Implications of Present-biased Preferences on Inheritance Taxes

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Abstract

We model an economy where present bias preferences affect the bequest leaving decision. Using Bequest in the Utility (BIU) setup, we show that the optimal inheritance tax rate under present bias can be derived in terms of estimable sufficient statistics. Further, this optimal tax rate decreases with the level of temptation and a subsidy can be optimal at any level of bequests received. We then use the standard Barro Becker Dynastic (BBD) setup to derive the expression for the optimal tax rates. We observe that if the agents internalize the taxes on the amount of bequest that they leave (sensitive generations), present bias and optimal taxes are negatively related as in BIU, that is, providing an incentive by extending subsidies or lowering taxes is recommended to curtail the effect of temptation. However, if agents ignore the taxes paid by their descendents on the inheritance left (ignorant generations) optimal tax rates increase with the level of present bias since present bias reduces the tax base and thus the rationale of providing incentives does not work anymore. A calibration exercise supports all these findings.

Keywords: Present - biased preferences, capital and inheritance taxes, wealth mobility

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1 Introduction

It is well accepted in the literature that present bias has serious implications on economic activities. It can lead to an excessive level of consumption at present and therefore affect some other crucial decisions. More specifically higher level of present consumption due to present bias can result in a significantly low bequest leaving. This paper analyzes the optimal inheritance tax when altruistic agents have present bias preferences.\(^1\) To the best of our knowledge, this is the first study that tries to capture the role of present bias in determining the optimal inheritance tax. This natural and important behavioral issue has somehow gone unnoticed in the literature. Recently Pavoni and Yazici (2017) also consider the importance of self control problems in the derivation of optimal tax issues where the severity of the problem depends on age but their focus is different. Relying on the quasi-hyperbolic discounting, they show that if agents’ ability to self-control increases concavely with age, savings should be subsidized and the amount should decrease with age.\(^2\) Our paper first establishes the relationship between the degree of present bias and the optimal inheritance tax through the reduced form expression of optimal inheritance tax that can be estimated. Thus, along with the analytical solutions, we provide a direction towards quantitatively evaluating the optimal inheritance tax in the presence of present bias. We notice that the relationship between the present bias and the optimal inheritance tax rates also depends on whether agents internalize the effect of the chosen tax rates on the amount of bequests they leave. Not only does this paper provide useful insights into the role that present bias plays in determining the optimal inheritance tax rate, it also connects to some of the prominent studies in the area of inheritance and capital taxation.\(^3\)

To model inheritance, we use setups that are prominent in the literature. Partic-

\(^1\)For a review of literature on intergenerational transfers and their taxation see Cremer and Pestieau (2006).

\(^2\)The importance of modeling policies in the presence of non-standard utility has been gaining momentum recently (see for e.g., Yu (2016), O’Donoghue and Rabin (2006), Gul and Pesendorfer (2007), Lockwood and Taubinsky (2017)).

\(^3\)Farhi and Gabaix (2015) construct a theory of optimal taxation with behavioral agents which incorporates behavioral biases such as misperceptions, internalities and mental accounting. Their analysis focuses on three major types of taxation, namely, Ramsey, Pigou, and Mirless. Chetty (2015) nicely argues that modern evidence calls for incorporating behavioral economics into the analysis of important economic questions. The paper puts forward a pragmatic perspective on behavioral economics that focuses on its value for improving policy decisions as well as empirical predictions. It mainly discusses three ways in which behavioral economics can contribute to public policy - first by offering new policy tools, second by improving the predictions about the effects of existing policies, and third by generating new welfare implications.
ularly we follow Piketty and Saez (2013) and also Farhi and Werning (2010). We use two standard frameworks to present altruism through inheritance. The first one is the ‘bequest in the utility’ (BIU) where agents care about the after-tax bequest they leave for their offspring. Later this has been extended to represent a Farhi and Werning (2010) (FW) economy. The second framework is the standard Barro-Becker dynastic (BBD) one. To capture present bias, we rely on temptation and self-control preferences as in Gul and Pesendorfer (2004).

In our dynamic stochastic model, agents are heterogeneous in terms of bequest motives and labor productivities. In all the cases presented below, to derive the optimal tax rates, the social planner maximizes the long-run steady-state welfare. Under this setup of BIU, when we determine the optimal tax rate, the bequest received by the agents are taken as given. Thus, the chosen tax rates affect only consumption and bequest leaving decisions but it can’t affect the bequest that has already been received by the agents. However under BBD where many generations appear directly in the setup, to derive the expressions for the tax rate, we make two different ass-

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4Our model can be made more complicated by including features like human capital with different sources of educational funding. While we expect that the qualitative results are unchanged, choosing a model similar to these two important papers helps us to compare our results with them.

5Since inheritance in this model can also be considered as physical capital, we can relate our findings to a rich set of results related to optimal capital taxation. Particularly, we observe that Chamley (1986) - Judd (1985) zero capital tax result holds true even when the preferences are subject to present bias. Using a different economy with physical capital and no altruistic motive, Krusell et al. (2010) show that a constant subsidy on capital is optimal in the presence of Gul and Pesendorfer (2004) preferences and therefore, the celebrated Chamley (1986) and Judd (1985) result does not hold in their setup. Under the limitation of linear tax and complete market, Bishnu and Wang (2013) show that the efficiency force for a negative capital tax may not be strong enough to reverse the politically-economic force for a positive redistributive taxation under temptation and self-control preferences. As Piketty and Saez (2013) pointed out that in a non-stochastic wage models like Chamley (1986) and Judd (1985), the feedback effect represented by the elasticity of the present discounted value of the tax base with respect to a future tax increase till the period of infinity, is infinite. This pushes the optimal tax rate to zero in the long run. Thus in our paper Chamley (1986) - Judd (1985) recommendation of zero tax on capital in the long run results remain valid under the present bias preferences. This has been shown separately in Appendix B.

6Following Strotz (1956) and Phelps and Pollak (1968), Gul and Pesendorfer (2004) proposed an alternative class of utility functions that provide a dynamically consistent model for addressing preference reversals created by self-control problems. Preferences are defined over consumption sets instead of consumption sequences. An individual’s actual choice is a compromise between the commitment utility (standard utility) and a temptation utility. In other words, individuals face a trade-off between short-term temptation and long-term interest. Contrary to time-inconsistent preferences (see Laibson (1997)), the main benefit of the self-control preferences proposed by Gul and Pesendorfer (2004) is that preferences remain perfectly time-consistent and permit commitment. There have been many extensions and applications of Gul and Pesendorfer (2004) (see for example Fudenberg (2006), Dekel et al. (2009), Stovall (2010), Dekel and Lipman (2012), DeJong and Ripoll (2007), and Estaban et al. (2007)).
sumptions regarding internalizing the chosen tax rates by other generations. First we assume that the agents fully internalize (sensitive generations) the effect of the chosen optimal tax rates when they make their bequest leaving decisions. Later we change this and assume that agents completely ignore (ignorant generations) the tax rates when taking their bequest leaving decisions. Interestingly, apart from different quantitative results, these two assumptions generate completely opposite qualitative results in terms of implications of present bias on the optimal tax rates.

We first present the results that we find under the BIU setup. Thereafter we present our findings in the BBD setup. One important view that emerges from the BIU setup is that the level of temptation and the optimal inheritance tax rate are inversely related, that is, optimal inheritance tax rate decreases with the level of temptation. In the extreme situation when temptation is severe, our theoretical model predicts that a subsidy can also be optimal at any level of bequest received. These results are robust to different specifications of the BIU model. Further, they hold true independent of the level of the elasticity of labor supply. Thus while the absence of temptation suggests that the optimal tax rate can be negative only at a higher level of bequests received, including temptation on the other hand guarantees that a subsidy can be optimal at any level of bequests received if the present bias is too high. That is, when the tax rates are otherwise positive, the presence of temptation recommends lowering it down by a significant amount. The negative relationship between the optimal tax rate and the level of temptation implies that, when agents are tempted, lowering the tax rate provides incentive to leave more bequests by making ‘succumbing to temptation less attractive’. A calibration exercise using the same micro-data from the United States that Piketty and Saez (2013) used shows that the effects of temptation is significant at any percentile of bequests received. Instead of emphasizing on the exact tax rates, the main takeaway from the calibration result is that the effects of present bias are significant.

Under BIU, in the Farhi and Werning (2010) economy when society puts a direct weight on the offspring, it is optimal to subsidize bequests. When it does not, it is optimal not to distort bequest. In a similar setup but with the added feature of present bias we show that if dynamic efficiency holds, a subsidy is the only optimal,

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7 Throughout the paper we have assumed that the labor supply is elastic. Deriving all the results under the assumption of inelastic labor is straightforward.

8 It is worth mentioning here that Piketty and Saez (2013) find that the optimal tax rate is very high (about 50% to 70%) for the bottom 70% of the population in terms of bequest received and then falls abruptly and becomes negative within the top 20% of inheritors (mainly for the top 10%).
regardless of the weights the social welfare function put on children.\textsuperscript{9} Thus the optimal zero tax result that holds in the absence of present biased is no longer valid. This is because temptation brings in a strong motive to reduce the tax or subsidize bequests, therefore supporting a case for subsidizing inheritance when society does not care about descendants directly and strengthening the case for subsidies when society cares about them directly. Note that the motive for reduction in taxes is so strong under temptation that any positive tax rate is never a solution.

Under the BBD setup where many generations are directly involved, we have an important observation. It appears that a decrease in tax rate with the level of temptation (as we have observed under BIU) is \textit{not} the only outcome. Here the results crucially depend on a particular fact: whether or not the other generations are reacting to the chosen tax rates. First we assume that while determining the tax rate for the present period, the government believes that all other generations including the generation that left bequest responded optimally to the chosen tax rates (\textit{sensitive} generations). However, on the other hand, if agents leave bequests independent of the optimal tax rate (\textit{ignorant} generations) to be imposed on the amount of bequest received by their children in the future, the outcome becomes exactly the opposite (even opposite to the result exists in the literature of capital tax). More precisely, when the government believes that the bequests left by the different generations do not respond to the optimal taxes to be paid on this in the future, the tax rate increases with the level of temptation. Thus if all other generations are insensitive and ignorant, in the long run steady state, incentivizing bequest leaving by reducing the tax rate or by providing subsidy further does not work at all. Intuitively, since the bequest level falls (hence the tax base decreases) when present bias increases, the only option that is left to the government in this scenario is to increase the tax rate with the degree of present bias to maintain the tax revenue at the same level. Hence we observe a positive relationship between the tax rate and the degree of temptation, that is, optimal tax rate increases with the level of present bias. A calibration exercise has been present for this result too. Suggesting a positive tax in the presence of present bias is undoubtedly an interesting outcome of the study since the suggestion goes against the standard belief that anything that leads to suboptimal should be subsidized. Whether agents leave bequests independent of the

\footnotesize{\textsuperscript{9}A point that is worth noting here is that both Piketty and Saez (2013) and Farhi and Werning (2014) find that inheritance tax rate increases with $r - g$. We find that the presence of present biased preferences does not change this relationship. However an increase in the level of temptation actually resists the increase in the optimal inheritance tax rate due to an increase in $r - g$.}
optimal tax rate or not is a matter of further empirical investigation, but certainly it is not a case that can be easily refuted. We should mention here that the paper by Pavoni and Yazici (2017) find optimal subsidies in the quantitative section. However, in a different paper Pavoni and Yazici (2016) show that when agents are altruistic, self-control problems may generate disagreement across generations and that in fact may imply a positive tax on bequests. Very recently Lockwood (2018) observes that when bequests are modelled as luxury goods, bequest motives significantly increase saving (and decrease purchases of long-term care insurance and annuities) and this may have serious implications on the policies of intergenerational transfers especially in the present of present bias.

The rest of the paper is organized as follows. Section 2 deals with the analysis under the assumption of bequest in the utility function, section 3 presents the analysis under dynastic utility, section 4 presents a calibration exercise, and finally section 5 concludes. All the proofs are presented in Appendix A.

2 Present bias in BIU

Here we present our results using a BIU setup. This setup where bequests appear directly in the utility function is one of the commonly used frameworks for presenting altruistic behavior. In line with Piketty and Saez (2013), we consider a dynamic economy with a discrete set of generations. The economy that we first produce here does not experience any growth, however, we incorporate growth later. Each generation has a unit mass (of measure 1) of agents who live for one period. In the next period, the next generation replaces the present generation. An individual agent $t_i$ from dynasty $i$ living in generation $t$ has exogenous pre-tax wage income $w_{ti}$ drawn from a stationary distribution. We assume that every agent has available labor time $l_{ti}$ and therefore the pre-tax wage income is $y_{Lti} = w_{ti}l_{ti}$ which they receive at the end of the period. Further, individual $t_i$ receives $b_{ti} \geq 0$ amount of bequests from generation $t-1$ at the beginning of period $t$. It is assumed that the initial distribution of the bequest, $b_{0i}$, is exogenously given. The agents receive an exogenous gross rate of return $R$ per generation on the amount of inheritance they receive. At the end of the period, agents allocate their lifetime resources, which consist of the net of tax labor income and capitalized bequest received, into consumption $c_{ti}$ and bequest left $b_{t+1i}$.

Both the labor tax and the tax on capitalized bequests are assumed to be linear.
Precisely, $\tau_{Lt}$ represents the labor tax rate and $\tau_{Bt}$ is the tax rate on capitalized bequests in period $t$. The lump-sum grant that the agents may also receive in period $t$ is represented by $E_t$. Agents receive utility from consumption, leisure, and the net-of-tax capitalized bequest left $b = Rb_{t+1i} (1 - \tau_{Bt+1})$. It should be noted that $\tau_{Bt}$ can well be interpreted as a capital tax in our model.

Like $w_{ti}$, the preferences are also drawn from an arbitrary stationary distribution. Thus, agents can draw any productivity and taste independent of parental taste and ability. Further, we assume that the agents suffer from temptation and self-control problem as in Gul and Pesendorfer (2004). Whenever they suffer from temptation, they consume more which naturally affects the amount of bequest left for the next generation. The decision problem of an individual $ti$ on the appropriate budget set (to be mentioned later) can be written as

$$\max_{c_{ti},b_{t+1i},l_{ti}} \left\{ V^{ti}(c_{ti}, b_{ti}, 1 - l_{ti}) + \tilde{V}^{ti}(\tilde{c}_{ti}, \tilde{b}_{ti}, 1 - \tilde{l}_{ti}) \right\} - \max_{\tilde{c}_{ti},\tilde{b}_{t+1i},\tilde{l}_{ti}} \tilde{V}^{ti}(\tilde{c}_{ti}, \tilde{b}_{ti}, 1 - \tilde{l}_{ti}), \quad (1)$$

where $\tilde{c}_{ti}$ represents the temptation consumption and $V^{ti}$ and $\tilde{V}^{ti}$ represent the commitment and temptation utilities, respectively. For any choice variables $c_{ti}, b_{t+1i}, l_{ti}$, the cost of disutility from self-control is given by

$$\max_{\tilde{c}_{ti},\tilde{b}_{t+1i},\tilde{l}_{ti}} \tilde{V}^{ti}(\tilde{c}_{ti}, \tilde{b}_{ti}, 1 - \tilde{l}_{ti}) - V^{ti}(c_{ti}, b_{ti}, 1 - l_{ti}).$$

Deriving the analytical results with this preference turns out to be very complicated. Thus we try to simplify the model keeping the basic idea intact. We have the following three key assumptions as follows:

- $\tilde{V}^{ti} = \lambda V^{ti}$ where $\lambda \geq 0$ is a scale parameter that measures the sensitivity to the temptation alternative. This assumption, appears in many papers including Bucciol (2012), Kumru and Thanopoulos (2008), Kumru and Thanopoulos (2011), DeJong and Ripoll (2007) among others, makes our model not only analytically tractable, but also, as mentioned by DeJong and Ripoll (2007), consistent with the balanced growth. As we have mentioned above, we extend the model to incorporate growth after presenting this setup without growth. In fact all our calibration results are based on the specification that incorporates growth.

- When agents are tempted towards consumption, due to a higher level of con-
sumption, the marginal utility is lower than when agents are free from temptation. Precisely, marginal utility from consumption under temptation is lower than under no temptation by the proportion of $\alpha \in (0, 1)$ and is the same for all $ti$, that is, $V_{c}^{ti} = \alpha V_{c}^{ti}$, $\alpha \in (0, 1)$ for all $ti$. A high value of $\alpha$ implies that the effect of temptation is significant.

- When the agents fully succumb to the temptation (the max case), they leave no bequests at all. Given this simplification, (1) takes the following form

$$\max_{c_{ti}, b_{t+1i}, l_{ti}} (1 + \lambda) V^{ti}(c_{ti}, \tilde{b}, 1 - l_{ti}) - \lambda V^{ti}(\tilde{c}_{ti}, \tilde{b} = 0, 1 - l_{ti}).$$

The above three assumptions jointly are capable of representing present bias (please see the example we provide below). It is easy to check that our usual no-temptation situation can be generated by setting $\lambda = 0$. We denote aggregate consumption in $t$, the labor income of generation $t$ and the aggregate bequest received in $t$ by $c_{t}$, $y_{Lt}$ and $b_{t}$ respectively. Although this paper focuses on inheritance tax, it is important to note here that the aggregate bequest flow in this model is the aggregate capital accumulation. The optimization problem of the individual incorporates the fact that the agents can choose the optimal amount of labor supply, along with the decision of consumption and the amount of bequests that they leave. Agent’s optimization problem can be formally written as

$$\max_{\{c_{ti}, l_{ti}, b_{t+1i}\}_{t=0}} (1 + \lambda) V^{ti}(c_{ti}, R (1 - \tau_{Bt+1}) b_{t+1i}, 1 - l_{ti}) - \lambda V^{ti}(\tilde{c}_{ti}, \tilde{b} = 0, 1 - l_{ti}) \quad (2)$$

subject to

$$c_{ti} + b_{t+1i} = R (1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) w_{ti} l_{ti} + E_{t},$$

$$\tilde{c}_{ti} = R (1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) w_{ti} l_{ti} + E_{t}.$$

Before we proceed to the derivation of the optimal inheritance tax rate, we present

10 The setup is complex and therefore the model we present is a bit restricted version of Gul and Pesendorfer (2004). A bit less restrictive version of our full - fledged model is the one where labor supply is exogenous. We have not presented in this paper but verified that all the results hold with the inelastic labor supply. Since elastic labor is more general case, we kept that as our main exercise.

11 A generic functional form for $\tilde{V}^{ti}$ can also be chosen. Even in that environment, the temptation part of the problem plays no role in determining the consumer’s actions in the first period. The problem of temptation notably affects consumer’s welfare. Hence, our choice of functional form for temptation is general enough to capture the effect of temptation.
a simple example to understand the mechanism that is at work here, with the above assumptions intact. We capture the fact that the future consumption enters with a lower marginal value in the evaluation of utility relative to the case without temptation. In this example, for simplicity, assume that the labor supply is inelastic with \( l_t = 1 \) and the utility function is quasilinear of the form \( V^{ti}(c_{ti}, b) = c_{ti} + \psi b \) so the agents solve the following problem akin to (2)

\[
\max_{\{c_{ti}, b_{t+1i}\}_{t=0}^{\infty}} (1 + \lambda) [c_{ti} + \psi b_{ti}] - \lambda c_{ti}
\]

subject to

\[
c_{ti} + b_{t+1i} = R (1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) w_{ti} + E_t,
\]

\[
\bar{c}_{ti} = R (1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) w_{ti} + E_t,
\]

where, after incorporating the budget constraints, the maximand above simplifies to

\[
(1 + \lambda) [c_{ti} + \psi b_{ti}] - \lambda [c_{ti} + b_{t+1i}]
\]

\[
= c_{ti} + (1 + \lambda) \psi b_{ti} - \lambda b_{t+1i}
\]

\[
= V^{ti}(c_{ti}, b_{ti}) - \lambda b_{t+1i} [1 - \psi R (1 - \tau_{Bt+1})].
\]

We have used the fact that \( b = R b_{t+1i} (1 - \tau_{Bt+1}) \). Given the temptation effect as shown above is in general non-zero for \( \lambda > 0 \), it is now transparent that the planner faces additional incentives in this environment to use \( \tau_{Bt+1} \) for redistribution through the inheritance tax. Temptation effect shown above also clearly increases with the parameter \( \lambda \). A first order condition with respect to \( b_{t+1i} \) clearly indicates that as \( \lambda \) rises, \( b_{t+1i} \) falls. We now proceed with the formal derivation of the tax rates for our original model.

Note that the first order condition for bequest left is given by

\[
V^{ti}_c = R (1 - \tau_{Bt+1}) V^{ti}_b.
\]

In this analysis, it is assumed that the economy converges to a unique, steady-state equilibrium independent of the initial distribution of bequests and that a steady-state equilibrium distribution of bequests and earnings exists. To derive the optimal tax rate, the government considers the long-run steady-state equilibrium of the economy.
where the choice of long-run economic policy \( E, \tau_L \) and \( \tau_B \) maximizes steady-state social welfare. Social welfare, denoted by \( SWF \), is the weighted sum of individual utilities with Pareto weights \( \omega_{ti} \geq 0 \), subject to a period-wise budget constraint. Formally, the government’s long run social welfare function can be written as

\[
SWF = \max_{\tau_L, \tau_B} \int_i \omega_{ti} \left[ (1 + \lambda) V^{ti} (R (1 - \tau_B) b_{ti} + (1 - \tau_L) y_{Lti} + E - b_{t+1i}, R (1 - \tau_B) b_{t+1i}, 1 - l_{ti}) - \lambda V^{ti} (R (1 - \tau_B) b_{ti} + (1 - \tau_L) y_{Lti} + E, b = 0, 1 - l_{ti}) \right]
\]

subject to \( E = \tau_B R b_t + \tau_L y_{Lt} \), with initial \( E \) as given.

We will show that the optimal inheritance tax rate depends on the size of behavioral responses to taxation through their measured elasticities, combination of social preferences, the distribution of bequests and earnings captured by distributional parameters and, importantly, by the temptation parameter represented by \( \lambda \) in this model. In our equilibrium, social welfare is constant over time.

We now focus on the elasticity parameters that will appear in the expression of the optimal \( \tau_B \). The long-run elasticities of aggregate bequest flow \( b_t \) with respect to the net-of-bequest tax rate \( 1 - \tau_B \) given \( E \) is represented by \( e_B \). Thus formally,

\[
e_B = \frac{d b_t}{d (1 - \tau_B)} \frac{1 - \tau_B}{b_t} \bigg|_{E}.
\]

The long-run elasticity of the aggregate labor supply with respect to the net-of-labor-tax rate \( 1 - \tau_L \), denoted by \( e_L \), is

\[
e_L = \frac{d y_{Lt}}{d (1 - \tau_L)} \frac{1 - \tau_L}{y_{Lt}} \bigg|_{E}.
\]

Next we define the distributional parameters that will also appear in the expression for \( \tau_B \). The social marginal welfare weight on individual \( ti \) is denoted by \( g_{ti} = \omega_{ti} V^{ti} c / \int_j \omega_{tj} V^{tj} c \) which is normalized to 1. Note that \( g_{ti} \) measures the social value of increasing consumption of an individual \( ti \) by one dollar relative to distributing one dollar equally across all individuals. With this \( g_{ti} \), the distributional parameters are defined as follows

\[
\bar{b}^{\text{received}} \equiv \int_i g_{ti} b_{ti}, \quad \bar{b}^{\text{left}} \equiv \int_i g_{ti} b_{t+1i}, \quad \text{and} \quad \bar{y}_L \equiv \int_i g_{ti} y_{Li}.
\]

where \( b_t = \int_i b_{ti} \). The social marginal weights for \( ti \) under temptation is \( \tilde{g}_{ti} = \)
Given the above assumption that $V_{c}^{\prime \prime} = \alpha V_{c}^{\prime}$, $\alpha \in (0, 1)$ for all $t_i$, it is easy to verify that $\bar{g}_{ti} = g_{ti}$ and therefore we guarantee

$$
\bar{b}^{\text{received}} = \bar{b}^{\text{received}}, \quad \bar{b}^{\text{left}} = \bar{b}^{\text{left}} , \quad \text{and} \quad \bar{y}_{L} = \bar{y}_{L},
$$

where $\bar{b}^{\text{received}} \equiv \int \bar{g}_{ti} b_{ti} / b_{ti}$, $\bar{b}^{\text{left}} \equiv \int \bar{g}_{ti} b_{t_i+1} / b_{t_i+1}$, and $\bar{y}_{L} \equiv \int \bar{g}_{ti} y_{Li} / y_{Ll}$.

Thus in this analysis, the social marginal welfare weight on individual $t_i$ remains unchanged in the presence of temptation, as do the distributional parameters. In this paper, we avoid the differential effects of temptation on agents due to varying levels of temptation at different levels of income or assets. To capture the pure effect of temptation, we do not focus on the additional source of heterogeneity due to temptation. Instead, we assume that independent of the level of assets or income, the level of temptation is same for everyone and the distributional parameters are unchanged. If the value of the variable is lower for those with higher social marginal weights, all of the above ratios are less than 1. Further, $\hat{e}_{B} = \bar{e}_{B}$ where $\hat{e}_{B}$ is the average of $e_{B_{ti}} = \frac{d b_{ti}}{d(1-\tau_{B})} \frac{1-\tau_{B}}{b_{ti}}$ weighted by $g_{ti} b_{ti}$, that is $\hat{e}_{B} \equiv \int g_{ti} b_{ti} e_{B_{ti}} / \int g_{ti} b_{ti}$ and $\bar{e}_{B}$ is the same expression under the temptation, that is $\bar{e}_{B} \equiv \int \bar{g}_{ti} b_{ti} e_{B_{ti}} / \int \bar{g}_{ti} b_{ti}$.

To derive the optimal tax rate, we consider a small reform $d\tau_{B} > 0$. A balanced budget condition is given by $dE = R b_{t} d\tau_{B} + \tau_{B} R db_{t} + y_{Ll} d\tau_{L} + \tau_{L} dy_{Ll}$. Using the elasticities defined above, under the balanced budget condition, we then have

$$
R b_{t} d\tau_{B} \left( 1 - \frac{e_{B_{ti}} \tau_{B}}{1-\tau_{B}} \right) + d\tau_{L} y_{Ll} \left( 1 - \frac{e_{L} \tau_{L}}{1-\tau_{L}} \right) = 0. \tag{5}
$$

Given $b_{t+1i}$ is chosen to maximize the agent’s utility and by applying the envelope theorem, the effect of reform $d\tau_{B}$ and $d\tau_{L}$ on the steady state social welfare is given by

$$
dSWF = (1 + \lambda) \int \omega_{ti} \left\{ V_{c}^{\prime \prime} \cdot ((1-\tau_{B}) R db_{ti} - R b_{ti} d\tau_{B} - y_{Ll} d\tau_{L})) - V_{c}^{\prime \prime} \cdot (R b_{t+1i} d\tau_{B}) \right\} \\
- \lambda \int \omega_{ti} V_{c}^{\prime \prime} \cdot ((1-\tau_{B}) R db_{ti} - R b_{ti} d\tau_{B} - y_{Ll} d\tau_{L}). \tag{6}
$$

11
At the optimum, \( dSWF = 0 \) implies that

\[
0 = (1 + \lambda) \int_i \omega_t \{ V^t_i \cdot ((1 - \tau_B) Rb_t - Rb_t d\tau_B - y_{Lt} d\tau_L) \} - V^{ti}_b \cdot (Rb_t+1 d\tau_B) \}
\]

\[
- \lambda \int_i \omega_t V^{ti}_c \cdot ((1 - \tau_B) Rb_t - Rb_t d\tau_B - y_{Lt} d\tau_L) .
\]

Our first proposition is as follows.

**Proposition 1** (a) For a given \( \tau_L \), the optimum tax rate \( \tau_B^{temp} \) which maximizes the long run steady state social welfare function with a period-wise budget balance is given by

\[
\tau_B^{temp} = \frac{1 - \left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right) \left[ \frac{\bar{b}^{received}}{\bar{y}_L} (1 + \hat{e}_B) + \frac{(1 + \lambda) \bar{b}^{left}}{R[1 + \lambda (1 - \alpha)] \bar{y}_L} \right]}{1 + e_B - \left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right) \frac{\bar{b}^{received}}{\bar{y}_L} (1 + \hat{e}_B)} .
\]

(b) To incentivize the leaving of bequests, the optimal tax rate should decrease with the level of temptation. Further, severe temptation may justify a subsidy at any level of bequest received.

The above result is interesting in its own right. First, when agents’ preferences are subject to temptation \((\lambda > 0)\), our reduced form expression of \( \tau_B^{temp} \) differs from \( \tau_B \). More precisely, in the presence of temptation, \( \tau_B^{temp} < \tau_B \). Thus, when individuals are tempted to consume more and leave a lower amount of bequests, the optimal inheritance tax rate should be lower than the rate under no temptation. This implies that, if the agents suffer from temptation, a higher tax rate is detrimental. In the presence of temptation, lowering the tax rate generates incentive to leave a higher amount of bequests by making surrendering to temptation less attractive. Further, it is clear from the above expression that the presence of acute temptation may also justify a subsidy in our analysis.

Another interesting observation can be made when we compare our expression of the optimal tax rate with the one derived by Piketty and Saez (2013) who recommended a subsidy at a higher percentile of bequest received. Our results confirm that potentially a subsidy can be recommended even for agents in a lower percentile in the presence of an acute lack of self-control. This is also clear from the calibration
exercise presented below in section 4. Thus, our study shows that depending on the severity of present bias, a subsidy may be required at any level of bequest received. It is clear from the above expression (8) that in the absence of temptation (when \( \lambda = 0 \)) \( \tau_B^{\text{temp}} \) coincides with the tax rate derived in Piketty and Saez (2013) with no temptation under the assumption that labor supply is perfectly inelastic.

It should be noted that when \( \alpha \) is very close to one, the difference between the marginal utilities under the commitment and temptation consumptions is very small. This implies that, for a given \( \lambda \), the agents leave no bequests, and therefore a subsidy can be recommended as an incentive-generating instrument. On the other hand, when \( \alpha \) is very small, the marginal utility under the temptation consumption is sufficiently lower than that from commitment consumption, which presents a case where the individual consumes more under temptation than without it. This means that they have already left a large amount of bequests, and that there is therefore no need for a subsidy. This implies that \( \tau_B^{\text{temp}} \) is now very close to \( \tau_B \). In this situation, an extra incentive to leave a higher amount of bequests by lowering the optimal tax is not needed. The only added feature is that the optimal tax rate under temptation now contains the elasticity of labor supply.\(^{12}\) This subsection can be seen as a special case where the generational discount rate \( \Delta = 1 \). To see a more general result, we assume that \( \Delta \leq 1 \) and calculate the optimal policy in the long run \((\tau_L, \tau_B)\). This derivation has been presented in Appendix C.

Next we extend the analysis to include labor-augmenting economic growth per generation at a rate of \( G > 1 \). We present the result under the assumption that we have a steady state where all of the variables, including the individual wage rate \( w_{ti} \), grow at the rate of \( G \). This makes it impossible for labor to be affected by the growth. Further, in this more realistic and complicated version of the analysis, as in Piketty and Saez (2013), we incorporate “wealth loving” motives which are important when any annuity market is not present or is imperfect. This supports the important observation that people often leave accidental bequests at the time of death. Thus, the assumption of a wealth-loving motive realizes that people may also leave bequests

\(^{12}\)This particular result about the inheritance tax has similar flavor to that of Krusell et al. (2010)’s qualitative result on capital tax. In a completely different model with non-altruistic agents and capital, they show that a subsidy on saving encourages agents to save more for future if their preferences are subject to temptation and self-control problem. In their long run, the optimal policy is a constant subsidy on capital. We show later that when generations are ignorant or the framework is Chamley (1986) - Judd (1985), our inheritance tax results in altruistic setup are completely opposite to Krusell et al. (2010) in spirit (discussed below in details).
for other reasons. In the presence of temptation, the wealth loving motive can play a crucial role. The expression that we derive in this more general setting is the one used for calibration.

We assume that individuals derive utility from four components: personal consumption, after-tax bequests, pretax bequests, and leisure. The function $V^t_i$ can be formally written as

$$V^t_i(c_{ti}, R \left(1 - \tau_{Bt+1}\right) b_{t+1} b_{t+1} + 1 - l_{ti}).$$

When agents do not care about the post-tax bequests, the tax rates do not affect their utility. However, those who receive the inheritance are definitively affected. The relative importance of altruism in bequests motives for individual $ti$ is measured by

$$\nu_{ti} \equiv R \left(1 - \tau_{Bt+1}\right) V^t_i b_{ti} / V^t_i c_{ti}$$

with a population average

$$\nu \equiv \int_t \nu_{ti} b_{t+1} / \int_t \nu_{ti} b_{t+1}.$$

This particular specification of the economy is used for the calibration exercise.

The problem of an individual under this setup can be written as

$$\max_{\{c_{ti}, l_{ti}, b_{t+1i}\}_{t=0}^{\infty}} (1 + \lambda) V^t_i(c_{ti}, R \left(1 - \tau_{Bt+1}\right) b_{t+1} b_{t+1} + 1 - l_{ti}) - \lambda V^t_i(\bar{c}_{ti}, \bar{b} = 0, b_{t+1} = 0, 1 - l_{ti})$$

subject to

$$c_{ti} + b_{t+1i} = R \left(1 - \tau_{Bt}\right) b_{ti} + (1 - \tau_{Lt}) y_{Lt} + E_t,$$

$$\bar{c}_{ti} = R \left(1 - \tau_{Bt}\right) b_{ti} + (1 - \tau_{Lt}) y_{Lt} + E_t.$$

The first order condition with respect to $b_{t+1i}$ is given by

$$V^t_c = R \left(1 - \tau_{Bt+1}\right) V^t_b + V^t_b.$$

Therefore the government’s long run social welfare function is as follows:

$$\text{SWF} = \max_{\tau_L, \tau_B} \int_t \omega_{ti} \left[ (1 + \lambda) V^t_i(R \left(1 - \tau_B\right) b_{ti} + (1 - \tau_L) y_{Lt} + E - b_{t+1i} b_{t+1i} + 1 - l_{ti}) - \lambda V^t_i(R \left(1 - \tau_B\right) b_{ti} + (1 - \tau_L) y_{Lt} + E, b = 0, b_{t+1i} = 0, 1 - l_{ti}) \right]$$
subject to $E = \tau_B R b_t + \tau_L y_{Lt}$. We derive
\[
dSWF = (1 + \lambda) \int \omega_t V_c^{tit} \cdot ((1 - \tau_B) R b_{tit} - R b_{tit} d\tau_B - y_{Lt,t} d\tau_L)
\]
\[
- (1 + \lambda) \int \omega_t V_b^{tit} \cdot R b_{t+1,t} d\tau_B - \lambda \int \omega_t V_c^{tit} \cdot (R (1 - \tau_B) db_{tit} - R b_{tit} d\tau_B - y_{Lt,t} d\tau_L)
\]
and present our next proposition below.

**Proposition 2** *(a) For a given $\tau_L$, the optimum tax rate $\tau_B^{temp}$ which maximizes the long run steady state social welfare with a period-wise budget balance is given by

\[
\tau_B^{temp} = 1 - \left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right) \frac{\bar{b}^{received}}{\bar{y}_L} (1 + \hat{e}_B) + \frac{G \nu (1 + \lambda) \bar{b}^{left}}{R [1 + \lambda (1 - \alpha)] \bar{y}_L} + \frac{G \nu (1 + \lambda) \bar{b}^{left}}{R [1 + \lambda (1 - \alpha)] \bar{y}_L}
\]

(b) To incentivize the leaving of bequests, the optimal tax rate should decrease with the level of temptation. Further, severe temptation may justify a subsidy at any level of bequest received.*

In this modified version of the tax rate too, the increase in the level of temptation suggests a lower optimal tax rate. All the discussions regarding Proposition 1 are also applicable here. The additional variables that appear here namely $G$ and $\nu$ negatively affect the optimal tax rate, as expected. Further, when comparing the above expression with (9), it is clear that $R$ has been replaced by $R/G$ since leaving a relative bequest $b_{t+1,t}/b_{t+1}$ now requires leaving a bequest $G$ times larger than leaving the same relative bequest $b_{t+1,t}/b_t$. Therefore the relative cost of taxation to bequest leavers is multiplied by $G$. These features of the model is not affected by the inclusion of the temptation parameter $\lambda$. As the gap between the return to capital and the growth rate increases, the optimal inheritance tax as well as inequality increases (see Piketty (2011), Piketty (2014)). Farhi and Werning (2014) provide a political economy model of bequests taxation where they show that their finding is broadly consistent with the above claim that higher values of $r - g$ result in higher and more progressive optimal taxes on bequests as well as higher level of wealth inequality. We show that while this inverse relationship between $R/G$ and the optimal inheritance tax rate is unchanged, at any level of $R/G$, temptation reduces the optimal tax rate independent of the level of bequests received.
The FW setup

Let us now discuss the link between this paper and Farhi and Werning (2010) in the presence of temptation and a lack of self-control. As mentioned by Piketty and Saez (2013), their results regarding a positive inheritance tax depend crucially on the fact that labor income is no longer the single source of resources in an individual’s life as in Farhi and Werning (2010). One more source of inequality is inheritance. We now compare our model with that of Farhi and Werning (2010) when this flow of inheritance is affected by the presence of temptation and self control behavior. In a two period model of Farhi and Werning (2010) in which each dynasty survives for two generations, working parents begin with no bequests but have earnings, whereas the children receive bequests but never work. While a formal extension of our model could include preferences $U^t(c, \bar{c}, \bar{b}, l_{ti}) = (1 + \lambda)V^t(c, \bar{b}, l_{ti}) - \lambda V^t(\bar{c}, \bar{b} = 0, l_{ti})$ for the parents and $V^t(c)$ for children, we refrain from this formal analysis. For a general case, Farhi and Werning (2010) focused on a weakly separable utility $V^t(u(c, \bar{b}), l_{ti})$ of parents with nonlinear taxation. By assuming the subutility $u(c, \bar{b})$ homogeneous of degree one in line with Piketty and Saez (2013), we can obtain the linear tax counterpart of their results. A crucial observation from this analysis is presented in the following proposition.

Proposition 3 Regardless of whether the social welfare function puts zero or positive direct weight on children, $\tau_B < 0$ is always the optimal.

We observe that when temptation is present and parents do not inherit any assets but take the decision of leaving bequests whereas children are the receiver without any work and bequest leaving decision, optimality always recommends a subsidy. The intuition behind this result is as follows. According to Farhi and Werning (2010) when society puts a direct weight on the offspring, it is optimal to subsidize bequests. When it does not, it is optimal not to distort bequest. However present bias, if exists, brings in a motive to subsidize bequests, therefore, implies a case for subsidizing inheritances when society does not care about descendants directly and strengthens the case for subsidies when society cares about descendants directly. Note that the motive for reduction in taxes is so strong under present bias that no positive tax rate is optimum.

This result has another important implication in the capital tax literature. In the Farhi and Werning (2010) type model with inheritance if preferences are subject to
present bias, a subsidy on capital is the only recommendation, no positive tax on capital is warranted.

3 Present bias in BBD

Since BBD is another significant way of modeling altruism, we present our results in this setup too. To derive the optimal tax results, one crucial assumption that we make here is that the generation that is leaving bequests has full knowledge of implementation of a tax in the future and more importantly, they act accordingly. In other words, the bequest leaving generation knows that the amount of assets that they leave will be subject to a tax and it will be collected from their next generation. This implies that $b_i$ is optimally chosen by the parental generation while leaving bequest knowing that an optimal tax set by the planner will be imposed on their children’s bequest income. This particular generation has been termed as sensitive generation. In this situation we observe that optimal inheritance tax decreases with the level of temptation. This qualitative part of the result is similar to what we have seen in the previous section. Later we change this assumption of a sensitive bequest leaving generation. Particularly, in subsection 3.2 we assume that the generation is ignorant of the chosen optimal tax rates, that means, they do not internalize the effect of the chosen tax rates on their bequest leaving behavior. Due to this change in assumption, the results in subsection 3.2 completely overturn all the previous results. When agents are ignorant we observe that as present bias increases, optimal tax rates also increases, that is, there is a positive relationship between the optimal tax rate and the level of present bias.

Further, using this BBD setup, we will revisit the celebrated Chamley (1986) - Judd (1985) zero tax results on capital tax as we have mentioned above (please see Appendix B).\textsuperscript{13}

\textsuperscript{13}The literature on capital tax is very rich. Some of the very recent papers that focus on this are Piketty and Saez (2013), Straub and Werning (2015), Saez and Stantcheva (2016), Moser and Olea de Souza e Silva (2017) among others. We must mention here that very recently, in an interesting study, Pavoni and Yazici (2017) show that the optimal taxation of savings of agents who face self-control problems and where the severity of the problem depends on age. Focusing on the quasi-hyperbolic discounting, they show that if agents’ ability to self-control increases concavely with age, savings should be subsidized and the amount should decrease with age.
3.1 Sensitive generations

We first present the basic of the BBD setup which is common to both this subsection and the next. Instead of enjoying utility directly from the net bequest left as in BIU, here an individual \( ti \) derives her utility from the utility of the next generation \( \mathcal{U}_{t+1}^{ti} \). This guarantees the following recursive structure of the utility function

\[
\mathcal{U}^{ti} = V^{ti}(c_{ti}, 1 - l_{ti}) + \delta \mathcal{U}_{t+1}^{ti} 
\]

where \( \delta \in (0, 1) \) represents the discount factor. When \( V^{ti} \) is assumed to follow the Gul-Pesendorfer preferences as discussed above, the utility of an individual \( ti \) can be written as

\[
\mathcal{U}^{ti} = (1 + \lambda) u^{ti}(c_{ti}, 1 - l_{ti}) - \lambda u^{ti}(\tilde{c}_{ti}, 1 - l_{ti}) + \delta \mathcal{U}_{t+1}^{ti}. \tag{10}
\]

We restrict ourselves to the same set of tax instruments. Individual maximizes utility as in (10) subject to a budget constraint \( c_{ti} + b_{t+1i} = R (1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t \) where \( E_t \mathcal{U}_{t+1}^{ti} \) is the expected utility of \( t+1i \) agent based on the information available in period \( t \). Thus the utility maximization problem is as follows:

\[
\max_{\{c_{ti}, l_{ti}, b_{t+1i}\}_{t=0}^{\infty}} \left\{ (1 + \lambda) \sum_{t=0}^{\infty} \delta^t E_t u^{ti}(c_{ti}, 1 - l_{ti}) - \lambda \sum_{t=0}^{\infty} \delta^t E_t u^{ti}(\tilde{c}_{ti}, 1 - l_{ti}) \right\}
\]

subject to

\[
c_{ti} + b_{t+1i} = R (1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t, \\
\tilde{c}_{ti} = R (1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t.
\]

The optimization problem formulated above can be rewritten as

\[
\max_{\{c_{ti}, l_{ti}, b_{t+1i}\}_{t=0}^{\infty}} \left\{ (1 + \lambda) \sum_{t=0}^{\infty} \delta^t E_t u^{ti}(R (1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t - b_{t+1i}, 1 - l_{ti}) - \lambda \sum_{t=0}^{\infty} \delta^t E_t u^{ti}(\tilde{c}_{ti}, 1 - l_{ti}) \right\}
\]

subject to

\[
c_{ti} + b_{t+1i} = R (1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t, \\
\tilde{c}_{ti} = R (1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t.
\]

The first order condition with respect to \( b_{t+1i} \) is therefore given by

\[
u^{ti}_c(c_{ti}, 1 - l_{ti}) = \delta R (1 - \tau_{Bt+1}) E_t u^{t+1i}_c(c_{t+1i}, 1 - l_{t+1i}). \tag{11}
\]

Note that since \( b_{t+1i} \) is known at the end of \( t \), (11) can be essentially expressed as

\[
\tilde{b}_{t+1}^{\text{left}} = \delta R (1 - \tau_{Bt+1}) \tilde{b}^{\text{received}}_{t+1}, \quad \text{where} \tilde{b}^{\text{received}}_{t+1} = \int_i \omega_{bi} u^{ti}_c(c_{ti}, 1 - l_{ti}) b_{ti}/b_t \int_i \omega_{bi} u^{ti}_c(c_{ti}, 1 - l_{ti}) b_{ti}/b_t, 
\]
\( l_{ti} \) and \( \bar{b}_{ti+1} = \int_i \omega_0 b_{ti} u_t^c(c_{ti}, 1 - l_{ti}) \) \( b_{ti+1} / b_{ti+1} \int_i \omega_0 b_{ti} u_t^c(c_{ti}, 1 - l_{ti}) \), where \( \omega_0 \) is any dynastic Pareto weights.\(^{14}\) Again we focus on the equilibrium where in the long run, individual outcomes are independent of the initial positions.

All other assumptions of the previous section are intact here. Further, as the periods in which individuals will leave no bequests are equally likely to the government, we assume that the government chooses \( \tau_B \) as if everyone inherits bequests in all periods. We solve the optimal tax rates here under the assumption of steady-state dynasty.\(^{15}\) The study reveals that the same negative relationship between the tax rate and the degree of temptation exists, although the magnitudes of the changes in the tax rate due to a change in the level of temptation are different.

Again we focus on the equilibrium where in the long run, individual outcomes are independent of the initial positions. Further, as the periods in which individuals will leave no bequests are equally likely to the government, we assume that the government chooses \( \tau_B \) as if everyone inherits bequests in all periods. We solve the optimal tax rates here under the assumption of steady-state dynasty.\(^{15}\) The study reveals that the same negative relationship between the tax rate and the degree of temptation exists, although the magnitudes of the changes in the tax rate due to a change in the level of temptation are different.

Again we focus on the steady state with a constant tax policy \( \tau_B, \tau_L \) and \( E \) such that the government budget constraint \( E = \tau_B R b_0 + \tau_L y_L \) holds in every period. When the optimal tax policy is calculated at the steady state, the equilibrium constant tax rates that obey the government’s balanced budget constraint maximize the social welfare. As before, this analysis also considers a small deviation in \( \tau_B \) so that \( dE = 0 \) holds. Here we have

\[
\text{SWF} = \max_{\tau_B} \left\{ (1 + \lambda) \sum_{t=0}^{\infty} \delta^t \int_i u_t^c (R(1 - \tau_B) b_{ti} + (1 - \tau_L) y_{Lti} + E - b_{ti+1}; 1 - l_{ti}) \right. \\
- \lambda \sum_{t=0}^{\infty} \delta^t \int_i u_t^c (R(1 - \tau_B) b_{ti} + (1 - \tau_L) y_{Lti} + E; 1 - l_{ti}) \\
\left. \right\}
\]

subject to a period-wise budget constraint. As usual the small reform of the tax rates on the steady state social welfare, assuming that \( b_{ti+1} \) and \( l_{ti} \) are chosen to maximize the individual utility, is given by

\[
d\text{SWF} = (1 + \lambda) \left[ \sum_{t=0}^{\infty} \delta^t \int_i u_t^c \cdot (R(1 - \tau_B) db_{ti} - Rb_{0i} d\tau_B) - \sum_{t=0}^{\infty} \delta^t \int_i u_t^c b_{ti+1} y_{Lti} d\tau_B \right] \\
\left. \right. \\
- \lambda \sum_{t=0}^{\infty} \delta^t \int_i u_t^c \cdot (R(1 - \tau_B) db_{ti} - Rb_{0i} d\tau_B) + \lambda \sum_{t=0}^{\infty} \delta^t \int_i u_t^c \cdot y_{Lti} d\tau_L \right].
\]

Here too we observe that as the level of temptation increases, the optimal inheritance tax rate under the dynastic setup, \( \tau_B^{\text{temp}} \), decreases.

**Proposition 4** (a) For a given \( \tau_L \), the optimum tax rate \( \tau_B^{\text{temp}} \) which maximizes the

\(^{14}\)In our paper we omit the case for general Pareto weights and focus on utilitarian weights, \( \omega_0 = 1, \forall i \).

\(^{15}\)An interesting situation that can arise in this setup is that the taxes are imposed at a later date however the agents react in advance. This has been presented in the Appendix B. Further, we use that framework to link our results with the capital tax results, mainly the celebrated Chamley (1986) and Judd (1985) result as mentioned in the footnote (9) in the Introduction.
long-run steady state social welfare with period-wise budget balance is given by

$$\tau_{B}^{\text{temp}} = 1 - \left[1 - \frac{e_L \tau_L}{1 - \tau_L}\right] \frac{(1 - \delta)\bar{b}^{\text{received}}}{\bar{y}_L} \left(1 + e_B\right) + \frac{1 + \lambda}{1 + \lambda (1 - \alpha)} \bar{b}^{\text{left}} \left(1 + \hat{\epsilon}_B\right)$$

(12)

(b) The optimal tax rate $\tau_{B}^{\text{temp}}$ should decrease with the level of temptation. Further, severe temptation may justify a subsidy at any level of bequest received.

### 3.2 Ignorant generations

Now we deviate from the assumption made in the previous subsection where the bequest leaving agents internalize the effect of taxes paid by their children on the inherited assets. Here we assume that the agents do not necessarily respond to the tax rates on inheritance that they leave, that is, bequest leaving decisions are independent of the taxes their children pay. This has been brought in our model by assuming $b_{ti}$ is given for all $ti$ instead of assuming $b_{ti}$ is optimally chosen. With this change in the assumption we observe that the relationship between the tax rate and the level of temptation is positive, that is, the qualitative result guarantees that as the level of temptation increases, the optimal tax rate actually increase. This is in contrast to the results we have observed so far; both in our paper and also in the literature of capital tax. Of course, the quantitative expression is also different from all the previous expressions.

Technically, this change in assumption appears through the envelope condition. If $b_{ti}$ is chosen optimally as is assumed previously, when envelope theorem is applied we omit the derivative with respect to $b_{ti}$. This is no more appropriate when the generations are assumed to be ignorant and therefore $b_{ti}$ is assumed to be given instead of optimally chosen. All other assumptions of the model, except the assumption mentions above, are valid here. We now proceed to derive the results. Our SWF is defines as

$$\text{SWF} = \max_{\tau_B, \tau_L} (1 + \lambda) \int_{i=0}^{\infty} \delta^i u^{ti} \left(R (1 - \tau_B) b_{ti} + (1 - \tau_L) y_{Lti} + E - b_{t+1i}, 1 - l_{ti}\right)
- \lambda \int_{i=0}^{\infty} \delta^i u^{ti} \left(R (1 - \tau_B) b_{ti} + (1 - \tau_L) y_{Lti} + E, 1 - l_{ti}\right)$$

20
subject to a period-wise budget constraint. Therefore,
\[
dSWF = (1 + \lambda) \int_i u_c^0 \cdot (R (1 - \tau_B) \, db_{0i} - Rb_{0i} \, d\tau_B) - (1 + \lambda) \sum_{t=0}^\infty \delta^{t+1} \int_i Ru_c^{t+1i} \cdot b_{t+1i} \, d\tau_B
\]
\[- (1 + \lambda) \sum_{t=0}^\infty \delta^t \int_i u_c^ti \cdot y_{Li}d\tau_L - \lambda \sum_{t=0}^\infty \delta^t \int_i u_c^ti \cdot (R (1 - \tau_B) \, db_{ti} - Rb_{ti} \, d\tau_B - y_{Li}d\tau_L).
\]

In this setup, the modified expression for $\tau_B^\text{temp}$ is given by
\[
\tau_B^\text{temp} = 1 - \left[ 1 - \frac{e_L \tau_L}{1 - \tau_L} \right] \frac{1 - \delta + \lambda (1 - \alpha - \delta) \left( 1 + \hat{e}_B \right)}{1 + \lambda (1 - \alpha)} \frac{1 + \lambda}{1 + \lambda (1 - \alpha) \bar{R} y_L}.
\] (13)

where it is shown in the proof that $\tau_B^\text{temp}$ increases with the level of temptation.

The result is intuitive. If the generations are ignorant about the tax rates while leaving bequests for their children, there is no point of subsidizing bequest. Subsidizing bequest makes succumbing to present bias less attractive but that works only when the agents are sensitive to the tax rates. When agents are ignorant, if they suffer from present bias the amount of bequest that they leave definitely falls. To keep the tax revenue at the same level, the only option that a planner has is to increase the tax rate since present bias reduces the tax base. This reduction in the tax base and therefore increase in the tax rate of course depend on the degree of present bias. Thus incentivizing bequest leaving by reducing the tax rate or by providing a subsidy on bequest which perfectly worked in the previous cases fails to work here. Hence the optimal tax rate increases with the level of present bias. This analysis has been summarized below as a proposition.

**Proposition 5** In the presence of present bias, incentivizing bequest works only when generations are sensitive to the chosen tax rates. If the generations are ignorant, there is no need to provide incentive to encourage bequest leaving by lowering the taxes or by providing a subsidy. In that scenario, since bequest leaving depletes with present bias and lowers the tax base, optimal tax rate has to increase with the degree of present bias.
4 Calibrations

This section aims to show the impact of various parameters on the optimal tax rate that supports our derived theoretical results. This paper’s major deviation from the existing literature is the assumption of a temptation economy under altruistic setup and it gets more complicated when the agents act in a certain way in response to the taxes paid by their children. In this part, we keep our presentation similar to Piketty and Saez (2013) so that the results can be compared but unlike them we do not provide any numerical result for the French economy since the order of magnitudes moves in the same direction as in the US economy.

4.1 Present bias in BIU

We use steady-state equilibrium tax rate presented in (9) to calculate the optimal tax rates for the US economy. In the benchmark model, following Piketty and Saez (2013) and Kopczuk and Lupton (2007), we set $e_B = \hat{e}_B = e_L = 0.2$, $\tau_L^{Temp} = 30\%$, $r - g = 2\%$, period of length $H = 30$ years, and $\nu = 0.7$. Note that $G = e^gH$ and $R = e^{rH}$ give $G/R = 1/e^{(r-g)H} = 0.55$. The values of the distributional parameters $\bar{b}^{received}$, $\bar{b}^{left}$, and $\bar{y}_L$ are taken from Piketty and Saez (2013).\footnote{Piketty and Saez (2013) used the joint micro-level distribution of bequests received $(b_{i,t})$, bequests left $(b_{i,t+1})$, and lifetime labor earnings $(y_{L,i})$ from the survey data (Survey for Consumer Finances 2010 for the US) to compute the values of distributional parameters $\bar{b}^{received}$, $\bar{b}^{left}$, and $\bar{y}_L$. To this end, they specified social weights $g_{ti}$ and considered percentile p-weights, which concentrate the weights $g_{ti}$ on percentile p of the distribution of bequests received. Consequently, for p weights, $\bar{b}^{received}$, $\bar{b}^{left}$, and $\bar{y}_L$ are the the average amount of bequests received, bequests left, and earnings relative to population averages among pth percentile bequest receivers. They computed the aforementioned distributional weights for individuals aged 70 or older. To estimate $\bar{b}^{received}$, retrospective questions about bequests and gift receipts were used. To estimate $\bar{b}^{left}$, questions about current net wealth were used. Finally, to estimate $\bar{y}_L$ questions regarding wages, self-employment, and retirement incomes were used. Married survey participants’ wealth was found by dividing household wealth by two. When individuals are married, received bequests were calculated by dividing the sum of bequests and gifts received by spouses. Piketty and Saez (2013) also stated the potential problems stemmed from using the survey data. The main problem was reporting bias, as survey participants often stated incorrect amounts for various reasons.} We also assume that distributional parameters and the interest rate $r$ are not affected by the level of the inheritance tax. There is a stream of literature that shows the link between capital (inheritance) taxation and capital accumulation (see for example Conesa et al. (2009)). Hence, a more complete analysis requires endogenizing some of the parameters including $r$. This is however out of the scope of this paper. Also, while a number of estimates exist for the value of the temptation strength parameter $\lambda$, we are not aware of any
Figure 1: Optimal linear inheritance tax rates, by percentile of $\bar{b}_{\text{received}}$ ($\lambda$ varies, $\alpha$ is fixed at 0.9)

estimate of parameter $\alpha$.\footnote{Huang et al. (2013) estimated $\lambda = 0.10$ by using National Income and Product Accounts data and estimated $\lambda = 0.24$ by using Consumer Expenditure Survey data, assuming that agents have self-control preferences in the form of $v(c) = \lambda u(c)$ and the risk aversion parameter is set to the unity.\footnote{Piketty and Saez (2013) reported that the optimum rate was about 50% for the lower 70% of the population in the US economy by setting $\nu = 1$. We set this to 0.70 in our benchmark economy, following Kopczuk and Lupton (2007).}} We thus conduct a number of experiments to show the impact of the parameters $\lambda$ and $\alpha$ on the optimal inheritance tax rates, and to explore the interaction between the parameter $\nu$ and temptation parameters.

Figure 1 examines the implications of the changes in the strength of temptation parameter $\lambda$ on the optimal inheritance tax rates from the perspective of each percentile $p$ of the distribution of the bequest received. We set $\alpha = 0.9$, and vary $\lambda$ by setting it to 0.1, 0.3, and 0.9, respectively. Since the optimum inheritance tax rate can be a quite large negative number for the higher percentiles, we set the lower bound at 20% for the ease of exposition. The optimal linear inheritance tax rate varies from 57% to 56% for the lower 75% of the population in a no temptation economy, in keeping with Piketty and Saez (2013).\footnote{Piketty and Saez (2013) reported that the optimum rate was about 50% for the lower 70% of the population in the US economy by setting $\nu = 1$. We set this to 0.70 in our benchmark economy, following Kopczuk and Lupton (2007).} When individuals face minor temptation, as captured by $\lambda = 0.1$, the optimal linear inheritance tax rates do not much deviate
from the case of no temptation. When individuals face mild temptation, as captured by \( \lambda = 0.3 \), the optimal tax rate varies from 50% to 51%, for the lower 75% of the population. This result clearly shows that the existence of temptation puts downward pressure on the optimal tax rate calculated for each percentile of the distribution of bequest received. When individuals face severe temptation, the optimal linear tax rates for each percentile of the distribution of bequest received decrease substantially. On average the optimal linear inheritance tax rates are lower by 18% for the lower 75% of the population in the severe temptation economy. These results show that, in a case of severe temptation, the optimal linear inheritance tax rates will be significantly lower. For the lower 75% of the population, the optimal inheritance tax rate decreases substantially and becomes negative for the upper 15% of the population in both temptation and no temptation economies. The optimal bequest tax rate is quite stable across the lower 70% because inherited wealth is highly concentrated.\(^{19}\) The lower 70% receive a very low amount of bequests (\( \bar{b}^{\text{received}} \) is quite close to 0%). The lower 50% of bequest receivers make approximately 90% - 95% of average earnings \( y_L \) but leave substantially smaller bequest at around 60% - 70% of the average bequest \( \bar{b}^{\text{left}} \). In both the economies, the lower 70% of the population leaves some amount of bequests but prefer higher inheritance tax rates to minimize their burdens on labor tax.

In our model, the strength of temptation is governed by two parameters, \( \alpha \) and \( \lambda \). In this experiment, we fix \( \lambda \) at 0.3 and vary the values of the parameter \( \alpha \) (see figure 2). For the given value of \( \lambda \), higher values of \( \alpha \) imply relatively severe temptation. Hence, when \( \alpha = 1 \), the optimal inheritance tax rate is significantly lower for all income percentiles compared to the cases in which \( \alpha \) takes relatively lower values. Both exercises support our theoretical findings and show that the existence of temptation puts a downward pressure on the optimal inheritance tax rate for all income percentiles. In the case of severe temptation implied by higher values of \( \alpha \) and \( \lambda \), the temptation economy prescribes significantly lower optimal linear inheritance tax rates.

\(^{19}\)This explanation follows Piketty and Saez (2013).
Figure 2: Optimal linear inheritance tax rates, by percentile of \( \overline{b}^{\text{received}} \) (\( \alpha \) varies, \( \lambda \) is fixed at 0.3)
## Table 1

This table presents simulations of the optimal inheritance tax rate $\tau_B$ using formula (9) for temptation and no-temptation economies. We set the labor income tax rate to 30%. Parameters $\bar{b}^{\text{received}}$, $\bar{b}^{\text{left}}$, and $\bar{y}_L$ are taken from Piketty and Saez (2013).

<table>
<thead>
<tr>
<th>Elasticty $e_B$</th>
<th>Temp</th>
<th>No Temp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$B$</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>$= 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= 0.2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= 0.5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= 1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Optimal tax for zero receivers (bottom 50%), $r - g = 2\% \ (G/R = 0.55)$, $\nu = 70\%$, $e_L = 0.2$

<table>
<thead>
<tr>
<th>Group</th>
<th>$r - g$</th>
<th>$\nu$</th>
<th>$e_L$</th>
<th>$\tau_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0-50</td>
<td>63%</td>
<td>70%</td>
<td>52%</td>
<td>42%</td>
</tr>
<tr>
<td>P51-70</td>
<td>62%</td>
<td>70%</td>
<td>52%</td>
<td>41%</td>
</tr>
<tr>
<td>P71-90</td>
<td>50%</td>
<td>60%</td>
<td>37%</td>
<td>24%</td>
</tr>
<tr>
<td>P91-95</td>
<td>-80%</td>
<td>-43%</td>
<td>-115%</td>
<td>-151%</td>
</tr>
</tbody>
</table>

2. Optimal linear tax rate for other groups by percentile of bequests received, $r - g = 2\% \ (G/R = 0.55)$, $\nu = 70\%$, $e_L = 0.2$

<table>
<thead>
<tr>
<th>Percentile</th>
<th>$\nu$</th>
<th>$e_L$</th>
<th>$\tau_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0-50</td>
<td>63%</td>
<td>70%</td>
<td>52%</td>
</tr>
<tr>
<td>P51-70</td>
<td>62%</td>
<td>70%</td>
<td>52%</td>
</tr>
<tr>
<td>P71-90</td>
<td>50%</td>
<td>60%</td>
<td>37%</td>
</tr>
<tr>
<td>P91-95</td>
<td>-80%</td>
<td>-43%</td>
<td>-115%</td>
</tr>
</tbody>
</table>

3. Sensitivity to capitalization factor, $\nu = 70\%$, $e_L = 0.2$

<table>
<thead>
<tr>
<th>$r - g$</th>
<th>$\nu$</th>
<th>$e_L$</th>
<th>$\tau_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$= 0% \ (G/R = 1)$</td>
<td>32%</td>
<td>46%</td>
<td>21%</td>
</tr>
<tr>
<td>$= 3% \ (G/R = 0.41)$</td>
<td>72%</td>
<td>78%</td>
<td>48%</td>
</tr>
</tbody>
</table>

4. Sensitivity to bequests motives, $r - g = 2\% \ (G/R = 0.55)$, $e_L = 0.2$

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>$e_L$</th>
<th>$\tau_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (100% bequest motives)</td>
<td>47%</td>
<td>58%</td>
</tr>
<tr>
<td>0 (no bequest motives)</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

5. Sensitivity to labor income elasticity, $r - g = 2\% \ (G/R = 0.55)$, $\nu = 70\%$

<table>
<thead>
<tr>
<th>$e_L$</th>
<th>$\tau_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>59%</td>
</tr>
<tr>
<td>0.5</td>
<td>68%</td>
</tr>
</tbody>
</table>
Next, we conduct a sensitivity analysis for temptation (Temp) vis-a-vis no temptation (No temp) economies. Table 1 presents simulations of the optimal inheritance tax rate $\tau_B$ using the equation (9) for temptation and no-temptation economies. The labor income tax rate was set to 30% and $\alpha = 0.9$ and $\lambda = 0.3$ for the temptation economy. In each experiment, we display optimal tax rates for $e_B = \hat{e}_B = 0, 0.2, 0.5,$ and 1. As expected, when $e_B$ approaches to 1, the optimal inheritance tax rates in both economies decrease. The tax rate is lower in the temptation benchmark economy than in the no temptation benchmark economy.

In both temptation and no temptation benchmark economies, we set $r - g = 2\%$ and $H = 30$ causing $G/R = 1/e^{(r-g)H} = 0.55$. Although narrowing the gap between the rates of return and growth leads to lower optimal rates in both economies, increasing this gap yields higher optimal rates. As shown above, the optimal rates in the temptation economy are relatively low.

In our benchmark economies, we set the bequest strength parameter to $\nu = 0.7$. When this is set to 1, optimal rates are relatively low compared to the benchmark economies. In contrast, when we assume a complete absence of bequest motives (i.e. $\nu = 0$), $e_B$ becomes the only limiting factor for tax rates in both temptation and no temptation economies. Hence, optimal tax rates are higher. This is the only case in which the existence of temptation does not affect the results.

The changes in the labor supply elasticity has a moderate effect. As it is expected, a higher labor supply elasticity prescribes higher taxes on inheritance, both under the economy with or without temptation. Exactly the opposite happens when it is lower.

There is an interesting interaction between the altruism parameter $\nu$ and the temptation parameters. In Figure 3, we set $\alpha = 0.9$, and $\lambda = 0.3$ and vary the value of the parameter $\nu$ to explore this interaction. The figure demonstrates that optimal inheritance tax rates are substantially lower in economies in which individuals are more altruistic and/or lack self-control. Interestingly, optimal rates in the temptation economy when $\nu = 0.7$ are almost identical to optimal rates in the no temptation economy when $\nu = 1$. This result shows that a high degree of substitution exists between altruism and temptation parameters.
4.2 Present bias in BBD (ignorant generations)

For the case of sensitive generation we observe that the quantitative prescriptions are certainly different but qualitatively tax rate and the degree of present bias are inversely related. Since we have presented the BIU setup above that explains the same qualitative results here we present the case of ignorant generation only. Particularly we use steady state equilibrium tax rate presented in (13) to calculate the optimal linear tax rates for the US economy: We use the same parameter values as above and set the time-discount parameter $\delta$ to 0.5 and $r = 2\%$ ($R = 1.82$) in the benchmark economy. All other parameters are set to the same values as in the previous section.

Figure 4 examines the implications of the changes in the strength of temptation parameter $\lambda$ on the optimal inheritance tax rates from the perspective of each percentile $p$ of the distribution of the bequest received. As in above, we set $\alpha = 0.9$, and vary $\lambda$ by setting it to 0.1, 0.3, and 0.9 respectively. In this economy, an increase in the strength of temptation puts an upward pressure on the optimal linear inheritance tax rates. This is the exact opposite of what we have shown in the previous subsection. In the case of severe temptation captured by setting $\lambda = 0.9$, the optimal
Figure 4: Optimal linear inheritance tax rates, by percentile of $b^{\text{received}}$ (λ varies, α is fixed at 0.8)

Linear inheritance tax rates on average 100% higher than that of the economy in which individuals do not face temptation.

We also run experiments varying the values of the parameter α by keeping the value of λ at 0.3. These experiments also verify the above results. Higher levels of temptation imposed by the higher values of the parameter α leads to higher optimal linear inheritance tax rates (see Figure 5).

Finally, as in the above section, we run a sensitivity analysis by varying the values of $r$ (we set $r$ to 1% and 3%), $\delta$ (we set $\delta$ to 0.3 and 0.7), and $e_L$ (we set $e_L$ to 0 and 0.5) by keeping the everything else is the same. In all cases, temptation economies prescribed higher optimal linear inheritance tax rates (see Table 2).
Figure 5: Optimal linear inheritance tax rates, by percentile of $\bar{b}^{\text{received}}$ ( $\alpha$ varies, $\lambda$ is fixed at 0.3)
Table 2: This table presents simulations of the optimal inheritance tax rate $\tau_B$ using formula (9) for temptation and no-temptation economies. We set the labor income tax rate to 30%. Parameters $\bar{b}^\text{received}$, $\bar{b}^\text{left}$, and $\bar{y}_L$ are taken from Piketty and Saez (2013).
5 Conclusion

We model an economy with inheritance where altruistic agents’ preferences are subject to temptation and self-control issues. First, using BIU setup, we derive the reduced form expression for the optimal inheritance tax rate and then we show that a negative relationship exists between the optimal inheritance tax rate and the level of temptation. This also leads to the feasibility of a subsidy at any percentile of bequest received when temptation is critical since a subsidy on inheritance provides an incentive to leave more bequest and makes a surrender to temptation less attractive. We then use the standard BBD setup to derive the expression for the optimal tax rates. We observe that if the agents are sensitive and respond to the taxes their next generation pays on the amount of bequest they leave, present bias and optimal taxes are negatively related as in BIU, that is, incentivizing bequest leaving through curtailing temptation works perfectly. However if the agents are ignorant, do not internalize the taxes paid by their descendents on the inheritance they leave, optimal tax rates increase with the level of present bias. This is because the present bias reduces the tax base and to generate the same revenue through taxation, the taxes have to increase with the level of present bias. In short, the rationale of providing incentives by reducing taxes or extending subsidies fails. A calibration exercise supports all these findings.
References


Appendix A

Proof of Proposition 1. (a) Equation (5) implies that

\[-y_L d\tau_L = \frac{Rb_t d\tau_B \left(1 - \frac{e_B \tau_B}{1 - \tau_B}\right)}{1 - \frac{e_L \tau_L}{1 - \tau_L}}.\]

Given (3), the above relationship, and dividing (7) by \(\int_i \omega_{ti} V_{c}^{ti}\) yields

\[\left(1 + \lambda\right) \left\{ -Rb_t d\tau_B (1 + e_{Bti}) + \frac{1 - \frac{e_B \tau_B}{1 - \tau_B}}{1 - \frac{e_L \tau_L}{1 - \tau_L}} Rb_t d\tau_B \frac{y_{Lt_i}}{y_{Lt}} - \frac{b_{ti+1i}}{1 - \tau_B} d\tau_B \right\} \]

\[= \lambda \int_i \omega_{ti} V_{c}^{ti} \left\{ -Rb_t d\tau_B (1 + e_{Bti}) + \frac{1 - \frac{e_B \tau_B}{1 - \tau_B}}{1 - \frac{e_L \tau_L}{1 - \tau_L}} Rb_t d\tau_B \frac{y_{Lt_i}}{y_{Lt}} \right\} \int_i \omega_{ti} V_{c}^{ti}.\]

Now by dividing the above equation by \(Rb_t d\tau_B\) and using the relationship \(V_{c}^{ti} = \alpha V_{c}^{ti}\), \(\alpha \in (0, 1)\), we get

\[(1 + \lambda) \left\{ -\bar{b}^{\text{received}} (1 + \tilde{e}_B) + \frac{1 - \frac{e_B \tau_B}{1 - \tau_B}}{1 - \frac{e_L \tau_L}{1 - \tau_L}} \bar{y}_L - \frac{\bar{b}^{\text{left}}}{R (1 - \tau_B)} \right\} \]

\[= \alpha \lambda \left\{ -\bar{b}^{\text{received}} (1 + \tilde{e}_B) + \frac{1 - \frac{e_B \tau_B}{1 - \tau_B}}{1 - \frac{e_L \tau_L}{1 - \tau_L}} \bar{y}_L \right\} \]

which, given \(\bar{b}^{\text{received}} = \bar{b}^{\text{received}}, \bar{b}^{\text{left}} = \bar{b}^{\text{left}}, \) and \(\bar{y}_L = \bar{y}_L\), guarantees that

\[\tau_B^{\text{temp}} = 1 - \left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right) \frac{\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \tilde{e}_B) + \frac{(1 + \lambda) \bar{b}^{\text{left}}}{R (1 + \lambda (1 - \alpha)) \bar{y}_L}\]

\[1 + e_B - \left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right) \frac{\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \tilde{e}_B)\]

(b) It is easy to verify that \(\frac{d\tau_B^{\text{temp}}}{d\lambda} < 0\) and hence the proof. ■

36
Proof of Proposition 2. (a) Setting $d\text{SWF} = 0$, using the envelope theorem, (5), and $V_{\tilde{c}_t} = aV_{c_t}$, and then dividing the equation by $Rb_t d\tau_B \int_i \omega_i V_{\tilde{c}_t}$ yields the following equation

$$0 = -(1 + \lambda (1 - \alpha)) \frac{1 + e_B}{b_t} (1 + e_{Bi}) + \left[1 + \lambda (1 - \alpha)\right]$$

$$\left(1 - \frac{e_B}{1 - \tau_B}\right) \int_i g_t y_{Li} - \frac{1 + \lambda}{R (1 - \tau_B)} \int_i g_t b_{i+1} V_{\tilde{c}_t}$$

Simplifying the above equation and using $b_{i+1} = Gb_t$ we get

$$(1 + \lambda (1 - \alpha)) \bar{b}^{\text{received}} (1 + \tilde{e}_B) + \frac{1 + \lambda}{R (1 - \tau_B)} \nu \bar{b}^{\text{left}} = \left[1 + \lambda (1 - \alpha)\right] \bar{y}_L \left(\frac{1 - e_B}{1 - \tau_B}\right) \left(\frac{1 - e_L}{1 - \tau_L}\right)$$

from which we derive the desired expression for the optimal tax rate

$$\tau_{B}^{\text{temp}} = \frac{1 - \left[1 - \frac{e_L}{1 - \tau_L}\right] \bar{b}^{\text{received}} (1 + \tilde{e}_B) + \frac{G\nu (1 + \lambda) \bar{b}^{\text{left}}}{R [1 + \lambda (1 - \alpha)] \bar{y}_L}}{1 + e_B - \left[1 - \frac{e_L}{1 - \tau_L}\right] \bar{b}^{\text{received}} (1 + \tilde{e}_B)}$$

(b) It is easy to verify that $\frac{d\tau_{B}^{\text{temp}}}{d\lambda} < 0$. Hence the proof.

Proof of Proposition 3. The proof is straightforward. With $u(c, b)$ homogeneous, bequest decisions are linear in lifetime resources, i.e., $b_{i+1} = s(1 - \tau_L) y_{Li}$ which guarantees $E(\omega_i V_{\tilde{c}_t} b_{i+1})/b_{i+1} = E(\omega_i V_{\tilde{c}_t} y_{Li})/y_{Li}$. This means that $\bar{b}^{\text{left}} = \bar{y}_L$. The level of $\lambda$ does not change this. Also, since there is only one dimension of inequality, bequest taxes are equivalent to labor taxes on distributional grounds, even under temptation. Hence, shifting from bequest taxes also has zero net effect on labor supply. Since parents receive nothing in this model, social welfare is only the parents’ welfare and $\bar{b}^{\text{received}} = 0$. Tax calculated in (15) given $\bar{b}^{\text{received}} = 0$ and $e_L = 0$ confirms that $\tau_B < 0$ since $(1 + \lambda)/1 + \lambda (1 - \alpha) > 1$. If children are also considered in the social welfare function and weights are put on them, $\bar{b}^{\text{received}} > 0$ which along with $\bar{b}^{\text{left}} = \bar{y}_L$ and $e_L = 0$ implies $\tau_B < 0$. Hence the proof.

Proof of Proposition 4. (a) First order condition of the individual utility maxi-
mization \( u_{c}^{t+1} \cdot b_{t+1} \) along with (5), applying the envelope theorem, and assuming \( u_{c}^{t} \alpha u_{c}^{t} \) yields

\[
dSWF = -(1 + \lambda (1 - \alpha)) \int_{t} u_{c}^{0} \cdot b_{0i}(1 + e_{Bi}) R d\tau_{B} - \frac{1 + \lambda}{1 - \tau_{B}} \sum_{t=0}^{\infty} \delta^{t} \int_{t} u_{c}^{t} \cdot b_{t+1} d\tau_{B} \\
+ (1 + \lambda (1 - \alpha)) R d\tau_{B} \frac{1 - e_{B\tau B}}{1 - \tau_{B}} \sum_{t=0}^{\infty} \delta^{t} \int_{t} u_{c}^{t} \cdot \frac{u_{c}^{t}}{y_{L} b_{t}}.
\]

Further, setting \( dSWF = 0 \) at the optimum \( \tau_{B} \) and dividing it by \( R b_{t} d\tau_{B} \int_{t} u_{c}^{t} \), (also note that in the steady state \( b_{t} = b_{0} \) and \( u_{c}^{t} = u_{c}^{0} \)) we get

\[
0 = -(1 + \lambda (1 - \alpha)) \int_{t} u_{c}^{0} \cdot b_{0i}(1 + e_{Bi}) \frac{1 + \lambda}{1 - \tau_{B}} \sum_{t=0}^{\infty} \delta^{t} \int_{t} u_{c}^{t} \cdot b_{t+1} + \\
(1 + \lambda (1 - \alpha)) \frac{1 - e_{BTB}}{1 - \tau_{B}} \sum_{t=0}^{\infty} \delta^{t} \int_{t} u_{c}^{t} \cdot \frac{u_{c}^{t}}{y_{L} b_{t}},
\]

where \( e_{Bi} = \frac{d b_{0i}}{d (1 - \tau_{B})} \frac{1 - \tau_{B}}{b_{0i}} \). This implies that

\[
0 = -(1 + \lambda (1 - \alpha))(1 - \delta) b_{\text{received}} (1 + \hat{e}_{B}) - \frac{1 + \lambda}{R (1 - \tau_{B})} b_{\text{left}} + (1 + \lambda (1 - \alpha)) \frac{1 - e_{B\tau B}}{1 - \tau_{B}} - \frac{\hat{e}_{B} y_{L}}{y_{L} b_{t}}.
\]

Simplifying this further yields

\[
\tau_{B}^{\text{temp}} = \frac{1 - \left[ 1 - e_{B\tau L} \frac{1 - \tau_{B}}{1 - \tau_{L}} \left( \frac{(1 - \delta) b_{\text{received}}}{y_{L}} + \frac{1 + \lambda}{1 + \lambda (1 - \alpha)} \frac{b_{\text{left}}}{R y_{L}} \right) \right]}{1 + e_{B} - \left[ 1 - e_{B\tau L} \frac{1 - \tau_{B}}{1 - \tau_{L}} \left( \frac{(1 - \delta) b_{\text{received}}}{y_{L}} (1 + \hat{e}_{B}) \right) \right]}.
\]

(b) It is easy to verify that \( \frac{d \tau_{B}^{\text{temp}}}{d \lambda} < 0 \). Hence the proof. ■

**Proof of Proposition 5.** Let us first present the steps to obtain the expression for \( \tau_{B}^{\text{temp}} \) with the assumption that \( b_{ii} \) is given. Once the expression for \( \tau_{B}^{\text{temp}} \) is derived, showing the rest is straightforward. Using the first order condition of the individual utility maximization \( u_{c}^{t+1} b_{t+1} = \frac{u_{c}^{t+1} b_{t+1}}{dR(1 - \tau_{B})} \) and \( R (1 - \tau_{B}) b_{t+1} = \)}
\[-Rb_t d\tau_B (1 + e_{Bi}),\] we get

\[
dSWF = -(1 + \lambda) \int_i u_c^{0i} \cdot b_{0i} (1 + e_{Bi}) Rd\tau_B - \frac{1 + \lambda}{1 - \tau_B} \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot b_{t+1} d\tau_B
\]

\[
+ (1 + \lambda) Rd\tau_B \frac{1 - \frac{e_{BTB}}{1 - \tau_B}}{1 - \frac{e_{LT_L}}{1 - \tau_L}} \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot \frac{y_{LT_i} b_t}{y_{Lt}}
\]

\[
+ \lambda \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot b_{t}(1 + e_{Bi}) Rd\tau_B - \frac{1 - \frac{e_{BTB}}{1 - \tau_B}}{1 - \frac{e_{LT_L}}{1 - \tau_L}} R d\tau_B \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot \frac{y_{LT_i} b_t}{y_{Lt}}
\]

where \(e_{Bi} = \frac{db_{ti}}{d(1 - \tau_B)} \cdot \frac{1 - \tau_B}{b_{0i}}\) and \(e_{Bi} = \frac{db_{ti}}{d(1 - \tau_B)} \cdot \frac{1 - \tau_B}{b_{t}}\). Setting \(dSWF = 0\) at the optimum \(\tau_B\), we get the following

\[
0 = -(1 + \lambda) \int_i u_c^{0i} \cdot b_{0i} (1 + e_{Bi}) \frac{R}{b_0 \int_i u_c^{0i}} - \frac{1 + \lambda}{R (1 - \tau_B)} \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot b_{t+1} \int_i u_c^{0i} +
\]

\[
(1 + \lambda) \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot \frac{y_{LT_i} b_t}{y_{Lt} \int_i u_c^{ti}}
\]

\[
+ \lambda \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot b_{t}(1 + e_{Bi}) \int_i u_c^{ti} - \frac{1 - \frac{e_{BTB}}{1 - \tau_B}}{1 - \frac{e_{LT_L}}{1 - \tau_L}} \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot y_{LT_i} \int_i u_c^{ti} \cdot \int_i u_c^{ti}.
\]

Since \(\bar{b}^{\text{left}} = \delta R (1 - \tau_B) \bar{b}^{\text{received}}\), from the above equation we have

\[
0 = -(1 - \delta + \lambda (1 - \alpha - \delta)) \bar{b}^{\text{received}} (1 + \tilde{e}_B) - \frac{1 + \lambda}{R (1 - \tau_B)} \bar{b}^{\text{left}} + \frac{(1 + \lambda (1 - \alpha)) \left(1 - \frac{e_{BTB}}{1 - \tau_B}\right)}{1 - \frac{e_{LT_L}}{1 - \tau_L}} \bar{y}_L
\]

which, after rearranging the terms, generates the following expression for the optimal tax rate

\[
\tau_B^{\text{temp}} = \frac{1 - \left[1 - \frac{e_{LT_L}}{1 - \tau_L}\right] \frac{1 - \delta + \lambda (1 - \alpha - \delta) 1 + \tilde{e}_B}{\delta} + \frac{1 + \lambda}{1 + \lambda (1 - \alpha) \frac{R \bar{y}_L}{e_B}}}{1 + e_B}.
\]
Now it is straightforward that

\[
\frac{d \tau_B^{\text{temp}}}{d \lambda} = - \left[ 1 - \frac{e_L \tau_L}{1 - \tau_L} \right] \frac{1 - \tau_L}{1 + e_B} \left[ \frac{\alpha \delta}{[1 + \lambda (1 - \alpha)]^2} \frac{1 + \hat{c}_B}{\delta} + \frac{\alpha}{[1 + \lambda (1 - \alpha)]^2} \right] \frac{\bar{b}^{\text{left}}}{R \overline{g}_L} \\
= \frac{1 - \frac{e_L \tau_L}{1 - \tau_L}}{(1 + e_B) R \overline{g}_L [1 + \lambda (1 - \alpha)]^2} \bar{b}^{\text{left}} \alpha \hat{c}_B > 0.
\]

Note that the optimal tax rates increase with the level of temptation. Hence the proof. ■
Appendix B

We first construct the case of period zero perspective under the BBD setup and then derive the long run optimal inheritance tax rate in the presence of present bias. As mentioned by Piketty and Saez (2013), under period zero perspective the bequest behavior of generations changes in advance due to the anticipation of changes in the tax rate, that is, a future tax change in date \( T \) does affect all the previous generations.

Before figuring out the exact expressions for the inheritance tax rate, we focus on some of the elasticities that will appear in our discussions. As in Piketty and Saez (2013), we divide \( e_B^{pdv} \), the elasticity of the present discounted value of the tax base with respect to a future tax increase into two parts - the usual part measures post-reform elasticity and the additional part under the period-zero case measures the anticipated pre-reform behavioral elasticities. Formally, \((1 - \delta) \sum_{t=1}^{\infty} \delta^{t-T} e_{Bt} \equiv e_B^{pdv} = e_B^{post} + e_B^{anticip.}\) with \( e_B^{post} = (1 - \delta) \sum_{t=0}^{\infty} \delta^{t-T} e_{Bt} \) and \( e_B^{anticip.} = (1 - \delta) \sum_{t=0}^{T-1} \delta^{t-T} e_{Bt} \) as \( e_B^{post} \) and \( e_B^{anticip.} \) are measured as the discounted average of the elasticities \( e_{Bt} \).

Given the elastic labor supply, the individual’s optimization problem can be written as

\[
\max_{\{b_{t+1}, l_t\}} \left\{ (1 + \lambda) \sum_{t=0}^{\infty} \delta^t E_t u^{l_t} \left( R (1 - \tau_{Bt}) b_{t_i} + (1 - \tau_{Lt}) y_{Lt_i} + E_t - b_{t+1}, 1 - l_t \right) \right. \\
- \lambda \sum_{t=0}^{\infty} \delta^t E_t u^{l_t} \left( R (1 - \tau_{Bt}) b_{t_i} + (1 - \tau_{Lt}) y_{Lt_i} + E_t, 1 - l_t \right) \}
\]

Then the government’s optimization problem can be written as

\[
SWF = \max_{\{\tau_{Bt}, \tau_{Lt}\}} \left\{ (1 + \lambda) \sum_{t=0}^{\infty} \delta^t \int_0^1 u^{l_t} (R(1 - \tau_{Bt}) b_{t_i} + (1 - \tau_{Lt}) w_{Li} l_t) + E_t - b_{t+1}, 1 - l_t \right. \\
- \lambda \sum_{t=0}^{\infty} \delta^t \int_0^1 u^{l_t} (R(1 - \tau_{Bt}) b_{t_i} + (1 - \tau_{Lt}) w_{Li} l_t) + E_t, 1 - l_t \}
\]

subject to a period-wise budget balance, \( \tau_{Bt} R b_t + \tau_{Lt} y_{Lt} = E_t \). As it is assumed that \( b_t \) changes in response to an anticipatory change in \( \tau_B \) to keep the budget balanced, it is necessary to change \( \tau_{Lt} \). This definitively changes the labor supply decision of individuals before and after tax changes and is captured in the following equations

\[
\forall t \geq T, \quad \tau_{Bt} R b_t + R b_t d \tau_B + \tau_{Lt} y_{Lt} + y_{Lt} d \tau_{Lt} = 0, \quad \text{and} \\
\forall t < T, \quad \tau_{Bt} R b_t + \tau_{Lt} y_{Lt} + y_{Lt} d \tau_{Lt} = 0.
\]
This generates the following two equations

\[ \forall t \geq T, \quad d\tau_L y_L = -\frac{1 - e^{\bar{\alpha}\tau_B}}{1 - e^{\bar{\alpha}\tau_L}} Rb_t d\tau_B, \]

\[ \forall t < T, \quad d\tau_L y_L = \frac{e^{\bar{\alpha}\tau_B}}{1 - e^{\bar{\alpha}\tau_L}} Rb_t d\tau_B. \]

The above relationship holds because we assume that a small change in \( \tau_B \) occurs on or after period \( T \), that is \( d\tau_B \) reform starts at \( T \). It can be shown that in this case

\[
dSWF = (1 + \lambda) \left[ -\sum_{t=T}^{\infty} \delta^t \int_i u_c^{ti}, Rb_t d\tau_B - \sum_{t=1}^{\infty} \delta^t \int_i u_c^{ti}, y_{L,t} d\tau_{L,t} \right]
- \lambda \left[ -\sum_{t=1}^{\infty} \delta^t \int_i u_c^{ti}, y_{L,t} d\tau_{L,t} \right] - \lambda \delta^T \int_i u_c^{T_i}, Rb_T d\tau_B.
\]

Using the usual process followed above, we have

\[
0 = (1 + \lambda) \left[ -\sum_{t=T}^{\infty} \delta^t \int_i u_c^{ti}, y_{L,t} d\tau_{L,t} \right] + \sum_{t=T}^{\infty} \delta^t \int_i u_c^{ti}, y_{L,t} d\tau_{L,t} \left[ \frac{e^{\bar{\alpha}\tau_B}}{1 - e^{\bar{\alpha}\tau_L}} - \sum_{t=1}^{T-1} \delta^t \int_i u_c^{ti}, y_{L,t} d\tau_{L,t} \frac{e^{\bar{\alpha}\tau_B}}{1 - e^{\bar{\alpha}\tau_L}} \right]
- \lambda \sum_{t=1}^{\infty} \delta^t \int_i u_c^{ti}, y_{L,t} d\tau_{L,t} \left[ \frac{e^{\bar{\alpha}\tau_B}}{1 - e^{\bar{\alpha}\tau_L}} \right] - \lambda \delta^T \int_i u_c^{T_i}, Rb_T d\tau_B.
\]

This equation can further be simplified to

\[
0 = (1 + \lambda(1-\alpha)) \left[ \bar{y}_L(1 - \delta) \sum_{t=T}^{\infty} \delta^t \frac{1 - e^{\bar{\alpha}\tau_B}}{1 - e^{\bar{\alpha}\tau_L}} - \bar{y}_L(1 - \delta) \sum_{t=1}^{T-1} \delta^t \frac{1 - e^{\bar{\alpha}\tau_B}}{1 - e^{\bar{\alpha}\tau_L}} \right] - (1 + \lambda) \bar{b} \text{ received}.
\]

With \( e_B^{\text{pdv}} = e_B^{\text{post}} + e_B^{\text{anticip}} \), where \( e_B^{\text{post}} = (1 - \delta) \sum_{t=T}^{\infty} \delta^t e_B, e_B^{\text{anticip}} = (1 - \delta) \sum_{t=1}^{T-1} \delta^t e_B, \) and \( e_L^{\text{pdv}} \) satisfies the following relationship

\[
1 - \frac{e_B^{\text{pdv}, \tau_B}}{1 - e_B^{\text{pdv}, \tau_L}} = (1 - \delta) \sum_{t=T}^{\infty} \delta^t \frac{1 - e_B^{\text{pdv}, \tau_B}}{1 - e_B^{\text{pdv}, \tau_L}} - (1 - \delta) \sum_{t=1}^{T-1} \delta^t \frac{e_B^{\text{pdv}, \tau_B}}{1 - e_B^{\text{pdv}, \tau_L}}.
\]

the above equation can be expressed as

\[
0 = (1 + \lambda(1-\alpha)) \bar{y}_L \frac{1 - e_B^{\text{pdv}, \tau_B}}{1 - e_B^{\text{pdv}, \tau_L}} - (1 + \lambda) \bar{b} \text{ received}.
\]

Using the first order condition of individual’s utility maximization \( \bar{b} \text{ received} \) equals \( \frac{\bar{b} \text{ received}}{\delta R(1 - \tau_B)} \).
we then get

\[ 0 = (1 + \lambda (1 - \alpha)) \bar{y}_L \left[ 1 - \frac{e_B \text{pdv} \tau_B}{1 - \tau_B} \right] - \left[ 1 - \frac{e_L \text{pdv} \tau_L}{1 - \tau_L} \right] \frac{(1 + \lambda) \bar{y}_{\text{left}}}{\delta R(1 - \tau_B)}. \]

This guarantees that the optimal tax rate \( \tau_B^{\text{temp}} \) is given by

\[
\tau_B^{\text{temp}} = 1 - \left[ 1 - \frac{e_B \text{pdv} \tau_L}{1 - \tau_L} \right] \frac{1 + \lambda}{1 + \lambda(1 - \alpha)} \frac{\bar{y}_{\text{left}}}{\delta R \bar{y}_L}. \tag{14}
\]

A few observations are immediate. First, note that \( \frac{d\tau_B^{\text{temp}}}{d\lambda} < 0 \) and this also guarantees that when \( e_B \text{pdv} \) is finite and \( \bar{y}_{\text{received}} < 1 \), a positive tax as recommended under that standard preferences is not necessarily the optimal. The negative relationship between the tax rate and the level of temptation persists, and, it is possible that a subsidy is optimal whenever the commitment consumption is different from that under temptation. Thus, the presence of present bias breaks the result that the optimal tax rate is always positive when \( \bar{y}_{\text{received}} < 1 \) (see Piketty and Saez (2013)).

Second, as mentioned above in the introduction, inheritance modeled in this way can also play the role of capital in our setup and therefore our inheritance tax results can be compared to the existing results in the literature of capital tax. The most important observation is that the celebrated Chamley (1986) - Judd (1985) result holds just as it holds in the absence of temptation and self control behavior. 20 Let us explain the reason behind holding the Chamley (1986) - Judd (1985) results in our framework. The term \( e_B \text{pdv} \) that appears in the denominator of the above expression (14) plays a crucial role here. As Piketty and Saez (2013) pointed out, the elasticity \( e_B \text{pdv} \) tends to infinity in the Chamley (1986) - Judd (1985) model with no uncertainty and, therefore, in the long run, zero tax results is obtained. Presence of present biased does not change this route. That means \( e_B \text{pdv} \) is infinite under the Chamley (1986) - Judd (1985) setup independent of the self-control problem and therefore the expression for the tax rate presented above in (14) goes to zero in the long run. Therefore, under the presence of temptation and self-control, the celebrated zero tax on capital result holds and optimality does not demand any subsidy.

20 In a model with capital Krusell et al. (2010) show that the Chamley (1986) and Judd (1985)’s zero capital income tax result does not hold when preferences are subject to temptation and self control. We are not in a position to directly compare these results since the setups are totally different and the only similarity is that both the papers use temptation and self control preferences.
Appendix C

Under the assumption of $\Delta \leq 1$, we first derive the expression for the optimal tax rate. While the expression for the optimal tax obviously different, there is no change in the qualitative results. Under this setup, the individual’s utility maximizing problem remains the same as follows

$$\max_{\{b_{t+1}, l_t\}_{t=0}^{\infty}} \left\{ (1 + \lambda) V^t_i (R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t - b_{t+1i}, R(1 - \tau_{Bt+1}) b_{t+1i}, 1 - l_{ti}) - \lambda V^t_i (R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t, b = 0, 1 - l_{ti}) \right\}.$$

The form of the first order condition with respect to $b_{t+1i}$ is therefore similar to the previous case:

$$V^t_i = R(1 - \tau_{Bt+1}) V^t_i.$$

The government’s problem under this new specification is given by

$$\text{SWF} = \max_{\tau_{Bt}, \tau_{Lt}} \left\{ (1 + \lambda) \sum_{t=0}^{\infty} \Delta^t \int_i \omega_i V^t_i (R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t - b_{t+1i}, R(1 - \tau_{Bt+1}) b_{t+1i}, 1 - l_{ti}) - \lambda \sum_{t=0}^{\infty} \Delta^t \int_i \omega_i V^t_i (R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t, b = 0, 1 - l_{ti}) \right\}.$$

In the long run as all the variables converge,

$$d\text{SWF} = (1 + \lambda) \left( \sum_{t=T}^{\infty} \Delta^t \int_i \omega_i V^t_i \cdot ((R(1 - \tau_B) db_{ti} - R b_{ti} d\tau_B - d\tau_{Lt} y_{Lti}) + \sum_{t=T-1}^{\infty} \Delta^t \int_i \omega_i V^t_i \cdot (-R b_{t+1i} d\tau_B) - \lambda \sum_{t=T}^{\infty} \Delta^t \int_i \omega_i V^t_i \cdot (R(1 - \tau_B) db_{ti} - R b_{ti} d\tau_B - d\tau_{Lt} y_{Lti}) \right) .$$

Assuming that the period-wise balanced budget holds, we can focus on a small reform $d\tau_B$ so that $d\tau_{Bt} = d\tau_B \forall t \geq T$ where $T$ is sufficiently large, keeping $dE_t = 0$. Unlike steady state maximization, in this case, it is necessary to sum all of the effects for $t \geq T$ that are not identical and reform at $T$ also affects those leaving bequests in generation $T - 1$. Before presenting the expression for the optimal tax rate in this environment, we define three average discounted elasticities as follows:

$$e_B = (1 - \Delta) \sum_{t=T}^{\infty} \Delta^{t-T} e_{Bt} \text{ and}$$

$$\hat{e}_B = (1 - \Delta) \sum_{t=T}^{\infty} \Delta^{t-T} \hat{e}_{Bt}, \text{ where } \hat{e}_{Bt} = \frac{\int_i g_{ti} b_{ti} \hat{e}_{Bti}}{\int_i g_{ti} b_{ti}} .$$
Discounted $e_L$ satisfies
\[
1 - \frac{e_B \tau_B}{1 - \tau_B} = (1 - \Delta) \sum_{t=T}^{\infty} \Delta^{t-T} \frac{1 - \frac{e_B \tau_B}{1 - \tau_B}}{1 - \frac{e_L \tau_L}{1 - \tau_L}}.
\]

Having this construction, we express the optimal inheritance tax rate under the social discounting as
\[
\tau_B^{\text{temp}} = 1 - \left[ 1 - \frac{e_L \tau_L}{1 - \tau_L} \right] \frac{\bar{b}^{\text{received}} / \bar{y}_L (1 + \hat{\epsilon}_B) + \frac{1 + \lambda}{1 + \lambda (1 - \alpha)} \hat{y}^{\text{left}}}{1 + \hat{\epsilon}_B - \left[ 1 - \frac{e_L \tau_L}{1 - \tau_L} \right] \frac{\bar{b}^{\text{received}} / \bar{y}_L (1 + \hat{\epsilon}_B)}{1 + \hat{\epsilon}_B}}.
\]  
(15)

To get the above expression we follow that same steps required to prove Proposition (1). By setting $d \text{SWF} = 0$, we have
\[
0 = -(1 + \lambda (1 - \alpha)) \sum_{t=T}^{\infty} \Delta^t \left[ \int_i \omega_i V_i^t (1 + e_B t) - \frac{1 - \frac{e_B \tau_B}{1 - \tau_B}}{1 - \frac{e_L \tau_L}{1 - \tau_L}} R_b d \tau_B y_L^t / y_L^t \right] (1 + \hat{\epsilon}_B) d \tau_B.
\]

Dividing the above expression by $R_b d \tau_B \int_i \omega_i V_i^t$ and using the fact that $g_{ti} = \frac{\omega_i V_i^t}{\int_i \omega_i V_i^t}$, we get
\[
0 = -(1 + \lambda (1 - \alpha)) \sum_{t=T}^{\infty} \Delta^t \bar{b}^{\text{received}} (1 + \hat{\epsilon}_B) + (1 + \lambda (1 - \alpha)) \frac{1 - \frac{e_B \tau_B}{1 - \tau_B}}{1 - \frac{e_L \tau_L}{1 - \tau_L}} \sum_{t=T}^{\infty} \Delta^t \bar{y}_L - \frac{1 + \lambda}{R (1 - \tau_B)} \sum_{t=T-1}^{\infty} \Delta^t \bar{b}^{\text{left}}.
\]

Further simplifying the above we get
\[
\tau_B^{\text{temp}} = 1 - \left[ 1 - \frac{e_L \tau_L}{1 - \tau_L} \right] \frac{\bar{b}^{\text{received}} / \bar{y}_L (1 + \hat{\epsilon}_B) + \frac{1 + \lambda}{1 + \lambda (1 - \alpha)} \hat{y}^{\text{left}}}{1 + \hat{\epsilon}_B - \left[ 1 - \frac{e_L \tau_L}{1 - \tau_L} \right] \frac{\bar{b}^{\text{received}} / \bar{y}_L (1 + \hat{\epsilon}_B)}{1 + \hat{\epsilon}_B}}.
\]