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Dyotona Dasgupta and Prabal Roy Chowdhury

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Indian Statistical Institute, Delhi
Economics and Planning Unit
7, S. J. S. Sansanwal Marg, New Delhi, 110016, India

Simultaneous Borrowing and Saving in Microfinance*

Dyotona Dasgupta[†] Prabal Roy Chowdhury[‡]

Abstract

This paper studies dynamic incentives provided by the microfinance institutions (MFIs) to ensure repayment. MFIs provide collateral-free loans, and yet observe near perfect repayment rate. In this paper, we provide an explanation of two widely practised mechanisms by MFIs – progressive lending i.e. increasing loan size over time and deposit collection. In our model, the MFI provides both credit and savings services. These help a strategic, poor borrower to accumulate a lumpsum amount and "graduate" to an improved lifetime utility which is not achievable when only credit is provided. These savings also act as an incentive device for repayment. We find that the optimal loan scheme is weakly progressive. It is "progressive with a cap" when the increase in utility from graduation is "modestly positive". Further, we show that, since the MFI is benevolent, an improvement in the borrower's outside option lengthens the time required to graduate which in turn reduces her welfare.

Keywords: Dynamic Incentives, Progressive Lending, Deposit Collection, Collateral Substitute, Graduation.

JEL Classification Number: O12, O16, D86, G21.

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† Corresponding author. Centre for Development Economics, Delhi School of Economics. Email: dasgupta.dyotona@gmail.com.

‡ Indian Statistical Institute, Delhi. Email: prabalrc1@gmail.com.

1 Introduction

Simultaneous borrowing and saving have been practised by many microfinance institutions (hereafter MFIs) (Grameen II, FINCA Nicaragua for example). However, the provision of simultaneous borrowing and saving may seem counter-intuitive since, with money being fungible, it would make more sense to just save the net amount instead. The existing literature mostly provides behavioral explanations for such a practice. In this paper, we provide a rationale which does not involve any behavioral anomaly. In particular, we develop a theoretical model where the MFI provides savings service along with credit, which facilitates an improvement of a poor borrower's lifetime utility beyond the level achievable, when only credit is provided. We find that the borrower's welfare maximizing loan sequence is progressive in nature, i.e. increasing over time contingent on successful repayment. To the best of our knowledge there is no paper which studies the impact of (compulsory) savings service on loan size. This paper helps us understand the relationship between compulsory savings and progressive lending. This is also important as progressive lending is practised by almost all the MFIs (Grameen II, FINCA Nicaragua for example), but there is hardly any theoretical paper which addresses this.[1]

Formally, we study a dynamic relationship between a poor borrower and a benevolent MFI[2] whose objective is to maximise the borrower's lifetime utility subject to a break-even condition. The borrower has access to a nonconvex technology which requires a fixed initial investment ($\bar{S}$ say). We may think of this $\bar{S}$ as the amount required to set up a small business, or to open a "Kirana" shop, or to buy a bicycle which helps her to go to a nearby city and sell her produce. When a poor borrower starts investing in this nonconvex technology we say that she has *graduated*. The problem is that her endowment is zero. Furthermore, she is subject to an *ex post* moral hazard problem in that she does not repay whenever she has an incentive to do so and it is *not* possible for the MFI to incentivise her to repay once she graduates. Therefore the MFI has to design a contract such that the borrower does not have any incentive to default and her utility is maximised.

The contract of the MFI involves both savings and credit: It provides several successive small loans that the borrower invests in a less productive technology (say traditional farming) and saves a part of the net return with the MFI. These savings accumulate over time and help her to graduate. In case of default the MFI confiscates these savings. This improves the borrower's incentive for repayment ensuring progressivity in the optimal loan scheme.

Specifically, we find that since graduation is welfare improving, at the optimum the borrower graduates *as soon as possible*: Along the equilibrium path, the MFI terminates the contract as soon as her total savings (along with interest) become $\bar{S}$. To reduce the time required to accumulate $\bar{S}$ the borrower saves the entire net return with the MFI. Further, the MFI lends in such a way that the net return is maximised, which requires lending the efficient amount[3] if that is incentive compatible, or the maximum amount that is. The optimum loan scheme is nondecreasing over time: When the increase in utility from graduation is *modestly positive*,[4] it is progressive with a

[1] There are a few papers which address progressive lending, however all of them involve default along the equilibrium path whereas we observe near perfect repayment rate in microfinance. More in Section 1.1.

[2] Though there is a growing concern about mission drift of MFIs, see de Quidt et al. (2018) for example, a significant portion of borrowers still take loans from the benevolent MFIs – the number of active borrowers of NBFI (Non-Bank Financial Institution) is 38,518400 vis-à-vis that of NGOs and Cooperative/Credit Union is 37,220400 (Source MIX (2017)). It is of course interesting to consider a motivated MFI which maximises the weighted sum of its own profit and the borrower's utility. To understand the implications of the possibility of graduation simply and clearly, we assume the MFI to be benevolent. We briefly discuss the case with profit maximizing MFI in Remark 3.

[3] The efficient amount is that amount for which the net return is maximised (formal definition later).

[4] Banerjee et al. (2015) in their famous six-country study find that increase in utility from microfinance is "modestly positive" and not "transformative". We use these two terms in this paper, precise parametric condition for increase

cap – it initially increases and then remains constant at the efficient level. When that increase is *transformative* the optimum loan scheme remains constant at the efficient level.

Intuitively, as time passes, the amount saved with the MFI increases, so the time remaining to graduate decreases, which implies that the present discounted payoff from graduation increases over time. Moreover, we assume that in the benchmark case, in case of default, the MFI confiscates her entire savings till date. So on the one hand gain from repayment increases, on the other hand loss from default increases. Hence the maximum incentive compatible loan amount increases over time which implies that the optimum loan scheme is nondecreasing. It remains constant when the efficient amount is incentive compatible from the very beginning. This happens when the increase in utility from graduation is *transformative*. When that increase in utility is *modestly positive*, the borrower does not repay the efficient amount initially. The utility from repayment increases over time and ultimately the efficient amount becomes incentive compatible. Hence under this parametric condition, the optimum loan scheme is progressive with a cap.

Next we extend the model in two directions. First, the borrower has access to another savings technology that is not controlled by the MFI. Second, in case of default the MFI cannot confiscate the borrower's entire savings with it. These extensions allow her to graduate even in case of default, making it more attractive. We find that under *some* parametric conditions – the "part" of savings the borrower gets back from the MFI in case of default is less than the interest rate on savings – the optimal loan scheme continues to be progressive. Intuitively, as time passes the borrower's present discounted value of lifetime utility from repayment increases – savings with the MFI increases and the time remaining to graduate decreases. But the present discounted value of lifetime utility from default also increases with time – savings with the MFI increases, hence the amount she gets back in case of default also increases and correspondingly, the time remaining to graduate decreases. The parametric restriction ensures that the increase in utility from repayment is higher than that from default which relaxes the incentive compatibility constraint. Hence the optimum loan scheme is progressive.

We then compare the optimum outcomes of the benchmark case with those of the extended case. We find that these extensions which improve the borrower's utility from default actually make her (weakly) worse off. This is of interest because while it is argued that access to savings technology improves the borrower's welfare but when that dampens her repayment incentive and the MFI is benevolent, it actually makes her worse off. Intuitively, these extensions strengthen the dynamic incentive compatibility constraints – that is at any instance the loan amount which is incentive compatible in the benchmark case may not remain the same in the extended case. This (weakly) increases the time required to graduate and hence the borrower's present discounted value of lifetime utility (weakly) decreases. In fact, the borrower gets strictly worse off whenever the efficient amount is not incentive compatible from the very beginning in the general framework.

To summarise, in this paper we seek to provide an explanation of two mechanisms which are widely used but less explored in the literature – progressive lending and savings acting as collateral substitute. The MFI provides both savings and credit services which help a poor borrower to accumulate a lumpsum amount required to graduate to an improved lifetime utility. So broadly this is a contribution to the argument that the MFIs should provide "credit plus" service (see Armendàriz and Morduch (2005) pp. 14-16 and chapter 6 for example). To the best of our knowledge, this is the first paper to study the effects of the possibility of graduation to a more productive activity on loan repayment.

in utility to be "modestly positive" or "transformative" is given later.

1.1 Related Literature

In literature, simultaneous borrowing and saving have mostly been explained behaviorally – Basu (2016), Laibson et al. (2003) for example. In particular, Basu (2016) develops a three period model where a sophisticated, present biased agent has an opportunity to invest at period 1. Without any commitment, due to present bias, he does not invest when time comes. To make him invest, his period zero self simultaneously borrow and save in a risky asset such that his wealth at that period remains the same. But the expected wealth at period 2 decreases. This makes the risk averse agent invest at period 1. Laibson et al. (2003) also assume that agents are present biased to explain 'debt-puzzle' – they borrow aggressively on credit cards, and simultaneously save for retirement. Baland et al. (2011) explain how poor people save and borrow simultaneously to pretend to be poor so that they do not have to lend to their poor relatives. Our explanation does not assume any behavioral anomaly, instead benevolent MFIs optimally choose to provide credit and savings service simultaneously in order to improve the welfare of the poor agents who are subject to *ex post* moral hazard.

There are a few papers to address progressive lending in microfinance. In a two period model Armendàriz and Morduch (2000) show how a *strategic* borrower's incentive to repay in the first period increases with an increase in the size of the second period loan. The equilibrium involves default in the second period though. In Ghosh and Ray (2016) progressive lending helps in weeding out the borrowers who never repay. Egli (2004) shows that progressive lending may fail to identify a "bad" type, since a bad borrower may camouflage herself as a "good" borrower (who always repays) in order to get a higher amount of loan later on which she defaults with certainty. Shapiro (2015) examines a framework with uncertainty over borrowers' discount rates. He shows that even in the efficient equilibrium almost all the borrowers default. However, that all these papers involve default along the equilibrium path. In contrast, the present paper seeks to explain progressive lending in a scenario where there is no default along the equilibrium path as we observe almost no default on MFI-loans.

2 Benchmark Case

2.1 Payoff, Technologies and Graduation

We study a dynamic relationship between a poor borrower who is subject to an *ex post* moral hazard problem and a benevolent MFI that operates under a zero profit condition. The borrower has access to a nonconvex technology $\langle V, \bar{S}\rangle$ where $\bar{S}(>0)$ is the required fixed initial investment, and V is the present discounted value of lifetime utility from investing in that technology (gross of $\bar{S}$). Her endowment is zero, so she cannot start the project on her own. The MFI could provide adequate credit to start the project, but once the borrower starts investing in $\langle V, \bar{S}\rangle$ technology, it is not possible to incentivise her to repay.[5]

Apart from zero endowment, the borrower also has limited access to technology: In this benchmark case, when her wealth is less than $\bar{S}$ she does not have access to any technology. The MFI can provide access to a savings and a deterministic neoclassical technology $f(\cdot)$. This technology $f(\cdot)$ does not require any minimum initial investment and satisfies the usual assumptions:

Assumption 1. $f(0) = 0,\ f'(\cdot) > 0,\ f''(\cdot) < 0,\ \lim\limits_{k\to 0} f'(k) = \infty$ *and* $\lim\limits_{k\to\infty} f'(k) = 0.$

[5]First, the MFI does not have any control over $\langle V, \bar{S}\rangle$ technology. Second, once the borrower's wealth becomes no less than $\bar{S}$ she gets access to a storage technology as well. Hence it is not possible to incentivise her to repay. This is a direct consequence of Bulow and Rogoff (1989) and Rosenthal (1991).

Now let the efficient scale of investment k^e solve $\underset{k}{\text{argmax}}\ [f(k) - k]$. Turning to the MFI-savings technology, given the zero profit condition of the MFI, the interest rate on savings is equal to the real rate of interest. Let r denote the real rate of interest and both the agents (the borrower and the MFI) discount the future at the rate r.

We also assume that the borrower's technology $\langle V, \bar{S}\rangle$ is more productive – the net gain from investing in that technology exceeds the present discounted net payoff from running the $f(\cdot)$ technology at its efficient level.

Assumption 2. $V - \bar{S} > \frac{1}{r}[f(k^e) - k^e]$.[6]

When a borrower starts investing in this nonconvex, more productive technology $\langle V, \bar{S}\rangle$ we say that she has *graduated*. The first-best thus involves providing $\bar{S}$ amount of loan at the very beginning so that she can graduate immediately. But as discussed above, that is not achievable – the borrower would default and the MFI would make a loss with certainty. The problem of the MFI thus is to design a dynamic self-enforcing scheme such that the borrower's lifetime utility is maximised and she chooses to repay always.

2.2 Contracts and Timeline

We consider an infinite horizon, continuous time framework, where $t \in [0, \infty)$. At $t = 0$, the MFI announces a contract $\langle \{\alpha_t\}_{t=0}^{T_M}, \{k_t\}_{t=0}^{T_M}, T_M\rangle$, where T_M is the "successful" termination date of this contract[7] and k_t and α_t, respectively, denote the loan amount and the part of net return which the borrower saves with the MFI, at any instance t, where $0 \leq t \leq T_M$. So, at any instance t, the borrower not only repays the amount k_t, but also saves a part α_t of her net return, i.e. $\alpha_t(f(k_t) - k_t)$, with the MFI. We shall consider α_t such that $0 \leq \alpha_t \leq 1$. Given limited liability α_t cannot be higher than 1, so this condition actually implies that dissaving is not allowed.[8]

The borrower either accepts or rejects this contract, with the game ending in case she rejects. If she accepts then the continuation game at any instance t, where $0 \leq t \leq T_M$, is as follows:

Stage 1: The MFI lends k_t, the borrower invests that amount in the $f(\cdot)$ technology which yields an instantaneous output of $f(k_t)$.

Stage 2: The borrower then decides whether to repay, or not:

(i) In case of repayment, she repays k_t, deposits a part α_t of her net return $\alpha_t[f(k_t) - k_t]$ with the MFI, consumes the rest instantaneously, and the game continues.

(ii) In case of default, she obtains her current gross income $f(k_t)$, that is we assume that the amount yielded cannot be seized. The MFI, however, terminates the contract, so that the borrower does not get any more loan from that instance onwards and withdraws her access to both $f(\cdot)$ and the savings technology. Moreover, the MFI confiscates her entire savings with it till date.

[6] Observe, $\frac{1}{r}[f(k^e) - k^e] = \int_0^\infty e^{-rt}[f(k^e) - k^e]dt$.

[7] A contract gets terminated in two ways – in case of default and in case the borrower always repays at this prespecified date T_M. To differentiate the latter from the former, when the contract gets terminated in the latter way, we say that it has been terminated "successfully".

[8] The assumption that $\alpha_t \geq 0$ is without loss of generality as at the optimum α_t takes the maximum value possible which is 1 (Lemma 2).

Finally, at the successful termination of the contract T_M, the borrower gets back her entire savings with the MFI till date, along with interest. If that amount is no less than $\bar{S}$ then she graduates immediately. Otherwise, given that she does not have access to any technology to transfer wealth from one instance to another she has to consume the entire amount immediately.

We solve for the subgame perfect Nash equilibrium (SPNE) of this game.

2.3 Analysis: Optimal Loan Scheme

Let us start with a brief overview of the results and the intuition behind them. Since graduation is welfare improving (Assumption 2), the benevolent MFI enables the borrower to graduate. In fact, due to the same reason its objective is to minimise the time required to graduate. In order to do that it (a) successfully terminates the contract and returns the borrower's savings as soon as that becomes $\bar{S}$ and (b) designs the contract such that the time required to accumulate $\bar{S}$ is the minimum, given that the borrower has incentive to repay. This in turn implies that the objective of the MFI is to maximise the instantaneous savings. Therefore, the MFI lends the efficient amount k^e whenever that is incentive compatible otherwise the maximum amount which is that and the borrower saves the entire net return. The last result is due to our assumptions of linear utility function and that discount factor is equal to the rate of interest on savings. Due to these assumptions the borrower is indifferent between consuming an amount now, and saving and consuming that amount (along with interest) later, but she is strictly better off as the time required to graduate decreases.[9]

For ease of exposition, we make the following assumption which ensures that the borrower cannot graduate in case of default.[10]

Assumption 3. $\bar{S} > f(k^e)$.

Now, there can be two cases – at the optimum the borrower may or may not graduate. In the former case, the problem of the MFI is to select a scheme $\langle \{\alpha_t\}_{t=0}^{T_M}, \{k_t\}_{t=0}^{T_M}, T_M \rangle$ which maximises the borrower's lifetime utility, subject to (i) the graduation condition (GC hereafter) which ensures that, by the end of the scheme, the accumulated savings exceeds $\bar{S}$ and (ii) the dynamic incentive compatibility constraints (DICs hereafter) which ensure that the borrower does not have any incentive to default at any instance t, where $0 \leq t \leq T_M$.

$$\underset{\langle \alpha_t, \{k_t\}_{t=0}^{T_M}, T_M \rangle}{\text{Maximise}} \quad \int_0^{T_M} e^{-rt}(1-\alpha_t)[f(k_t)-k_t]dt + e^{-rT_M}\left[\int_0^{T_M} e^{r(T_M-t)}\alpha_t[f(k_t)-k_t]dt - \bar{S} + V\right]$$

$$\text{Subject to: GC: } \int_0^{T_M} e^{r(T_M-t)}\alpha_t[f(k_t)-k_t]dt \geq \bar{S},$$

$$\text{DIC: } \int_t^{T_M} e^{-r(t'-t)}(1-\alpha_{t'})[f(k_{t'})-k_{t'}]dt' + e^{-r(T_M-t)}\left[\int_0^{T_M} e^{r(T_M-t')}\alpha_{t'}[f(k_{t'})-k_{t'}]dt' - \bar{S} + V\right]$$

$$\geq f(k_t), \quad \forall t \leq T_M.$$

Let us briefly explain these two constraints. The left hand side of the GC is the total savings till T_M, which needs to be at least $\bar{S}$, so that the borrower can graduate at T_M. On the other hand, the DIC states that at any $t \leq T_M$ the borrower's present discounted value of lifetime utility from repayment is higher than that from default. The borrower's present discounted value of lifetime

[9] We conjecture that qualitatively all the results go through even if we assume strictly concave utility function, we discuss that briefly in Remark 2.

[10] We relax this assumption in the general case (section 3) and in Remark 1 we briefly discuss what happens if we relax this assumption in this benchmark case.

utility from repayment, i.e. the left hand side of the DIC has two components: the first term denotes the present discounted value of her utility from consumption till the T_M^{th} instant (evaluated at t) and the second term denotes the present discounted (again evaluated at t) value of her utility at the T_M^{th} instant when she gets back her entire savings (along with interest) and graduates, $V - \bar{S}$ being the net gain from graduation. Finally, the right hand side of the DIC is the borrower's utility at t from default, i.e. her instantaneous return $f(k_t)$. Observe, due to our assumption 3 and the fact that the borrower does not have access to any storage technology, she cannot graduate in case of default. Thus her present discounted value of lifetime utility from default at t is her utility from consumption.

Similarly, in case the borrower does not graduate, the problem of the MFI is to

$$\underset{\langle \alpha_t, \{k_t\}_{t=0}^{T_M}, T_M \rangle}{\text{Maximise}} \quad \int_0^{T_M} e^{-rt}(1-\alpha_t)[f(k_t) - k_t]dt + e^{-rT_M} \int_0^{T_M} e^{r(T_M - t)} \alpha_t [f(k_t) - k_t]dt$$

Subject to:

$$\text{DIC: } \int_t^{T_M} e^{-r(t'-t)}(1-\alpha_{t'})[f(k_{t'}) - k_{t'}]dt' + e^{-r(T_M-t)} \int_0^{T_M} e^{r(T_M-t')} \alpha_{t'}[f(k_{t'}) - k_{t'}]dt' \geq f(k_t), \quad \forall t \leq T_M.$$

In our first proposition we show that at the optimum the borrower always graduates. This is a direct consequence of our assumption 2 – graduation is welfare improving. The borrower's lifetime utility increases as she graduates, so the MFI, being benevolent, designs the optimum contract in such a way that the borrower can graduate. We prove the result by contradiction, we start with an optimum contract where the borrower does not graduate and show that given Assumptions 1, 2 and 3, it is possible to construct another DIC contract where the borrower graduates and her present discounted value of lifetime utility is higher. So, the original contract cannot have been optimum. For the rest of the paper, we restrict ourselves to the class of contracts where k_t and α_t are continuous in t. All the proofs can be found in Appendix A.

Proposition 1. *Let Assumptions 1, 2 and 3 hold. At the optimum the borrower always graduates.*

In fact in the next lemma we show that at the optimum the borrower graduates as soon as possible, i.e. she graduates as soon as her savings becomes $\bar{S}$. This is because graduation is welfare improving (Assumption 2) and beyond $\bar{S}$, V is independent of the wealth with which the borrower graduates.

Lemma 1. *Let Assumptions 1, 2 and 3 hold. Optimally, the contract is terminated as soon as the borrower accumulates enough savings to graduate i.e. start the $\langle V, \bar{S} \rangle$ technology: so the graduation constraint GC binds at the optimum.*

Given Proposition 1 and Lemma 1, the problem of the MFI can be expressed as follows:

$$\underset{\langle \{\alpha_t\}_{t=0}^{T_M}, \{k_t\}_{t=0}^{T_M}, T_M \rangle}{\text{Maximise}} \quad \int_0^{T_M} e^{-rt}(1-\alpha_t)[f(k_t) - k_t]dt + e^{-rT_M} V$$

$$\text{Subject to GC: } \int_0^{T_M} e^{r(T_M-t)} \alpha_t [f(k_t) - k_t]dt = \bar{S}, \tag{2.1}$$

$$\text{DIC: } \int_t^{T_M} e^{-r(t'-t)}(1-\alpha_{t'})[f(k_{t'}) - k_{t'}]dt' + e^{-r(T_M-t)} V \geq f(k_t), \quad \forall t \leq T_M. \tag{2.2}$$

Let the optimum scheme be denoted by $\langle \{\alpha_t^*\}_{t=0}^{T_M^*}, \{k_t^*\}_{t=0}^{T_M^*}, T_M^* \rangle$, where T_M^* is the time required to save $\bar{S}$ under this scheme, i.e.

$$\int_0^{T_M^*} e^{r(T_M^*-t)} \alpha_t^* [f(k_t^*) - k_t^*] dt = \bar{S}.$$

Before proceeding further let us introduce the following technical definition.

Definition 1. *Given a scheme* $\langle \{\alpha_t\}_{t=0}^{T_M}, \{k_t\}_{t=0}^{T_M}, T_M \rangle$*, let* $k_{It}(\langle \{\alpha_t\}_{t=0}^{T_M}, \{k_t\}_{t=0}^{T_M}, T_M \rangle)$ *denote the maximum loan amount at* t*, such that DIC at* t *holds.*

In the next lemma we identify the optimum loan amount and the part of net return to be saved at any instance t, where $0 \leq t \leq T_M^*$. We find that the MFI optimally chooses a contract such that the borrower's instantaneous savings is the maximum. It involves maximizing the instantaneous net return $f(k_t) - k_t$, given DIC, and setting α_t as high as possible.

Lemma 2. *Let Assumptions 1, 2 and 3 hold. The MFI chooses the optimal scheme* $\langle \{\alpha_t^*\}_{t=0}^{T_M^*}, \{k_t^*\}_{t=0}^{T_M^*}, T_M^* \rangle$ *such that the instantaneous savings* $\alpha_t^*[f(k_t^*) - k_t^*]$ *is the maximum:*

(a) It lends the efficient amount whenever that is DIC, otherwise it lends the maximum amount which is DIC; formally $k_t^* = \min\{k_{It}, k^e\}$ *for all* t*,*

(b) The borrower saves her entire net return with the MFI, formally $\alpha_t^* = 1$ *for all* t*.*

This is quite intuitive. First, given our assumptions that the discounting rate of the borrower and the interest rate on savings both equal r, and that her utility function is linear, she is indifferent between consuming an amount now, and saving and consuming that amount (along with interest) later. Second, given Assumption 2 and Lemma 1, her utility increases as the time required to save $\bar{S}$ decreases. These imply that the objective of the MFI is to maximise the instantaneous savings $\alpha_t[f(k_t) - k_t]$ which involves $k_t = k^e$ and $\alpha_t = 1$. So at the optimum, the MFI lends k^e whenever that is incentive compatible, otherwise, it lends k_{It} and sets $\alpha_t = 1$.

The only potential problem we need to address here is that this increase in instantaneous savings, especially setting $\alpha_t = 1$ may affect the DICs adversely. If it does so, then this lemma is not so obvious, but fortunately, that is not the case. This is again because of the fact that borrower is indifferent between consuming an amount now, and saving and consuming that amount later. In fact, observe an increase in the instantaneous savings relaxes the DICs.

We summarise the above discussion in the following proposition which gives us the MFI's optimum contract in this benchmark case.

Proposition 2. *Let Assumptions 1, 2 and 3 hold. The optimal scheme* $\langle \{\alpha_t^*\}_{t=0}^{T_M^*}, \{k_t^*\}_{t=0}^{T_M^*}, T_M^* \rangle$ *satisfies the following:*

(a) The borrower graduates and moreover she graduates as soon as the minimum required amount $\bar{S}$ *is accumulated,*

(b) The MFI lends the efficient amount k^e*, unless constrained by the incentive condition,*

(c) The borrower saves the maximum possible amount with the MFI at all t*.*

2.4 The Time Path of The Optimal Loan Scheme

We next characterise the time path of the optimal loan scheme. We demonstrate that the optimal loan scheme is (weakly) increasing over time. Further, we find that when the increase in utility from graduation is not too large (defined formally later), the optimal loan amount initially increases and then remains constant (at the efficient level k^e) over time. For ease of reference, we call such a scheme *progressive with a cap*. This is of interest given that in reality (*a*) almost all the MFIs practise lending that is "progressive with a cap" and (*b*) Banerjee et al. (2015) suggest that the increase in utility from any microfinance scheme is "modestly positive, but not transformative". So the prediction of this model conforms with the empirical findings. We also find that when this increase in utility from graduation is "transformative", the optimal loan scheme remains "constant" at the efficient level k^e. Again for ease of reference, we call such a scheme *constant*. Similarly, a loan scheme which keeps on increasing over time is termed *strictly progressive*.

Finally, we say that increase in utility from graduation is "modestly positive" when

$$\frac{f(k^e) - k^e}{rf(k^e)}V - \bar{S} < \frac{f(k^e) - k^e}{r}.$$

Otherwise, we say that increase in utility from graduation is "transformative". Now we are in a position to state the main result of this section:

Proposition 3. *(The Dynamics of the Optimal Loan Scheme): Let Assumptions 1, 2 and 3 hold.*

A. The optimal loan scheme is weakly progressive.

B. The optimal loan scheme is either progressive with a cap or constant:

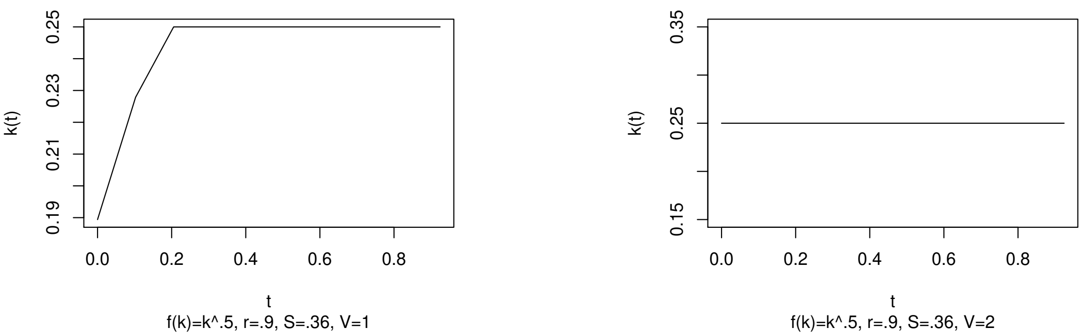
(i) The optimal loan scheme is "progressive with a cap" if and only if increase in utility from graduation is "modestly positive".

(ii) The optimal loan scheme is "constant" if and only if increase in utility from graduation is "transformative".

Intuitively, with the passage of time, on the one hand the borrower's savings increase, so that the loss from default increases, whereas on the other hand, the graduation date gets closer, so the present discounted value of lifetime utility from repayment increases. This ensures that the DICs get relaxed over time. Given Proposition 2(*b*), this implies that the optimal loan scheme is weakly progressive. So, there can be three cases – the optimal loan scheme is strictly progressive, progressive with a cap or constant.

When the increase in utility from graduation is *transformative*, then the present discounted value of lifetime utility from repayment is very high. This makes the efficient level of investment k^e incentive compatible from the very beginning. Thus in this case the optimum loan scheme is a constant. Correspondingly, when the increase in utility from graduation is *modestly positive*, it is not that attractive. This makes incentive for repayment weak – the efficient amount k^e is not incentive compatible, at least initially. Whether that would at all become incentive compatible or not depends on the parametric condition. It turns out that when $\bar{S}$ is not small, formally when assumption 3 is satisfied, the efficient amount k^e becomes incentive compatible towards the end.[11] Hence, this gives us the interesting result – when increase in utility from graduation is *modestly positive* the optimal loan scheme is "progressive with a cap".

[11] In other words, Assumption 3 precludes the possibility of optimal loan scheme to be strictly progressive.



(a) Moderate V: Progressive with a cap **(b) Large V: Constant Loan Scheme**

Figure 1: The Optimal Loan Schemes under Different Parametric Values

Remark 1. (Relaxation of Assumption 3) Suppose Assumption 3 is relaxed and consider the case where $f(k^e) \geq \bar{S}$. Recall at the optimum $k_t \leq k^e$. Therefore, at any $t \in [0, T_M]$ there can be two contingencies – $f(k^e) \geq f(k_t) \geq \bar{S}$ and $f(k_t) < \bar{S}$. This changes the DIC (utility from default is $f(k_t) - \bar{S} + V$ if $f(k_t) \geq \bar{S}$ and $f(k_t)$ otherwise.) Now observe in the former case, if the borrower defaults, her present discounted value of lifetime utility would be higher than that from repayment (recall, in case of repayment she graduates as soon as her savings becomes $\bar{S}$). So, she would never repay such a k_t. Thus, when $\bar{S} \leq f(k^e)$ the efficient amount k^e never becomes DIC which implies strict progressivity in the optimal loan scheme – it keeps on increasing over time without reaching the efficient amount. For ease of exposition, in the benchmark case we make this assumption and relax it in the next section.

Remark 2. (Concave utility function of the borrower) How robust is the analysis if the utility function of the borrower is strictly concave? Given there is no uncertainty, concavity in utility function implies that the borrower has a preference for consumption smoothing over time. We conjecture that the optimal loan scheme would continue to be progressive. We do not allow for dissaving which implies that the borrower saves only when she graduates. In both the cases an amount which is incentive compatible at some instance remains incentive compatible in the future as well. Hence the conjecture.

Remark 3. (Profit-maximizing MFI) As discussed in the Related Literature section, Liu and Roth (2017) show that in their framework a profit-maximizing MFI optimally designs a contract such that a poor borrower can never graduate. In our framework, when the MFI is a profit-maximizer, whether a borrower would be able to graduate or not is an open question. Intuitively graduation may or may not happen because there are two opposing forces in play. On the one hand, as a borrower graduates, the MFI loses a client. So depending on its outside option, availability of a new borrower for example, the MFI may choose to not give up a potential source of revenue.[12] On the other hand, when a borrower graduates a higher amount of loan becomes DIC towards the end in comparison to the case where she never graduates (as present discounted value of lifetime utility from repayment is higher in the former case). Since the profit of the MFI is increasing in loan amount, the MFI may want her to graduate. If the latter effect dominates then at the optimum the borrower does graduate, otherwise she does not. Nevertheless, we conjecture that the optimum

[12] Also note that even if new borrowers are available, profit from a new borrower can be substantially lower than that from an old borrower.

loan scheme continues to be progressive. While a complete analysis is beyond the scope of this paper, it can be shown that this is indeed true. Intuitively, since we do not allow for dissaving at the optimum the MFI takes savings only if the borrower graduates. In both the cases an amount which is incentive compatible at some instance remains incentive compatible in the future as well. Hence the conjecture.[13]

3 General Framework: Allowing Another Savings Institution and A General Confiscation Rule

In this section, we introduce two changes in our basic framework which improve the borrower's utility from default: First, the borrower has access to a savings technology on her own – she can save with a Savings Institution (SI hereafter), at the same instantaneous interest rate provided by the MFI.[14] Second, the MFI cannot confiscate her entire savings with it even in case of default – it has to return at least $\underline{\gamma}$ part of that (along with interest), where $0 < \underline{\gamma} < 1$, is exogenously given. These enable a borrower to graduate even in case of default – she can save a part of the amount with which she defaults and graduate using that. Thus incentive for repayment is harder to satisfy here. The objective is to check whether our central result – progressivity in loan size, survives or not. Also, we relax Assumption 3, that is we allow for the case where $\bar{S} \leq f(k^e)$.

3.1 Contracts and Timeline

At $t = 0$, the MFI announces a contract $\langle \{\alpha_{st}\}_{t=0}^{T_{sM}}, \gamma, \{k_{st}\}_{t=0}^{T_{sM}}, T_{sM} \rangle$,[15] where like before k_{st}, α_{st} and T_{sM} denote the loan amount, the part of the net return to be saved with the MFI at the instance t and the 'successful' termination date of the contract, respectively. γ denotes the part of savings, the borrower gets back from the MFI, in case of default; $\gamma \in [\underline{\gamma}, 1]$. Hence, in case of default, like before, the MFI terminates the credit contract and withdraws access to both the production and savings technologies provided by the MFI, but unlike the benchmark case it returns a part of the her savings with it till date $S_t^D \equiv \gamma \int_0^t e^{r(t-t')} \alpha_{st'} [f(k_{st'}) - k_{st'}] dt'$.

The individual either accepts or rejects the MFI contract, with the game ending in case she rejects. If she accepts, then in the next stage at $t = 0$, she chooses $\langle \{ \{\sigma_t^R\}_{t=0}^{T_{sM}}, \sigma_t^D \}, \{ T_B^R, T_B^D(t) \} \rangle$, where σ_t^R denotes the part she wants to save with the SI at any arbitrary t, after repaying and saving with the MFI (or getting back her savings from the MFI which happens at T_{sM}), and σ_t^D denotes the part of $f(k_{st}) + S_t^D$ she wants to save with the SI after defaulting at t; where $0 \leq t \leq T_{sM}$ and $0 \leq \sigma_t^R, \sigma_t^D \leq 1$. Similarly, T_B^R denotes the date at which she withdraws her savings from the SI in case she always repays (and saves), and $T_B^D(t)$ denotes the date of withdrawal of savings from the SI, in case she defaults at t.[16]

[13] In the section marked For the Referee, we provide an argument for progressivity.

[14] Perhaps, it is more natural to assume that the interest rate provided by the SI is lower than that provided by the MFI, but we assume them to be equal as this is a robustness check exercise, and the borrower's outside option (weakly) increases with the interest rate provided by the SI.

[15] To distinguish the notations from the benchmark case, we subscript the variables of this general framework with 's', where s denotes the fact that in this framework we are allowing for a savings technology other than MFI.

[16] Observe that we have not considered the case where the borrower withdraws savings from the SI multiple times. Also we have assumed $\sigma_t^R, \sigma_t^D \leq 1$, these imply that we have not allowed dissavings in this framework also. However, due to our assumptions that the utility function is linear and that the future is discounted in the same way as the interest rate on savings, these are without loss of generality. Furthermore, at this point we are not imposing that $T_B^R \geq T_{sM}$ or $T_B^D(t) \geq t$; if $T_B^R < T_{sM}$ then $\{\sigma_t^R\}_{t=T_B^R}^{T_{sM}} = 0$ and if $T_B^D(t) < t$ then $\sigma_t^D = 0$ and $\{\sigma_{t'}^R\}_{t'=T_B^D(t)}^{t} = 0$.

Given the MFI-contract, and the borrower's strategy, the continuation game at any instance t, where $0 \leq t \leq T_{sM}$, is as follows:

Stage 1: The MFI lends k_{st} which the borrower invests in the $f(\cdot)$ technology and gets $f(k_{st})$ instantaneously.

Stage 2: She then decides whether to repay, or not:

(i) In case she decides to repay, she returns k_{st} to the MFI, deposits $\alpha_{st}[f(k_{st}) - k_{st}]$ with the MFI, and $\sigma_t^R(1-\alpha_{st})[f(k_{st})-k_{st}]$ with the SI, with all deposits attracting interest at the instantaneous rate r. She consumes the rest instantaneously and the game continues.

(ii) In case of default the MFI terminates the contract, and withdraws the borrower's access to both $f(\cdot)$ and the MFI-savings technology. She obtains her current gross income $f(k_{st})$ and unlike the benchmark case, she also gets back S_t^D – a part of her savings with the MFI till date (along with interest). She, then, saves a part σ_t^D of $f(k_{st}) + S_t^D$ with the SI and consumes the rest instantaneously.
She withdraws her savings from the SI at $T_B^D(t)$, if any.

In case of successful termination of the contract at T_{sM}, the borrower gets back her entire savings along with interest, from the MFI. She saves a part $\sigma_{T_{sM}}^R$ of that return with the SI and consumes the rest instantaneously. She withdraws her savings from the SI, at T_B^R, if any.
Finally, she graduates, if she wishes to and has at least $\bar{S}$ amount with her.

We now define a term "money in hand" which we will be using repeatedly in the subsequent analysis. *Money in hand* at any t denotes the entire amount to which the borrower has access at t.

- At the time of successful termination of the contract T_{sM} money in hand includes her entire savings with the MFI as well as that with the SI, formally

$$\int_0^{T_{sM}} e^{r(T_{sM}-t)}\alpha_{st}[f(k_{st}) - k_{st}]dt + \int_0^{T_{sM}} e^{r(T_{sM}-t)}\sigma_t^R(1-\alpha_{st})[f(k_{st}) - k_{st}]dt.$$

- In case of repayment at t, where $t \leq T_{sM}$, money in hand includes the amount she has after repaying and saving with the MFI at that instance as well as her savings with the SI till date, formally

$$(1-\alpha_{st})[f(k_{st}) - k_{st}] + \int_0^t e^{r(t-t')}\sigma_{t'}^R(1-\alpha_{st'})[f(k_{st'}) - k_{st'}]dt'.$$

- In case of default at t, where $t \leq T_{sM}$, money in hand includes gross return $f(k_{st})$, the savings she gets back from the MFI i.e. S_t^D and her savings with the SI till date, formally

$$f(k_{st}) + \gamma \int_0^t e^{r(t-t')}\alpha_{st'}[f(k_{st'}) - k_{st'}]dt' + \int_0^t e^{r(t-t')}\sigma_{t'}^R(1-\alpha_{st'})[f(k_{st'}) - k_{st'}]dt'.$$

- Finally at any t after the termination date, money in hand includes her savings with the SI.

Also observe, T_B^R and $T_B^D(t)$ are well defined only when she has positive amount of savings with the SI. Formally, when there exists an interval $[\underline{t}, \bar{t}] \subseteq [0, T_{sM}]$ such that $[\underline{t}, \bar{t}]$ has a positive measure and the borrower's savings with the SI is positive for all $t \in [\underline{t}, \bar{t}]$.

3.2 Analysis: The Borrower's Problem

We begin by solving the borrower's problem. Given any MFI-contract $\langle\{\alpha_{st}\}_{t=0}^{T_{sM}},\gamma,\{k_{st}\}_{t=0}^{T_{sM}},T_{sM}\rangle$, her objective is to choose her strategy: {Repay, Default} and $\big\langle\big\{\{\sigma_t^R\}_{t=0}^{T_{sM}},\sigma_t^D\big\},\big\{T_B^R,T_B^D(t)\big\}\big\rangle$ to maximise her present discounted value of lifetime utility. Given Assumption 2, that is graduation is welfare improving and MFI-contract her objective is to choose her strategy such that the time required for graduation is minimised. Now it is clear that, she would withdraw her savings from the SI as soon as money in her hand becomes $\bar{S}$. More specifically, if money in her hand at the termination date of the MFI-contract, irrespective of whether that was terminated successfully or because of default, is no less than $\bar{S}$, she withdraws her savings from the SI immediately, otherwise she waits till her savings becomes $\bar{S}$ and withdraws immediately.

Turning to the choice of optimal $\{\sigma_t^R\}_{t=0}^{T_{sM}}$ and σ_t^D, note that it is weakly dominant to always save as much as possible, as that may decrease the time required for graduation. More specifically, due to our assumptions of linear utility function and that she discounts future in the same way as the interest rate on savings, she is indifferent between consuming an amount now, and saving and consuming that amount (along with interest) later. But she gets strictly better off if that savings decrease the time required for graduation. Hence, the borrower is weakly better-off when she saves the maximum amount possible with the SI, she is strictly better off if that decreases the time required for graduation.

Finally, it may happen that her choice of $\big\langle\big\{\{\sigma_t^R\}_{t=0}^{T_{sM}},\sigma_t^D\big\},\big\{T_B^R,T_B^D(t)\big\}\big\rangle$ does not affect the time of graduation. Then she is indifferent between saving and not saving with the SI. Even if she saves, the time of withdrawal of her savings from the SI does not affect her present discounted value of lifetime utility. Given these indifferences, without loss of generality, we assume that she withdraws her savings from the SI, if any, at the termination date of the MFI-contract and till then saves the maximum amount possible with the SI. Hence, the next proposition.

Proposition 4. *Let Assumption 2 hold. Given any MFI-contract $\langle\{\alpha_{st}\}_{t=0}^{T_{sM}},\gamma,\{k_{st}\}_{t=0}^{T_{sM}},T_{sM}\rangle$, the borrower chooses $\big\langle\big\{\{\sigma_t^R\}_{t=0}^{T_{sM}},\sigma_t^D\big\},\big\{T_B^R,T_B^D(t)\big\}\big\rangle$ such that she can graduate as soon as possible.*

Therefore, the borrower does not consume anything before graduating. Hence, given any MFI-contract $\langle\{\alpha_{st}\}_{t=0}^{T_{sM}},\gamma,\{k_{st}\}_{t=0}^{T_{sM}},T_{sM}\rangle$, the borrower's present discounted value of lifetime utility (evaluated at $t=0$) from repayment is

$$e^{-rT_B^{R^*}}\left[\int_0^{T_{sM}} e^{r(T_B^{R^*}-t)}[f(k_{st})-k_{st}]dt-\bar{S}+V\right]. \tag{3.1}$$

Similarly, her present discounted value of lifetime utility (evaluated at $t=0$) from default at t, where $0<t\le T_{sM}$ is

$$e^{-rT_B^{D^*}(t)}\Bigg[e^{r(T_B^{D^*}(t)-t)}\Big[\int_0^t e^{r(t-t')}\big(1-\alpha_{st'}\big)[f(k_{st'})-k_{st'}]dt' \\ +\int_0^t e^{r(t-t')}\gamma\alpha_{st'}[f(k_{st'})-k_{st'}]dt'+f(k_{st})\Big]-\bar{S}+V\Bigg],$$

where the first term i.e. $\int_0^t e^{r(t-t')}\big(1-\alpha_{st'}\big)[f(k_{st'})-k_{st'}]dt'$ represents the amount she had already saved with the SI till t and the next two terms i.e. $\int_0^t e^{r(t-t')}\gamma\alpha_{st'}[f(k_{st'})-k_{st'}]dt'+f(k_{st})$ represent the amount she saves at t. Hence, her present discounted value of lifetime utility (evaluated at $t = 0$) if she defaults at $t \in (0, T_{sM}]$ is

$$e^{-rT_B^{D^*}(t)}\left[e^{r(T_B^{D^*}(t)-t)}\Big[\int_0^t e^{r(t-t')}\big(1-\alpha_{st'}(1-\gamma)\big)[f(k_{st'})-k_{st'}]dt' + f(k_{st})\Big] - \bar{S} + V\right]. \tag{3.2}$$

3.3 Analysis: The Optimal Contract

Next, we characterise the optimal contract. Given the optimal strategy of the borrower, the problem of the MFI is to choose $\langle\{\alpha_{st}\}_{t=0}^{T_{sM}}, \gamma, \{k_{st}\}_{t=0}^{T_{sM}}, T_{sM}\rangle$ such that the borrower's present discounted value of lifetime utility is maximised and the borrower always repays. As observed above given Assumption 2, the objective of the MFI boils down to minimizing the time required to graduate, provided that the borrower repays.

The optimal contract in this framework is very similar to that in the benchmark case, we discuss that as we go along. But an interesting point to note here is that the MFI may choose to terminate the contract, successfully, before the borrower's total savings become $\bar{S}$. Since, the borrower has access to a savings technology on her own, she would be able to save and graduate even in that case. However, that would increase the time required to graduate vis-à-vis the case where the MFI lends till the time of graduation. Hence, the MFI would terminate a contract before her total savings become $\bar{S}$ only if lending till the end is not incentive compatible.

Note that it is not obvious whether a contract where the MFI lends till the borrower's total savings become $\bar{S}$ is DIC or not, because of the following reason. Lending till the end results early graduation. But that is true not only when the borrower repays but also when she defaults – her savings with the MFI increases over time, so the amount she gets back in case of default also increases over time. So on the one hand, lending till the end improves the incentive to repay. On the other hand, towards the end of the contract incentives to default also increases. Despite this trade-off, we argue by contradiction that it is possible to construct a DIC contract where the MFI lends till the borrower's total savings become $\bar{S}$ so that she graduates immediately after the successful termination of the contract. This new contract provides her higher utility, hence, the original contract cannot have been optimum. The formal proof can be found in Appendix A.

Now given assumption 2, since the objective of the MFI is to minimise the time required for graduation, it terminates the contract as soon the borrower's total savings become $\bar{S}$. The proof is very similar to that of Lemma 1, so we skip that. The following lemma characterises the "successful" termination date.

Lemma 3. *Suppose Assumptions 1 and 2 hold. At the optimum, the borrower graduates as soon as the MFI terminates the contract.*

Now we introduce the following observation which argues that, in any DIC contract, the money in the borrower's hand, in case of default at any t, where $0 < t < T_{sM}$, must be less than $\bar{S}$. The reason being – given Proposition 4, we know that otherwise, in case of default at such a t the borrower would graduate immediately, with no less than $\bar{S}$ amount, whereas, in case of repayment she graduates at $T_{sM} > t$ with exactly $\bar{S}$. So, DIC at t will definitely be violated. Hence, the observation.

Observation 1. *Let, $\langle\{\alpha_{st}\}_{t=0}^{T_{sM}}, \gamma, \{k_{st}\}_{t=0}^{T_{sM}}, T_{sM}\rangle$ be a DIC contract. The money in the borrower's hand in case of default at any t, where $0 < t < T_{sM}$, must be less than $\bar{S}$. The money in her hand in case of default at T_{sM} can be no higher than $\bar{S}$.*

So given Proposition 4, DIC at any t, where $0 < t \leq T_{sM}$, boils down to

$$e^{-r(T_{sM}-t)}V \geq e^{-r(T_B^{D^*}(t)-t)}V.$$

This further implies that in a DIC contract, in case of default the borrower cannot graduate at an earlier date than that in case she repays always.

Now like before we introduce the following technical definition.

Definition 2. *Given a scheme* $\langle\{\alpha_{st}\}_{t=0}^{T_{sM}}, \gamma, \{k_{st}\}_{t=0}^{T_{sM}}, T_{sM}\rangle$*, let* $k_{sIt}\big(\langle\{\alpha_{st}\}_{t=0}^{T_{sM}}, \gamma, \{k_{st}\}_{t=0}^{T_{sM}}, T_{sM}\rangle\big)$ *denote the loan amount at* t *for which the DIC at* t *binds.*

In the next lemma we identify the optimum loan amount k_{st}, the part of net return α_{st} to be saved with the MFI at any instance t, where $0 \leq t \leq T^*_{sM}$ and the part of savings γ the borrower gets back from the MFI in case of default. We find that the MFI optimally chooses a contract such that the borrower's instantaneous savings is the maximum. It involves maximizing the instantaneous net return $f(k_{st}) - k_{st}$, setting α_{st} as high as possible and γ as low possible.

Lemma 4. *Let Assumptions 1 and 2 hold. The optimal scheme* $\langle\{\alpha^*_{st}\}_{t=0}^{T^*_{sM}}, \gamma^*, \{k^*_{st}\}_{t=0}^{T^*_{sM}}, T^*_{sM}\rangle$ *satisfies the following:*

(a) It lends the efficient amount whenever doing so is DIC, otherwise it lends the maximum amount which is DIC. Formally, $k^*_{st} = \min\{k_{sIt}, k^e\}$ *for all* t*.*

(b) The borrower saves her entire net return with the MFI: $\alpha^*_{st} = 1$ *for all* t*.*

(c) In case of default, the MFI confiscates the maximum amount of savings: $\gamma^* = \underline{\gamma}$*.*

The intuition behind this result is in a similar vein – given Assumption 2 the objective of the MFI is to minimise the time required to graduate which in turn requires maximising the instantaneous savings. Thus at the optimum it lends to maximise the net return, given DIC which implies lending the efficient amount k^e whenever that is incentive compatible otherwise the maximum amount which is that.

Two points to note are as follows – First, *given* any loan scheme the choice of α_{st} does not affect the borrower's savings. This is because, the borrower saves the rest, whatever she has after repaying and saving with the MFI, with the SI. Moreover, the interest rates provided by the MFI and SI are equal. But low α_{st}, that is higher savings with the SI affects DIC adversely – the maximum loan amount which is DIC decreases with decrease in α_{st}. Hence, the optimal loan amount and correspondingly the instantaneous savings (weakly) decrease with decrease in α_{st}. Therefore, the MFI chooses α_{st} as high as possible. Given the limited liability condition it chooses $\alpha^*_{st} = 1$.

The second point of interest follows from the same line of argument. The MFI chooses γ as low as possible because increase in γ means she gets higher amount of savings from the MFI in case of default. This improves her deviation payoff and that in turn adversely affects DIC. Hence, the instantaneous savings (weakly) decreases with increase in γ. Therefore the MFI chooses γ as low as possible: $\gamma^* = \underline{\gamma}$.

We summarise the optimal contract in the following proposition.

Proposition 5. *Let Assumptions 1 and 2 hold. The optimal scheme* $\langle\{\alpha^*_{st}\}_{t=0}^{T^*_{sM}}, \gamma^*, \{k^*_{st}\}_{t=0}^{T^*_{sM}}, T^*_{sM}\rangle$ *satisfies the following:*

(a) The MFI terminates the contract as soon as the borrower's savings becomes $\bar{S}$ *and she graduates immediately, that is at* T^*_{sM}*,*

(b) It lends the efficient amount k^e, unless constrained by the incentive condition,

(c) In case of repayment, the borrower saves her entire net return with the MFI,

(d) In case of default, the MFI confiscates the borrower's savings as much as it can.

Observe, the optimum loan contract is unique.

3.4 The Time Path of the Optimal Loan Scheme

We next turn to the dynamics of the optimal loan size. Like in the benchmark case, here, we characterise the time path of the optimal loan scheme. Unlike the benchmark case, the optimal loan scheme is not always (weakly) progressive, specifically it need not be (weakly) progressive when $\underline{\gamma}$ is very high. We provide a sufficient condition which ensures that DIC gets relaxed over time and hence the optimal loan amount (weakly) increases over time.

Assumption 4. $r \geq \underline{\gamma}$.

Intuitively, the borrower saves her entire net return with the MFI. Now with marginal increase in time her savings increase by the rate of interest i.e. r. The time remaining to graduate decreases and the borrower's present discounted value of lifetime utility increases marginally by the future discount rate which is again r. Due to this, the maximum incentive compatible loan amount increases by $rf(k_{sIt})$. But in case of default the borrower also gets back a part of her savings with the MFI which increases with time. To be precise, with marginal increase in time the amount she gets back from the MFI marginally increases by $\underline{\gamma}[f(k_{st}) - k_{st}]$. This dampens her incentive to repay. Assumption 4 ensures that the DICs get relaxed over time.

Therefore, given this assumption the optimal loan scheme is weakly progressive. Depending on the value of $\bar{S}$, there can be three cases – (a) The loan scheme is strictly increasing, (b) progressive with a cap and (c) constant. Here $\bar{S}$ plays such a crucial role because now the borrower can graduate on her own in case of default. Thus when $\bar{S}$ is "low", the efficient loan amount never becomes incentive compatible and the optimal loan scheme is strictly increasing. Converesly, when $\bar{S}$ is "large", the efficient loan amount is always incentive compatible and hence the optimal loan scheme remains constant at the efficient level throughout. Finally, when $\bar{S}$ is "moderate" the optimal loan scheme is progressive with a cap – initially increases till it reaches the efficient amount and then remains constant at that.

We say that $\bar{S}$ is "low" when $(1-\underline{\gamma})\bar{S} < f(k^e)$. Similarly we say that it is "moderate" when $\frac{f(k^e)-k^e}{r\bar{S}+f(k^e)-k^e}\bar{S} \leq f(k^e) \leq (1-\underline{\gamma})\bar{S}$, and "large" when $f(k^e) \leq \frac{f(k^e)-k^e}{r\bar{S}+f(k^e)-k^e}\bar{S}$.

Now we characterise the time path of the optimal loan scheme.

Proposition 6. *(The Dynamics of the Optimal Loan Scheme): Let, $\langle\{\alpha^*_{st}\}_{t=0}^{T^*_{sM}}, \gamma^*, \{k^*_{st}\}_{t=0}^{T^*_{sM}}, T^*_{sM}\rangle$ be the optimal contract and Assumptions 1, 2 and 4 hold.*

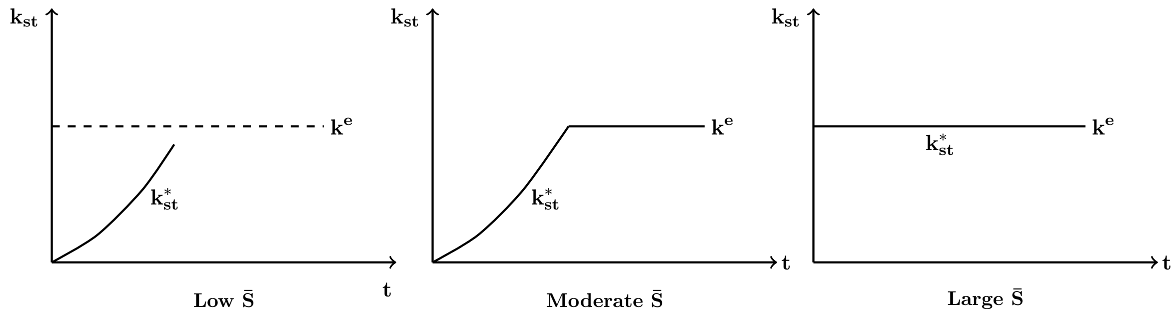
A. Weakly Progressive: The optimal loan scheme is always weakly progressive.

B. Depending on the value of $\bar{S}$ the optimal loan would be strictly progressive, progressive with a cap or constant.

(i) Strictly Progressive: The optimal loan scheme is strictly progressive if and only if $\bar{S}$ is low,

(ii) Progressive with a cap: The optimal loan scheme is progressive with a cap if and only if $\bar{S}$ is moderate,

(iii) Constant: The optimal loan scheme is constant if and only if $\bar{S}$ is large.



4 Comparison Between the Benchmark and the General Case: Welfare Implication

In this section, we compare the optimal outcomes and the borrower's welfare in the general case with those in the benchmark case. The borrower's outside option is higher in the general case – she can save on her own and gets back a part of her savings with the MFI in case of default. Apparently it may seem that it would improve her welfare, but actually it makes her worse off.

The reason is that in this framework, the MFI is benevolent and the only problem here is that the borrower is strategic and does not repay whenever she has an incentive to do so. Under this general framework, her deviation payoff is larger which decreases her incentive to repay. The optimal loan amount and hence the instantaneous savings must then be (weakly) lower in the general case. This increases the time required for graduation. Hence, the borrower is weakly worse off in the general case. She is strictly worse off whenever $\bar{S}$, the fixed initial investment required to start the technology $\langle V, \bar{S} \rangle$ is not large, ensuring that providing a loan of k^e from $t = 0$ is not incentive compatible.

Interestingly observe, along the equilibrium path the borrower actually does not save with the SI or does not default (and hence does not get back any savings from the MFI before the successful termination date). Hence these extensions only improve her deviation payoff and that makes her (weakly) worse off.

Proposition 7. *Let Assumptions 1, 2, 3 and 4 hold. The borrower's present discounted value of lifetime utility is weakly lower in the general case than that in the benchmark case. It is strictly lower if and only if the investment required to start the technology $\langle V, \bar{S} \rangle$ is not large:*

$$f(k^e) > \frac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e}\bar{S}.$$

This is not to claim that developmet of savings intitutions should not be encouraged. In fact, papers like Burgess and Pande (2005), Ashraf et al. (2006), Dupas and Robinson (2013a,b) find positive impacts of savings on the poor people. What Proposition 7 shows however is that there can be some unintended consequences of doing so in this scenario.

5 Conclusion

Many scholars Armendàriz and Morduch (2005), Roodman (2009) among others argue that MFIs should provide not only credit but also other financial services like savings, insurance etc. In fact Rhyne (November 2, 2010) directly links *Andhra-crisis* to lack of deposit collection. Many MFIs are broadening their initial focus on microcredit to include the provision of savings (and other) products (Karlan et al., 2014).

In this paper we develop a theoretical model where the MFI provides not just credit but also access to other services in particular a savings facility. This savings service coupled with the credit service help a poor borrower to accumulate a lumpsum amount which enables her to graduate. Thus we provide *one* explanation where savings coupled with credit indeed improve borrower's utility beyond the level achievable when only credit is provided. We find that the optimal loan scheme is (weakly) progressive and when the increase in utility from graduation is "modestly positive" the optimal loan scheme is progressive with a cap – loan size initially increases and then remains constant at the efficient level of investment which conforms with reality.

Appendix A

Proof of Proposition 1. We show that given Assumptions 1, 2 and 3, at the optimum the borrower graduates. We show this by contradiction.

Suppose not and there exists an optimum contract $\langle \{k_t^*\}_{t=0}^{T_M^*}, \{\alpha_t^*\}_{t=0}^{T_M^*}, T_M^* \rangle$[17] such that the borrower does not graduate. So, her present discounted value of lifetime utility is

$$\int_0^{T_M^*} e^{-rt}(1-\alpha_t^*)[f(k_t^*)-k_t^*]dt + e^{-rT_M^*}\int_0^{T_M^*} e^{r(T_M^*-t)}\alpha_t^*[f(k_t^*)-k_t^*]dt$$

And, DIC at any t, where $0 \leq t \leq T_M^*$, is

$$\int_t^{T_M^*} e^{-r(t'-t)}(1-\alpha_{t'}^*)[f(k_{t'}^*)-k_{t'}^*]dt' + e^{-r(T_M^*-t)}\int_0^{T_M^*} e^{r(T_M^*-t')}\alpha_{t'}^*[f(k_{t'}^*)-k_{t'}^*]dt' \geq f(k_t^*).$$

Now there can be two cases (a) $\exists \tilde{t} \leq T_M^*$ such that $\int_0^{\tilde{t}} e^{r(\tilde{t}-t)}[f(k_t^*)-k_t^*]dt \geq \bar{S}$ and (b) there does not exist any such $\tilde{t}$. We argue that in each of the cases, it is possible to construct another DIC contract such that the borrower graduates and her lifetime utility is higher, so the original contract cannot have been optimum.

Case (a) $\exists \tilde{\mathbf{t}} \leq \mathbf{T_M^*}$ **such that** $\int_0^{\tilde{\mathbf{t}}} \mathbf{e^{r(\tilde{t}-t)}[f(k_t^*)-k_t^*]dt} \geq \bar{\mathbf{S}}$

Define a new contract $\langle \{\hat{k}_t\}_{t=0}^{\hat{T}_M}, \{\hat{\alpha}_t\}_{t=0}^{\hat{T}_M}, \hat{T}_M \rangle$ as follows

$$\begin{cases} \hat{k}_t = k_t^* \text{ and } \hat{\alpha}_t = 1 & \forall t \in [0, \hat{T}_M] \\ \text{and } \hat{T}_M \text{ is such that} & \displaystyle\int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(k_t^*)-k_t^*]dt = \bar{S}. \end{cases}$$

Observe, by Intermediate Value Theorem such a $\hat{T}_M$ exists and $\hat{T}_M \leq T_M^*$. Under this new contract, the borrower graduates at $\hat{T}_M$ as that gives her higher utility:

$$\int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(\hat{k}_t)-\hat{k}_t]dt - \bar{S} + V > \int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(\hat{k}_t)-\hat{k}_t]dt.$$

The left hand side is her lifetime utility if she graduates and the right hand side is that from consuming the amount immediately. We get this inequality because $V - \bar{S} > 0$. Now we show that her present discounted value of lifetime utility under this new contract is higher than that from the original contract. Her present discounted value of lifetime utility under this new contract is

[17] Here we want to point out that none of the results depend on our restriction $\alpha_t \geq 0$, i.e. even if we allow for dissaving all the results go through. This is of particular interest because this restriction implies that in case the borrower does not graduate i.e. $T_M = \infty$ there will be no savings. But if we allow for dissaving, the MFI may take deposits even when $T_M = \infty$ and return at certain instances which may relax DIC and improve borrower welfare. However, this assumption is without loss of generality because even if we allow for dissaving, the MFI would enable the borrower to graduate as soon as possible and set α_t as high as possible. The proof is available on request.

$$
\begin{aligned}
& e^{-r\hat{T}_M}\left[\int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(\hat{k}_t)-\hat{k}_t]dt-\bar{S}+V\right] \\
> & e^{-r\hat{T}_M}\left[\int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(\hat{k}_t)-\hat{k}_t]dt\right]+e^{-r\hat{T}_M}\frac{1}{r}[f(k^e)-k^e] \\
= & e^{-r\hat{T}_M}\left[\int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(k_t^*)-k_t^*]dt\right]+e^{-r\hat{T}_M}\int_0^{\infty} e^{-rt}[f(k^e)-k^e]dt \\
\geq & \int_0^{\hat{T}_M} e^{-rt}[f(k_t^*)-k_t^*]dt+\int_{\hat{T}_M}^{T_M^*} e^{-rt}[f(k_t^*)-k_t^*]dt \\
= & \int_0^{T_M^*} e^{-rt}[f(k_t^*)-k_t^*]dt \\
= & \int_0^{T_M^*} e^{-rt}(1-\alpha_t^*)[f(k_t^*)-k_t^*]dt+e^{-rT_M^*}\int_0^{T_M^*} e^{r(T_M^*-t)}\alpha_t^*[f(k_t^*)-k_t^*]dt.
\end{aligned}
$$

where the first inequality is coming from Assumption 2 and the second inequality is coming from the definition of k^e.[18] The last expression is the borrower's present discounted value of lifetime utility under the original contract. Now it is immediate that DICs at all t where $0 \leq t \leq \hat{T}_M$ are satisfied. So, the original contract cannot have been optimum.

Case (b) $\nexists \tilde{\mathbf{t}} \leq \mathbf{T}_\mathbf{M}^*$ **such that** $\int_\mathbf{0}^{\tilde{\mathbf{t}}} \mathbf{e}^{\mathbf{r}(\tilde{\mathbf{t}}-\mathbf{t})}[\mathbf{f}(\mathbf{k}_\mathbf{t}^*)-\mathbf{k}_\mathbf{t}^*]\mathbf{dt} \geq \bar{\mathbf{S}}$

An optimum contract must be non-trivial in that there must exist finite $t \leq T_M^*$ such that $k_t^* > 0$. To construct the new contract, we follow the algorithm below. Consider any finite T and define $\hat{t} = \left\{\text{minimum } t \in [0,T] | f(k_t^*) - k_t^* \geq f(k_{t'}^*) - k_{t'}^* \ \forall t' \in [0,T]\right\}$.[19] In other words, $f(k_{\hat{t}}^*) - k_{\hat{t}}^* > f(k_t^*) - k_t^* \ \forall t \in [0,\hat{t})$ and $f(k_{\hat{t}}^*) - k_{\hat{t}}^* \geq f(k_t^*) - k_t^* \ \forall t \in [\hat{t},T]$. Now compute

$$
\int_0^{\hat{t}} e^{r(T-t)}[f(k_t^*)-k_t^*]dt+\int_{\hat{t}}^{T} e^{r(T-t)}[f(k_{\hat{t}}^*)-k_{\hat{t}}^*]dt.
$$

(A) If that amount is no less than $\bar{S}$, stop the algorithm and define the new contract as follows:

$$
\begin{cases}
\hat{k}_t = k_t^* \quad \forall t \in [0,\hat{t}) \quad \text{and } \hat{k}_t = k_{\hat{t}}^* \ \ \forall t \in [\hat{t},\hat{T}_M] \\
\hat{\alpha}_t = 1 \quad \forall t \in [0,\hat{T}_M] \\
\text{and } \hat{T}_M \text{ is such that} \quad \displaystyle\int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(\hat{k}_t)-\hat{k}_t]dt = \bar{S}.
\end{cases}
$$

(B) If that amount is less than $\bar{S}$, increase T and follow the same algorithm until we get (A). Observe, since we consider only non-trivial contracts, that such a T exists follows from the the Intermediate Value Theorem since,

$$
\underset{T\to 0}{\text{Limit}} \int_0^{T} e^{r(T-t)}[f(\hat{k}_t)-\hat{k}_t]dt = 0 \quad \text{and} \quad \underset{T\to\infty}{\text{Limit}} \int_0^{T} e^{r(T-t)}[f(\hat{k}_t)-\hat{k}_t]dt = \infty.
$$

[18] Recall, k^e solves $\underset{k}{\text{argmax}}\,[f(k)-k]$.

[19] k_t is continuous in t and $[0,T]$ is bounded, so from Weierstrass Theorem such a t exists.

Also observe, since $\nexists \tilde{t} \leq T_M^*$ such that $\int_0^{\tilde{t}} e^{r(\tilde{t}-t)}[f(k_t^*) - k_t^*]dt \geq \bar{S}$, $\hat{t} < \hat{T}_M$.

Mimicing the argument in case (a) it can be shown that under this new contract the borrower graduates at $\hat{T}_M$. Now we show that her present discounted value of lifetime utility under this new contract is higher than that from the original contract. Her present discounted value of lifetime utility under this new contract is

$$
\begin{aligned}
& e^{-r\hat{T}_M}\left[\int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(\hat{k}_t) - \hat{k}_t]dt\right] + e^{-r\hat{T}_M}(V - \bar{S}) \\
> & e^{-r\hat{T}_M}\left[\int_0^{\hat{t}} e^{r(\hat{T}_M-t)}[f(\hat{k}_t) - \hat{k}_t]dt + \int_{\hat{t}}^{\hat{T}_M} e^{r(\hat{T}_M-t)}[f(\hat{k}_t) - \hat{k}_t]dt\right] + e^{-r\hat{T}_M}\int_0^{\infty} e^{-rt}[f(k^e) - k^e]dt \\
\geq & \int_0^{\hat{t}} e^{-rt}[f(k_t^*) - k_t^*]dt + \int_{\hat{t}}^{\hat{T}_M} e^{-rt}[f(k_t^*) - k_t^*]dt + \int_{\hat{T}_M}^{T_M^*} e^{-rt}[f(k_t^*) - k_t^*]dt \\
= & \int_0^{T_M^*} e^{-rt}[f(k_t^*) - k_t^*]dt \\
= & \int_0^{T_M^*} e^{-rt}(1 - \alpha_t^*)[f(k_t^*) - k_t^*]dt + e^{-rT_M^*}\int_0^{T_M^*} e^{r(T_M^*-t)}\alpha_t^*[f(k_t^*) - k_t^*]dt.
\end{aligned}
$$

where again the first inequality is coming from Assumption 2, the second inequality is coming from the construction and the definition of k^e.[20] The last expression is the borrower's present discounted value of lifetime utility under the original contract. Now it is immediate that DICs at all t where $0 \leq t \leq \hat{T}_M$ are satisfied. So, the original contract cannot have been optimum. ■

Proof of Lemma 1. Let $\langle \{\alpha_t\}_{t=0}^{T_M}, \{k_t\}_{t=0}^{T_M}, T_M \rangle$ be an optimum contract, and suppose to the contrary the accumulated savings at T_M exceeds $\bar{S}$. We construct a new contract $\langle \{\hat{\alpha}_t\}_{t=0}^{\hat{T}_M}, \{\hat{k}_t\}_{t=0}^{\hat{T}_M}, \hat{T}_M \rangle$ such that both the constraints are satisfied and the borrower's present discounted value of lifetime utility is higher under the new scheme, so that the original contract cannot have been the optimum. The new scheme is as follows:

$$
\begin{cases}
\hat{\alpha}_t = \alpha_t \quad \forall t \in [0, \hat{T}_M], \\
\hat{k}_t = k_t \quad \forall t \in [0, \hat{T}_M], \text{ and} \\
\hat{T}_M = T_M - \Delta, \;\; \text{such that } \Delta > 0 \text{ and } \displaystyle\int_0^{\hat{T}_M} e^{r(\hat{T}_M-t)}\hat{\alpha}_t[f(\hat{k}_t) - \hat{k}_t]dt \geq \bar{S}.
\end{cases}
$$

In step 1, we show that the borrower's present discounted value of lifetime utility is higher under this new contract and then in step 2, we show that the new contract is DIC.

Step 1. The borrower's present discounted value of lifetime utility, if she always repays, is

$$
\int_0^{T_M} e^{-rt}(1 - \alpha_t)[f(k_t) - k_t]dt + e^{-rT_M}\left[\int_0^{T_M} e^{r(T_M-t)}\alpha_t[f(k_t) - k_t]dt - \bar{S} + V\right]
$$

Due to our assumptions that future is discounted similarly as the interest on savings and that the

[20] By construction, $f(k_t^*) - k_t^* = f(\hat{k}_t) - \hat{k}_t \;\forall t \in [0, \hat{t}]$ and $f(k_t^*) - k_t^* \leq f(\hat{k}_t) - \hat{k}_t \;\forall t \in (\hat{t}, \hat{T}_M]$. And from the definition of k^e, $\int_{\hat{T}_M}^{\infty} e^{-rt}[f(k^e) - k^e]dt \geq \int_{\hat{T}_M}^{T_M^*} e^{-rt}[f(k_t^*) - k_t^*]dt$.

borrower's utility function is linear, observe it can be written as

$$\int_0^{T_M} e^{-rt}[f(k_t) - k_t]dt + e^{-rT_M}\Big[V - \bar{S}\Big]. \tag{5.1}$$

This is essentially saying that the borrower is indifferent between consuming an amount now, and saving and consuming that amount (along with interest) later.
Partially differentiating (5.1) with respect to T_M from the left we get:

$$e^{-rT_M}[f(k_{T_M}) - k_{T_M}] - re^{-rT_M}(V - \bar{S}) = -re^{-rT_M}\Big[V - \bar{S} - \frac{[f(k_{T_M}) - k_{T_M}]}{r}\Big] < 0,$$

where the inequality follows since given Assumption 2 and the definition of k^e

$$V - \bar{S} > \frac{f(k^e) - k^e}{r} \geq \frac{f(k_{T_M}) - k_{T_M}}{r}.$$

Step 2. Finally, we argue that the DICs for the new scheme $\langle\{\hat{\alpha}_t\}_{t=0}^{\hat{T}_M}, \{\hat{k}_t\}_{t=0}^{\hat{T}_M}, \hat{T}_M\rangle$ hold for all $t \leq \hat{T}_M$. Consider some $t \leq \hat{T}_M$, the L.H.S. of the DIC at t equals

$$\begin{aligned}
& \int_t^{\hat{T}_M} e^{-r(t'-t)}(1-\hat{\alpha}_{t'})[f(\hat{k}_{t'}) - \hat{k}_{t'}]dt' + e^{-r(\hat{T}_M - t)}\Bigg[\int_0^{\hat{T}_M} e^{r(\hat{T}_M - t')}\hat{\alpha}_{t'}[f(\hat{k}_{t'}) - \hat{k}_{t'}]dt' - \bar{S} + V\Bigg] \\
= & \int_t^{\hat{T}_M} e^{-r(t'-t)}(1-\alpha_{t'})[f(k_{t'}) - k_{t'}]dt' + e^{-r(\hat{T}_M - t)}\Bigg[\int_0^{\hat{T}_M} e^{r(\hat{T}_M - t')}\alpha_{t'}[f(k_{t'}) - k_{t'}]dt' - \bar{S} + V\Bigg] \\
> & \int_t^{T_M} e^{-r(t'-t)}(1-\alpha_{t'})[f(k_{t'}) - k_{t'}]dt' + e^{-r(T_M - t)}\Bigg[\int_0^{T_M} e^{r(T_M - t')}\alpha_{t'}[f(k_{t'}) - k_{t'}]dt' - \bar{S} + V\Bigg] \\
\geq & f(k_t) = f(\hat{k}_t)
\end{aligned}$$

where the first equality follows from construction, the second inequality follows from the construction, in particular $\hat{T}_M < T_M$, and the argument in Step 1, and the final inequality follows from the DICs for the original scheme. Hence, the original scheme cannot have been the optimum. ■

Proof of Lemma 2. Let $\langle\{\alpha_t^*\}_{t=0}^{T_M^*}, \{k_t^*\}_{t=0}^{T_M^*}, T_M^*\rangle$ be an optimum contract. In Step 1, we show that $k_t^* = \min\{k_{It}, k^e\}$ for all t, and then in Step 2, we show that $\alpha_t^* = 1$ for all t, where $0 \leq t \leq T_M^*$.

Step 1. $\mathbf{k_t^* = \min\{k_{It}, k^e\}}$ for all t

Observe that $k_t^* \leq k_{It}$, $\forall t \leq T_M^*$ (otherwise, given the definition of k_{It}, DIC at t will be violated). Next, consider the set $\mathcal{M} = \{t \leq T_M^* :$ Either $k_t^* < \min\{k_{It}, k^e\}$, or $k_t^* \in (k^e, k_{It}]\}$. In order to prove this lemma, it is sufficient to show that the measure of the set $\mathcal{M}$ is zero.[21] Suppose not. Then $\exists \mathcal{M}'$ and $T_M' < T_M^*$ such that (i) $\mathcal{M}' \subsetneq \mathcal{M}$, (ii) $t \leq T_M'$ for all $t \in \mathcal{M}'$, and (iii) the measure of $\mathcal{M}' > 0$.

We then construct another scheme $\langle\{\alpha_t^*\}_{t=0}^{T_M}, \{k_t'\}_{t=0}^{T_M^*}, T_M^*\rangle$ such that:

$$k_t' = \begin{cases} k_t^*, & \text{when } t \notin \mathcal{M}' \text{ and } t \leq T_M^*, \\ \dfrac{k_t^* + \min\{k_{It}, k^e\}}{2}, & \text{when } t \in \mathcal{M}' \text{ and } k_t^* < \min\{k_{It}, k^e\}, \\ \dfrac{k^e + k_t^*}{2}, & \text{when } t \in \mathcal{M}' \text{ and } k_t^* \in (k^e, k_{It}]. \end{cases}$$

[21] Given our assumption of continuous k_t, this ensures that $k_t^* = \min\{k_{It}, k^e\}$ for *all* t.

Hence by construction $\forall t \in \mathcal{M}'$, $[f(k_t^{'}) - k_t^{'}] > [f(k_t^*) - k_t^*]$.[22] Further, $\forall t \leq T_M^*$ and $t \notin \mathcal{M}'$, we have $k_t^{'} = k_t^*$, and thus $[f(k_t^{'}) - k_t^{'}] = [f(k_t^*) - k_t^*]$. Therefore,

$$\int_0^{T_M^*} e^{r(T_M^*-t)}\alpha_t^*[f(k_t^{'}) - k_t^{'}]dt > \int_0^{T_M^*} e^{r(T_M^*-t)}\alpha_t^*[f(k_t^*) - k_t^*]dt = \bar{S},$$

where the inequality is strict since $\mathcal{M}'$ has a positive measure. Now we check the DICs of the new contract.

i) DIC at any $t \notin \mathcal{M}'$ is satisfied because we started with a DIC contract and the continuation payoff at t under this new contract is no less than that under the original contract whereas the deviation payoff has not changed.

ii) Consider the change at t where $k_t^{'} = \dfrac{k_t^* + \min\{k_{It}, k^e\}}{2}$. The continuation payoff as well as the deviation payoff at t have increased, but that does not violate DIC because $k_t^{'} \leq k_{It}$.

iii) Similarly, consider the change at t where $k_t^{'} = \dfrac{k^e + k_t^*}{2}$. The continuation payoff at t has increased but the deviation payoff at t has decreased. So, DIC at such a t is satisfied.

Next since time is continuous, $\exists \Delta^{'} > 0$ such that

$$\int_0^{T_M^*-\Delta^{'}} e^{r(T_M^*-\Delta^{'}-t)}\alpha_t^*[f(k_t^{'}) - k_t^{'}]dt \geq \bar{S} \text{ and } \Delta^{'} < T_M^* - T_M^{'}.$$

Finally, mimicing the argument of Step 2 of the proof of Lemma 1, it can be shown that, for $\Delta^{'}$ small enough, DICs are not violated in this new scheme. Thus, for $\Delta^{'}$ small enough, we have constructed another scheme $\langle\{\alpha_t^*\}_{t=0}^{T_M^*-\Delta^{'}}, \{k_t^{'}\}_{t=0}^{T_M^*-\Delta^{'}}, T_M^* - \Delta^{'}\rangle$ that satisfies the DICs and the GC, and ends earlier than T_M^*. Hence, given Lemma 1, the scheme $\langle\{\alpha_t^*\}_{t=0}^{T_M^*}, \{k_t^*\}_{t=0}^{T_M^*}, T_M^*\rangle$ cannot have been optimal, which is a contradiction.

Step 2. $\alpha_{\mathbf{t}}^* = \mathbf{1}$ for all t. The proof is immediate from the argument above. ■

Proof of Proposition 3. To prove this proposition we introduce the following two lemmas.

Lemma 5. *Let Assumptions 1, 2 and 3 hold.*

(i) The optimal loan scheme is always weakly progressive.

(ii) The optimal loan scheme can never be strictly progressive.

Proof. (i)Note that k_{It} is strictly increasing over time. This follows since $f(k_{It}) = e^{-r(T_M^*-t)}V$ $\forall t \leq T_M^*$, so differentiating it with respect to t we get $re^{-r(T_M^*-t)}V > 0$. So given Lemma 2, the optimal loan scheme is also strictly increasing over time unless it becomes constant at the efficient amount k^e. Hence, the optimal loan scheme is weakly progressive.

(ii) To prove the claim, we need to show that any progressive loan scheme must be capped at k^e that is k^e becomes DIC at some $t < T_M^*$. For that it is sufficient to argue that $k_{IT_M^*} > k^e$. This follows since $f(k_{IT_M^*}) = V$[23] and from assumption 3 we have $f(k^e) < \bar{S}$. □

[22] Recall k^e maximises $f(k) - k$. Now first consider any $t \in \mathcal{M}'$ and $k_t^* < \min\{k_{It}, k^e\}$, $f(k_t) - k_t$ increases as we increase k_t. Next consider any $t \in \mathcal{M}'$ and $k_t^* \in (k^e, k_{It}]$, $f(k_t) - k_t$ increases as we decrease k_t.

[23] Recall, k_{It} denotes the maximum amount of loan which is DIC at t, where $0 \leq t \leq T_M$. Now, DIC at T_M^* is $V \geq f(k_{T_M^*})$, hence $f(k_{IT_M^*}) = V$

Hence, the optimum loan scheme can be either progressive with a cap or constant. Next, we characterise the corresponding parametric conditions. We find that the optimal loan scheme is "progressive with a cap" if and only if the increase in utility from graduation is *modestly positive.* In that event, the efficient level k^e cannot be sustained from the very beginning. Thus the loan amount keeps on increasing till it reaches k^e and remains constant thereafter. Finally, when this increase in utility from graduation is *transformative* k^e becomes DIC from the very beginning, hence the optimal loan amount remains constant at k^e. We prove these formally in the following lemma.

Lemma 6. *Let Assumptions 1, 2 and 3 hold.*

(i) The optimal loan scheme is progressive with a cap if and only if the increase in utility from graduation is not too large:

$$\frac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e} V < f(k^e),$$

(ii) Otherwise, the optimal loan scheme is constant.

Proof. From the preceding lemma we know that the optimal loan scheme is either progressive with a cap or constant:

(i) If $k_{I0} < k^e$ then from Lemma 2, and the argument in Lemma 5, the optimal loan scheme must be "progressive with a cap".

(ii) And similarly if $k_{I0} \geq k^e$ then the optimal loan scheme must be constant at k^e.

So we characterise the parametric conditions under which the optimal loan scheme is progressive with a cap, that is $k_{I0} < k^e$. We show that

$$k_{I0} \geq k^e \text{ if and only if } \frac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e} V \geq f(k^e).$$

- Suppose $k_{I0} \geq k^e$: This implies k^e is DIC at $t = 0$. So from Lemma 5 we have $k_t^* = k^e \ \forall t \in [0, T_M^*]$. Then the graduation constraint GC can be written as $\int_0^{T_M^*} e^{r(T_M^* - t)}[f(k^e) - k^e]dt = \bar{S}$, which in turn implies that $e^{-rT_M^*}V = \dfrac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e} V$.

  Now observe, $f(k_{I0}) = e^{-rT_M^*}V$. Hence, $k_{I0} \geq k^e$ implies $\dfrac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e} V \geq f(k^e)$.

- Similarly $f(k^e) \leq \dfrac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e} V$ implies $k_{I0} \geq k^e$. Hence, the lemma. □

Hence, the proposition. ■

Proofs of the General Framework.

Proof of Proposition 4. For this we need the following two lemmas. The first one characterises $\langle T_B^R, T_B^D(t) \rangle$ the time of withdrawal of savings from the SI in case of repayment and default at t, where $0 \leq t \leq T_{sM}$. The second lemma characterises $\langle \{\sigma_t^R\}_{t=0}^{T_{sM}}, \sigma_t^D \rangle$ where σ_t^R denotes the part she wants to save with the SI at any arbitrary t, after repaying and saving with the MFI (or getting back her savings from the MFI which happens at T_{sM}), and σ_t^D denotes the part of $f(k_{st}) + S_t^D$ she wants to save with the SI after defaulting at t; where $0 \leq t \leq T_{sM}$ and $0 \leq \sigma_t^R, \sigma_t^D \leq 1$.

Lemma 7. *Let Assumption 2 hold. Given any MFI-contract* $\langle \{\alpha_{st}\}_{t=0}^{T_{sM}}, \gamma, \{k_{st}\}_{t=0}^{T_{sM}}, T_{sM}\rangle$

1. *Suppose the borrower always repays, the optimum* $T_B^{R^*}$ *satisfies the following:*

   i) *When money in her hand at the termination date of the contract* T_{sM} *is at least* $\bar{S}$*, she withdraws her savings from the SI at that termination date, that is* $T_B^{R^*} = T_{sM}$

   ii) *When money in her hand at* T_{sM} *is less than* $\bar{S}$*, she chooses* $T_B^{R^*}$ *in such a way that at* $T_B^{R^*}$ *her savings with the SI becomes exactly equal to* $\bar{S}$*.*

2. *Suppose the borrower defaults at some* t*, where* $0 \leq t \leq T_{sM}$*, the optimum* $T_B^{D^*}(t)$ *satisfies the following:*

   i) *When money in her hand at the termination date of the contract* t *is at least* $\bar{S}$*, she withdraws her savings from the SI at that termination date, that is* $T_B^{D^*}(t) = t$

   ii) *When money in her hand at* t *is less than* $\bar{S}$*, she chooses* $T_B^{D^*}(t)$ *in such a way that at* $T_B^{D^*}(t)$ *her savings with the SI becomes exactly equal to* $\bar{S}$*.*

Proof. This proof follows from the facts that graduation is welfare improving so the borrower wants to graduate as soon as possible and that she is indifferent between consuming an amount now, and saving and consuming that amount (along with interest) in future. Fomally, we show this in three steps.

Step 1. We show that the borrower prefers to save and graduate over consuming that amount. Denote the money in the borrower's hand at the termination date T, irrespective of whether the contract was terminated successfully or due to default, by M and the time of graduation by τ, where $\tau \geq T$.
If she does not graduate her present discounted value of lifetime utility at T is M. If she graduates at τ her present discounted value of lifetime utility at T is

$$e^{-r(\tau-T)}\Big[e^{r(\tau-T)}M + (V - \bar{S})\Big].$$

Given Assumption 2, $V - \bar{S} > 0$ hence her utility is higher when she graduates.

Step 2. We show that her welfare increases as the time of graduation decreases. For that, we show that the borrower's present discounted value of lifetime utility is decreasing in τ. Differentiating present discounted value of the borrower's lifetime utility at T from graduation with respect to τ we get $-re^{-r(\tau-T)}(V - \bar{S}) < 0$. Therefore, she optimally chooses the time of withdrawal of her savings from the SI as soon as that becomes $\bar{S}$.

Step 3. We show that when her savings with the SI is not required to graduate, time of withdrawal of her savings from the SI does not affect her utility. Given our assumptions that the borrower's utility function is linear and that the future is discounted in the same way as the interest rate this is immediate. Recall without loss of generality, we assume that in such cases the borrower chooses the termination date of the contract as the time of withdrawal of her savings from the SI. From these three steps the lemma is immediate. □

Lemma 8. *Given an MFI-contract* $\langle \{\alpha_{st}\}_{t=0}^{T_{sM}}, \gamma, \{k_{st}\}_{t=0}^{T_{sM}}, T_{sM}\rangle$*, the borrower saves as much as she can with the SI, in case of repayment as well as in case of default:*

1. $\sigma_t^{D^*} = 1$ *for all* t*, where* $0 \leq t \leq T_{sM}$*,*

2. $\sigma_t^{R^*} = 1$ *for all* t*, where* $0 \leq t \leq T_{sM}$*.*

Proof.

1. For that we fix the borrower's strategy in case of repayment at $\langle \sigma_t^R, T_B^R \rangle$ and from Lemma 7 we know that $T_B^{D^*}(t)$ satisfies the following:

   i) $T_B^{D^*}(t) = t$ when the amount with which she defaults is at least $\bar{S}$

   ii) Otherwise $T_B^{D^*}(t)$ is such that

   $$e^{r(T_B^D(t)-t)} \Bigg[\sigma_t^D \Big[f(k_{st}) + \gamma \int_0^t e^{r(t-t')} \alpha_{st'} [f(k_{st'}) - k_{st'}] dt' \Big] + \int_0^t e^{r(t-t')} \sigma_{t'}^R (1 - \alpha_{st'}) [f(k_{st'}) - k_{st'}] dt' \Bigg] = \bar{S}.$$

   In (i) she graduates immediately, using the amount she gets from the lender, this is same as saying that she saves the entire amount with the SI and withdraws immediately. To keep the notation similar we say that in this case $\sigma_t^{D^*} = 1$.

   Now for (ii) – as argued above, a borrower is indifferent between consuming an amount now, and saving and consuming that amount later (along with interest). However, increase in σ_t^D decreases $T_B^{D^*}(t)$ which implies that the borrower graduates at an earlier date. Hence, her present discounted value of lifetime utility increases with increase in σ_t^D. Given limited liability $\sigma_t^{D^*} = 1$.

2. Now we show that $\sigma_t^{R^*} = 1$ for all $t \in [0, t_{sM}]$. Observe that following repayment, at any $t < T_{sM}$ there can be two cases: The borrower defaults at some $\tau \in (t, T_{sM}]$ or she repays at all $t \leq T_{sM}$. So we consider both the cases.

   (i) Suppose the borrower defaults at some $\tau \in (t, T_{sM}]$. Recall, the borrower's optimum strategy in case of default at τ is given by $\sigma_\tau^{D^*} = 1$ and $T_B^{D^*}(\tau)$ as characterised in Lemma 7.

   Now as observed above, the borrower's present discounted value of lifetime utility is decreasing in $T_B^{D^*}(\tau)$. Next given a contract, the borrower's savings with the SI (weakly) increases as σ_t^R increases and that (weakly) decreases $T_B^{D^*}(\tau)$. So given limited liability constraint, the measure of the set Ω^D is zero; where $\Omega^D = \{t \leq \tau : \sigma_t^{R^*} < 1\}$.

   (ii) The borrower repays at all $t \leq T_{sM}$. From Lemma 4 and the argument made above it is obvious that the measure of the set Ω^R is zero; where $\Omega^R = \{t \leq T_{sM} : \sigma_t^{R^*} < 1\}$. Hence, the lemma. □

Hence, the proposition. ■

Proof of Lemma 3. To prove this lemma we need to show that at the optimum, (a) the MFI provides loans at all instances till the borrower graduates i.e. she graduates at the successful termination date of the contract T_{sM}^* and (b) the graduation constraint binds. We prove (a) and skip (b) as that is very similar to the proof of Lemma 1.

(a) Suppose not, the optimal contract be $\langle \{\alpha_{st}^*\}_{t=0}^{T_{sM}^*}, \gamma^*, \{k_{st}^*\}_{t=0}^{T_{sM}^*}, T_{sM}^* \rangle$ and the borrower graduate at some $T = T_B^{R^*} > T_{sM}^*$. We construct another DIC contract $\langle \{\hat{\alpha}_{st}\}_{t=0}^{\hat{T}_{sM}}, \hat{\gamma}, \{\hat{k}_{st}\}_{t=0}^{\hat{T}_{sM}}, \hat{T}_{sM} \rangle$ such that the borrower's present discounted value of lifetime utility is higher under this new contract. So the original contract cannot have been optimum. The new contract $\langle \{\hat{\alpha}_{st}\}_{t=0}^{\hat{T}_{sM}}, \hat{\gamma}, \{\hat{k}_{st}\}_{t=0}^{\hat{T}_{sM}}, \hat{T}_{sM} \rangle$ is as follows

$$\begin{cases} \hat{T}_{sM} = T^*_{sM} + \Delta \quad \text{where } \Delta \text{ is such that } \displaystyle\int_0^{\hat{T}_{sM}} e^{r(\hat{T}_{sM}-t)}[f(\hat{k}_{st}) - \hat{k}_{st}]dt < \bar{S} \\ \hat{k}_{st} = k^*_{st} \ \ \forall t \in [0, T^*_{sM}] \quad \text{and} \quad \hat{k}_{st} = k^*_{sT^*_{sM}} \forall t \in (T^*_{sM}, \hat{T}_{sM}] \\ \hat{\alpha}_{st} = \alpha^*_{st} \ \ \forall t \in [0, T^*_{sM}] \quad \text{and} \quad \hat{\alpha}_{st} = \alpha^*_{sT^*_{sM}} \forall t \in (T^*_{sM}, \hat{T}_{sM}] \\ \hat{\gamma} = \gamma^*. \end{cases}$$

We first show that this new contract provides higher utility. Recall, given the borrower's optimum strategy she graduates at $\hat{T}_B^{R^*}$ where it is given by $\int_0^{\hat{T}_{sM}} e^{r(\hat{T}_B^{R^*}-t)}[f(\hat{k}_{st}) - \hat{k}_{st}]dt = \bar{S}$ So, she is better off under this new scheme as she graduates at an earlier date. But she becomes better off not only when she repays always, but also when she defaults at some $t \in (T^*_{sM}, \hat{T}_{sM}]$, so it may not be obvious that DICs at all $t \in (T^*_{sM}, \hat{T}_{sM}]$ are satisfied. However, we show that DICs at all $t \in [0, \hat{T}_{sM}]$ are satisfied which implies that the original scheme cannot have been optimum.

DICs of the new contract at any $t \in [0, T^*_{sM}]$ are satisfied because we started with a DIC contract and under this new scheme at any $t \in [0, T^*_{sM}]$, the present discounted value of lifetime utility from repayment has increased whereas that from default has not changed. Now we argue that DIC at any $\tilde{t} \in (T^*_{sM}, \hat{T}_{sM}]$ is also satisfied. For that it is sufficient to show that the borrower's total savings till that $\tilde{t}$ is higher than the money in her hand in case of default.[24] So we want to show that

$$\int_0^{\tilde{t}} e^{r(\tilde{t}-t)}[f(\hat{k}_{st}) - \hat{k}_{st}]dt + f(\hat{k}_{s\tilde{t}}) - \hat{k}_{s\tilde{t}} \geq \int_0^{\tilde{t}} e^{r(\tilde{t}-t)}[1 - \hat{\alpha}_{st}(1-\hat{\gamma})][f(\hat{k}_{st}) - \hat{k}_{st}]dt + f(\hat{k}_{s\tilde{t}})^{[25]}$$

$$\Rightarrow \int_0^{T^*_{sM}} e^{r(\tilde{t}-t)}\alpha^*_{st}(1-\gamma^*)[f(k^*_{st}) - k^*_{st}]dt + \int_{T^*_{sM}}^{\tilde{t}} e^{r(\tilde{t}-t)}\alpha^*_{st}(1-\gamma^*)[f(k^*_{st}) - k^*_{st}]dt \geq k^*_{sT^*_{sM}}. \quad (5.2)$$

where the second expression follows from the construction.

We establish this from the dynamic incentive compatibility constraint, of the original contract, at T^*_{sM}. That implies money in hand at T^*_{sM} in case of repayment must be no less than that in case of default. Otherwise given the borrower's strategy she would graduate at an earlier date[26] in case of default which will violate DIC at T^*_{sM}. So, we have

$$\begin{aligned} &\int_0^{T^*_{sM}} e^{r(T^*_{sM}-t)}[f(k^*_{st}) - k^*_{st}]dt + f(k^*_{sT^*_{sM}}) - k^*_{sT^*_{sM}} \\ &\geq \int_0^{T^*_{sM}} e^{r(T^*_{sM}-t)}[1 - \alpha^*_{st}(1-\gamma^*)][f(k^*_{st}) - k^*_{st}]dt + f(k^*_{sT^*_{sM}}) \\ \Rightarrow &\int_0^{T^*_{sM}} e^{r(T^*_{sM}-t)}\alpha^*_{st}(1-\gamma^*)[f(k^*_{st}) - k^*_{st}]dt \geq k^*_{sT^*_{sM}}. \end{aligned}$$

[24] It is sufficient because in case of default the borrower has that money in hand only, to graduate – she saves that amount with the SI and graduates as soon as that becomes $\bar{S}$. Now in case of repayment she also gets loan till $\hat{T}_{sM}$. Hence the amount which helps her to graduate not only includes savings till that instance, but also the net return at each instance in future.

[25] Recall money in hand in case of default at $\tilde{t}$ is

$$f(\hat{k}_{s\tilde{t}}) + \int_0^{\tilde{t}} e^{r(\tilde{t}-t)}\hat{\gamma}\hat{\alpha}_{st}[f(\hat{k}_{st}) - \hat{k}_{st}]dt + \int_0^{\tilde{t}} e^{r(\tilde{t}-t)}(1-\hat{\alpha}_{st})[f(\hat{k}_{st}) - \hat{k}_{st}]dt.$$

[26] That is $T_B^{R^*} < T_B^{D^*}(T^*_{sM})$.

Now expression (5.2) is immediate as $\tilde{t} > T^*_{sM}$ and $\int_{T^*_{sM}}^{\tilde{t}} e^{r(\tilde{t}-t)}\alpha^*_{st}(1-\gamma^*)[f(k^*_{st})-k^*_{st}]dt > 0$.

(b) The problem of the MFI thus becomes

$$\underset{\langle \{\alpha_{st}\}_{t=0}^{T_{sM}},\gamma,\{k_{st}\}_{t=0}^{T_{sM}},T_{sM}\rangle}{\text{Maximise}} \quad e^{-rT_{sM}}\Big[\int_0^{T_{sM}} e^{r(T_{sM}-t)}[f(k_{st})-k_{st}]dt - \bar{S} + V\Big]$$

$$\text{Subject to: } \mathrm{GC}_R: \int_0^{T_{sM}} e^{r(T_{sM}-t)}[f(k_{st})-k_{st}]dt \geq \bar{S},$$

$$\text{DIC:} \quad \forall t \leq T_{sM}; \quad e^{-r(T_{sM}-t)}\Big[\int_0^{T_{sM}} e^{r(T_{sM}-t')}[f(k_{st'})-k_{st'}]dt' - \bar{S} + V\Big]$$

$$\geq e^{-r(T_B^{D^*}(t)-t)}\Bigg[e^{r(T_B^{D^*}(t)-t)}\Big[\int_0^t e^{r(t-t')}\big(1-\alpha_{st'}(1-\gamma)\big)[f(k_{st'})-k_{st'}]dt' + f(k_{st})\Big] - \bar{S} + V\Bigg].$$

We want to show that GC_R binds. This proof is very similar to that of Lemma 1, so we skip it here. ■

Proof of Observation 1. Suppose not. Then $\exists t \in [0, T_{sM})$, such that money in the borrower's hand in case she defaults at t is no less than $\bar{S}$, that is, $\int_0^t e^{r(t-t')}\big(1-\alpha_{st'}(1-\gamma)\big)[f(k_{st'})-k_{st'}]dt' + f(k_{st}) \geq \bar{S}$. Given Lemma 4, in case of default she immediately graduates and her utility is

$$\int_0^t e^{r(t-t')}\big(1-\alpha_{st'}(1-\gamma)\big)[f(k_{st'})-k_{st'}]dt' + f(k_{st}) - \bar{S} + V$$

which is higher than $e^{-r(T_{sM}-t)}V$ – present discounted value of lifetime utility from repayment. Hence, DIC at t cannot be satisfied.

Following the same argument, we get that the money in her hand in case of default at T_{sM} can be no higher than $\bar{S}$. ■

Proof of Proposition 6.

A. From Lemma 4 we know $k^*_{st} = \min\{k_{sIt}, k^e\}$, so all we need to show is that, given assumption 4, k_{sIt} is (weakly) increasing in t, where $0 \leq t < T^*_{sM}$.

Now, given observation 1 and the borrower's strategy identified in Proposition 4, we can write the following:

$$e^{r(T_B^{D^*}(t)-t)}\Big[\int_0^t e^{r(t-t')}\underline{\gamma}[f(k^*_{st'})-k^*_{st'}]dt' + f(k^*_{st})\Big] = \bar{S}$$

$$\Rightarrow \quad e^{-r(T_B^{D^*}-t)} = \frac{1}{\bar{S}}\Big[\int_0^t e^{r(t-t')}\underline{\gamma}[f(k^*_{st'})-k^*_{st'}]dt' + f(k^*_{st})\Big]$$

So, DIC at any $t \in [0, T^*_{sM})$ can be written as

$$e^{-r(T^*_{sM}-t)}V \geq \frac{1}{\bar{S}}\Big[\int_0^t e^{r(t-t')}\underline{\gamma}[f(k^*_{st'})-k^*_{st'}]dt' + f(k^*_{st})\Big]V$$

From this we can write for any $t \in [0, T^*_{sM})$

$$f(k_{sIt}) = \bar{S}e^{-r(T^*_{sM}-t)} - \int_0^t e^{r(t-t')}\underline{\gamma}[f(k^*_{st'}) - k^*_{st'}]dt' \tag{5.3}$$

So, to prove that the optimum loan scheme is weakly progressive it is sufficient to show that the R.H.S of (5.3) is increasing in t. Differentiating (5.3) with respect to t we get

$$\begin{aligned}
&r\bar{S}e^{-r(T^*_{sM}-t)} - r\int_0^t e^{r(t-t')}\underline{\gamma}[f(k^*_{st'}) - k^*_{st'}]dt' - \underline{\gamma}[f(k^*_{st}) - k^*_{st}] \\
&= rf(k_{sIt}) - \underline{\gamma}[f(k^*_{st}) - k^*_{st}] > 0.
\end{aligned}$$

where the inequality is coming from Assumption 4 and $k^*_{st} = \min\{k_{sIt}, k^e\}$.

B. This implies that the optimal loan scheme is either constant or progressive which may or may not be capped.

(i) Given Lemma 4 and part A of this proposition, a loan scheme is "strictly progressive" if and only the efficient loan amount is not incentive compatible even at T^*_{sM} i.e. $k_{sIT^*_{sM}} < k^e$. Now, observe $f(k_{sIT^*_{sM}}) = (1-\underline{\gamma})\bar{S}$. Hence the loan scheme is "strictly progressive" if and only

$$(1-\underline{\gamma})\bar{S} < f(k^e).$$

This implies that the loan scheme is either "progressive with a cap" or "constant" over time if and only $(1-\underline{\gamma})\bar{S} \geq f(k^e)$. Mimicing the steps used in the Proof of 3, it can be shown that

(ii) The optimal loan scheme is "constant" if and only if $f(k^e) \leq \dfrac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e}\bar{S}$.

(iii) It is "progressive with a cap" if and only if $\dfrac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e}\bar{S} \leq f(k^e) \leq (1-\underline{\gamma})\bar{S}$. ■

Proof of Proposition 7. To prove this proposition we first introduce the following observation.

Observation 2. *The necessary and sufficient condition for the constant scheme, in the general case is implied by that in the benchmark case.*

Proof. The necessary and sufficient condition for k^e to be DIC from the very first instance, in the benchmark case, is $f(k^e) < \dfrac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e}V$ whereas that in the general case is given by $f(k^e) \leq \dfrac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e}\bar{S}$. Since $V > \bar{S}$, this observation is immediate. □

Recall, we say that he investment required to start the technology $\langle V, \bar{S}\rangle$ is not large when

$$f(k^e) > \frac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e}\bar{S}.$$

Now we prove the proposition in three steps. In the first step, we show that at any t, where $0 \leq t \leq T^*_M$, the optimal loan amount is weakly lower in the general case in comparison to that in the benchmark case. It is strictly lower if and only if $\bar{S}$ is not large. The next two steps are obvious, in the second step we show that the time required to graduate is weakly higher in the general case, it is strictly higher if and only if $\bar{S}$ is not large. Finally in the third step, we show that the borrower's present discounted value of lifetime utility is weakly lower in the general case than that in the benchmark case. It is strictly lower if and only if $\bar{S}$ is not large.

Step 1. Since the deviation payoff is higher in the general case where the borrower gets back a part of her savings with the MFI till date, and can graduate by saving that amount with the SI, the amount which is DIC in the general case is also DIC in the benchmark case.

Now, the optimum loan amount is the minimum of the efficient amount and the maximum amount which is DIC, so when k^e is not DIC in the benchmark case it is not DIC in the general case as well, so in that case, the optimum loan amount is higher in the benchmark case than that in the general case.

Also, when the efficient amount is DIC in the benchmark case but not in the general case, the optimum loan amount is higher in the former case.

The optimum loan amounts are equal only in those instances where k^e is DIC in the general case. So, the optimal loan schemes under these two cases are identical when k^e is DIC at $t = 0$ in the general case. Recall, that happens if and only if $f(k^e) \leq \frac{f(k^e) - k^e}{r\bar{S} + f(k^e) - k^e}\bar{S}$. Therefore, the optimal loan amount at any t, where $0 \leq t \leq T_M^*$, is weakly lower in the general case in comparison to that in the benchmark case.

Step 2. This proof follows from the preceding step.

Step 3. Since the time required to graduate is weakly lower in the benchmark case than that in the general case and the borrower's present discounted value of lifetime utility increases with a decrease in the time required to graduate, the result is immediate. ■

Appendix B

In this Appendix we provide some evidence that support various modelling assumptions made in the paper. We prepare this Appendix using the Global Outreach and Financial Performance Benchmark Report 2015 (MIX (2017)), data from MIX website, data from the websites of different MFIs, quotes from different books, journal articles etc.

A. Outreach. "In FY 2015, 1033 institutions reported an outreach of *116.6 million borrowers* who have access to credit products, corresponding to a *gross loan portfolio of USD 92.4 billion...* and *98.4 million depositors* and *account for USD 58.9 billion of deposits*". In Table 1 we provide some more details. *Source:* MIX (2017).

B. Near Perfect Repayment Rate in Microfinance. Table 2 shows that repayment rates are very high. *Portfolio at Risk (PAR)* is one of the indicators of repayment rate. Low PAR indicates high repayment rate. *Source:* MIX (2017).

C. Progressive Lending with a Cap. Almost all the MFIs practise progressive lending. Many of those MFIs set caps as well – loan size cannot increase beyond that. Here we provide some examples from India, Bangladesh and Vietnam – top three MFIs by number of active borrowers. Table 3 shows that all the top five MFIs (by number of active borrowers) of India practise "progressive lending with a cap". Table 4 shows that all the top five MFIs (by number of active borrowers) of Bangladeh practise "progressive lending with a cap". Vietnam is the third largest country by active borrowers and Vietnam Bank of Social Policies is the largest MFI. In their website it is not mentioned whether they practise progressive lending or not, but each of the products offered by them has a cap. Table 5 documents that.

D. Savings. We then discuss deposit collection in various parts of the World.

South Asia. Due to regulation, deposit collection in India is low. In fact, Kline and Sadhu (2015) point out "No microfinance institution registered as an NBFC, currently accepts deposits because regulation requires that institutions must obtain an investment grade rating, which no microfinance institution has obtained." In table 6 we document the savings products offered by SEWA Bank, the largest Indian MFI (by number of depositors).[27] In table 7 savings products offered by the top five MFIs in Bangladesh are documented.

LAC is covered in table 8. Colombia, Peru and Bolivia are top three countries by number of depositors in Latin America and Carribean (LAC).

EAP, Africa and ECA are covered in table 9. Philippines, Indonesia and Vietnam are top three countries by the number of depositors in East Asia and the Pacific (EAP). Nigeria and Mongolia are top countries by the number of depositors in Africa and Eastern Europe and Central Asia (ECA) respectively.

A. Global Outreach

Table 1: Global Outreach: Borrower-Depositor *Source* MIX (2017)

Regions	Number of Active Borrowers ′000	Percentage of Total Borrowers	Number of Depositors ′000	Percentage of Number of Depositors	Deposits USD m	Percentage of Total Deposits
Africa	5,778.2	5%	17,928.0	18%	9,212.1	16%
EAP	16,257.5	14%	16,117.9	16%	7,687.2	13%
ECA	3,082.6	3%	5,091.0	5%	7,664.3	13%
LAC	22,495.3	19%	23,708.6	24%	27,293.1	46%
MENA	2,148.4	2%	465.1	0%	251.0	0%
South Asia	66,929.3	57%	35,109.2	36%	6,885.8	12%
Grand Total	116,691.3	100%	98,419.8	100%	58,993.6	100%

B. Near Perfect Repayment Rate in Microfinance – Evidence

Table 2: Near Perfect Repayment Rate in Microfinance – Evidence

Regions	Percentage of Total Borrowers	Percentage of Gross Loan Portfolio (GLP)[†]	**Portfolio at Risk> 30 Days (PAR)[‡]**
Africa	5%	9%	**10.60%**
East Asia and the Pacific (EAP)	1%	16%	**3.40%**
Eastern Europe and Central Asia (ECA)	3%	11%	**10.00%**
Latin America and the Carribean (LAC)	19%	42%	**5.40%**
Middle East and North Africa (MENA)	2%	1%	**3.60%**
South Asia	57%	20%	**2.60%**

[†] "Gross Loan Portfolio (GLP)": All outstanding principals due for all outstanding client loans. This includes current, delinquent, and renegotiated loans, but not loans that have been written off.
[‡] "Portfolio at Risk (PAR)": is one of the indicators of repayment rate. PAR [xx] days is defined as the value of all loans outstanding that have one or more installments of principal past due more than [xx] days.
Source: MIX (2017): Global Outreach and Financial Performance Benchmark Report 2015.

[27] We do not consider Bandhan here, as it has become a bank now and in the website it is not mentioned which savings products are for poor people.

C. Progressive Lending with a Cap – Evidence

Table 3: India – The Largest Country by Number of Active Borrowers

MFI	No. of Active Borrowers '000	Gross Loan Portfolio (GLP) m	Description			
			Product Name	Progressive Lending?	Maximum Loan Amount INR	Reference/url (accessed on 31st October, 2019)*
Bandhan	–	2,596.22	Suchana	Yes	25,000	https://www.bandhanbank.com/Microloans.aspx
			Srishti	Yes	1,50,000	
Jana Small Finance (Formerly known as Janalakshmi)	5,888.75	1,974.73	Small Batch Loans	Yes	50,000	http://www.janalakshmi.com/products-services/loans-for-individuals *(Accessed in January, 2018 before it became a Bank.)
			Jana Kisan Loan	Yes	1,00,000	
Bharat Financial Inclusion Limited (Formerly known as SKS Microfinance Limited)	5,323.06	1,413.30	Income Generation Loans (IGL) – Aarambh	Yes	29,565	http://www.bfil.co.in/our-products/
			Mid-Term Loans (MTL) – Vriddhi	Yes	15,010	
			Long Term Loans (LTL)	Yes	49,785	
Share	3,740.00	251.68	General Loans	Yes	60,000	http://www.sharemicrofin.com/products.html
			Micro Enterprise Loans	Yes	2,50,000	
Shree Kshethra Dharmasthala Rural Devt. Project (SKDRDP)	3,013.18	986.55	Pragathi Nidhi Programme	Yes	50,000 (collateralized thereafter)	Rao (2005) and https://skdrdpindia.org/programmes/microfinance/

India: No. of active borrowers 43,153,000 and gross loan portfolio 14,901m. Top 5 MFIs from India by the no. of active borrowers, except Bandhan as number of active borrowers is not available in MIX Market data, however it is well known that this is the largest MFI in India (Gross Loan Portfolio is the maximum). Source MIX (2017).

Table 4: Bangladesh – The Second Largest Country by Number of Active Borrowers

MFI	No. of Active Borrowers ’000	Gross Loan Portfolio (GLP) m	Description			
			Product Name	Progressive Lending?	Maximum Loan Amount	Reference/url (accessed on 31st October, 2019)
Grameen	7,290.00	1,498.47	Basic Loan	Yes	No (but an individual gets a loan as long as she is below poverty line)	http://www.grameen.com/wp-content/uploads/bsk-pdf-manager/GB-2015_33.pdf https://grameenfoundation.org/sites/default/files/books/GrameenGuidelines.pdf
ASA	6,794.85	1,919.02	Primary Loan	Yes Constant at the max. when the economic potential is large. Otherwise increasing.	BDT 99,000	http://www.asa.org.bd/FinancialProgram/LoanProducts
			Special Loan		BDT 10,00,000	
BRAC	5,356.52	1,768.61	Microloans (DABI)	Yes	USD 2,500	http://www.brac.net/images/factsheet/MF_Briefing_Doc_English.pdf
			Small enterprise loans (PROGOTI)	Yes	USD 13,000	
			Agriculture Loan	Yes	USD 1,500	
BURO Microfinance Program	996.22	406.58	Micro-Enterprise Loan	Yes	BDT 300,000	https://www.burobd.org/microfinance-loan-product.php?id=11
			Agriculture Loan	Yes	BDT 50,000	
Thengamara Mohila Sabuj Sangha (TMSS)	739.80	231.92	Loan for Enterprise Advancement and Development (LEAD)	Yes	BDT 10,00,000	http://tmss-bd.org/loan-for-enterprise-advancement-and-development-lead
			Rural Micro Credit (Jagaron)	Not Mentioned		http://tmss-bd.org/annual-report-2016
			Ultra Poor Program (Buniad)			
			Micro Enterprise SME Program (Agroshar)			

Bangladesh: No. of active borrowers 25,671,000 and gross loan portfolio 7,206m. Top 5 MFIs from Bangladesh by the no. of active borrowers. Source MIX (2017).

Table 5: Vietnam – The Third Largest Country by Number of Active Borrowers

Vietnam Bank of Social Policies (VBSP)			
Product Name	Maximum Loan Amount	Progressive Lending?	Reference/url (accessed on 31st October, 2019)
Poor Households Lending	VND 30 million/household	Not Mentioned	http://eng.vbsp.org.vn/poor-households-lending.html
Job Creation	Enterprises: VND 500,000,000/project. Households: VND 20,000,000/household		http://eng.vbsp.org.vn/job-creation.html
Overseas Workers	VND 30,000,000/labor		http://eng.vbsp.org.vn/overseas-workers-lending.html
Business& Production Households in Disadvantaged Areas	Generally VND 30 million. In some specific cases, loan amount can be over VND 30,000,000 to under VND 100,000,000		http://eng.vbsp.org.vn/business-production-households-in-disadvantaged-areas.html
Small and Medium Enterprises	VND 500,000,000/enterprise		http://eng.vbsp.org.vn/small-and-medium-enterprises.html
Extremely Disadvantaged Ethnic Minority Households	VND 5,000,000		http://eng.vbsp.org.vn/extremely-disadvantaged-ethnic-minority-households.html

Vietnam: No. of active borrowers 7,394,000 and gross loan portfolio 7,937m. VBSP is the largest MFI by the no. of active borrowers: No. of active borrowers 6,784740 and gross loan portfolio 6,911.69m. VBSP is the largest single microcredit lender in the world (Haughton and Khandker (2016)).
Source: MIX (2017).

D. Savings

Demand for Savings Service among Poor People and Lack of that

- "The commitment savings account gives you the chance to make a really long-term high-value swap, suitable for family ambitions like education, marriages and jobs for the youngsters, land and housing, and more distant anxieties like how to survive after you are too old and weak to work." (Rutherford (2009))).
- "Poor people save even at negative interest rate" (for example with Jyothi in India (Rutherford (2009))), and with the Susu men in Africa (Besley (1995)).

Deposit Collecting MFIs

- "...(M)any MFIs have become true microbanks, doing both credit and voluntary savings. Their savings accounts take various forms. Some are completely liquid, allowing deposits and withdrawals of any amount at any amount, or nearly. Others are time deposits, like certificates of deposit, which are locked up for agreed periods and pay higher interest in return. In between there are semi-liquid accounts.... which limit the number, amount, or both of transactions per month through rules of penalties." (Roodman (2009) p 261.)
- "...Some forced savings are taken directly out of the loan amount before disbursal; as of 2003, for example, the Bolivian village banking MFI Crédito con Educacíon Rural withheld

10 − 20 percent of a loan upfront. In contrast, FINCA Nicaragua took forced savings equal to 32 percent of the loan amount incrementally, like loan payments, at successive group meetings. Some MFIs allow clients to withdraw forced savings when they are done paying off the associated loan, others not until the client leaves the program altogether.

This counterintuitive combination of saving and borrowing accelerates loan repayment so that toward the end of a loan cycle, the MFI is actually in debt to its clients...." (Roodman (2009) p 124.)

- Village Banking Institutions "typically require each village bank member to save. These forced savings are often a significant percentage of the amount the member has borrowed from the VBI. For example, forced savings range from 10 to 32 percent of the amount borrowed in the four leading Latin American VBIs analyzed in this study. Forced savings serve at least two major purposes. First, they act as cash collateral... The second purpose of forcing village bank members to save is to introduce them to the discipline and habit of saving and to the possibilities that having a sizable savings balance could open up for them. For example, a sizable pool of savings could be used for emergencies, to pay school fees and other large household expenditures, to buy tools or machinery, or to start another business" (Westley (2004)).
- "Thus, in effect, the funds serve as a form of partial collateral." (Morduch (1999)).
- "(C)ollateralizing mandatory savings could offer a win-win solution for both lender and borrower by providing the MFP" (Microfinance Providers) "with security while at the same time building the asset base of the client." (Aslam and Azmat (2012)).

Table 6: South Asia: India – Savings Services provided by the Top MFI (by no. of depositors)

MFI No. of Depositors Deposits (USD m)	Product	Terms	Reference/url (accessed on 31st October, 2019)
Shri Mahila Sewa Sahakari Bank Ltd. 2,00,660 14.08	Regular Savings Product		https://www.sewabank.com/saving.html
	Fixed Deposit		https://www.sewabank.com/fixed-deposit.html
	Chinta Nivaran Yojana (Worry Riddance Scheme)	Deposits are made every month up to Five Years. In any emergency, after one year of joining in the scheme they can get an overdraft loan.	https://www.sewabank.com/recurring.html
	Kishori Gold Yojana	To encourage member to save money for special occasion. This was aimed at meeting expenses towards buying gold and gold ornaments during the wedding of their progeny.	
	Mangal Prasang Yojana	Help members during wedding of their sons and daughters.	
	Ghar Fund Yojana (Housing Fund Scheme)	To enable the member to have a house of their own. Maturity after 5/10 years	
	National Pension Scheme		https://www.sewabank.com/pension.html

India: No. of active depositors 374,000 and total deposit USD 329.65m. (We have not considered Bandhan here, as it has become a Bank now and in the website it is not mentioned which savings products are for the poor people.) *Source:* MIX (2017).

Table 7: South Asia: Bangladesh – Savings Services provided by the Top Five MFIs

MFI	Product	Terms	Reference/url (accessed on 31st October, 2019)
Grameen Deposits (USD m) 2,604.93	Personal Savings	**Weekly compulsory savings.** Withdrawal at any time is allowed.	Alam and Getubig (n.d.), Rutherford (2010)
	Grameen Pension Scheme (GPS)	**For five to ten years.** Higher interest rate. Not restricted to retirement needs: Many younger families see the program as a means to save for medium-term expenses, such as school fees or weddings in the future for recently born children.	
ASA No. of Depositors 7,843960 Deposits (USD m) 826.34	Regular Savings: Clients belonging to Loan Programs need to deposit a regular fixed amount.	**Min. savings: Tk. 10 per week and Tk. 50 per month for primary loan; Tk. 50 per week and Tk. 100 per month for special loan.Members may withdraw from their savings any time maintaining a balance of at least 10% of their loan outstanding.**	http://www.asa.org.bd/FinancialProgram/SavingsProducts
	Voluntary Savings: Excess of Mandatory/Regular savings is treated as voluntary savings.	May deposit any amount above their *mandatory weekly savings.* Members may withdraw from their savings anytime **maintaining a balance of at least 10% of their loan outstanding.**	
	Long Term Savings: Any client can participate in this product.	Members deposit from Tk. 50 to Tk. 1000. Members can withdraw from their savings anytime at an interest rate calculated on monthly basis. **For withdrawal before maturity she is given lower rate of return.**	
	Capital Buildup Savings Fund	Weekly premium is BDT 10 or monthly premium BDT 50.The duration of CBSF is 400 weeks. For withdrawal before its maturity the borrower is given interest benefit on deposited amount at a special rate. On death of a borrower his/her family is given twice the deposited amount as security.	
BRAC No. of Depositors 5,957950 Deposits (USD m) 635.14	General Savings		http://www.brac.net/program/microfinance/
	*Safe*save	Longer-term "commitment savings" account. Deposit regularly for a defined term of up to ten years and receive higher rates of interest.	
BURO Bangladesh No. of Depositors 1,449090 Deposits (USD m) 128.14	General Savings	The general savings account is like a current account, where customers can save or withdraw on demand.	https://www.burobd.org/microfinance-savings-product.php?id=12
	Contractual Savings	A way of building up useful lump sums: This savings can be invested or used for social obligations such as marriages,funeral or children's education. Higher interest than general savings. **In the contractual savings account clients agree to regularly deposit a set amount for a set period of time after which they can withdraw the entire amount plus the interest.**	
TMSS No. of Depositors 879600 Deposits (USD m) 73.10	General Savings, Special Savings, Monthly Savings		http://tmss-bd.org/annual-report-2016

Bangladesh: No. of active depositors 24,353,000 and total deposit USD 4,884m. *Source:* MIX (2017).

Table 8: Evidence for Savings in Latin America and the Caribbean (LAC)

Country No. of Depositors Deposits (USD m)	MFI No. of Depositors Deposits (USD m)	Product	Terms	Reference/url (accessed on 31st October, 2019)
Colombia 7,274,000 4,598	Banco Caja Social 4,655,300 3,352.44	Data not found	–	–
Peru 5,835,000 9,476	MiBanco 631,770 4,655.30	MiBanco (has) moved strongly into savings		Roodman (2009)
Bolivia 3,992,000 6,741	BancoSol 847,660 1,114.13	Liquid Savings Product: Cuenta de Ahorro		https://translate.google.com/translate?hl=en&sl=es&tl=en&u=https%3A%2F%2Fwww.bancosol.com.bo%2Fahorros
		Semi-liquid Savings Product: Cuenta de Mayor	**A minimum balance has to be maintained. The maximum number of withdrawal is 4 per month.**	
		Fixed Term Deposit: Deposito A Plazo Fizo		
		Sol Seguro: "(N)icely combines the virtues of insurance with an incentive to save." Some other products include Savings for children: Solecito (0-12 years) and SolGeneracion(13-17 years)		

Table 9: Evidence for Savings contd.

Region	Country No. of Depositors Deposits (USD m)	MFI No. of Depositors Deposits (USD m)	Product	Terms	Reference/url (accessed on 31st October, 2019)
East Asia and The Pacific (EAP)	Philippines 7,244,000 734	ASA Philippines 1,532,700 135.40	Capital Build-Up (CBU)	CBU is an alternative micro savings service for clients designed to promote the idea of poor families saving for the future in order to meet family emergencies and other needs. Withdrawable at any time.	http://asaphil.org/about/who-we-are/primary-services.aspx
			Locked-in Capital Build Up (LCBU)	**LCBU is fixed and mandatory, and serves as a monitoring tool of a client's performance and a basis for determining a client's loan renewal and any increase in loan amount. Non-withdrawable although it is 100% refundable.**	
	Indonesia 782000 29	Bank of Rakyat Indonesia (BRI) Unit Desa – –	"The global goliath of microsavings, BRI, offers all three" (liquid savings, locked-up savings and in between).		Roodman (2009)
	Vietnam 556000 3,404	Vietnam Bank of Social Policies (VBSP) — 2,463.85	Savings Deposit through savings and credit group (SCG) (An individual gets a loan from VBSP only if she is a member of SCG and saves)		http://eng.vbsp.org.vn/terms-savings-deposits.html http://eng.vbsp.org.vn/demand-savings-deposits.html
			Demand savings deposit		
			Term savings deposit		
Africa	Nigeria 4,240,000 184	Life Above Poverty Organization (LAPO) Microfinance Bank 2,631,980 90.97	Offers different savings product including – Regular Savings, Savings Plan Account, Term Deposit Savings, Voluntary Savings, Individual Savings, Festival Savings, My Pikin Savings		Roodman (2009) and http://product.lapo-nigeria.org/
Eastern Europe and Central Asia (ECA)	Mongolia 2,987,000 2,404	Khan Bank 2,397,570 2,001.23	Data not found	–	–

For the Referee

Proof of Remark 3. We prove the progressivity of the optimal loan scheme in a restrictive framework where (a) dissaving is not allowed and (b) the interest rate z charged by the MFI on loan repayment is time-invariant and exogenously given.[28]

Let Assumptions 1, 2 and 3 hold and the optimal scheme be $\langle\{\alpha_t^*\}_{t=0}^{T_M^*}, \{k_t^*\}_{t=0}^{T_M^*}, T_M^*\rangle$. We want to show that the optimal loan scheme (weakly) increases over time. Note that in this framework T_M^* can be ∞ which implies that the borrower never graduates. The profit of the MFI from this borrower at any $t \leq T_M^*$ is $(z-1)k_t$ and that is increasing in k_t.

Let us denote the borrower's present discounted value of lifetime utility from repayment at any t by V_t, where $0 \leq t \leq T_M$. To show that the optimal loan size is (weakly) progressive it is sufficient to show that the measure of either $[\underline{t}, \hat{t}]$ or $(\hat{t}, \bar{t}]$ such that $k_{t'}^* > k_{t''}^*$ $\forall t' \in [\underline{t}, \hat{t}]$, $t'' \in (\hat{t}, \bar{t}]$ and $\bar{t} \leq T_M^*$ is zero. We prove this by contradiction.

Suppose not. Measure of both $[\underline{t}, \hat{t}]$ and $(\hat{t}, \bar{t}]$ are positive. We construct another DIC contract such that the profit of the MFI is higher under this new contract, so the new contract cannot have been optimal. Before that consider DIC at any $t' \in [\underline{t}, \hat{t}]$: $V_{t'} \geq f(k_{t'}^*)$ and similarly DIC at any $t'' \in (\hat{t}, \bar{t}]$: $V_{t''} \geq f(k_{t''}^*)$. Now as observed above the profit of the MFI is increasing in k_t, so $k_{t'}^* > k_{t''}^*$ and DICs imply $V_{t'}^* > V_{t''}^*$ $\forall t'$ and t'', where $t' \in [\underline{t}, \hat{t}]$ and $t'' \in (\hat{t}, \bar{t}]$ and $\bar{t} \leq T_M^*$.

The new contract $\langle\{\hat{\alpha}_t\}_{t=0}^{\hat{T}_M}, \{\hat{k}_t\}_{t=0}^{\hat{T}_M}, \hat{T}_M\rangle$ is as follows:

$$\begin{cases} \hat{k}_t = k_t^* \text{ and } \hat{\alpha}_t = \alpha_t^* & \forall t \leq \hat{t} \\ \hat{k}_t = k_{\hat{t}}^* \text{ and } \hat{\alpha}_t = \alpha_{\hat{t}}^* & \forall t \in (\hat{t}, \bar{t}] \\ \hat{k}_t = k_{t-\hat{t}}^* \text{ and } \hat{\alpha}_t = \alpha_{t-\hat{t}}^* & \forall t \in (\bar{t}, \hat{T}_M] \\ \hat{T}_M = T_M^* + [\bar{t} - \hat{t}] & \text{if } T_M^* = \text{ finite} \\ \hat{T}_M = \infty & \text{otherwise.} \end{cases}$$

First observe the profit of the MFI is higher here. Now from the observation above that $V_{t'}^* > V_{t''}^*$ $\forall t'$ and t'', where $t' \in [\underline{t}, \hat{t}]$, $t'' \in (\hat{t}, \bar{t}]$ and $\bar{t} \leq T_M^*$, and the construction it is easy to check that the new contract is DIC $\forall t \leq \hat{T}_M$: For that first observe DICs at all $t \leq \hat{t}$ are satisfied because the original contract is DIC and the utility from default at any such t is the same whereas that from repayment has increased. Second, DICs at any $t \in (\hat{t}, \bar{t}]$ is satisfied because the original contract is DIC at $\hat{t}$. Finally, DICs at any $t \in (\bar{t}, \hat{T}_M]$ is satisfied because the original contract is DIC at $t \in (\hat{t}, T_M^*]$.■

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[28] These two restrictions are *with* loss of generality. First, not allowing dissaving is with loss of generailty because in case the borrower does not graduate, this restriction implies that at the optimum there will be no savings at all. That may impose a serious restriction as we are ruling out the possibility of cycles in the optimum loan scheme. Second, time-independent, exogenous interest rate is with loss of generailty because this is actually a choice variable of the MFI, even if we assume that there is an exogenously given upper bound on the interest rate, the profit-maximizing MFI may optimally choose that to be lower than the upper bound at some instances.

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