Fairness is flexible: A study of competing focal points*

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Abstract

While several allocation rules known in the cooperative bargaining literature, implicitly allow for various notions of fairness, there is no consensus about which notion of fairness exactly prevails in a given contextual environment involving a bargain. We look at three broad classes of fairness concepts and provide useful insights about which fairness notion to expect in a given scenario. We generate a unique dataset through our experiment involving dialogue-based bargaining and show that the size of the divisible pie itself provides strong hints about which fairness solution to expect.

Keywords: fairness, cooperative bargaining, experiment

JEL classifications: C91, C70, D63, D64

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1. Introduction

We address one of the most fundamental questions about the (non-)uniqueness of fairness ideals. The idea that negotiated outcomes between two or more agents should be *fair* is universally agreed upon. This idea in itself, yet remains insufficient in predicting real-life negotiation which often witnesses several competing ideals of fairness. The fact that the ideals of fairness need not be unique, motivates our current investigation. For example, two agents who can individually earn $100 and $50 can come together and earn $300. One can argue that a fair split will require that the joint pie is divided equally (each agent gets $150). One can also argue that a fair split should be according to (in proportion to) the agents' individual earning capacities ($200 and $100). Yet another way to describe a fair split is where only the surplus beyond the individual earnings is shared equally ($175 and $125). To the best of our knowledge, this is a first attempt at a conclusive understanding of which fairness ideal to expect in a given context.

We formally examine the above three classes of fairness ideals in the spirit of Moulin (2003). Our aim is to provide useful insights on exactly which fairness ideal to expect in a given negotiation-environment. We demonstrate that each of the above notions of fairness can become more relevant than the others according to the specifics of a bargain - which fairness ideal is more apt than the others, is essentially a question of which one acts as a natural focal-point in a given context.

Ken Binmore (1994, 1998) argues that *fairness* is nature's solution to the allocation problem of limited resources. The problem of resource allocation is two-fold: the first level addresses ways to achieve (higher) efficiency (i.e. the movement from lower to higher payoff frontiers – a class of folk theorems can guarantee the sustainability of the higher frontiers); and the second level addresses the problem of picking an allocation among a set of competing allocations that are efficient. The focus of this paper is on the second level that has witnessed contributions in collective decision making in the form of bargaining problems (e.g. Nash) and those that involve the aggregation of individual preferences (e.g. Arrow), among other approaches to collective decision making that aim to achieve outcomes deemed apt by each consequential agent (e.g. Coase).

Fairness is often a crucial starting point to the development of such approaches. For example, implicit notions of fairness are embedded in Nash's (1950) *axiom of symmetry*, Arrow's *axiom of non-dictatorship*, and can generally be thought to be implicit in the preferences of an impartial arbitrator who must decide the outcome of negotiation between two agents (Thomson, 1994). Fairness, as a key requirement, however, faces a central problem – there is no unique way to define fairness. Consequently, notions of fairness are not devoid of context. As an extreme example, Birkeland and Tungodden (2014), show that bargaining outcome could be sensitive to the fairness motivation of the negotiating agents – disagreement on what a fair outcome is, may result in a disagreement outcome (i.e. a loss of efficiency).

We present the findings of an experiment, in which each treatment uniquely corresponds to exactly one of the three families of fairness solutions (discussed in the opening note) that is deemed most suitable. In other words, we explicitly show that in one treatment, one family of fairness ideals is the most suitable, in a second treatment, a second family of fairness ideals is the most suitable and so on. Therefore, each family of fairness solutions has a predictive capacity that is restricted to only, and exactly one treatment.

As a first step, we will establish that the pie-size (the only source of change between our treatments) is itself a good indicator of which fairness ideal will be empirically dominant. In this sense, our study is related to the research of List and Cherry (2008); and Andersen et al (2011), where it is shown that the monies transferred by the more consequential subjects respond less-than-proportionately to the stakes in hand. For example, Andersen et al (2011) show that as the pie-size increases, the proportional/relative offers made by the proposers of an ultimatum game diminish. These results can be explained by Rabin's (1993) attempt to incorporate fairness in game theory, in his stylized requirement that the willingness (of the responder) to punish (by rejecting the offer) should diminish with higher stakes. In general, it is agreed that fairness considerations can play an important role in the determination of bargaining outcomes when stake sizes vary (Karagözoğlu and Urhan, 2017). We go a step further to single out the fairness ideal that is the most dominant (in comparison with other fairness ideals) in explaining observed outcome in each context.

It is clear that fairness considerations matter to agents on any matter involving an allocation of resources. Fehr and Schmidt (2003) explicitly argue that many people are strongly motivated by concerns for fairness and reciprocity - not just material self interest. Therefore, even if an agent does not feel strongly about fairness considerations, he may want to make higher offers to mitigate the chances of rejection by another agent who is known to value fairness strongly (Carpenter, 2003).

The task of unifying fairness ideals is challenging because it is well recognized that fairness perceptions frequently respond to changes in strategic environments (Schmitt, 2004). This malleability of ideals makes it possible for agents to (unknowingly) have self-serving biases in the idea of fairness (Babcock and Loewenstein, 1997). However, when notions of fairness are well-defined, any deviation(s) from the same often trigger feelings of shame and guilt for the deviating agents. These notions of fairness can be so powerful that if an agent is known to treat other interacting agents in a fair manner, then it is an immediate guarantee of loyalty and reciprocated fairness for this agent from the receivers of fair treatment (Chiu et al, 2009).

In conclusion, fairness is a key to pro-sociality (Henrich et al, 2010; Charness and Rabin, 2002), and is a driver of societal and institutional progress (Janssen, 2000), and can even shape regulatory stance (Banerjee, 2015). Therefore, we can consequently benefit from a unifying fairness ideal, in which many contextual ideas of fairness are nested. The additional merit of our study is that we look at unstructured or free-form bargaining - which is what we observe in the real world.

2. The theory

In what follows, we now formalize the motivating example in the introductory note in the spirit of Moulin (2003), which in turn, will help us motivate the rationale behind our treatment groups.

2.1. The formulation

Two individuals X and Y (both from the same homogenous population) have the following two options.

Option 1: Individually earn $d(x)$ and $d(y)$, respectively.

Option 2: Cooperate and generate a pie of size $z > d(x) + d(y)$, and share the same. Their respective shares are x and y (both non-negative), so that $x + y = z$.

In the event Option 2 is chosen, Moulin (2003) offers the following three classes of solutions (the last one is an intermediate between the first two extremes).

1. *Uniform Gains (UG):* X and Y share z equally – i.e. $x = y = z/2$.

2. *Proportional Gains (PRO):* *X* and *Y* share *z* in proportion to their individual earnings

– i.e. $x/y = d(x)/d(y)$.

3. *Equal Surplus (ES):* *X* and *Y* share *z*, such that the gains from cooperation are matched

– i.e. $x – d(x) = y – d(y)$.

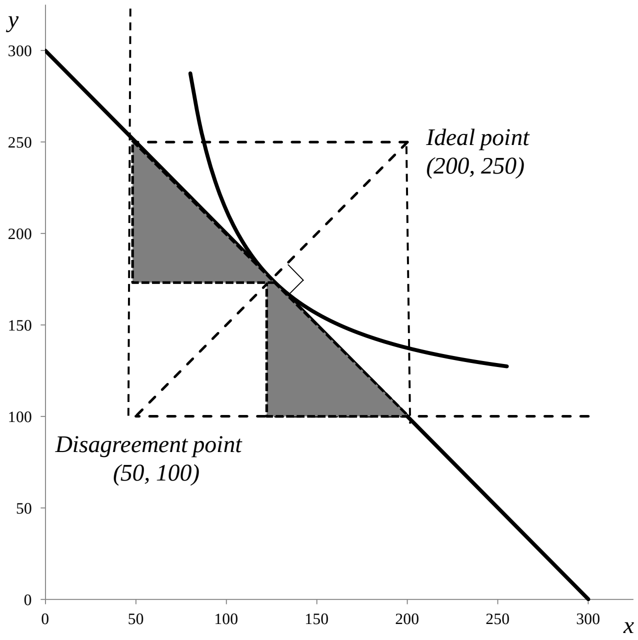
It is clear that when $d(x) = d(y)$, all the three solution concepts yield the same result. The blur between the fairness ideals occur when $d(x)$ and $d(y)$ are different.

2.2. A discussion

In what follows, without loss of generality, we assume that $d(y) > d(x)$. Now each of the three solution concepts can be thought of as a fair way to distribute *z*. For example, fixing $d(x) = 50$, $d(y) = 100$, and $z = 300$, will give us $(x = 150, y = 150)$ under the *Uniform Gains* (*UG*) protocol; $(x = 100, y = 200)$ under the *Proportional Gains (PRO)* protocol; and $(x = 125, y = 175)$ under the *Equal Surplus (ES)* protocol. It is clear that the *ES* allocation rules will always remain somewhere midway between the *UG* and the *PRO* protocols for any value of *z*, $d(x)$, and $d(y)$.

Before proceeding further, it helps to clarify that each of the above three formalizations of fairness (*UG, PRO* and *ES*) includes a class of allocation rules/bargaining solutions that have implicit notions of fairness. For example, in Figure 1, the *ES* fairness ideal introduced above can be interpreted as the outcome of Nash (1950) bargaining (which maximizes $[x - d(x)][y - d(y)]$, subject to $x + y = z$); the Kalai-Smorodinsky (1975) solution (KS hereafter, which requires (x, y) to be along the line that joins the disagreement point to the *ideal point*); the (discrete) Raiffa (1953) solution (which bisects the line segment of Pareto optimal points above each agents' disagreement payoffs); the Equal Area solution (EA hereafter which equates the surpluses (shown by the shaded area) given up by each agent in order to reach an agreement); the Yu (1973) solution (which chooses *(x, y)* closest to the *ideal point* of KS).[1] All these bargaining solutions are individually discussed in Thomson (1994), and have been presented in Figure 1 for the values for disagreement payoffs and the pie size assumed in the previous paragraph. Thus, since the *ES*, as a fairness rule, is a class of bargaining solutions covering the Nash, KS, Yu, and Raiffa solutions (among others), we do not need separate discussions around these solutions of cooperative bargaining game theory.[2] What follows now is a brief discussion about the *UG* and *PRO* fairness rules.

Figure 1: ES is a class of bargaining solutions



[1]In Figure 1, the set of feasible alternatives is shown by the region bounded by the axes (which represent the monetary payoffs for *X* and *Y*) and the negative-45° line (which is the set of Pareto efficient points). The *ideal point* for each agent can be thought of as that point the agent will be able to achieve if he/she were a dictator. It is more formally defined as the most favorable Pareto optimal point subject to the requirement that each other agent receives at least their disagreement payoff levels.

[2]Similarly, note that the *UG* rule is a class of solutions covering the Egalitarian-type solutions discussed in Thomson (1994). Interestingly, for $z = 180$, it also covers the dictatorial solution of Thomson, 1994, where agent *X* is the dictator.

The *Uniform Gains* protocol is the egalitarian solution discussed in Thomson, 1994, is the first extreme, where the differences between *X* and *Y*, if any, are inconsequential in terms of the final solution. For instance, the idea that all the rich and poor are equal in the eyes of the law is thought of a *fair* way to disseminate justice. The *Uniform Gains* solution may however, be questionable for it completely ignores (as it should) systematic differences between the agents in question. In our example, since *Y* can individually earn twice that of *X*, (i.e., clearly $d(y) = 100 = 2d(x)$) then, *Y* may feel entitled to a higher share in *z*, since *Y* is (say) more efficient than *X*. This is true of the world we live in – Cardenas and Carpenter (2008), for example, point out that the perception of how deserving recipients of ultimatum games are, is a strong predictor of altruism. We turn to this idea of fairness now.

Proportional Gains (*PRO*) requires that the agents share the pie in proportion to their perceived individual capacities (in this case, 2:1), and has appealed to theorists who have modeled the same. These interpretations are consistent with Aristotle's idea of fairness which should be proportional to some measure of agents' need, ability, effort and status (additionally see Harsanyi 1962, 1966 for one of the first theoretical approaches to bargaining). The effects of ability, status, and effort on bargaining outcomes have been frequently demonstrated experimentally (Hoffman et al 1994; Ball et al 2001).

In short, all the classes of fairness rules require a sense of equality. Under the *UG* protocol, it is the equality of the *shares* (of the total pie-size); under the *PRO* protocol, it is the equality of *proportions* (between the outcomes of agreement and disagreement); and under the *ES* protocol, it is the equality of *gains* (from cooperation – i.e. the transition from disagreement to agreement). We are now in a position to describe our experiment designed to disentangle one fairness solution concept from another.

3. The experiment

3.1. An overview

A total of 452 undergraduate and post-graduate students, aged between 18 to 28 years, from five institutes across India were the subjects of our experiment. Each individual received a show-up fee of INR 200, in addition to which, they retained the amount they could bargain for themselves in the experiment. They were randomly assigned to one of four treatments, after which they took a test. After the test, each treatment group was divided into two sub-groups of top half and bottom half performers – that is, the tests were graded and ranked according to the subjects' performances, and then split into a top-half group and a bottom-half group in each treatment. Finally, in each treatment, each subject among the top half performers was randomly paired with a subject among the bottom half performers for the purpose of bargaining. As we will see below, the only distinguishing characteristic between our treatments is the pie-size. What follows is a description of the treatments.

Treatment 180 (T180): In this treatment of a total of 116 individuals (58 pairs), each subject from the top half (Y) is paired with a subject in the bottom half (X) and each pair formed of individuals X and Y are asked to split INR 180 (= z) among themselves.[3] They are given a time period of ten minutes to reach an agreement, failing which, the outcome is treated as a disagreement, in which case the high-ranker in each pair is given INR 100 (= $d(y)$), and the low ranker is given INR 50 (= $d(x)$).[4] Both negotiating agents in a pair knew their own and each other's disagreement payoffs. Note that, in this treatment, the size of the pie is only marginally higher than the sum of the disagreement payoffs ($d(x) + d(y)$ = INR 150). We

[3] As of September 1, 2019, US$1 = INR 72.

[4] The suitability of ten minutes was determined from prior pilot studies.

observe that *X* and *Y* share the pie-size of INR 180 mostly according to the *PRO* rule in this treatment. There were no disagreements.

Treatment 300 (T300): In this treatment of a total of 110 individuals (55 pairs), *X* and *Y* are asked to split a sum of INR 300 (= z) between themselves. Everything else remains the same including the disagreement payoffs. In this treatment too, we saw no disagreements and a majority bargained according to the *PRO* rule.

Treatment 600 (T600): In this treatment of 108 individuals (54 pairs), *X* and *Y* are asked to split a sum of INR 600 (= z) between themselves. Everything else remains the same including the disagreement payoffs. Note that in this treatment, the gains from cooperation are fairly high ($z - d(x) - d(y)$ = INR 450). We observe that a majority of the agents *X* and *Y* go for the *UG* solution. There were no disagreements.

Treatment 900 (T900): In this treatment of 118 individuals (59 pairs), *X* and *Y* are asked to split a sum of INR 900 (= z) between themselves. Everything else remains the same. Note that in this treatment, the gains from cooperation are the largest ($z - d(x) - d(y)$ = INR 750). We observe that a majority of the agents *X* and *Y* settle by the *UG* solution. There were no disagreements.

3.2. Further details

In each treatment, bargaining happened between *X* and *Y* over Skype with rank revealing IDs such as Rank.001, Rank.002 and so on. This helped in preserving anonymity, which is desirable because the knowledge of who each subject was paired with could mitigate the effect of the test. Further, it retained a key feature of real-life bargaining and negotiation processes – *dialogue*. In the real world, economic agents give away apt reactions through either their vocal tone or facial expressions when they are pleased or displeased with the direction of negotiations. Our subjects were found to frequently use apt emoticons according to whether they felt that the bargains suggested by their partners were fair or unfair. The process of communication used text-chat instead of voice-chat to mitigate the possibility of any identification, since the subjects came from a homogeneous population (i.e. the same institution) and were likely to be friends. The subjects were instructed to bargain only in English.[5]

Finally, the test (given in the appendix) was a compilation of 20 extremely difficult questions for which, a time limit of 10 minutes was given. Each question was followed by four possible answers of which, only one was correct. There was no negative marking and the instructions explicitly required the subjects to maximize their total score of right answers. The extreme difficulty level coupled with the limited time to solve would have ensured that the subjects were forced to resort to random marking of the answers.[6] Thus, effectively, each question had a one-fourth probability of being answered correctly. Clearly, on an average, therefore, we should expect one-fourth of the given questions to be correctly answered. We see that the students got an average score of 4.96 out of 20, which is not significantly different from what is expected. Therefore, we are confident that the ranking on the basis of the test is as good as random. Thus, having the actual rank of bargaining agents as a potential determinant of bargaining outcomes is least likely to be correlated with unobserved ability - moreover we explicitly examine the same by checking for possible correlation between our test ranks and the actual academic performances of our subjects in their respective courses provided by the institutes where we undertook our experiments (practically negligible R-squared of 0.008).

[5]The alternative language was Hindi. The issue is that speaking in the first-person language reveals the gender of the individual. Male agents for example will say "I *think* this is a fair split" differently from female agents.

[6]For example, the second question in the test (called Monila's urn) is based on the Fermat's last theorem, which took over three centuries for mathematicians to solve.

3.3. Testable hypotheses

We now set up regression functions that models the (functions of) expected share of the high-ranked agents conditional on the pie size as follows:

$$E(y|z) = \alpha_0 + \alpha_1 z \quad (1)$$

This leads us to the following hypotheses of interest for our chosen values of $d(x) = 50$ and $d(y) = 100$

1. *Hypothesis UG* (H_{UG}): $\alpha_0 = 0$ and $\alpha_1 = 1/2$

2. *Hypothesis PRO* (H_{PRO}): $\alpha_0 = 0$ and $\alpha_1 = 2/3$

3. *Hypothesis ES* (H_{ES}): $\alpha_0 = 25$ and $\alpha_1 = 1/2$

It should be noted that for the treatment T180, the *UG* rule requires that agent *Y* be given at least his/her disagreement payoff level, so that $y = 100$ and $x = 80$ (Moulin, 1988). This is discussed more closely and in greater detail when we discuss our non-parametric sampling methodology and the empirical analyses that follow. The power analyses (deferred to the appendix) presented in relation to this, accounts for the potential existence of mass-points when the underlying random variable is non-Gaussian. For now, we note that our game-structure itself gives strong hints about how to conduct power analyses for sample sizes.

4. Descriptive statistics

4.1. An overview

Table 1 presents a brief summary of the four treatments. Most of the participants were female (just over 55%). The high-ranked subjects were able to negotiate, on an average about 58% of the respective pie-sizes (Table 1 reports the figures by treatment). Apart from T180, no other treatment saw individual shares exceeding two-thirds of the given pie-size. The average time to negotiate remained between three and four minutes in each treatment.

Table 1. Summary of the treatments

	T180	T300	T600	T900
Observations	116	110	108	118
Number of pairs	58	55	54	59
Number of males	52	52	47	50
Mean Share (High-ranked)	0.640	0.600	0.547	0.524
(Standard Error)	(0.005)	(0.009)	(0.008)	(0.005)
Maximum Share (High-ranked)	0.694	0.667	0.667	0.667
Average time taken (Seconds)	214.586	219.382	199.370	194.424
(Standard Error)	(17.145)	(18.149)	(16.747)	(16.533)

Figure 2: Distribution of shares of high-ranked individuals

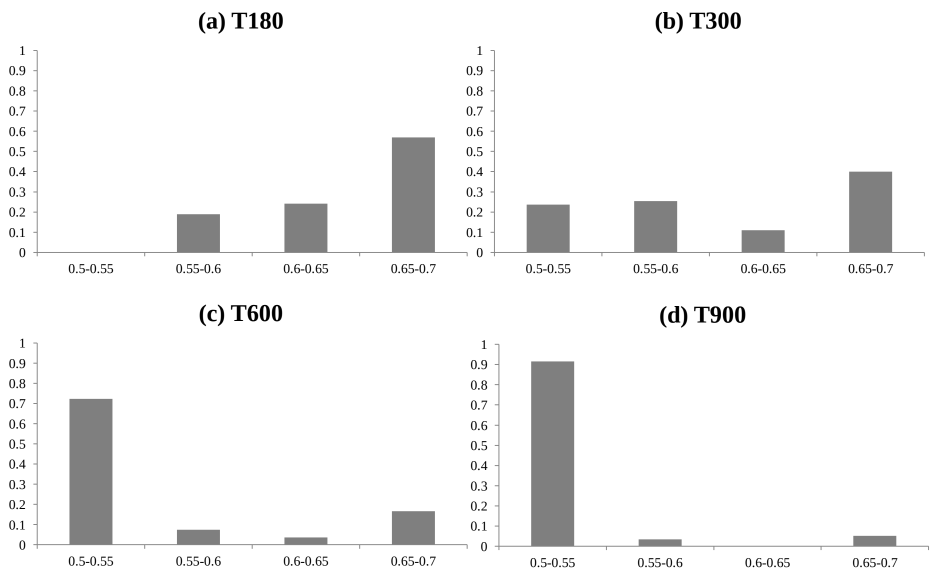
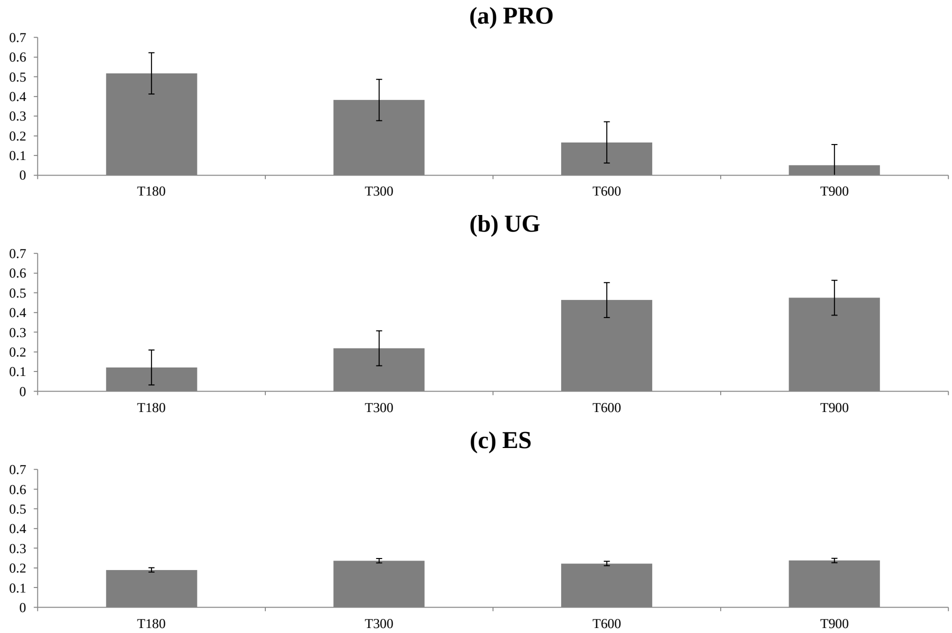


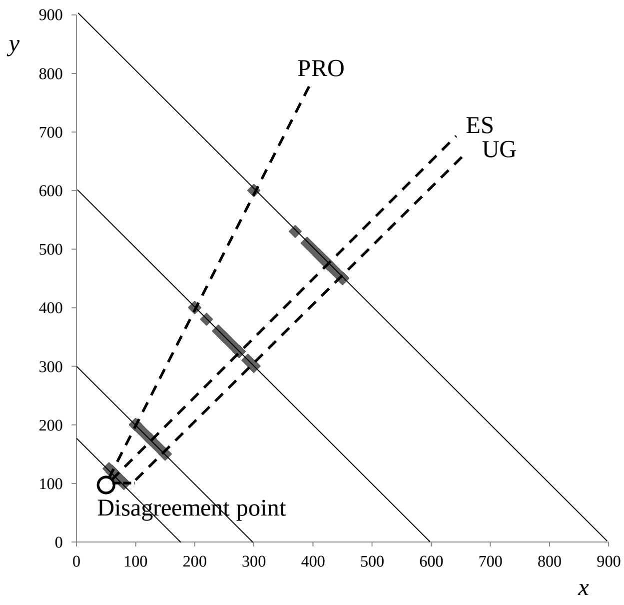
Figure 2 displays the distribution of shares of the high-ranked individuals in each treatment (the distribution of the shares of the low-ranked individuals is a mirror-image of this). It is immediately seen that the density of shares gravitate (away from about two-thirds) towards half as the pie-size increases from T180 (panel a) to T300 (panel b) to T600 (panel c) to T900 (panel d). From Figure 3, it becomes immediately clear why this is so. We see in Figure 3, that the results are primarily driven by focal-points (which become statistical mass-points, and therefore define the modal class of observations - all this in turn, influences the mean). If we, for now, accept the exact values assumed by our hypotheses (*UG*, *PRO* and *ES*) as focal points, then it will help to look into the proportion of individuals agreeing on each focal-point in each treatment. This is the purpose of Figure 3 below.

Figure 3: Fraction of focal point agreements



We begin by pointing out that focal points are *exact* values that are immediate and easy to spot and calculate. For example, in T180, an agreement like *(x, y) = (62, 118)*, would not qualify as a focal point even though it is very close to the *PRO* solution of *(60, 120)*, which becomes easy to spot and calculate since it easily replicates the ratio of the disagreement payoffs. Panel (a) in Figure 3 reports the proportion of negotiating pairs who exactly went for the *PRO* rule in each treatment. For example, among the total number of negotiations in T180, about 52% settled on the *PRO* rule. The corresponding figures for T300, T600 and T900 are 38%, 17% and 5% respectively, which makes it clear that lesser and lesser proportion of negotiating pairs deemed the *PRO* rule to be apt as the pie-size increased. The exact opposite can be argued of the *UG* rule in which case, the proportion of negotiating agents who deem the *UG* rule to be apt increases with the pie size (12% in T180, to 22% in T300, to 46% in T600, to 47% in T900). The proportion of negotiating pairs that saw the *ES* rule as apt remained relatively stable (in comparison to the above rules) around 20% to 25% throughout the four treatments. Before we examine these focal points any closer, it will help to put data and theory together - we look at the scatter-plot of negotiated outcomes against the requirements of the *UG, PRO* and *ES* fairness rules in Figure 4 below. The four negative-45° lines are the set of Pareto optimal points on our payoff-frontiers corresponding to our four treatments. The disagreement point *(d(x), d(y)) = (50, 100)* is marked with ○.

Figure 4: Scatter-plot of negotiated outcomes against fairness ideals



The set of *PRO* agreements are along the line $y = 2x$ as shown above. Similarly, the set of *ES* agreements are along the line $y = x + 50$, and the set of *UG* agreements are along the line $y = x$ above $x = 100$ (up to which y remains *100* - since the *UG* rule requires that the agents share the pie equally subject to each agent receiving at least the disagreement level payoff). It is immediately seen that the scatter gravitates towards equality as the pie-size increases in the north-east direction. For the sake of robustness, in the subsection that follows, we impose stringent requirements on what qualifies as a focal point.

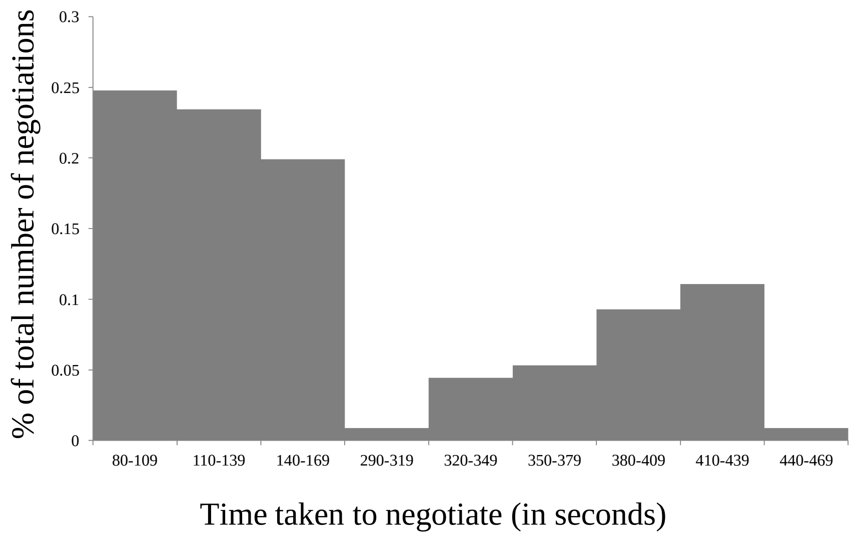
4.2. A closer look at focal points

The natural choice for focal points in each treatment is immediately driven by the ease with which they are spotted instinctively - and since focal points are natural enough to be instinctively spotted, they are also likely to lead to relatively quicker agreements between bargaining agents. The process of bargaining itself (in the real world) can be costly and stressful (Banerjee, 2015), and therefore there is value in spotting focal points that facilitate quicker agreements between agents. As a first step, we look at the distribution of the time taken to reach a negotiation between the pairs of agents (Figure 5). Since our subjects engaged in chat-based negotiation over Skype, we were able to record the time the negotiation started and compare the same to the exact time when they struck a deal. Figure 5 shows the distribution of the time taken (in seconds) by our negotiating agents to strike a deal.

The distribution of the time taken to negotiate (Figure 5) clearly shows that a significant proportion (32% or about one-third) of our negotiating pairs did bargain hard and argue till the very end, averaging over six and a half minutes (more precisely, 390.19 seconds) to reach an agreement (this makes up the right-cluster in our distribution). A majority (the remaining

68%, or about two-thirds, making up the left-cluster in our distribution) of our negotiating pairs however, chose to strike a deal quickly averaging at about two minutes (more precisely, 121.14 seconds). Thus, Figure 5 shows a clear distinction between pairs that arrived at quick decisions and the pairs that did not.[7] Since we are looking at pairs that engage in a natural process of social contemplation (through a verbal exchange which we call *dialogue*), and telling them apart from pairs that display a natural social preference embedded in their instincts, our work is also related to that of Rubinstein (2016), which classify agents as contemplative or instinctive depending on their individual response times in their choice(s) of action.

Figure 5: Distribution of time taken to reach agreements



It should be noted that not all negotiations that ended quickly ended as focal-point agreements (for example, there were negotiating pairs where, after one of the agents proposed a focal-point split, the bargaining agents quickly negotiated their way to a final settlement that remained around, but different from the initial focal point identified. Similarly, not all

[7] Indeed, any value between 180 to 270 seconds can comfortably act as a natural split between the quick and the not-so-quick negotiating pairs.

focal-point negotiations ended quickly (for example, many bargaining pairs negotiated their way to finally reach splits that looked like focal-points). Table 2, displays this clearly. Overall, about 59% of all agreements were focal point agreements. It is also clear from Table 2 that people generally prefer quicker negotiations.

Table 2: Distribution of focal-point agreements by time taken

	Focal-point agreement	Non-focal-point agreement	**Total**
Quicker negotiation	106	48	**154**
Longer negotiation	28	44	**72**
Total	**134**	**92**	**226**

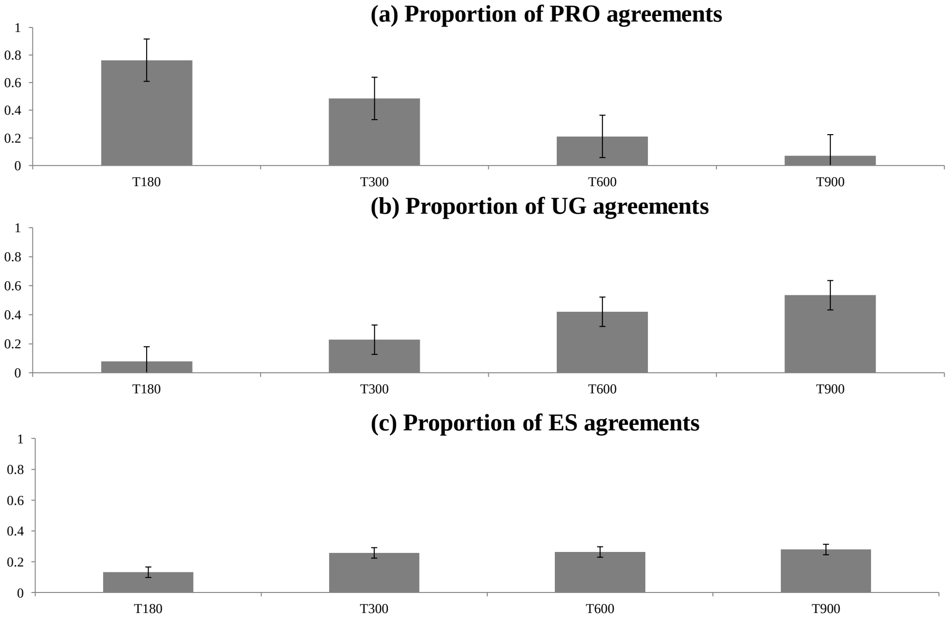
We now accept the following two criteria in order to identify focal points (in the interest of more stringent requirement for a negotiation to qualify as focal point):

1. Easy to locate, identify, and calculate

2. Reduce the time to negotiate.

In Figure 6, we look at focal-point agreements as a proportion of the quickest (that make up the left cluster) conversations in each treatment. For example, from Panel (a), we learn that of all the negotiations that concluded quickly in T180, 76% were *PRO* outcomes. Similarly 49%, 21%, and 7% of the quickest negotiations in T300, T600 and T900 respectively, were *PRO* outcomes. It is clear that as the pie-size increased, lesser and lesser number of bargaining pairs felt that the *PRO* rule was apt. Similarly, the fact that among the quickest negotiations in T180, T300, T600 and T900, respectively 8%, 23%, 42% and 53% were according to the *UG* rule. Clearly, more and more negotiating pairs saw this to be more apt as the pie size increased. As the pie size increased from T180 to T300 to T600 and finally to T900, 13%, 26%, 26% and 28% of the quickest decisions were as per the *ES* fairness rule.[8]

Figure 6: Fraction of focal points amongst quickest decision



Thus, it is relatively easy to see that some fairness rules are more apt than others for a given pie size potentially because they naturally stem from instincts, revealing joint/social preferences.

In the section that follows, we will put forward a behavioral explanation for why some focal points are naturally more attractive to negotiating instincts. When we present the central conclusions, we will propose a general fairness solution that unifies all the fairness rules discussed so far (and by extension, the bargaining solutions from cooperative bargaining game theory) and then argue that *general* social preferences are different from *aggregate* social preferences.

[8] This is not surprising since in T180, an allocation like *(x, y) = (65, 115)* does not look like a focal point. The *PRO* fairness rule which is close to this would have looked more natural.

5. Empirical Strategy and Results

5.1. Determination of sample-sizes

We non-parametrically determine our sample-sizes for each treatment without making any assumption on the distribution of the underlying outcome variables (negotiated outcomes). Since our final hypotheses of interest are based on the shares of the high-ranked individuals in a bargaining pair, our sampling methodology is centered around the same (this prepares us for our final analyses). In general, it is desirable that sampling methodology be consistent with the technique of estimation (as is often the case with the purer sciences including experimental physics and biology).

Our choice(s) of null and alternate hypotheses for power analyses in sample size determination come directly from theory. The knowledge that *ES* always lies between the *PRO* and the *UG* rules aids our two-fold strategy for determining sample sizes for each treatment: first we take the H_{UG} to be the null and H_{ES} to be the alternate hypothesis; and second we treat the H_{ES} to be the null and H_{PRO} to be the alternate hypothesis.[9] For each treatment, these test of hypotheses are conducted for a given (lower bound of the) power of our tests, based on which we get two sample sizes (one from testing H_{UG} against H_{ES} and the other from testing H_{ES} against H_{PRO}), and we choose the larger of the two sample sizes as the desired sample size for that particular treatment. This prepares us for all contingencies in relation to statistical inference.

Our method of determining sample sizes is consistent (in fact, intertwined) with our technique of estimation which is primarily based on the shares of the high-ranked individuals in each negotiating pair. We account for the possibility of mass-points in our method of

[9] Note that this is necessary since, it is impossible to convincingly define the power of our test for three competing hypotheses together. Calculating sample sizes from competing pairs of hypotheses prepares us for the worst case and guarantees a minimum power as shown in the appendix.

determining sample sizes and make no assumption on the distribution of the underlying random variable(s) associated with observed negotiated outcomes. The exact details of arriving at the sample sizes are detailed, and are therefore, deferred to the appendix.

5.2. Hypotheses tests and regression results

In what follows, we use the information presented in Table 1, and use our empirical confidence intervals to see if they include the hypothesized values of the *UG*, *ES* and the *PRO* fairness rules under each treatment. Table 3 below presents the same.

Table 3. Observed data against fairness rules

Treatment	**N**	**Mean**	**95% CI**	**PRO**	**ES**	**UG**
(1)	(2)	(3)	(4)	(5)	(6)	(7)
T180	58	0.640 (0.005)	0.629 – 0.650	2/3 ☒	23/36 ☑	5/9 ☒
T300	55	0.600 (0.008)	0.582 – 0.617	2/3 ☒	7/12 ☑	1/2 ☒
T600	54	0.547 (0.008)	0.530 – 0.563	2/3 ☒	13/24 ☑	1/2 ☒
T900	59	0.524 (0.005)	0.514 – 0.534	2/3 ☒	19/36 ☑	1/2 ☒

Column (1) above enlists the treatments, and in column (2), we report the number of pairs in each treatment. Column (3) reports the mean share of the high-ranked individual *Y* observed in each of the treatments followed by the standard errors in the parentheses, which in turn are used for the confidence intervals in column (4). Columns (5), (6) and (7) show the expected share for the high-ranked subject *Y* in each treatment under the *PRO*, *ES*, and the *UG* allocation rules, followed by a ☒ or a ☑, depending on whether observations in the relevant treatment groups are inconsistent or consistent with the allocation rules in question (i.e. the

95% confidence intervals around the observed mean in the given treatment contains the predicted value of the said allocation rule). For example, the entry of 2/3 under the *PRO* rule in column (5) is clearly outside the 95% confidence intervals (shown in column 4) around the observed mean share of 60% in T300. Thus, we reject the hypothesis that the *PRO* rule explains the observed shares in T300, and represent the same with a ☒ sign (underneath the expected value under the *PRO* rule). Since each of the columns (5) and (7), has at least one ☒, it means that all of the observed experimental data is not *strictly* being explained by any of these rules.

In a nutshell, we see in Table 3 that even though the *PRO* fairness rule was the most popular (modal) choice in the T180 and the T300 treatments, it lies outside the 95% confidence interval around the observed means of those two treatments. Similarly, even though the *UG* fairness rule was the most popular choice in the T600 and the T900 treatments, it remains significantly distinct from the average choice of those treatment groups. The *ES* fairness rule, however, remains only insignificantly far from the observed mean in each treatment despite not being the most popular choice in any. In what follows, we formally examine this using the following regression equation in accordance with our testable hypotheses.

$$NegotiAmt_i = \alpha_0 + \alpha_1 PieSize_i + \alpha_2 T180_i + \boldsymbol{W_i \beta} + \varepsilon_i \qquad (2)$$

where $NegotiAmt_i$ (associated with the coefficient α_1) is the amount that the high-ranked agent Y negotiates (with the low-ranked agent X) for himself/herself in the ith pair; $PieSize_i$ is the size of the pie that our agents bargain over; $\boldsymbol{W_i}$ (associated with the coefficient vector $\boldsymbol{\beta}$) is a vector of other covariates that could potentially influence our bargaining outcomes; α_0 is the constant of regression and ε_i is the error specific to the negotiating pair. Lastly, we include a treatment dummy for T180 ($T180_i$ equals 1 if the ith bargaining pair belongs to T180, and 0 otherwise) as per the requirements of our hypotheses in Section 3. Since, if the *UG* solution were to explain the data, then agent *Y* must (on an average) get half the share in T300, T600, and T900 and an additional INR10 in T180, for he/she must at least earn his/her disagreement payoff subject to negotiation. The treatment dummy $T180_i$ solves this problem because, together with $\alpha_2 = 10$, the hypothesized values, $\alpha_0 = 0$, $\alpha_1 = 1/2$ continue to represent the *UG* fairness rule. Panels 1, 2 and 3 in Table 4 summarize the results from this regression.

In panel 1, we show the regression results without any additional controls. The F-tests for H_{UG} and H_{PRO} (shown in the lower panel) suggest that our observed regression coefficients are significantly different from what is required by the *UG* and the *ES* fairness rules. At this point we do not have sufficient evidence against the *ES* fairness rule, so we do not reject that. It seems that if Thomson's (1994) *impartial arbitrator* cared for *majority decision rule*, then he/she would specifically recommend the *PRO* rule or the *UG* rule depending on the pie-size - i.e. he/she would look for *general* preference (he/she will observe the mode and pick the modal preference). However, if he/she believed in some form of a social *aggregation* of the preferences of all agents (say, as in Arrow), then he/she would propose the *ES* fairness rule.

In panel 2, we introduce additional controls for the individual rank of the high-ranked individual, how many ranks ahead is he/she of his/her negotiating opponent (Rank Difference), the gender of the high-ranked individual, that of his/her negotiating opponent (opponent's gender), the high-ranked agent's academic aptitude and the time taken to negotiate (in seconds). In panel 3, in addition to these covariates, we include institution dummy variables. We see that the higher the rank of the high-ranked individual, the greater is his expected negotiated amount (note that Rank 1 is better than Rank 10, so higher values of the Rank variable correspond to lower ranks, thereby explaining the observed negative coefficients). We also see that the greater the rank difference between the two agents in a pair, the greater, on an average, is the gap between their respective shares (the share of *Y* moves upward, so that of *X* must necessarily move downward given the pie-size).

Table 4. Pie-Size as a determinant of the share of high-ranked agents

Dependent variable: y	(1)	(2)	(3)
Pie-Size (z)	0.487***	0.486***	0.484***
	(0.009)	(0.007)	(0.007)
T180	-7.189*	-8.512***	-8.850***
	(3.716)	(2.917)	(2.917)
Rank		-2.269***	-1.925***
		(0.660)	(0.668)
Rank Difference		2.455***	2.677***
		(0.524)	(0.573)
Gender (Male = 1)		1.385	-1.504
		(3.478)	(3.829)
Opponent's Gender (Male = 1)		-1.103	-2.596
		(3.304)	(3.357)
Academic Aptitude		0.528	0.618
		(0.549)	(0.540)
Time Taken		0.010	0.011
		(0.011)	(0.012)
Constant	34.724***	19.055*	19.668*
	(4.926)	(10.473)	(0.071)
N(umber of pairs)	226	226	226
Institution dummies	No	No	Yes
F-Test for H_{UG}	$F(3, 223) = 139.61$	$F(3, 217) = 15.55$	$F(3, 213) = 15.36$
(p-value)	(0.0000)	(0.0000)	(0.0000)
F-Test for H_{PRO}	$F(2, 223) = 521.23$	$F(2, 217) = 380.76$	$F(2, 213) = 408.73$
(p-value)	(0.0000)	(0.0000)	(0.0000)
F-Test for H_{ES}	$F(2, 223) = 2.10$	$F(2, 217) = 3.43$	$F(2, 213) = 3.79$
(p-value)	(0.1251)	(0.0342)	(0.0241)
Decision rule	Reject H_{UG} and H_{PRO}	Reject all	Reject all

Notes: ***, **, and * mark out significance at 1%, 5% and 10%; Robust standard errors in parentheses.

To get uniformity in academic scores (which were in the form of aggregate percentages or as GPAs with different denominators for each institution as per its own norms), we split this data (for each institution individually) into ten groups of roughly equal size. Thus, for each institution we coded those in the top ten percent as 10, the next ten percent as 9 and so on till the bottom ten percent (coded as 1).[10] The fact that this variable called Academic Aptitude is not correlated with our test-ranks also verifies that our assignment of ranks (based on the test) is indeed random.

Finally, with the inclusion of other covariates (panels 2 and 3) in our regression specification, we find evidence even against the *ES* solution at the 5% level (see the F-tests in the lower panel). Note that since our reported p-values are upper bounds on the actual p-values (see Appendix), it is clear that our standard errors are also upper bounds on the 'true' (and unobserved) standard errors (in general the smaller the standard errors, the smaller are the p-values). Before we propose a fairness rule that unifies all the rules discussed here, it will help to look at a behavioral explanation behind the observations we make.

5.3. Why some focal points are naturally more apt than the others - A behavioral explanation

Before we formally present our explanation, it will help to consider two thought experiments. In the first, *ceteris paribus*, suppose we had a treatment with a pie-size equaling INR153 - so that this treatment was called T153. The *UG,* the *ES* and the *PRO* rules will respectively

[10] This data on academic scores were recorded right at the end of our experiment under the conditions of strict anonymity. The arrangement was that our subjects were identified throughout by their experiment reference identities (also on the first page of the test on the appendix). These IDs were then handed over to our local institute contacts who provided us the academic scores against the IDs on our dataset without revealing any names. This data was recorded right at the end of our experiment and we do not retain any copies of our subjects' workings on our test (as per the requirement that all information that could identify our subjects be destroyed). We are not even aware if the percentages/GPAs we received were cumulative or just the scores for the last semester/trimester our subjects had been a part of - it just remained consistent for all the agents who came from the same institution.

suggest *(x, y) = (53, 100), (x, y) = (51.5, 101.5),* and *(x, y) = (51, 102)*. In all these cases, the actual ratio *y/x* is remarkably close to *2/1*. Thus, the ratio between our disagreement payoffs serves as a natural point of reference for bargaining making it a strong focal point. In general, whenever the joint surplus is not significantly different from the sum of our agents' individual earning capacities, we expect the ratio of our disagreement points to be a strong predictor of our negotiated outcomes. This is what we observe in T180, where the size of the surplus is only INR30 more than the sum of our disagreement payoffs.

In the second thought experiment, we look at another extreme example. Suppose agents *X* and *Y* could earn \$1 and \$2 by themselves. Suppose that they could earn a joint sum of \$1 billion *only if* they worked together. How likely will they be to split the joint pie in the ratio 2:1 (the ratio of their disagreement payoffs)? We emphasize that the size of the joint pie is so enormously far from the (sum of) individual payoffs that *nothing* about the individual payoffs (let alone the ratio between them) any longer matters. In this case, therefore the joint pie is more likely to be split equally. This is indeed what we observe in T600 and T900.

The above two thought experiments motivate us to propose the following fairness rule in which the agents weigh the size of they are jointly earning against their own individual payoffs. If we define 'surplus from cooperation' *s* to be the difference between a given pie-size and the sum of individual earning capacities, i.e. *s = z – [d(x) + d(y)],* then our allocation rule is given as follows:

$$\frac{y}{x} = \left[\frac{s}{a} + d(y)\right] / \left[\frac{s}{a} + d(x)\right] \qquad (3)$$

where a is nonnegative constant. The intuition based on the two thought experiments is simple - ff the gains from cooperation (s) are low in relation to the disagreement payoffs, then the subjects will still tend to use the *PRO* rule and share in proportion to their disagreement payoffs. However, if the gains from cooperation are so high that the effect of individual capacities blur away, then there will be a convergence toward equality. More formally, the *PRO* rule is attained such that

$$y/x = \lim_{s\to 0}[(\tfrac{s}{a}) + d(y)]/[(\tfrac{s}{a}) + d(x)] = d(y)/d(x)$$

and the *UG* rule is attained such that

$$y/x = \lim_{s\to \infty}[(\tfrac{s}{a}) + d(y)]/[(\tfrac{s}{a}) + d(x)] = 1.$$

Our fairness rule above unifies all the fairness solutions known in the literature. In fact, different values of *a* specifically characterize different fairness solutions. For example $a = 0$ gives us *exactly* the *UG* fairness rule; $a = 2$ gives us *exactly* the *ES* fairness rule; $a = \infty$ gives us *exactly* the *PRO* fairness rule; $a = 1$ gives us a modified version of the Kalai-Smorodinsky solution where the final outcome is observed along the line joining the disagreement point to the *ideal point* of each agent if the other became absent immediately after the joint surplus was created (in Figure 1, this *ideal point* will be *(300, 300)* instead of *(200, 250)* shown).

Table 5. Observed data against fairness rules and our allocation rule

Treatment (1)	**N** (2)	**Mean** (3)	**95% CI** (4)	**PRO** (5)	**ES** (6)	**UG** (7)	**Our allocation rule**
T180	58	0.640	0.629 – 0.650	0.667	0.638	0.556	0.641
T300	55	0.600	0.582 – 0.617	0.667	0.583	0.500	0.590
T600	54	0.547	0.530 – 0.563	0.667	0.542	0.500	0.546
T900	59	0.524	0.514 – 0.534	0.667	0.528	0.500	0.530

Our experimental data shows that the observed share of the high-raked individual gets closer to 50% as the pie size z (and therefore s) increases. This convergence to equality is different from what the *ES* rule suggests. For final robustness, we estimate the value of a from our data and examine if the 95% confidence interval around our point estimate $\hat{a}$ excludes the values that correspond to the *UG*, *ES* and the *PRO* fairness rules (as per what we should expect from the F-tests corresponding to the regressions in panels 2 and 3 in Table 4). The point estimate $\hat{a} = 2.28$, with a 95% confidence interval of 2.15 to 2.42 excludes the values assumed by the *ES* ($a = 2$), the *UG* ($a = 0$) and the *PRO* ($a = \infty$) rules. In Table 5, we display the estimated shares according to our allocation rule in comparison to the actual observations and the predicted values under the *ES*, *UG* and the *PRO* rules. Thus, we have proposed an allocation rule that not only unifies the individual fairness rules discussed, but also refines the predictive capacity of fairness rules.

5. Conclusion

We have addressed one of the most frequently arising issues in the understanding of bargaining outcomes in relation to the uniqueness of fairness ideals. In the experiment, it was observed that the amount of stakes involved may in itself provide useful hints on what is perceived as fair by the agents, as the disagreement payoffs (and hence their sum) are dwarfed in relation to the pie-sizes, our agents move away from choosing *PRO* toward *UG* to determine the final outcome(s). We also proposed a solution concept that unifies all the three fairness rules and individually displays better predictive capacity in comparison to all the fairness rules discussed. Our results are primarily driven by focal points which we identify using the following two criteria

- Its precise and easy number (a value like 112.5 is unlikely to be a focal point)

- It reduces the time taken to bargain

The most famous example of the *Equal Surplus* allocation rule is the Nash bargaining (Nash, 1950) solution, which maximizes $(x - d(x))(y - d(y))$ with respect to x and y subject to the constraint: $x + y = z$, and leads to a first order condition: $x - d(x) = y - d(y)$, which is, in fact, the very requirement of the *ES* protocol. These solutions can also be perceived to be fair since there is a sense of equality in gains.

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APPENDIX A

Test

Instructions: You have 15 minutes to complete this test. There are 20 questions: Each question (marked 1, 2, 3, etc.) is immediately followed by four options (marked a, b, c, and d). Only one of the options correctly answers the associated question. Your task is to mark a tick on what you believe to be the correct answer and maximize your score. Each correct entry carries one point. There is no negative marking. You may begin. All the best.

Name:

Gender (M/F):

Course:

Please leave the following spaces blank.

Time:

Score:

Experimental Reference ID:

1. A truel is similar to a duel, except that there are three participants rather than two. One morning Mr. Black, Mr. Grey, and Mr. White decide to resolve a conflict by truelling with pistols until only one of them survives. Mr. Black is the worst shot, hitting his target on average only one time in three. Mr. Grey is a better shot hitting his target two times out of three. Mr. White is the best shot hitting his target every time. To make the truel fairer, Mr. Black is allowed to shoot first, followed by Mr. Grey (if he is still alive), followed by Mr. White (if he is still alive) and round again (and again) until only one of them survives. Where should Mr. Black aim his first shot?

(a) He should aim at Mr. White

(b) He should aim at Mr. Grey

(c) He should shoot himself

(d) He should shoot in the air

2. Two urns contain the same total number of balls - each ball is either black or white, and these urns have different compositions of black and white balls. There is at least one ball of each color in each urn. From each urn, n (≥ 3) balls are drawn with replacement. We are interested in the number of drawings and the composition of black and white balls in the two urns, such that the probability that all the balls drawn from the first urn are white, is equal to, the probability that either all balls drawn from the second urn are white or all are black. Which of the following statements is true?

(a) This will never be possible

(b) Number of white balls in the first urn must be greater than the number of both white and black balls in the second urn

(c) Number of black balls in the second urn ≥ number of white balls in the first urn ≥ number of white balls in the second urn

(d) Number of white balls in the second urn ≥ number of white balls in the first urn ≥ number of black balls in the second urn

Answer the next two questions (3 and 4) based on the information in the following question:

3. If $N_K = \{1, \dots, K\}$, then how many sets $X = \{x_i \in N_{K^*} | i \in N_{K_*}\}$ solve the following problem when $K^* > K_* > 2$?

$$\text{maximize:} \quad [max(X) - min(X)] - [max(X \setminus \{max(X)\}) - min(X \setminus \{min(X)\})]$$

(a) There exists only one unique set solving the above problem

(b) $K^* - K_*$ sets

(c) There are exactly two sets that solve the above problem

(d) $K^* - K_* + 1$ sets

4. The maximum value in the above problem is

(a) $K^* - K_* - 1$

(b) $K^* - K_*$

(c) $K^* - K_* + 1$

(d) $K^* - K_* + 2$

Answer the next three questions (5 to 7) based on the information in the following question:

5. Let the function $f: (1, \infty) \mapsto (0, \infty)$ satisfy the property $f(xy) = f(x) + f(y)$; $\forall x, y \in (1, \infty)$, we look at the set of equations below

$$f(y) = f(2) + f(x)$$

$$yf(x) = xf(y)$$

the pair (x, y) that solves the above set of equations is

(a) not unique, there are infinitely many such pairs

(b) the information is insufficient to even determine if (x, y) is unique or not

(c) $x = 2, y = 4$

(d) unique, but there is insufficient information to arrive at the actual values of x and y

6. Now, alter the domain of the function f to $[1, \infty)$, and its range to $[0, \infty)$. Define a function

$$g: (-\infty, \infty) \mapsto (0, \infty)$$

which satisfies the property $g(x + y) = g(x)g(y)$. The value of $f(g(0)) + g(f(1))$ always equals

(a) 0

(b) 0.5

(c) 1

(d) Cannot be determined

7. Alter again the domain of f above to $\mathbb{R} - \{0\}$ and its range to $(-\infty, \infty)$. Consider the following statements.

Statement 1: $f((1/x)) = -f(-x)$

Statement 2: $f(-1) = 0$

Mark the correct option.

(a) Only Statement 1 is true

(b) Only Statement 2 is true

(c) Both the statements are true

(d) Neither of them is true

Answer the following two questions (8 and 9) based on the following information. Jack is captured by a tribe. Whether or not he gets to live is decided by the tribe members based on the outcome of the following exercise. There are 50 black and 50 white balls, which Jack must distribute between two identical and opaque boxes (that the tribe provides to him) in any way he wishes, but with the requirement that each ball must be put into one of the two boxes. The tribe then secretly allocates the balls among the two boxes as instructed by Jack and closes them before putting them in front of him. Jack gets to randomly pick a box before they blindfold him and make him draw a ball from it. If the ball is white, he survives, otherwise they execute him.

Answer the following two questions.

8. Jack's maximum probability of survival is

(a) $1/2$

(b) $74/99$

(c) $3/4$

(d) $71/100$

9. If Jack were offered five boxes instead of just two above, then his maximum probability of survival will

(a) definitely increase

(b) definitely decrease

(c) remain the same

(d) well ... cannot say

10. Which of the following events is more likely than the others?

(a) Getting at least 1 six when 6 dice are rolled

(b) Getting at least 2 sixes when 12 dice are rolled

(c) Getting at least 3 sixes when 18 dice are rolled

(d) All the three events above are equally likely

11. A professor chooses two consecutive numbers from the following set *{1, 2, 3, ... , 10}*. A is told the first number and B, the other. The following conversation takes place:

A: I do not know your number.

B: Neither do I know your number.

A: Now I know.

In how many ways can the professor choose the numbers so that this exact conversation between A and B is possible?

(a) *1*

(b) *5*

(c) 2

(d) *4*

12. The number of linear functions $f: \mathbb{R} \mapsto \mathbb{R}$ that satisfy the property $f(x + f(x)) = x$ is

(a) *1*

(b) *2*

(c) *3*

(d) *4*

13. You want to find someone whose birthday matches yours. What is the least (expected) number of strangers whose birthdays you need to ask to have a (greater than) 50% chance of finding a match? (Assume a year of 365 days.)

(a) *23*

(b) *183*

(c) *253*

(d) *364*

14. Alice was first to arrive at a theatre with 98 seats. She forgot her seat number and picks a random seat for herself. After this, every single person who get to the theatre sits on his seat if its available else chooses any available seat at random. Charles is last to enter the theatre and 97 seats were occupied. With what probability does he get to sit in his own seat?

(a) *1*

(b) *1/2*

(c) *1/3*

(d) *1/4*

15. Shuffle an ordinary deck of 52 playing cards containing four aces. Then turn up cards from the top until the first ace appears. On the average, how many cards are required to produce the first ace?

(a) *10.6*

(b) *9.6*

(c) *13.0*

(d) *13.4*

16. If a stick is broken in two at random, what is the average (expected) ratio of the length of the smaller piece to the larger?

(a) *0.333*

(b) *0.386*

(c) *0.301*

(d) *0.441*

17. A player tosses a coin from a distance of about five feet onto the surface of a table ruled in one-inch squares. If the coin (3/4 inches in diameter) falls entirely inside a square, the player wins a holiday package; otherwise he loses. If the penny lands on the table, what is his probability of winning?

(a) *1/2*

(b) *1/4*

(c) *1/8*

(d) *1/16*

18. To encourage Bob's promising tennis career, his father offers him a prize if he wins (at least) two tennis sets in a row in a three-set series to be played with his father and a club champion alternately: father-champion-father or champion-father-champion according to Bob's choice. The champion is a better player than Bob's father. Which series should Bob choose (assume that the outcome of each game in a given series in independent of another)?

(a) father-champion-father

(b) champion-father-champion

(c) He will be indifferent between the two

(d) There is no definite answer

19. For any function f with $f' > 0$, and $f'' < 0$, the maximum value of $f(x)f(1-x)$ is attained at

(a) the maximum of $f[x(1-x)]$

(b) $x = 1/2$

(c) both (a) and (b) above

(d) Cannot be determined

20. A three-man jury has two members, each of whom independently has a probability p of making the correct decision and a third juror who flips a coin for each decision (majority rules). A one man jury has the probability p of making the correct decision. Which jury has the better probability of making the correct decision?

(a) Both of them are equally good

(b) The three-man jury is better than the one-man jury

(c) The one-man jury is better than the three-man jury

(d) There is no conclusive answer

APPENDIX B

Working of sample sizes for our treatment groups

Let the *i*th pair of shares be (x_i, y_i), where $x_i + y_i = 1$. Thus, we are expressing *x* and *y* as a fraction of the pie size in this entire section. For any pair *i*, the share *y* of the high-ranked individual *Y* is sufficient to uniquely characterize the shares of both the agents.[11] Therefore we define the negotiated outcome of any bargaining pair as $z_i = y_i$, and then let $\bar{Z} = \frac{Z_1 + \ldots + Z_n}{n}$ (where *n* is the number of observed pairs). Thus, $\bar{Z}$ measures the average share of the high-ranked individuals.

In what follows, we will demonstrate the process of calculation explicitly for *T300* and just state the desirable sample-size values for our other treatments. Our aim is to answer how large our sample sizes should be to tell our fairness solution concepts apart. In each stage we will suppose that the population mean our random variable $\bar{Z}$ is μ.

We first consider the test of the null hypothesis that $\mu = \mu_0 = 1/2$ (i.e. the *UG* fairness solution is indeed what the population of pairs choose on average). The question is: what would be the minimum sample that is required for such a test to have reasonable power against an alternative hypothesis that the population mean is $\mu = \mu_1 = 7/12 > \mu_0 = 6/12$ (i.e. the *ES* fairness solution is indeed what the population of pairs choose on average)?[12] We do not make any assumption(s) on the distribution of Z_i (and therefore $\bar{Z}$) under the null or the alternate hypothesis.

Let α be the (maximum permissible) size of the type-I error. Let *c* be a non-negative constant such that $P(\bar{Z} - \mu_0 > c \mid \mu = \mu_0) \leq \alpha$. In other words, the null is rejected whenever $\bar{Z} > \mu_0 + c$.

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[11] Clearly, for example, from $y = 0.6$, we immediately know that $x = 0.4$since, $x + y = 1$.

[12] Note that for a pie-size equalling *300,* the *ES* fairness rule suggests that *X* gets 125 and *Y* gets *175*. In proportions this translates to $x = 5/12$ and $y = 7/12$, which is the chosen value for μ_1 here.

To determine c as a function of α and n, we note the following inequalities (the first one of which is $P(\bar{Z} \le \mu_0 + c) \ge P(\mu_0 - c \le \bar{Z} \le \mu_0 + c) \ge P(\mu_0 - c < \bar{Z} < \mu_0 + c)$).

$$P(\bar{Z} \le \mu_0 + c) \ge P(\mu_0 - c < \bar{Z} < \mu_0 + c); \{\because \text{LHS spans more values}\}$$

$$P(\mu_0 - c < \bar{Z} < \mu_0 + c) = P(|\bar{Z} - \mu_0| < c) \ge 1 - \frac{\sigma_Z^2}{nc^2}; \{\because \text{Chebyshev's inequality}\}$$

We combine the two inequalities above as follows

$$P(\bar{Z} \le \mu_0 + c) \ge 1 - \frac{\sigma_Z^2}{nc^2}$$

$$\Rightarrow \quad P(\bar{Z} - \mu_0 > c | \mu = \mu_0) \le \frac{\sigma_Z^2}{nc^2}$$

$$\Rightarrow \quad \text{P(Type I error)} \le \frac{\sigma_Z^2}{nc^2} = \alpha$$

$$\Rightarrow \quad c = \frac{\sigma_Z}{\sqrt{\alpha n}} \qquad \text{(A.1)}$$

Thus, the probability of a Type-I error does not exceed α when $c = \frac{\sigma_Z}{\sqrt{\alpha n}}$. Now we turn to Type II error (which should not exceed β).

$$P(\text{Type II error}) = P(\bar{Z} < \mu_0 + c | \mu = \mu_1)$$

Now $\mu_0 = 0$, and we substitute for c from (A.1), we get

$$P(\text{Type II error}) = P(\bar{Z} < \mu_0 + \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1)$$

Note that for any k, we know from Chebyshev's inequality that

$$P(\mu_1 - k < \bar{Z} < \mu_1 + k | \mu = \mu_1) \ge 1 - \frac{\sigma_Z^2}{nk^2}$$

We now take $k = \mu_1 - \mu_0 - c = \mu_1 - \mu_0 - \frac{\sigma_Z}{\sqrt{\alpha n}}$ in the above inequality (with $\mu_0 = 0$) to get

$$P(\frac{\sigma_Z}{\sqrt{\alpha n}} < \bar{Z} < 2\mu_1 - \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1) \ge 1 - \frac{\sigma_Z^2}{nk^2} \qquad \text{(A.2)}$$

But

$$P(\bar{Z} \ge \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1) \ge P(\frac{\sigma_Z}{\sqrt{\alpha n}} < \bar{Z} < 2\mu_1 - \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1); \{\because \text{LHS spans more values}\}$$

(A.3)

The LHS above spans more values since:

$P(\overline{Z} \geq \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1) \geq P(\overline{Z} > \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1) \geq P(\frac{\sigma_Z}{\sqrt{\alpha n}} < \overline{Z} < 2\mu_1 - \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1)$

On combining the inequalities (A.2) and (A.3), we get

$$P(\overline{Z} \geq \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1) \geq 1 - \frac{\sigma_Z^2}{nk^2}$$

$$\Rightarrow \quad 1 - P(\overline{Z} \geq \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1) \leq 1 - (1 - \frac{\sigma_Z^2}{nk^2}) = \frac{\sigma_Z^2}{nk^2}$$

$$\Rightarrow \quad P(\overline{Z} < \frac{\sigma_Z}{\sqrt{\alpha n}} | \mu = \mu_1) \leq \frac{\sigma_Z^2}{nk^2}$$

$$\Rightarrow \quad P(\text{Type II error}) \leq \frac{\sigma_Z^2}{nk^2} = \beta \qquad \text{(A.4)}$$

Thus, the probability of a Type II error does not exceed β when $\frac{\sigma_Z^2}{nk^2} = \beta$. Substituting for $k = \mu_1 - \mu_0 - c = \mu_1 - \mu_0 - \frac{\sigma_Z}{\sqrt{\alpha n}}$, we get

$$\beta = \frac{\sigma_Z^2}{n(\mu_1 - \mu_0 - \frac{\sigma_Z}{\sqrt{\alpha n}})^2} \qquad \text{(A.5)}$$

$$\Rightarrow \quad n = \frac{\sigma_Z^2}{(\mu_1 - \mu_0)^2}\left(\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}}\right)^2 \qquad \text{(A.6)}$$

In this expression, we fix the probabilities of α and β, (the upper bounds on our Type I and II errors) to be 0.05 and 0.20 respectively. With $\mu_0 = 6/12$ and $\mu_1 = 7/12$, the only limitation is that we do not know the value of σ_Z.[13] To estimate σ_Z, we use a pilot study in which, $\hat{\sigma}_Z = \hat{\sigma}_y = 0.06$.[14] Using this value gives us $n^* = 23.33 \approx 24$ pairs (48 subjects). Note that c equals 0.05 for this value of n. In other words, if the mean is indeed $\mu_0 = 1/2$ (as per the *UG* rule which requires that $x = y = 50\%$), then with 24 negotiating pairs, the probability that we will observe an average $\bar{z} > \mu_0 + c = 0.55$ (i.e. the average agent X gets at most 45%, and the

[13] According to Thompson (2012), an "aspect of sample size formulas such as these is that they depend on the population variance, which generally is unknown. In practice, one may be able to estimate the population variance using a sample variance from past data from the same or a similar population."

[14] Note that Var[z] = Var[y]. Thus, $\sigma_Z = \sigma_y$.

average agent Y gets at least 55%) will be *at most* 5%.[15] This sums up how we work out the sample size when we take our null hypothesis to be H_{UG}, and our alternate hypothesis to be H_{ES}.

Similarly, in T300, if we took our null hypothesis to be H_{ES}, and our alternate hypothesis to be H_{PRO}, then our values for μ_0 and μ_1 will respectively be *7/12* (as calculated above), and *2/3* (= *8/12*).[16] For our choice of values of α (= 0.05), β (= 0.20) and, $\hat{\sigma}_Z$ (= 0.06), we again get n^* = $23.33 \approx 24$ pairs (48 subjects). In general, these two sample sizes need not be equal for any given treatment, in which case, we choose the larger of the two values of n^* as our chosen sample size for that treatment. Table A.1. below, summarizes the above calculations for all our treatments. For each treatment (column), for the ease of comparison, we have represented the hypothesized mean values with a common denominator. For example, under H_{PRO}, the hypothesized value of the average share of the high-ranked agent Y, is 2/3 - this is written as 24/36 for T180, 8/12 for T300, 16/24 for T600, and 24/36 for T900 (our μ_1 entries for H_{ES} vs H_{PRO}). We do the same for H_{UG} (representing 1/2 as 6/12, 12/24 and 24/36 for T300, T600 and T900), with the exception of T180, where the high-ranked agent Y should earn at least his/her disagreement payoff of 100.

The intuition behind why the sample-sizes vary for different pairs of competing hypotheses across different treatments is noteworthy. Our desired sample-size expression is inversely related to the number of standard deviations that can be fitted between the means assumed under the two hypotheses (also known as Cohen's d - see Cohen, 1977). The gap between the population mean values assumed under H_{UG} and H_{ES} ($\frac{23}{36} - \frac{20}{36} = \frac{3}{36}$) in T180 is wider than the

[15] In fact, the probability of our type-I error will be significantly less than 5% (note that Chebyshev's bounds are never very tight). The exact same argument holds for our type-II error as well (our power is way higher than 80%)

[16] Note that for a pie-size equalling *300*, the *PRO* fairness rule suggests that X gets 100 and Y gets *200*. In proportions this translates to $x = 1/3$ and $y = 2/3$, which is the chosen value for μ_1 here.

gap between the population means assumed under H_{UG} and H_{ES} ($\frac{24}{36} - \frac{23}{36} = \frac{1}{36}$) in the same treatment. Thus, clearly more data is required in the latter to tell the competing hypotheses apart. The logic is that the standard error of the sample-mean is inversely related to the (square-root of) the sample-size. This argument extends to the varying calculated sample-sizes for all the other treatments.

Table A.1. Sample size determination for $\alpha = 0.05$, and $\beta = 0.20$

	T180	**T300**	**T600**	**T900**
μ_0 (H_{UG} vs H_{ES})	20/36	6/12	12/24	18/36
μ_1 (H_{UG} vs H_{ES})	23/36	7/12	13/24	19/36
μ_0 (H_{ES} vs H_{PRO})	23/36	7/12	13/24	19/36
μ_1 (H_{ES} vs H_{PRO})	24/36	8/12	16/24	24/36
Pilot $\hat{\sigma}_Z$	0.03	0.06	0.04	0.03
n^* (H_{UG} vs H_{ES})	6	24	42	53
n^* (H_{ES} vs H_{PRO})	53	24	5	3
Chosen n^* (greater of the above two)	53	24	42	53