Centralization vs. Delegation: A Principal-Agent Analysis

Dushyant Kumar (Indian Statistical Institute)

Abstract

We study two types of organizational structures, namely centralization and delegation. Centralization refers to a contractual relationship where the principal contracts with all the agents directly. Delegation refers to a contractual relationship where the principal contracts with some agents (a proper subset, to be precise) and give them the right to contract with others. We allow for the collusion among agents. Both these contracts are quite common in production networks, supply chain managements and procurements. We provide an intuitive model for the collusion among agents. We show that the collusion among agents maybe beneficial for the principal also. Hence the whole issue of justifying delegation as counter-strategy to collusion is somewhat misplaced. We provide a sufficient condition under which delegation outperforms centralization. We provide the first-order conditions which characterize Nash equilibrium outcomes for both, centralization with and without collusion cases.
1 Introduction

In a principal-agent setup, centralization refers to a contractual relationship where the principal contracts with all the agents directly. Delegation or decentralization refers to a contractual relationship where the principal contracts with some agents (a proper subset, to be precise) and asks them (or better say, give them the right) to contract with others. Centralization versus decentralization is a really old debate in social science and it is still wide open. In the context of organisational structures or contracts, this is a widely studied question. The literature tries to find the optimal organisational design. This literature tries to understand the internal organisation of the firm, kind of trying to explain the *black box*.

Both centralized, as well as delegated contracts are quite common. For centralized contracts, consider the case of bundled goods. Often different brands come together and sell their products as a bundle. Here the customer is principal and different brands or companies are agents. So, the consumer can be thought of as contracting with all the agents, i.e., brands or companies. So different brands of the bundle are reliable separately. There is no main contractor or subcontractor in this case.

For delegated contracts, consider the personal computer industry. When someone buys a laptop, notebook or all-in-one PC, often it comes with several loaded softwares (Windows 7, MS-Office, e.t.c), which the computer manufacturer procure from different companies/suppliers. Even for hardware, often different companies supply sub-parts. So, if someone is buying a Lenovo notebook, it is not the case that Lenovo Corp. is manufacturing all major parts itself. It procures processor, motherboard, audio devices, e.t.c from different companies like Intel, Asus, AMD, SRS, e.t.c. In this case the customer can be thought of as the principal, the Lenovo Corp. as the main contractor and others like Intel, Asus, Gigabyte, Seagate, Microsoft, e.t.c as sub-contractors. The customer signs the explicit contract only with the Lenovo Corp. So, if there is any problem with, say, the audio of the laptop, one goes to the Lenovo service centre not the audio device producer, maybe SRS. Similar kind of contracts are found in auto-mobile industry. Company
like Tata Motors, Ford Motor Company, Volkswagen Group and others also follow similar organisational setup for production.

There are many other examples of centralized, as well as delegated contracts. However, in a standard theoretical framework, it is hard to justify the presence of delegated contracts. In a principal-agent framework, with no-collusion among agents, the revelation principle insures that centralized contracts can always achieve whatever delegated contracts can. This is a standard result in the literature. Here the centralized contracts can be thought of as direct contracts and the delegated contracts can be thought of as indirect contracts.

Recently many authors have tried to look at the possibility of collusion among agents as a justification for the existence of delegation. The literature tries to identify conditions under which delegation outperforms centralization. Here, we first explore the centralized contracts, with and without collusion among agents. Then we proceed to study the delegated contracts. We provide a sufficiency condition under which delegation outperforms centralization. As is true with most of the literature related to information economics, this one is also mainly divided along two lines: adverse selection and moral hazard models. A general result is yet to emerge.

One crucial aspect of this literature is the way to capture the collusion among agents. Most of the literature seems to capture it through some enforceable side-contract among agents. However the literature is more or less silent on the issue of enforceability as well as the details of these side-contracts. Most of the papers deal in somewhat restrictive structure, like either adverse selection or moral hazard, discrete costs, two agents, special information structure, etc. This paper is aimed to contribute to the existing literature which deals with the comparative analysis of centralized contracts and delegated contracts in the presence of collusion. It is an one-principal two-agents model. But the methodology allows for the any number of agents. It deals in a framework where adverse selection and moral hazard both exist. However it is a particular type of adverse selection-moral hazard
setup. In this setup, adverse selection and moral hazard are deterministically related to each other. So possible set of manipulation on one dimension (say, AS) is restricted by the other (MH). We describe this formally in next section. In future works, we would like to extend it to a more general setup. Both, the types and efforts, of agents, are continuous. We start with analyzing centralization with perfect collusion among agents.

Our motivation for starting with the perfect collusion, is a case study of the Boeing Corporation. It is related to the Boeing 787 Dreamliner project. Boeing 787 is a mid-size aircraft which was supposed to be a game-changer in the aircraft industry. Commercially it is a huge success. It has been in news for many wrong reasons like technical glitch, slow delivery, etc. To some extent, these problems are due to the changes made in supply-chain management for this project. However, we are interested in these changes for a different reason. To our interest, the Boeing Co. has changed the role of suppliers dramatically for this project. It has decreased the number of suppliers directly contracting (with the Boeing Corporation). These suppliers are now called global partners and they share the responsibilities to manage and extend the supply chain. They contract with other small suppliers and supply subsystems instead of the parts. Another interesting point is, the Boeing Co. is promoting regular meetings and collaborations among suppliers. It is like they are promoting collusion among agents.

There are others cases also where the procurer seems to encouraging collaborations among suppliers. These cases suggest that the procurer might also benefit from efficient collusion among suppliers. We have shown that this indeed is the case. We capture collusion among agents by their ability to manipulate types’ representations and efforts in a coordinated way. Moreover they can manipulate their cost-accounts’ books. This notion of collusion allows for cost-synergies in production of inputs which might benefit the principal as well.

To better understand these synergies, consider a simple example. Suppose an academic institute wants to build a computer lab and for that it
wants to procure computer systems, hardware as well as software. It has a budget of 100 and it values one computer system at 38. These are common knowledge. There are two suppliers, $A_1$ and $A_2$. $A_1$ supplies the hardware and $A_2$ supplies the software. The production costs of both hardware as well as software (for one computer system) are 10. To simplify things further, assume that the academic institute doesn’t have any information about the production costs of $A_1$ and $A_2$. Now consider a situation where both $A_1$ and $A_2$ quote a price of 19. They will be both supplying 2 units and their profits will be 18 each. The institute payoff will be zero. This is a Nash equilibrium in non-collusion scenario. Now allow for collusion between $A_1$ and $A_2$. If they can bring down the aggregate price to 33 instead of 38, they can earn a total profit of 39 instead of 36. The institute’s payoff also increases to 15. Under Collusion, this is possible. Moreover, under collusion this is the unique Nash equilibrium. *The institute (the principal, in our model) also gains from the collusion among suppliers (the agents)*. Notice here we don’t need perfect collusion or the agents to have perfect information about each other. This is one kind of synergy. As explained above, similarly there can be other synergies also. In the context of consumer goods, it can be thought of as the phenomenon of bundled goods. Many times, brands come together to offer discounts and all. Here the customer is principal and these brands suppliers. It can be argued that there exist situations where it is a win-win situation for both.

Most of the literature focus on restricting the collusion among agents. The implicit assumption behind this approach is that collusion among agents is bad for the principal. Here we would like to stress on the point that while forming coalition, the agents try to maximize there own utilities. The agents’ and principal’s interest need not always be in conflict. We provide an appropriate way to model collusion among the agents and we derive a sufficiency condition under which the principal prefers collusion among agent.

We also differ from the existing literature on the information structure. we assume that people at the same level of hierarchy know better about each-other as compared to people across hierarchy. So in this framework,
the agents know more about each-other as compared to what the principal
knows about agents. This seems a reasonable assumption in any institutional structure. Notice that in both the examples of delegation that we have
given above, it can be argued safely that contractors/suppliers/agents know
about each-others more than the customer/principal knows about them. In
any society, people from same income-level/profession/background interact
more with each-other, in general and hence know more about each-other as
compared to someone from different income-level/profession/background.
Similar information structures are used in evolutionary literature, literature
on lobbying and tournaments, etc. Dubey and Sahi (2012) analyses the op-
timal prize allocation technique. They use an information structure where
the principal doesn’t have any information about the agents’ skill however
the agents knows about each-other skills.

We derive first-order-conditions for the optimality for both, centraliza-
tion with and without collusion. This approach is useful in many other contexts as well. It provides a convenient way to derive Nash equilibrium
outcome in different cases.

1.1 Related Literature

Recently there has been a lot of work in this area. In mechanism design and
contract theory literature, there are several papers along the lines of ad-
verse selection and moral hazard frameworks. Lafont and Martimort (1998)
is closely related to our work here in terms of modeling the collusion among
the agents. This paper deals in an adverse selection framework where agents
supply perfectly complementary goods and their costs take just two possi-
ble values. Here the collusion among agents is organized by a third party,
who cares for both agents symmetrically. Authors find that in this setup,
collusion does not have any bite on the organisational efficiency. This result
crucially depends on just two possible realization of costs. They find that
both centralization and delegation perform equally well. This result depends
on their assumption of perfect complementarity and two cost types. They
also consider limited communication case. In the presence of both limited
communication and collusion possibilities, delegation strictly dominates centralization, if presence of limited communication restrict the centralization to treat both agents symmetrically. Baron and Besanko (1999) is also somewhat similar in formulation.

Another closely related paper is Mookherjee and Tsumagari (2004). It deals in a one-principal two-agents framework with adverse selection and collusion among agents. The principal and any particular agent share a common belief about the other agent’s type. In the given setup, it is shown that delegating to one agent the right to subcontract with other agent always earns lower profit for the principal compared to the centralized setup. Here the collusion among agents consists of coordinating cost reports, reallocation of production assignments and payments received by the principal. This is done through an enforceable side-contract among agents which is not observable to the principal. The principal cannot observe production reallocation but can verify aggregate output. For side-contract, all the bargaining power rest with one particular agent. That particular agent make a take-it-or-leave-it offer to the other agent. So while comparing with delegation to that particular agent, centralization can achieve this outcome by offering a null contract to the other agent.

Che and Kim (2006) nicely sums up the issue of collusion having a detrimental effect on the principal’s utility, in adverse selection framework. The principal can attain the second best outcome if the following three conditions are satisfied: 1. correlation of the colluding agent’s type satisfy a sort of full rank condition, 2. transferable utility, and 3. agents take participation decision before collusion decision.

Baliga and Sjostrom (1998), Itoh (1993) and others address similar issues in a moral hazard framework.

There are several related works on collusion in industrial organisation literature. In a oligopolistic framework, Salvo and Vasconcelos (2012) have shown that collusion among the producers can increase the consumer wel-
fare for a significant range of parameters. In their competition regulation related writings, Farrell and Shapiro have argued that a merger among two firms need not always increase the price. A merger definitely decreases competition but it also bring synergies. So, the net effect depends on which of these two effect dominates.

Here, we try to combine the insights from these two literature (mechanism design-contract theory and organisation theory).

2 The Model

Consider a procurement setup. There are three strategic players in this setup: one principal ($P$) and two agents ($A_1$ and $A_2$). Here the principal ($P$) wants to procure two goods $q_1$ and $q_2$ from two agents $A_1$ and $A_2$, respectively. First the principal announces a menu of contracts (explained later in the section). Then the agents choose the ones that maximize their respective utilities. The agents supply the goods accordingly. The principal then reimburses as per the contract.

Let us now introduce some notations. Let $V(q_1,q_2)$ denote the value that the principal gets out of this procurement process. Let the agent $A_i$’s cost of producing $q_i$ be denoted by $C_i(\beta_i, e_i, q_i)$. Here, $\beta_i$ is the technological/productivity parameter of agent $A_i$. It can also be referred as agent’s type. $e_i$ is agent $A_i$‘s effort which results in cost reduction. Finally, the effort ($e_i$) entails disutility to the agent $A_i$, let us denote it by $\Psi_i(e_i)$. Let $U_i$ denote the utility of agent $A_i$.

We shall carry the following assumption throughout this paper.

Assumption 1.
(i) $V(q_1,q_2)$ is twice differentiable. Both inputs are essential i.e., $V(0,q_2) = V(q_1,0) = 0$. Moreover $V(q_1,q_2)$ is increasing and concave in the inputs $q_1$ and $q_2$.
(ii) $C_i(\beta_i, e_i, q_i)$ is twice differentiable with $C_{\beta_i} > 0$, $C_{e_i} < 0$, $C_{q_i} > 0$.
(iii) $\Psi_i(e_i)$ is twice differentiable with $\Psi_i' > 0, \Psi_i'' > 0, \Psi_i''' \geq 0$. 

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We assume that the principal observes the total cost $C_i$ for both the agents but cannot observe either an agent’s type or effort. An agent knows her type before signing the contract. For principal, $\beta_i$ is drawn randomly from some cumulative distribution function $F(\beta_i)$ with support $[\beta_i, \beta_i]$ and with density function $f(\beta_i)$. So, it is a restricted adverse selection-moral hazard (AD-MH) setup. In the literature, it has been shown that this setup is qualitatively equivalent to the adverse selection setup. We persist with this setup as it is more suitable and intuitive for the modeling of collusion among agents. Also later on we would like to extend the analysis to a general AD-MH setup.

We work with the following payment protocol for agent $A_i, \ i = 1, 2$: the principal reimburses total cost $C_i$ and in addition transfers some amount $t_i$. The transfer of $t_i$ can be thought of as compensation towards the agent’s effort as putting effort entails disutility or cost for the agent. We now write down the utility function of both, the principal and the agents, taken to be risk neutral. The utility function of the principal is given by:

$$V(q_1, q_2) - \Sigma_i(t_i + C_i)$$

Whereas the utility function of the agents are given by:

$$U_i = t_i - \Psi_i(e_i)$$

Now we define minimum effort function $E_i(\beta_i, C_i, q_i)$ from total cost function in the following manner:\footnote{We use Assumption 1, particularly $C_{e_i} < 0$, and implicit function theorem to derive function $E_i(\beta_i, C_i, q_i)$.}

$$C_i = C_i(\beta_i, E_i(\beta_i, C_i, q_i), q_i)$$

Given the production technology, to produce a given output, a particular type agent also need to put some effort. The cost of production depends on the amount of effort put. Given our specifications, higher the effort, lower
the cost; lower the effort, higher the cost. \( E_i \) is the minimum effort that a \( \beta_i \) type agent require to put in order to produce the output \( q_i \) at the cost \( C_i \). Here we have, from Assumption 1,

\[
E_{\beta_i}^i > 0, E_{C_i}^i < 0, E_{q_i}^i > 0.
\]

Now we turn to first solve the complete information case. This will serve as a benchmark case for the rest of the analysis.

3 Perfect Information Case

Suppose the principal knows agents’ types perfectly and can monitor their efforts perfectly too. Then the principal will simply maximize the following objective function with respect to quantities and efforts level and get it implemented by agents just subject to the individual rationality constraints (of agents).

\[
\max V(q_1, q_2) - \sum_i (t^i + C^i) = V(q_1, q_2) - \sum_i (U^i + \Psi_i(e_i) + C_i)
\]

subject to \( U^i \geq 0 \) for \( i = 1, 2 \).

Since \( U_i \) appears with a negative sign in the objective function, following hold:

**Lemma 1** Let Assumption 1 hold. The principal’s utility maximization implies \( U^i = 0 \) for \( i = 1, 2 \).

*Proof:* Let \( U^i > 0 \) for at-least one \( i \). So, \( t^i > \psi_i(e_i) \) as \( U^i = t^i - \psi_i(e_i) > 0 \). Notice this is a complete information case, hence the principal can perfectly observe and monitor \( e_i \). Let the principal decrease \( t^i \) to \( t^i = \psi_i(e_i) \). This increases the principal’s utility while the agent’s IR constraint continues to hold. Hence in equilibrium, \( U^i = 0 \) for \( i = 1, 2 \). 

First-order conditions for maxima will be given by following set of equations:

\[
V_{q_i} = C_{q_i} \text{ for } i = 1, 2.
\]
\[ \Psi_i' = -C_{e_i}^i \text{ for } i = 1, 2. \] (4)

These are standard marginal cost equals marginal benefit equations. We will have standard second-order sufficiency conditions. We now turn to the analysis of asymmetric information case.

4 Asymmetric Information

Now we consider the asymmetric information case described earlier in the introduction part. Here principal has several options regarding organisational design. We will be considering two particular (extreme) organisational setups:

- **Centralization**: The principal contracts with both agents directly.
- **Delegation**: The principal contracts with agent $A_i$ and gives him the right to contract with agent $A_j$.

So one natural question to ask is, from principal’s point of view which of the above setup is better? If we assume that agents behave in non-cooperative way or they don’t collude in the process of their interaction with the principal, the answer to the above question is straightforward. Myerson (1982) ensures that centralization can always achieve the payoff (for the principal) whatever delegation can. This is an implication of generalized revelation principle. Here centralized contracts can be viewed as direct contracts whereas delegated contracts can be viewed as indirect contracts. Once we allow for collusion among agents, we can’t claim the same.

4.1 Centralization with Collusion

In this subsection we analyze the centralized contracts, with collusion. In literature, one crucial and debated aspect is ‘how to capture the collusion in such framework’. Here we will be trying to capture the notion of perfect collusion in one particular way which seems quite reasonable. Suppose there is one third party 'A' who knows both agents’ type perfectly and can monitor their efforts perfectly too. This third party manages the coalition among agents. This third party can be thought of as trade union, labour union
or industry federation. So from principal’s point of view, it is like he/she is contracting with this third party A for the input vector \((q_1, q_2)\). This is somewhat realistic in the context of labour union literature (bargaining models).

Let us denote \(B = (\beta_1, \beta_2), e = e_1 + e_2\) and \(Q = (q_1, q_2)\). Then if A can be characterized by total cost function \(C(B, e, Q)\), this ‘centralization with perfect collusion’ analysis can be seen as one principal-one agent problem which can be solved relatively easily and neatly.

We now turn to write down the game form for centralization with collusion case:

**Stage 1.** The principal offers a contract which is essentially a transfer schedule \(t(C, Q)\), where \(C\) is the cost observed by the principal and \(Q\) is the quantity of good produced.

**Stage 2.** Agents’ types are realized and they take the participation decisions. If both agents decide to participate, the game continues to the next step, otherwise the game ends with principal and agents getting their outside options.

**Stage 3.** The agents cooperatively decide the optimal \((Q, e)\) for the given \(t(C, Q)\). If they fail to reach the agreement they choose the quantities and efforts non-cooperatively.

**Stage 4.** The principal reimburses the realized costs and makes transfers. Payoffs for principal and agents are realized accordingly.

**Stage 3** captures the collusion process here. Without collusion, the agents can misreport their types and efforts independently, subject to the principal observing the total costs. With collusion they can misreport in a coordinated way. Moreover the principal can no longer observe their total costs \((C^1\text{ and } C^2)\) separately, the principal only observe \(C^1 + C^2\). So the agents can misreport their types \((\beta_i)\) and efforts \((e_i)\) subject to the principal
observing only $C^1 + C^2$. Hence here the agents or the coalition forming authority $(A)$ can divert agent $A_i$’s effort to the $A_j$’s production activity. For simplicity, we assume here that $A_i$’s effort and $A_j$’s effort are homogeneous. So one unit of $A_i$’s effort diverted to $q'_j$’s production activity works just like an extra unit of $A_j$’s effort in cost-reduction in $q'_j$’s production.

Under this setup $A$ (on behalf of the agents) will like to assign the efforts, $e_1$ and $e_2$, in the optimal way. For every level of total effort $(e)$, $e_1$ and $e_2$ will solve the following problem (for a given transfer schedule):

$$
\max_{(e_1,e_2)} t^1 + t^2 - \Psi_1 - \Psi_2
$$

subject to $e_1 + e_2 \equiv e$.

This will give an aggregate cost function $C(B,e,Q)$, as well as aggregate disutility function $\Psi(e)$. Now for some class/family of cost functions, we get aggregate cost function as $C(f(\beta_1, \beta_2), e, Q)$. In these cases, we can simply take $f(\beta_1, \beta_2) = \beta$ as the A’s type. Separable cost functions are one such family of cost functions.

Suppose the aggregate cost function takes the form $C(\beta, e, Q)$ where $\beta = f(\beta_1, \beta_2)$. As earlier, define $E(\beta, C, Q)$ as

$$
C \equiv C(\beta, E(\beta, C, Q), Q).
$$

The principal will maximize his net expected payoff

$$
\int_\beta [V(Q) - U - \Psi(e) - C] f(\beta) d\beta.
$$

subject to individual rationality (IR) and incentive compatibility (IC) constraints. For simplicity we assume that outside options for both agents are zero. So IR constraints take the form

$$
U(\beta) \geq 0 \ for \ all \ \beta.
$$

As is standard in the literature, the IC constraints come from the agents’ optimization exercise. It is just like a standard Stackleberg case where
principal incorporates agents’ behaviour in his optimization. A derivation of this IC constraints is done in the Appendix. IC constraints take the form

$$\dot{U}(\beta) = -\Psi'(e)E_{\beta}(\beta, C, Q).$$ (5)

Intuitively, an agent of type $(\beta - \Delta \beta)$ can produce an output vector $Q$ at the same cost as that of the agent with type $\beta$ by decreasing her effort by $\Delta e = E_{\beta} \Delta \beta$. So above constraint implies that any agent don’t have incentive to do it. Carroll (2012) shows that in our setup, the above local incentive compatibility implies full (global) incentive compatibility. He particularly uses the convexity of domains (types) and quasilinear preferences to show this.

**Lemma 2** Let Assumption 1 hold. Then the following is true:

1. $\dot{U}(\beta) < 0$.

2. The set of individual rationality (IR) constraints reduces to a single constraint: $U(\beta) = 0$.

**Proof:** Consider the agents’ optimization problem. Suppose an agent of (true) type $\beta$ announces his/her type $\alpha$. The agents will announce his/her true type if it solves the following problem

$$\max_{\alpha} U(\alpha/\beta) = t(\alpha) - \Psi(E(\beta, C(\alpha), Q(\alpha)))$$

Evaluating the first-order conditions at $\alpha = \beta$, gives

$$\dot{U}(\beta) = -\Psi'(e)E_{\beta} < 0$$

Here, from Assumption 1, $\Psi'(e) > 0$ and $E_{\beta} > 0$. Hence $\dot{U}(\beta) < 0$.

Second part of the lemma is an implication of the first part. $U(\beta) = 0$ and $\dot{U}(\beta) < 0$ ensures that the IR constraints for all the types are satisfied.

Now using Lemma 2, the principal’s problem becomes

$$\max_{Q, e, U} \int_{\beta} [V(Q) - U - \Psi(e) - C]f(\beta)d\beta$$
subject to $\dot{U}(\beta) = -\Psi'(e)E_{\beta}(\beta, C, Q)$
and $U(\beta) = 0$.

This can be thought as an optimal control problem with $Q(\beta)$ and $e(\beta)$ as control variables and $U(\beta)$ as state variable. We setup the Hamiltonian and use first-order necessary and second-order sufficiency conditions for maximization. We solve it in the the Appendix, here we are just stating the first-order conditions that govern the optimal choice of quantity and effort level.

The first-order conditions take the form

$$\Psi'(e) = -C_e - \frac{F'(\beta)}{f(\beta)} \left[ \Psi''(e)E_{\beta} + \Psi'(e)E_{\beta C}C_e \right]$$ \hfill (6)

$$V_{q_i} = C_{q_i} + \frac{F'(\beta)}{f(\beta)} \Psi'(e) \frac{dE_{\beta}}{dq_i} \text{ for } i = 1, 2. \hfill (7)$$

These first-order conditions characterizes the efforts and quantities level in the equilibrium. These are kind of modified marginal cost equals marginal benefit equations. The second term on the right-hand side of the both equations captures the distortion due to the informational asymmetries.

We can summarize above findings in the following proposition:

**Proposition 1** Let Assumption 1 hold. Above equations 6 and 7 gives the Nash equilibrium outcome $(e^*, q^*_i)$, for the centralization with collusion case.

### 4.2 Benevolent Principal

In the above analysis we have assumed that the principal does not care about the agents’ welfare at all. In some cases this might not be an appropriate assumption. The government sector is a major example. Even in the private sector, employers don’t ignore employees’ welfare entirely. So let us consider the case where the principal does care about the agents’ welfare. Allowing
for such behaviour, the principal’s objective function changes to
\[
\max_{Q,e,U} \int_{\beta} [V(Q) - (1 - \lambda)U - \Psi(e) - C] f(\beta) d\beta
\]
where \(\lambda\) is kind of a motivation parameter (Besley and Ghatak (2005)).

It can also be referred as the degree of careness (by the principal) of agents’ welfare. So the initial analysis is one special case where \(\lambda\) is equal to zero.

The first-order conditions become (derivation is shifted to the Appendix):
\[
\Psi'(e) = -C_e - \frac{(1 - \lambda)F(\beta)}{f(\beta)} [\Psi''(e)E_\beta + \Psi'(e)E_\beta C_e] \\
V_{q_i} = C_{q_i} + \frac{(1 - \lambda)F(\beta)}{f(\beta)} \Psi'(e) \frac{dE_\beta}{dq_i} \text{ for } i = 1, 2.
\]

So, as \(\lambda \to 1\), i.e., principal assigns same weight to his/her payoff as that of agents’ payoffs, effort and quantity level tend to the first best level showed earlier. The second term in the right hand side of both the equations are distortions due to asymmetric information. These distortions disappear as \(\lambda \to 1\). The joint net payoff of principal and agents is maximized when the distortions are set to zero. In a sense here the principal internalizes the distortion. We can summarize this in the following proposition:

**Proposition 2** Let Assumption 1 hold. As \(\lambda \to 1\), the centralized contract attains the outcome of the complete information case.

This result hold for both, with and without collusion case.

### 4.3 Coalition Formation

In our setup of coalition design, the aggregate rationality of the coalition formation implies individual rationality for agents also. If as a group \(A_1\) and \(A_2\) can generate some net surplus from colluding, \(A\) can always distribute it in such a way that both agents will be better off. So the rationality constraint for the coalition formation takes the form
\[
U_{\text{collusion}} \geq U_{\text{noncollusion}}^1 + U_{\text{noncollusion}}^2
\]
Further this rationality of coalition-formation will always be weakly satisfied because A always has an option to dictate agents to opt for the non-cooperative behaviour.

While maximizing principal’s utility, the literature has generally focuses on the ways to restrict collusion among agents. The implicit assumption behind this approach is that collusion among agents hurts principal. Here we would like argue and demonstrate that this need not be the case always. We would like to stress the point that the agents’ objective is to maximize their own utilities, not to hurt principal’s utility. So there might be cases where the principal also prefers collusion to non-collusion. This is possible because in case of collusion, there might be cost synergies \((C \rightarrow \min(C_1+C_2))\) and the principal might also get benefited from it. This is applicable in other contexts as well, like consumer-producer case.

Formally, consider the following procurement setup. The principal \(P\) wants to procure two goods, \(q_1\) and \(q_2\), from two agents, \(A_1\) and \(A_2\), respectively. For simplicity, we modified the timings as the following: first the agents quote their prices, \(P_1\) and \(P_2\), and then the principal chooses the quantities, \(q_1\) and \(q_2\). The value that principal gets out of this procurement, is denoted by \(V(q_1, q_2)\). \(V\) is increasing and concave in inputs, \(q_1\) and \(q_2\). Let us denote cost of production for agent \(A_1\) by \(C_1(q_1)\) and that of agent \(A_2\) by \(C_2(q_2)\). In this section, we have eliminated \(\beta\) and \(e\) from the cost functions. This is to simplify the analysis. Let us denote utilities of principal and agents by \(U^P\) and \(U^i\), where \(i = 1, 2\), respectively.

\[
U^P = V(q_1, q_2) - P_1q_1 - P_2q_2.
\]

\[
U^i = P_i q_i - C_i(q_i) \quad \text{for} \quad i = 1, 2.
\]

First, we solve the principal’s optimization problem for given \(P_1\) and \(P_2\). Then we incorporate this in agents’ utilities and solve the agents’ optimization problem.
Principal’s optimization:
\[
\max_{q_1, q_2} V(q_1, q_2) - P_1 q_1 - P_2 q_2.
\]

The interior solution must satisfy the following first-order necessary conditions:
\[
V_{q_1} - P_1 = 0
\]
\[
V_{q_2} - P_2 = 0
\]

We can solve these two first-order conditions to derive \( q_i^*(P_1, P_2) \) for \( i = 1, 2 \). The second-order sufficiency condition is satisfied here, since we have assumed concavity of the value function \( V \).

For agents’ optimization, first we solve it for without collusion case. Both agents’ maximize their utilities independently.

Agents’ optimization:
\[
\max_{P_i} P_i q_i^*(P_1, P_2) - C_i(q_i^*(P_1, P_2)) \quad \text{for} \quad i = 1, 2.
\]

The interior solution must satisfy the following first-order necessary condition:
\[
q_i^*(P_1, P_2) + P_i \frac{\partial q_i^*(P_1, P_2)}{\partial P_1} - \frac{\partial C_i}{\partial q_i} \frac{\partial q_i^*(P_1, P_2)}{\partial P_1} = 0
\]

Here if \( q_i^*(P_1, P_2) > 0 \) and \( \frac{\partial q_i^*(P_1, P_2)}{\partial P_i} < 0 \) (strictly positive demand and negative own price effect), we have \( P_i - \frac{\partial C_i}{\partial q_i} > 0 \).

For maximization, the following second-order sufficiency condition also need to be satisfied:
\[
\frac{\partial^2 U_i}{\partial P_i^2} < 0.
\]

Let us denote the solution of this exercise as \( \hat{P}_i \).

Now we solve the agents’ optimization for perfect collusion case. Here agents’ maximize their joint profit cooperatively:
\[
\max_{P_1, P_2} \sum_{i} P_i q_i^*(P_1, P_2) - C_i(q_i^*(P_1, P_2))
\]
The interior solution must satisfy the following first-order necessary conditions:

\[
\frac{\partial U^1}{\partial P_1} + \frac{\partial U^2}{\partial P_1} = 0 \tag{9}
\]

\[
\frac{\partial U^2}{\partial P_2} + \frac{\partial U^1}{\partial P_2} = 0 \tag{10}
\]

Again we will have corresponding second-order sufficiency conditions.

We analyze these two first-order conditions 9 and 10 at the price vector \((\hat{P}_1, \hat{P}_2)\). First, we do it for equation 9. Similar analysis hold for 10. At this price vector, the first part of the left-hand-side (LHS) of the equation is zero, from equation 8. If we assume strictly positive demand for both goods, negative own price effect and negative cross price effect \(\left(\frac{\partial q^*_2}{\partial P_1} < 0\right)\), the second part is negative. So the LHS is negative at the price vector \((\hat{P}_1, \hat{P}_2)\). Moreover, at this price vector,

\[
\frac{\partial^2(U^1 + U^2)}{\partial P_1^2} < 0 \quad \text{if} \quad \frac{\partial^2 q^*_1}{\partial P_1^2} < 0.
\]

Let us denote the price vector, which satisfy equations 9 and 10, as \((\bar{P}_1, \bar{P}_2)\). So, if we have strictly positive demand for both goods, negative own price effect and negative cross price effect and \(\frac{\partial^2 q^*_1}{\partial P_1^2} < 0\), these conditions imply \(\bar{P}_i < \hat{P}_i\) for \(i = 1, 2\). Lower prices imply that the principal’s utility is higher when agents collude perfectly. All these conditions are satisfied for \(V(q_1, q_2) = q_1^{\alpha_1}q_2^{\alpha_2}\), where \(\alpha_1 + \alpha_2 < 1\).

We can summarize this in the following proposition:

**Proposition 3** Let Assumption 1 hold. Let \(V(q_1, q_2) = q_1^{\alpha_1}q_2^{\alpha_2}\), where \(\alpha_1 + \alpha_2 < 1\). Then the principal prefers perfect collusion compared to no collusion (among agents). This is true for all \(V(q_1, q_2)\), for which we have strictly positive demand for both goods, negative own price effect and negative decreasing cross price effect.

A somewhat similar arguments can also be found in the literature of corruption. Bag (1997) explore the ways to control corruption in hierarchies. However in it’s conclusion, the author mentions about not modeling the cost
and benefits of corruption. Bac and Bag (2006) revisits this issue. Here the authors do the cost-benefit analysis and identify the conditions under which the collusion among agent and supervisor benefit the principal.

In a recent work Deltas, Salvo and Vasconcelos (2012) has shown that collusion among oligopolist producers can increase the consumer surplus. It is a different setting, but the arguments are on the somewhat similar lines. These kind of arguments are also present in the literature on mergers and anti-competition policies. In Farrell and Shapiro (2008), the authors talk about cost synergies or efficiencies. They proposes the idea of net upward pricing pressure. The merger generate two effects: lesser competition and cost savings (better coordination). The first one tends to increase the pricing whereas the second one acts in opposing direction. If the second factor dominates then net upward pricing pressure can be negative and in these cases the merger can beneficial instead of being harmful.

4.4 Centralization without Collusion

Under centralization, we can have the outcome either with or without collusion. Even if beneficial, the without collusion outcome can occur (because of coordination failures, among other reasons). In the no-collusion case, the timings of the game changes to the following:

Stage 1. The principal offers a contract which is essentially a transfer schedule $t(C, Q)$, where $C$ is the cost observed by the principal and $Q$ is the quantity of goods produced.

Stage 2. Agents’ types are realized and they take the participation decisions. If both agents decide to participate, the game continues to the next step, otherwise the game ends with principal and agents getting their outside options.

Stage 3. Both agents independently decide the their optimal $(q_i, e_i)$ for the given $t(C, Q)$. They produce and supply their optimal quantities.
Stage 4. The principal reimburses the realized costs and makes transfers. Payoffs for principal and agents are realized accordingly.

For non-collusion case, the optimization problem becomes

$$\max_{Q,e,U} \int_{\beta_1} \int_{\beta_2} \left[ V(Q) - U^1 - U^2 - \Psi_1(e_1) - \Psi_2(e_2) - C^1 - C^2 \right] f(\beta_1, \beta_2) d\beta_1 d\beta_2$$

subject to

$$\dot{U}^1(\beta_1) = -\Psi_1'(e_1)E^1_{\beta_1}$$
$$\dot{U}^2(\beta_2) = -\Psi_2'(e_2)E^2_{\beta_2}$$
$$U(\beta_1) = 0$$
$$U(\beta_2) = 0$$

This is a two dimensional optimal control problem. We use the Hamiltonian technique to solve this. We derive the first-order conditions for the optimality in the Appendix. For without collusion case, it is a well established result in the literature that the centralization perform (at-least weakly) better than any other forms of organisational structure. It is based on the application of generalized revelation principle. However the technique that we use here to derive equilibrium outcomes \((e^*_i, q^*_i)\) can be quite useful in many contexts. It provides a convenient way to derive Nash equilibrium outcome.

5 Delegation

In our setup, delegation corresponds to the case where the principal contracts with one agent, giving him the right to contract with the other agent. Recall the examples of the personal computer industry and the automobile industry given in the introduction section. As assumed throughout the paper, we work with the information structure where the agents at the same hierarchy knows more about each-other than across hierarchy.

Consider the setup that we have used in the coalition formation subsection. Suppose the principal delegate, say agent \(A_1\), the right to contract with agent \(A_2\). Now agent \(A_1\) will choose the price vector which maximizes his profit. Here \(A_1\) can set the other agent’s (\(A_2\’s\)) utility to zero by just
paying him the production cost of $q_2$, given the information structure that we have assumed. Given this, $A_1$ will choose the price vector $(\bar{P}_1, \bar{P}_2)$, as it maximizes the joint profit. So under delegation, principal’s utility is given by:

$$U^P(\bar{P}_1, \bar{P}_2) = V(q_1^*(\bar{P}_1, \bar{P}_2), q_2^*(\bar{P}_1, \bar{P}_2)) - \bar{P}_1 q_1^*(\bar{P}_1, \bar{P}_2) - \bar{P}_2 q_2^*(\bar{P}_1, \bar{P}_2) \tag{11}$$

Under centralization, the principal’s utility will be $U^P(\bar{P}_1, \bar{P}_2)$, if there is perfect collusion among agents. In absence of perfect collusion, the principal’s utility will be different. Under centralization, we might not have perfect collusion among agents due to various coordination reasons. Under the conditions described in the subsection coalition formation, i.e. strictly positive demand for both goods, negative own price effect and negative cross price effect and $\frac{\partial^2 q_2^*}{\partial P_1^2} < 0$, we know that the principal’s utility is maximized at the price vector $(\bar{P}_1, \bar{P}_2)$. In particular, this is true for $V(q_1, q_2) = q_1^{\alpha_1} q_2^{\alpha_2}$, where $\alpha_1 + \alpha_1 < 1$. So for $V(q_1, q_2) = q_1^{\alpha_1} q_2^{\alpha_2}$, where $\alpha_1 + \alpha_1 < 1$, delegation outperforms centralization, at-least weakly.

Under centralization, we can have the outcome either with collusion or without collusion. Whereas under delegation, we will always have the outcome with collusion. So for the cases where the collusion is beneficial for the principal, delegation outperforms the centralization.

**Proposition 4** Let Assumption 1 hold. Let $V(q_1, q_2) = q_1^{\alpha_1} q_2^{\alpha_2}$, where $\alpha_1 + \alpha_1 < 1$. Then delegation outperforms centralization, at-least weakly. This is true for all $V(q_1, q_2)$, for which we have strictly positive demand for both goods, negative own price effect, and negative and decreasing cross price effect.

One variant of the above information structure can be that the people in the same hierarchy incur a lower cost to acquire the knowledge about each other. This setup is similar to the one used by Fahad Khalil in his several works with his co-authors (Cremer, Khalil, and Rochet (1998), Cremer and Khalil (1992)). This variant is more suitable for exposition purpose. In present context, one agent’s cost to get the information about the other
agent is lower than the principal’s cost to know about that agent. In this context, what delegation does is to initiate the process of one agent (main contractor) investing to acquire the information about the other agent (sub-contractor). This avoids the possibility of co-ordination problem (which may arise in centralization case). So, in the context of the Boeing example, delegation acts to improve the efficiency of the collusion. In the context of delegation, the coalition formation authority $A$ acts as a captive institution for the main contractor. In a asymmetric setup, it is better to delegate to the agent whose marginal cost to acquire information about other agent is lower.

6 Conclusion

In this paper, we have studied centralized and delegated contracts, in a procurement setup. Here we have allowed for the possibility of collusion among agents. We have explored centralized contracts with perfect collusion among agents. Collusion among agents is captured by their ability to misreport their types and efforts in a coordinated way. Using dynamic optimization technique, we have provided first-order conditions which characterize Nash equilibrium outcomes, for both, centralization with and without collusion cases.

While comparing centralization, with and without collusion cases, we found that collusion among agents can benefit the principal as well, in some cases. Hence, in general, we should be looking at the cost-benefit analysis of collusion rather than just ways to block collusion. This is an important insight often overlooked in the literature. This insight should be be kept in mind, particularly while framing rule and regulation governing mergers and acquisitions. Farrell and Shapiro have highlighted this through their writings on competition regulation policies.

We have used this insight in the comparative analysis between centralization and delegation. Using this, we get a sufficiency condition under which delegation outperforms centralization. Delegation removes possibility
of coordination failure among the agents.
Appendix

IC Constraints (for lemma 2)

Consider the agents’ optimization problem. Suppose an agent of type $\beta$ announces his/her type $\alpha$. The agents will announce his/her true type if it solves the following problem

$$\max U(\alpha/\beta) = t(\alpha) - \Psi(E(\beta, C(\alpha), Q(\alpha)))$$

Evaluating the first-order conditions at $\alpha = \beta$ gives

$$\frac{dt(\beta)}{d\beta} - \Psi'[E_c\frac{dC}{d\beta} + \sum_i E_{q_i}\frac{dq_i}{d\beta}] = 0$$

Using $U(\beta) = t(\beta) + \Psi(E(\cdot))$, we get,

$$\dot{U}(\beta) = \frac{dt(\beta)}{d\beta} - \Psi'[E_c\frac{dC}{d\beta} + \sum_i E_{q_i}\frac{dq_i}{d\beta}] - \Psi'(e)E_\beta$$

$$\Rightarrow \dot{U}(\beta) = -\Psi'(e)E_\beta < 0$$

Dynamic Optimization: finite horizon, continuous time

Dynamic optimization techniques are very much used in areas like macroeconomics, economics of growth, etc. Here we are providing a basic outline of the technique suited for the concerned problem in the paper. For the detailed analysis and proof, any standard textbook of dynamic optimization (Pontryagin et al. (1962), Chiang (1993)) can be consulted. The following borrows heavily from Lorenzoni (2009).

Suppose the instantaneous payoff is given by $f(t, x(t), y(t))$, where $x(t) \in X$ and $y(t) \in Y$. $t$ denotes the time element. Here $x(t)$ is state variable and $y(t)$ is control variable. The agent chooses or controls $y(t)$ to maximize the
payoff. The state variable depends on the agent’s choice of control variable and represent the dynamics of the system. In a typical macroeconomics example, the consumption choices serve as the control variable and the capital serves as the state variable. The capital formation dynamics captures the state of the economy. In the current scenario, the effort \( e \) will be the choice variable of the agent and the utility \( u \) acts as the state variable. The state variable dynamics act as the constraint to the optimization problem:

\[
\dot{x}(t) = g(t, x(t), y(t))
\]

We assume that both \( f \) and \( g \) are continuously differentiable functions. The problem is to maximize

\[
\int_0^T f(t, x(t), y(t)) \, dt
\]

subject to the constraint \( \dot{x}(t) = g(t, x(t), y(t)) \) for all \( t \in [0, T] \) and given the initial condition \( x(0) \).

We use the Hamiltonian technique to setup the Hamiltonian as,

\[
H(t, x(t), y(t), \lambda(t)) = f(t, x(t), y(t)) + \lambda(t)g(t, x(t), y(t))
\]

**Necessary condition for optimality:**
If \( x^* \) and \( y^* \) are optimal, continuous and interior then there exists a continuously differentiable function \( \lambda(t) \) such that

\[
H_y(t, x^*(t), y^*(t), \lambda^*(t)) = 0
\]

\[
\dot{\lambda}(t) = -H_x(t, x^*(t), y^*(t), \lambda^*(t))
\]

\[
\dot{x}^*(t) = -H_\lambda(t, x^*(t), y^*(t), \lambda^*(t))
\]

and, \( \lambda(T) = 0 \).

**Sufficiency condition for optimality:**
Define

\[
M(t, x(t), \lambda(t)) = \max_y H(t, x(t), y, \lambda(t))
\]

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If \( x^* \) and \( y^* \) are two the continuous functions that satisfy above necessary conditions for some continuous function \( \lambda(t) \), \( X \) is a convex set and \( M(t, x(t), \lambda(t)) \) is concave in \( x \) for all \( t \in [0, T] \), then \( x^* \) and \( y^* \) are optimal.

This technique can be extended to suite the contexts (multiple control variables, multi-dimensional optimization) applicable in our problems.

**FOC for principal’s problem (for proposition 1)**

\[
\max \int_\beta [V(Q) - U - \Psi(e) - C] f(\beta) d(\beta)
\]

subject to

\[
\dot{U}(\beta) = -\Psi'(e) E_\beta \\
U(\beta) = 0
\]

The Hamiltonian becomes

\[
H = (V(Q) - U - \Psi(e) - C) f(\beta) - \mu(\beta) \Psi'(e) E_\beta
\]

So the first-order conditions become,

\[
\frac{\partial H}{\partial U} = -\dot{\mu}(\beta)
\]

\[
\Rightarrow f(\beta) = \dot{\mu}(\beta)
\]

integrating both sides and using \( \mu(\beta) = 0 \) (Since \( U(\beta) > 0 \)) gives

\[
\mu(\beta) = F(\beta)
\]

Now using \( \mu(\beta) = F(\beta) \) in the \( H \), the other first-order conditions \( \frac{\partial H}{\partial q_i} = 0 \) and \( \frac{\partial H}{\partial q_i} = 0 \) for \( i = 1, 2 \) straightway give the earlier stated first-order conditions.

\[
\Psi'(e) = -C_e - \frac{F(\beta)}{f(\beta)} [\Psi''(e) E_{\beta} + \Psi'(e) E_{\beta C} C_e]
\]

\[
V_{q_i} = C_{q_i} + \frac{F(\beta)}{f(\beta)} \Psi'(e) \frac{dE_{\beta}}{dq_i} \text{ for } i = 1, 2.
\]
For the second-order sufficiency conditions to be met, we need to assume the followings: the domain of $u$ is convex and the max functions (as defined in the above section) are concave in $u$.

**Two dimensional optimal control problem**

(for centralization without collusion, subsection 1.4.4)

We have to solve the following problem:

$$\max \int_{\beta_1} \int_{\beta_2} P(U, e_1, Q) f(\beta_1, \beta_2) d\beta_1 d\beta_2$$

subject to

$$\Delta \beta_2 \dot{U}_1 \beta_1 = \Delta \beta_2 g_1(\beta_1, e_1, q_1)$$

$$\Delta \beta_1 \dot{U}_2 \beta_2 = \Delta \beta_1 g_2(\beta_2, e_2, q_2)$$

where, $\Delta \beta_i$ is the range of $\beta_i$. The Lagrange method can be used to get the solution of above problem.

Let's denote $P(U, e_1, Q) f(\beta_1, \beta_2) + \mu_1(\beta_1) g_1(\beta_1, e_1, q_1) + \mu_2(\beta_2) g_2(\beta_2, e_2, q_2)$ by $H$. Then $\ell$ becomes

$$\ell = \int_{\beta_1} \int_{\beta_2} P(U, e_1, Q) f(\beta_1, \beta_2) d\beta_1 d\beta_2 + \int_{\beta_1} \mu_1(\beta_1) [g_1(\beta_1, e_1, q_1) - \dot{U}_1 \beta_1] \Delta \beta_2 d\beta_1 + \int_{\beta_2} \mu_2(\beta_2) [g_2(\beta_2, e_2, q_2) - \dot{U}_2 \beta_2] \Delta \beta_1 d\beta_2$$

$$\Rightarrow \ell = \int_{\beta_1} \int_{\beta_2} [P(U, e_1, Q) f(\beta_1, \beta_2) + \mu_1(\beta_1) g_1(\beta_1, e_1, q_1) + \mu_2(\beta_2) g_2(\beta_2, e_2, q_2)] d\beta_1 d\beta_2 - \int_{\beta_1} \mu_1(\beta_1) \dot{U}_1 \beta_1 \Delta \beta_2 d\beta_1 - \int_{\beta_2} \mu_2(\beta_2) \dot{U}_2 \beta_2 \Delta \beta_1 d\beta_2$$

Now,

$$\frac{d[U_1 \mu_1]}{d\beta_1} = \dot{U}_1 \mu_1 + \dot{\mu}_1 U_1$$

$$\Rightarrow \int_{\beta_1} \frac{d[U_1 \mu_1]}{d\beta_1} d\beta_1 = \int_{\beta_1} \dot{U}_1 \mu_1 d\beta_1 + \int_{\beta_1} \dot{\mu}_1 U_1 d\beta_1$$

$$\Rightarrow \int_{\beta_1} \dot{U}_1 \mu_1 d\beta_1 = \text{constant} - \int_{\beta_1} \dot{\mu}_1 U_1 d\beta_1$$

Let's denote $P(U, e_1, Q) f(\beta_1, \beta_2) + \mu_1(\beta_1) g_1(\beta_1, e_1, q_1) + \mu_2(\beta_2) g_2(\beta_2, e_2, q_2)$ by $H$. Then $\ell$ becomes
\ell = \text{constant} + \int_{\beta_1}^{\beta_2} \left[ H + \hat{\mu}_1 u_1 + \hat{\mu}_2 u_2 \right] d\beta_1 d\beta_2

Now using the pointwise maximisation, we get the first-order conditions as

\frac{\partial H}{\partial e_i} = 0 \text{ for } i = 1, 2.

\frac{\partial H}{\partial q_i} = 0 \text{ for } i = 1, 2.

\frac{\partial H}{\partial u_i} + \mu_i = 0 \text{ for } i = 1, 2.

Again we need to make similar assumptions as above for the second-order sufficiency conditions to be met.

Now we use this technique to derive the solution for the centralisation without collusion case. First, we form the Hamiltonian

\[ H = [V(Q) - u_1 - u_2 - \Psi_1(e_1) - \Psi_2(e_2) - C_1 - C_2] f(\beta_1, \beta_2) - \mu_1(\beta_1) \Psi_1'(e_1) E_{\beta_1}^1 - \mu_2(\beta_2) \Psi_2'(e_2) E_{\beta_2}^2 \]

The first-order condition for the optimality becomes:

\[ \Psi_i'(e_i) = -C_i e_i - \frac{f(\beta_i)}{f(\beta)} \left[ \Psi_i'(e_i) E_{\beta_i}^1 + \Psi_i'(e_i) E_{\beta_i}^1 C_i e_i \right] \text{ for } i = 1, 2. \]

\[ V_q = C_i e_i + \frac{f(\beta_i)}{f(\beta)} \Psi_i'(e_i) \frac{dE_{\beta_i}^1}{dq_1} \text{ for } i = 1, 2. \]
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