_{strong}\}$ by choosing a strong tie. The first inequality (line 4) indicates that the wage that they can get from a strong tie is higher than the average of abilities $\theta \geq \alpha_{weak}$ because this includes some workers with strictly lower abilities $\theta \in [\alpha_{weak}, min\{s^*, \alpha_{strong}\})$. The second inequality (line 5) indicates that the average of abilities $\theta \geq \alpha_{weak}$ is weakly higher than any wage that workers can get from choosing a weak tie. This follows from the fact that everyone with abilities $\theta \in [\alpha_{weak}, \theta^*)$ choose weak ties and only β of workers with abilities $\theta \geq \theta^*$ choose weak ties.

Thus, the threshold ability is $\theta^* = min\{\widehat{\theta}, s^*, \alpha_{strong}\}$.

(2) If the threshold signal is lesser than the informativeness level of the weak tie $s^* \leq \alpha_{weak}$, then there is no Cost Efficient Equilibrium.

Workers with abilities $\theta \in [0, s^*)$ are indifferent between the two type of ties, and workers with abilities $\theta \geq s^*$ get same wage offers. Thus, any equilibrium involves all such workers $\theta \geq s^*$ preferring same type of tie. However, no such equilibrium exists. Suppose, all such workers prefer and use weak ties. They will get wage equal to the average of abilities $\theta \geq s^*$, i.e. $\frac{1}{1-s^*} \int_{s^*}^{1} \theta d\theta$. Then, $\theta > \frac{1}{1-s^*} \int_{s^*}^{1} \theta d\theta$ ability workers can profitably deviate by using strong ties to get a wage higher than the average of abilities $\theta \geq s^*$. Similarly, suppose all such workers prefer strong ties. Only those with access to strong ties will be able to use it. Then, the wage from weak and strong tie is the same, i.e. $\frac{1}{1-s^*} \int_{s^*}^{1} \theta d\theta$. As before, $\theta > \frac{1}{1-s^*} \int_{s^*}^{1} \theta d\theta$ ability workers can profitably deviate by using weak ties to get a wage higher than the average of abilities $\theta \geq s^*$. ■

Proof of proposition 3

(1) Each firm's weakly dominant strategy is to set $r_j = r_{max}$.

Each firm's weakly dominant strategy is to maximize the set of ability levels that it can infer. A firm's ability to infer ability levels is increasing in reputation cost level. To see this, note that the threshold signal (following similar arguments as in the motivating example) in this extension is

$$
\begin{aligned}
s^* &= \frac{1}{2\left(\frac{r_j}{\gamma(t)}\right)}\left[\frac{1-e^{2\left(\frac{r_j}{\gamma(t)}\right)}}{e^{2\left(\frac{r_j}{\gamma(t)}\right)}}\right]+1 \\
&= \frac{1}{2\left(\frac{r_j}{\gamma(t)}\right)e^{2\left(\frac{r_j}{\gamma(t)}\right)}}-\frac{1}{2\left(\frac{r_j}{\gamma(t)}\right)}+1
\end{aligned}
$$

And the threshold signal is increasing in reputation cost level,

$$s^{*'}(r_j) = e^{2(r_j/\gamma(t))}\,(2/\gamma(t))\left(e^{2(r_j/\gamma(t))}-(2/\gamma(t))\,r_j-1\right) > 0.$$

This follows from the fact that $\left(e^{2(r_j/\gamma(t))}-(2/\gamma(t))\,r_j-1\right) > 0$ for all $r_j > 0$. Thus, it is each firm's weakly dominant strategy to set its reputation cost to the maximum level $r_j = r_{max}$.

(2) If $r_1 = r_2 = r_{max}$, then there are two cases two consider.

Case 1 - For the interval of ability levels that firms can infer.

A firm incurs loss by setting wage above the ability level $w_j(\rho,t) > \theta$, so no firm will set such wage. I will first show that $w_1(\rho,t) = w_2(\rho,t)$. Suppose that $w_2(\rho,t) < w_1(\rho,t)$.

If $w_2(\rho,t) < w_1(\rho,t) = \theta$, then firm 1 can profitably deviate by setting wage $w_1(\rho,t) = w_2(\rho,t) < \theta$.

If $w_2(\rho,t) < w_1(\rho,t) < \theta$, then firm 2 can profitably deviate by setting wage $w_2(\rho,t) = w_1(\rho,t) < \theta$.

Thus, $w_1(\rho,t) = w_2(\rho,t)$. If $w_1(\rho,t) = w_2(\rho,t) \neq \theta$, then each firm can profitably deviate by offering slightly higher wage than the other firm.

Case 2 - For the interval of ability levels that firms cannot infer.

A firm incurs loss by setting wage above the expected ability level $w_j(\rho,t) > E[\theta|s \geq s^*, t]$, so no firm will set such wage. Using similar arguments as case 1, $w_1(\rho,t) = w_2(\rho,t) = E[\theta|s \geq s^*, t]$ for all such ability levels. ■

Proof of proposition 4

Each unemployed worker (θ, T, g) chooses tie type $t \in T$ and the occupation type $occ \in \{occ_m, occ_h\}$ that maximizes his payoff,

$$Max_{(t,occ)}\left\{w(\rho(s,t),t,\beta_g,occ) - 1_{occ_h}k\right\}$$

If $w(\rho(s,t),t,\beta_g,occ_h) \geq E[\theta]$ for some tie type $t \in T$, then unemployed worker can get higher payoff at human capital intensive occupation than the maximum payoff at manual labor occupation $w(\rho(s,t), t, \beta_g, occ_h) - k > \theta_{low}$. Thus, he will choose human capital intensive occupation.

If $w(\rho(s,t), t, \beta_g, occ_h) < E[\theta]$ for both tie types $t \in T$, then unemployed worker gets negative payoff by working at human capital intensive occupation $w(\rho(s,t), t, \beta_g, occ_h) - k < 0$, and non-negative payoff by working at manual labor intensive occupation $w(\rho(s,t), t, \beta_g, occ_m) \in [0, \theta_{low}]$. Thus, he will choose manual labor intensive occupation. ■

Following proposition 3, the threshold ability is $\theta^* = \widehat{\theta}$ where $\widehat{\theta} = \{\theta : \theta = E[\theta | s \geq \alpha_{weak}, t_{weak}]\}$. The next lemma describes four properties of $\widehat{\theta}$ which I use to prove proposition 5.

Lemma 1. $\widehat{\theta} = \{\theta : \theta = E[\theta | s \geq \alpha_{weak}, t_{weak}]\}$ satisfies the following five properties: (1) $\frac{\partial \widehat{\theta}}{\partial \alpha} > 0$, (2) $\frac{\partial \widehat{\theta}}{\partial \beta} < 0$, (3) $\widehat{\theta} < 0.75$, (4) if $\beta = 1$, then $\widehat{\theta} = \alpha_{weak}$, and (5) if $\alpha_{weak} = 0$ and $\beta = 0$, then $\widehat{\theta} = 0.5$.

(1) The first two properties follows directly from the equation which solves for $\widehat{\theta}$

$$\widehat{\theta} = \{\theta : \theta = E[\theta | s \geq \alpha_{weak}, t_{weak}]\} \Longleftrightarrow \widehat{\theta} = \frac{1}{2\left[\beta\widehat{\theta} + (1-\beta) - \alpha_{weak}\right]} \left[\beta(\widehat{\theta})^2 + (1-\beta) - \alpha_{weak}^2\right]$$

$$\Longleftrightarrow \beta(\widehat{\theta})^2 + 2\left[(1-\beta) - \alpha_{weak}\right]\widehat{\theta} - \left[(1-\beta) - \alpha_{weak}^2\right] = 0 \tag{13}$$

Applying the Implicit Function Theorem with
$F(\widehat{\theta}, \alpha_{weak}, \beta) = \beta(\widehat{\theta})^2 + 2\left[(1-\beta) - \alpha_{weak}\right]\theta - \left[(1-\beta) - \alpha_{weak}^2\right] = 0$ gives the following.

$$\frac{\partial \widehat{\theta}}{\partial \alpha_{weak}} = -\frac{2\left[\alpha_{weak} - \widehat{\theta}\right]}{2\beta\widehat{\theta} + 2\left[(1-\beta) - \alpha_{weak}\right]} > 0$$

The numerator is negative since $\alpha_{weak} < \widehat{\theta}$. The denominator is positive as follows.

$$2\beta\widehat{\theta} + 2\left[(1-\beta) - \alpha_{weak}\right] > 2\beta\alpha_{weak} + 2\left[(1-\beta) - \alpha_{weak}\right] = 2(1-\alpha_{weak})(1-\beta) > 0$$

where the first inequality follows from $\alpha_{weak} < \widehat{\theta}$.

(2) Similarly,

$$\frac{\partial \widehat{\theta}}{\partial \beta} = -\frac{\left(\widehat{\theta}^2 - 2\widehat{\theta} + 1\right)}{2\beta\widehat{\theta} + 2\left[(1-\beta) - \alpha_{weak}\right]} < 0$$

The numerator is positive since $\widehat{\theta} \in [0,1]$. The denominator is positive as explained above.

(3) By property 1 and 2, $\widehat{\theta}$ is maximized by choosing maximum value of α_{weak} and minimum value of β. Since $\beta = 0$ is the minimum value and $\alpha_{weak} < 0.5$, then the equation (15) which solves for $\widehat{\theta}$ implies $\widehat{\theta} < 0.75$.

(4) Using quadratic formula, equation (13) implies

$$\widehat{\theta} = \frac{-(1-\alpha_{weak}-\beta) + \sqrt{1-\alpha_{weak}}\sqrt{1-\alpha_{weak}(1-\beta)-\beta}}{\beta} \tag{14}$$

If $\beta = 1$, then $\widehat{\theta} = \alpha_{weak}$.

(5) If $\alpha_{weak} = 0$ and $\beta = 0$, then equation (13) implies$2\widehat{\theta} - 1 = 0 \Longleftrightarrow \widehat{\theta} = 0.5$. ■

Proof of proposition 5

Let $\widehat{\theta}_{white}$ be the threshold ability for whites, $\widehat{\theta}_{black}$ be the threshold ability for blacks, and $s^* > \widehat{\theta}_{black} \geq \widehat{\theta}_{white}$ by proposition 3.

(1) All black workers with ability levels $\theta \geq \alpha_{weak}$ choose human capital intensive occupation. In the manual labor intensive occupation occupation, only workers with ability levels $\theta < \alpha_{weak}$ can be compared. The firm can infer these ability levels with strong and weak ties. Therefore, the black and white workers get the same wage, and the wage gap between whites and their black counterparts is zero.

$$WG_m = \frac{1}{\alpha_{weak}} \int_0^{\alpha_{weak}} [w_{white}(\theta,t) - w_{black}(\theta,t)]\, d\theta = \frac{1}{\alpha_{weak}} \int_0^{\alpha_{weak}} [\theta - \theta]\, d\theta = 0.$$

For a given informativeness level of weak tie α_{weak}, let β^* be the value for the proportion of white workers with access to strong tie such that $\widehat{\theta}_{white} = 0.5$. If $\alpha_{weak} = 0.5$, then $\beta^* = 1$ by property (4) of lemma 1. Since $\alpha_{weak} < 0.5$, property (1) of lemma 1 implies that $\widehat{\theta}_{white} < 0.5$ if $\beta_{white} = 1$. Then, property (2) of lemma 1 implies that there exists a threshold value $\beta^* < 1$ such that $\widehat{\theta}_{white} = 0.5$.

If $\beta_{white} > \beta^*$, then $\widehat{\theta}_{white} < 0.5$ follows from property (2) of lemma 1. If $\widehat{\theta}_{white} < 0.5$, then below average ability white workers will not be able to enter human capital intensive occupation through weak ties, and the Black-White wage gap is higher in the human capital intensive occupation (which is the occupation with higher mean wage) as follows.

$$
\begin{aligned}
WG_h &= \frac{1}{0.5}\int_{0.5}^{1}\left[w_{white}(\theta, t_{strong}) - w_{black}(\theta, t_{weak})\right] d\theta \\
&= \frac{1}{0.5}\left[\int_{0.5}^{s^*}\left(\theta - \widehat{\theta}_{black}\right) d\theta + \int_{s^*}^{1}\left(\frac{1+s^*}{2} - \widehat{\theta}_{black}\right) d\theta\right] \\
&= \frac{1}{0.5}\left[\frac{(s^*-0.5)}{2}\left(s^* + 0.5 - 2\widehat{\theta}_{black}\right) + \frac{(1-s^*)}{2}\left(1 + s^* - 2\widehat{\theta}_{black}\right)\right] \\
&= \left(1 - \widehat{\theta}_{black}\right) - 0.5^2 > 0
\end{aligned}
$$

The first equality follows from the fact that only above average ability whites workers enter human capital intensive occupation through strong ties. In the human capital intensive occupation occupation, only above average ability workers can be compared. The second equality follows from the fact that for $\theta \in [0.5, s^*)$, firm can infer the ability levels of the white workers, and for $\theta \geq s^*$ firm cannot infer the ability levels of the white workers. The final inequality follows from the property (3) of lemma 1 $\widehat{\theta}_{black} < 0.75$.

(2) If $\alpha_{weak} = 0$, then $\beta^* = 0$ by property (5) of lemma 1.

(3) If $\alpha_{weak}^{'} > \alpha_{weak}$ and $\beta_{white} = \beta^*$, then property (1) of lemma 1 implies that $\widehat{\theta}_{white}^{'} > \widehat{\theta}_{white} = 0.5$. Then, property (2) of lemma 1 implies that $\beta^{*'} > \beta^*$. ■

Proof of theorem 2

As before, all black workers with ability levels $\theta \geq \alpha_{weak}$ choose human capital intensive occupation. In the manual labor intensive occupation occupation, only workers with ability levels $\theta < \alpha_{weak}$ can be compared. The firm can infer these ability levels with strong and weak ties. Therefore, the black and white workers get the same wage, and the wage gap between whites and their black counterparts is zero.

$$
WG_m = \frac{1}{1-\alpha_{weak}}\int_{0}^{\alpha_{weak}}\left[w_{white}(\theta, t) - w_{black}(\theta, t)\right] d\theta = \frac{1}{1-\alpha_{weak}}\int_{0}^{\alpha_{weak}}[\theta - \theta]\, d\theta = 0.
$$

For the given informativeness level of weak tie α_{weak}, if $\beta_{black} \leq \beta^* < \beta_{white}$, then $\widehat{\theta}_{white} < 0.5$ and $\widehat{\theta}_{black} \geq 0.5$. As a result, below average ability white workers will not be able to enter human capital intensive occupation through weak ties, but below average ability black workers will enter human capital intensive occupation through weak ties. Thus, the Black-White wage gap is higher in the human capital intensive occupation (which is the occupation with higher mean wage) as follows.

Observe that

$$
\begin{aligned}
& \sum_{(t_{white},t_{black})\in\{t_{weak},t_{strong}\}^2}\left[\int_{0.5}^{1} f(t_{white}|occ_h,\theta)f(t_{black}|occ_h,\theta)d\theta\right] \\
= & \left[\int_{0.5}^{1} f(t_{strong}|occ_h,\theta)f(t_{strong}|occ_h,\theta)d\theta\right]+\left[\int_{0.5}^{1} f(t_{strong}|occ_h,\theta)f(t_{weak}|occ_h,\theta)d\theta\right] \\
= & \left[\int_{\widehat{\theta}}^{1} \beta_{white}\beta_{black}d\theta\right]+\left[\int_{0.5}^{\widehat{\theta}_{black}} \beta_{white}(1)d\theta+\int_{\widehat{\theta}_{black}}^{1} \beta_{white}(1-\beta_{black})d\theta\right] \\
= & (0.5)\beta_{white}
\end{aligned}
$$

where the first equality follows from the fact that $f(t_{weak}|occ_h,\theta)=0$ for whites.

$$
\begin{aligned}
WG_h &= \sum_{(t_{white},t_{black})\in\{t_{weak},t_{strong}\}^2}\int_{0.5}^{1}\left[w_{white}(\theta,t)-w_{black}(\theta,t)\right]\frac{f(t_{white}|occ_h,\theta)f(t_{black}|occ_h,\theta)}{(0.5)\beta_{white}}d\theta \\
&= \frac{1}{(0.5)\beta_{white}}\int_{\widehat{\theta}_{black}}^{1}\left[w_{white}(\theta,t_{strong})-w_{black}(\theta,t_{strong})\right]\beta_{white}\beta_{black}d\theta \\
&+ \frac{1}{(0.5)\beta_{white}}\int_{0.5}^{1}\left[w_{white}(\theta,t_{strong})-w_{black}(\theta,t_{weak})\right]\beta_{white}f(t_{weak}|occ_h,\theta)d\theta \\
&= \frac{1}{(0.5)\beta_{white}}\int_{0.5}^{1}\left[w_{white}(\theta,t_{strong})-w_{black}(\theta,t_{weak})\right]\beta_{white}f(t_{weak}|occ_h,\theta)d\theta \\
&= \frac{\beta_{white}}{(0.5)\beta_{white}}\int_{0.5}^{1}\left[w_{white}(\theta,t_{strong})-w_{black}(\theta,t_{weak})\right]f(t_{weak}|occ_h,\theta)d\theta \\
&= \frac{1}{(0.5)}\left[\int_{0.5}^{\widehat{\theta}_{black}}\left(\theta-\widehat{\theta}_{black}\right)(1)d\theta+\int_{\widehat{\theta}_{black}}^{s^*}\left(\theta-\widehat{\theta}_{black}\right)\left(1-\beta_{black}\right)d\theta\right] \\
&+ \frac{1}{(0.5)}\left[\int_{s^*}^{1}\left(\frac{1+s^*}{2}-\widehat{\theta}_{black}\right)\left(1-\beta_{black}\right)d\theta\right] \\
&= \frac{1}{(0.5)}\left[\left(1-\beta_{black}\right)\left(1-\widehat{\theta}_{black}\right)^2-\left(\widehat{\theta}_{black}-0.5\right)^2\right] \\
&> 0
\end{aligned}
$$

In the human capital intensive occupation occupation, only above average ability workers can be compared. The third equality follows from the fact that both white and black workers get the same wages when they both use strong ties, i.e. $w_{white}(\theta,t_{strong})=w_{black}(\theta,t_{strong})$. The final inequality is $WG_h>0\iff\widehat{\theta}_{black}<\frac{0.5+\sqrt{1-\beta_{black}}}{1+\sqrt{1-\beta_{black}}}$ and it follows from $\alpha_{weak}<0.5$. To see this, note that property (1) of lemma 1 implies $\frac{\partial\widehat{\theta}_{black}}{\partial\alpha_{weak}}>0$. So there is some $\widetilde{\alpha}_{weak}$ at which the left hand side (equation 16) of the inequality $\widehat{\theta}_{black}<\frac{0.5+\sqrt{1-\beta_{black}}}{1+\sqrt{1-\beta_{black}}}$ equals the right hand side. For $\alpha_{weak} < \widetilde{\alpha}_{weak}$, this inequality is satisfied. It is easy to verify that $\widetilde{\alpha}_{weak} = 0.5$, and this condition is satisfied from assumption that weak ties can't infer between below and above average ability workers $\alpha_{weak} < 0.5$. ■

Proof of proposition 6

At equilibrium, wage offer from a weak tie is

$$
\begin{aligned}
E[\theta|s \geq \alpha_{weak}, t_{weak}] &= \int_{\alpha_{weak}}^{1} \theta \frac{f(t_{weak}|\theta)}{\int_{\alpha_{weak}}^{1} f(t_{weak}|\theta)d\theta} d\theta \\
&= \frac{1}{\int_{\alpha_{weak}}^{\theta^*}(\beta_{weak}+\beta_{both})d\theta + \int_{\theta^*}^{1}\beta_{weak}d\theta}\left[\int_{\alpha_{weak}}^{\theta^*}(\beta_{weak}+\beta_{both})\theta d\theta + \int_{\theta^*}^{1}\theta\beta_{weak}d\theta\right] \\
&= \frac{1}{2\left[\beta_{both}\left(\theta^* - \alpha_{weak}^2\right) + \beta_{weak}\left(1-\alpha_{weak}\right)\right]}\left[\beta_{both}\left((\theta^*)^2 - \alpha_{weak}^2\right) + \beta_{weak}\left(1-\alpha_{weak}^2\right)\right]
\end{aligned}
$$

As before, setting $\widehat{\theta} = \{\theta : \theta = E[\theta|s \geq \alpha_{weak}, t_{weak}]\}$ gives

$$
\beta_{both}(\theta^*)^2 + 2\theta\left[\beta_{weak}\left(1-\alpha_{weak}\right) - \beta_{both}\alpha_{weak}\right] - \beta_{weak}\left(1-\alpha_{weak}^2\right) - \beta_{both}\alpha_{weak}^2 = 0
$$

Applying the Implicit Function Theorem with $F(\widehat{\theta}, \alpha_{weak}, \beta_{weak}, \beta_{both}) = \beta_{both}(\widehat{\theta}^*)^2 + 2\widehat{\theta}\left[\beta_{weak}\left(1-\alpha_{weak}\right) - \beta_{both}\alpha_{weak}\right] - \beta_{weak}\left(1-\alpha_{weak}^2\right) - \beta_{both}\alpha_{weak}^2 = 0$ gives the following.

$$
\frac{\partial\widehat{\theta}}{\partial\alpha_{weak}} = -\frac{2\widehat{\theta}\left[\beta_{weak}+\beta_{both}\right] - 2\alpha_{weak}\left[\beta_{both}-\beta_{weak}\right]}{2\widehat{\theta}\beta_{both} + 2\left[(1-\alpha_{weak})\beta_{weak} - \alpha_{weak}\beta_{both}\right]} > 0
$$

The numerator is negative as follows.
$2\widehat{\theta}\left[\beta_{weak}+\beta_{both}\right] - 2\alpha_{weak}\left[\beta_{both}-\beta_{weak}\right] < -4\alpha_{weak}\left[\beta_{weak}\right] < 0$
where the first inequality follows from $\alpha_{weak} < \widehat{\theta}$.
The denominator is positive as follows.

$$
2\widehat{\theta}\beta_{both} + 2\left[(1-\alpha_{weak})\beta_{weak} - \alpha_{weak}\beta_{both}\right] > -2\alpha_{weak}\left[\beta_{weak}\right] + 2\beta_{weak} = 2(1-\alpha_{weak})\beta_{weak} > 0
$$

where the first inequality follows from $\alpha_{weak} < \widehat{\theta}$.
Similarly,

$$\frac{\partial \widehat{\theta}}{\partial \beta_{weak}} = -\frac{2\widehat{\theta}(1-\alpha_{weak}) - (1-\alpha_{weak}^2)}{2\widehat{\theta}\beta_{both} + 2\left[(1-\alpha_{weak})\beta_{weak} - \alpha_{weak}\beta_{both}\right]} > 0$$

The numerator is negative as follows.

$$2\widehat{\theta}(1-\alpha_{weak}) - (1-\alpha_{weak}^2) = -2\widehat{\theta}(1-\alpha_{weak})\alpha_{weak} < 0$$

The denominator is positive as explained above.

For a given informativeness level of weak tie α_{weak}, let β^* be the value for the proportion of workers with access to weak tie such that $\widehat{\theta} = 0.5$. If $\beta_{weak} > max\{\beta^*, \beta_{strong}\}$, then $\widehat{\theta} > 0.5$, and

$$\begin{aligned}
f(t_{weak}) &= \int_0^1 f(t_{weak}|\theta)d\theta \\
&= \int_0^{\widehat{\theta}} (\beta_{weak} + \beta_{both})d\theta + \int_{\widehat{\theta}}^1 \beta_{weak}d\theta \\
&= \beta_{weak} + \widehat{\theta}\beta_{both} \\
&> \beta_{strong} + (1-\widehat{\theta})\beta_{both} \\
&= \int_0^{\widehat{\theta}} \beta_{strong}d\theta + \int_{\widehat{\theta}}^1 (\beta_{strong} + \beta_{both})d\theta \\
&= \int_0^1 f(t_{strong}|\theta)d\theta. \quad \blacksquare
\end{aligned}$$

Proof of proposition 7

(1) At a cost efficient equilibrium, the wage offer from direct application is equal to the threshold ability $w_d = E[\theta|d] = \theta^*$. It is easy to verify that the $E[\theta|d] = \frac{1}{2[\beta\theta^* + (1-\beta)]}\left[\beta\theta^{*^2} + (1-\beta)\right]$, which doesn't depend on δ. As a result, the threshold ability doesn't depend on δ, and it is characterized (with $\alpha_{weak} = 0$) as before $\beta\theta^{*2} + 2(1-\beta)\theta^* - (1-\beta) = 0$. Thus, I can apply results obtained in lemma 1.

Since the firm has minimal informativeness level $\alpha_d = 0$, $\beta^* = 0$ by property (5) of lemma 1. If $\beta > 0$, then $\theta^* < E[\theta] = 0.5$. As a result, for above average ability workers, returns to ties are always positive,

$$w_{t-d}(\theta > 0.5) = [E[\theta|t, \theta > 0.5] - w_d] = \int_{0.5}^1 \theta \frac{f(t|\theta)}{\int_0^1 f(t|\theta)d\theta} d\theta - \theta^* > 0.$$

(2) (i) For below average ability workers, returns to ties depend on β.

$$\begin{aligned}
w_{t-d}(\theta < 0.5) &= [E[\theta|t, \theta < 0.5] - w_d] \\
&= \int_0^{0.5} \theta \frac{f(t|\theta)}{\int_0^1 f(t|\theta)d\theta} d\theta - \theta^* \\
&= \frac{1}{\int_0^{\theta^*} \theta\beta d\theta + \int_{\theta^*}^{0.5} \theta\beta(1-\delta)d\theta} \left[\int_0^{\theta^*} \theta\beta d\theta + \int_{\theta^*}^{0.5} \theta\beta(1-\delta)d\theta \right] - \theta^* \\
&= \frac{1}{2\left[0.5 - \delta\theta^*\right]} \left[0.5^2 - \delta{\theta^*}^2 \right] - \theta^* \\
&= \frac{1}{2\left[0.5 - \delta\theta^*\right]} \left[0.5^2 + \delta{\theta^*}^2 - \theta^* \right].
\end{aligned}$$

By property (2) of lemma 1, $\frac{\partial\theta^*}{\partial\beta} < 0$. Moreover,

$$\begin{aligned}
\frac{\partial\left[E[\theta|t, \theta < 0.5] - w_d\right]}{\partial\theta^*} &= \frac{\partial E[\theta|t, \theta < 0.5]}{\partial\theta^*} \\
&= \frac{1}{\left(2\left[0.5 - \delta\theta^*\right]\right)^2} \left\{ \left[0.5^2 - \delta{\theta^*}^2 \right] (-\delta) - 2\delta\theta^* \right\} \\
&< 0.
\end{aligned}$$

This follows from the fact that $\theta^* < E[\theta] = 0.5$. Thus, $\frac{\partial[E[\theta|t,\theta<0.5]-w_d]}{\partial\beta} = \frac{\partial[E[\theta|t,\theta<0.5]-w_d]}{\partial\theta^*}\frac{\partial\theta^*}{\partial\beta} > 0$.

Since $\theta^* < E[\theta] = 0.5$, then

$$\begin{aligned}
\frac{\partial\left[E[\theta|t, \theta < 0.5] - w_d\right]}{\partial\delta} &= \frac{\partial E[\theta|t, \theta < 0.5]}{\partial\delta} \\
&= \frac{\theta^*\left(0.5 - \theta^*\right)}{\left(2\left[0.5 - \delta\theta^*\right]\right)^2} \\
&> 0.
\end{aligned}$$

By property (2) of lemma 1, $\frac{\partial\theta^*}{\partial\beta} < 0$. If $\beta = 1$, then $\theta^* = 0$. Thus, $\theta^* \in [0, 0.5)$.

If $\delta = 0$, then $\theta^* = 0.25 \implies [E[\theta|t, \theta < 0.5] - w_d] = 0$. If $\delta = 1$, then $\theta^* = 0.5 \implies [E[\theta|t, \theta < 0.5] - w_d] = 0$.

Since $\theta^* \in [0, 0.5)$, $\frac{\partial[E[\theta|t,\theta<0.5]-w_d]}{\partial\delta} > 0$, then for each $0 \leq \delta < 1$, there exists a β^{**} such that $[E[\theta|t, \theta < 0.5] - w_d] = 0$. If $\beta \approx \beta^{**}$, then for below average ability workers, average returns to tie is small and insignificant $[E[\theta|t, \theta < 0.5] - w_d] \approx 0$.

(ii) If $\beta < \beta^{**}$, then the average returns to tie is negative. $[E[\theta|t, \theta < 0.5] - w_d] < 0$.

(iii) Since $\frac{\partial[E[\theta|t,\theta<0.5]-w_d]}{\partial\beta} > 0$ and $\frac{\partial[E[\theta|t,\theta<0.5]-w_d]}{\partial\delta} > 0$, if $\delta' < \delta$, then ${\beta^{**}}' > \beta^{**}$. ■

Appendix B. Cost Efficient Equilibrium

I focus on an appealing class of pure strategy Perfect Bayesian Equilibria, which I call the Cost Efficient Equilibria. This section of the appendix provides a formal justification for my focus on the Cost Efficient Equilibria.

I consider recommendation strategies which *reveal maximum information* in that starting from the lowest ability level, the employed worker reveals information to the firm until he either runs out of recommendation values or the signal reaches his informativeness level. Let $[0, \theta_{max}]$ be the interval of ability levels that a firm can infer from an employed worker who reveals maximum information. The next proposition shows that the cost efficient recommendation strategy reveals maximum information in the cheapest way possible.

Proposition 8. If an employed worker's recommendation strategy ρ is cost efficient, then

(a) it reveals maximum information $[0, \theta_{max}]$, and

(b) for any other recommendation strategy which also reveals maximum information $\rho' \neq \rho$, an employed worker incurs lower reputation costs by revealing maximum information through the cost efficient recommendation strategy, i.e. $(\rho - E[\theta|s,t]^2) \leq (\rho' - E[\theta|s,t]^2)$ for all $s \in [0,1]$, and the inequality is strict for at least one $\widetilde{s} \in [0,1]$.

Proof of proposition 8

(a) This follows from point (2) of cost efficient recommendation strategy's definition. Since recommendations are increasing in signals for $s < min\{s^*, \alpha_t\}$ where $\rho(s^*, t) = 1$, the employed worker's recommendations reveal his signal value to the firm $\{s' : \rho(s', t) = \rho(s, t)\} = \{s\}$ for all such signals. Thus, the employed worker reveals information to the firm until he either runs out of recommendation values or the signal reaches his informativeness level.

(b) For the lowest signal value, the employed worker incurs zero reputation costs.

The employed worker reveals information by increasing recommendation values as signal value increases. Since reputation costs depend on the distance between the recommendation value and the signal value, revealing information by increasing recommendation values minimizes reputation costs.

For the set of signals at the top, the employed worker reveals his information to the firm by minimizing reputation costs. ■

Remark on reputation costs functional form

Let $1_{\rho>E[\theta|s,t]}$ be an indicator function which takes value of one if recommendations are higher than the expected ability value and zero otherwise. If reputation costs only come from recommendations that are higher than the expected ability value $1_{\rho>E[\theta|s,t]}(\rho - E[\theta|s,t])^2$, then Cost Efficient Recommendation Strategy still reveals maximum information in the cheapest way possible. Thus, all results of this paper will continue to hold in such variation of the model.

To see this, observe that the initial value condition is still the cheapest way to reveal the lowest signal. For the interval of signals that firm can infer, firm sets wage equal to signal $w(\rho(s,t),t) = s$. If $\rho(s,t) \leq s$, then FOC of the employed worker's maximization problem implies $w_1(\rho,t) = 0 \iff \frac{1}{\rho_1(s,t)} = 0$. Then, firm can't infer any signals above the lowest signal because $\rho_1(s,t) \to \infty$. Thus, recommendation strategy must have $\rho(s,t) > s$ for firm to infer signals above the lowest signal. As a result, FOC of the employed worker's maximization problem (and the corresponding differential equation) implies recommendations are in increasing in signals for such interval of signals. And the rest of the arguments follow from proof of proposition 6.