Linking consumption externalities with optimal accumulation of human and physical capital and intergenerational transfers*

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Abstract

This paper opens a new perspective from which one can explain the presence of government intervention in education even in the absence of human capital externality. It argues that consumption externalities can provide rationale for government intervention in education. Within the context of overlapping generations economy, it has also been shown that competitive equilibrium either underaccumulates both physical and human capital or overaccumulates both. Thus the result rules out the possibility of competitive equilibrium deviating from the social optimum in its allocation of physical and human capital in opposite directions. Immediate policy issues have also been discussed.

Keywords: Consumption externality, human capital, education subsidy

JEL Classification: E6, E21, H52, H55
1 Introduction

In a recent contribution, Docquier et al. [21] have argued that in an economy with human capital externality, for a certain range of values of the social weight that the planner assigns to the future generations, in the long run, competitive equilibrium experiences an overinvestment in physical capital along with an underinvestment in education compared to what the planner would have liked. The claim that there exists a possibility where the levels of accumulation of the two types of capital (human and physical) differ from the social optimum in opposite directions raises some concern. If we include a large number of economies in the analysis, it would be much more likely to observe that human and physical capital move in the same direction. Though this is a major concern, the immediate motivation behind this study comes from an observation which is recently being empirically debated. While government involvement in primary and secondary education is almost universal, its presence in higher education (especially, in the form of education loans and subsidies) is also fairly common. The most widely discussed rationale behind government intervention is the alleged external benefits from education: “existing graduates [will have] more graduates to talk to” (Layard [31]). In other words, if there are spillovers from education, then the argument is that those producing it should not bear the full cost of it (for if they were forced to, they would “undereducate” themselves). Such a human capital (measured by education) externality argument features prominently in some of the most influential work in growth theory from the last three decades. While human capital spillover in the production of output has featured in Lucas [33], Romer [37], in Tamura [40] spillover in the human capital accumulation has been considered.\footnote{1} In an intergenerational setup, the implication of human capital externality is that a smart generation will produce a smarter future generation. From a policy-making perspective, the human capital externality and its size is important because it determines the extent to which education and job training should be subsidized (Heckman and Klenow [28] and Heckman [27]). Yet, the empirical backing for such an externality is not as strong as one would think.\footnote{2} These observations help motivate a fairly broad question:

\footnote{1}{The idea of human capital externality was also used by Tamura [41] to explain the diffusion of the Industrial Revolution. More recently Tamura [42] uses the externality induced by human capital on mortality to explain the timing and diffusion of the Demographic Transition to the Industrial Revolution. In all these models human capital externality helps explain the observed phenomenon and implies that equilibrium allocations are not efficient.}

\footnote{2}{Studies involving the estimation of private return (see Card [16] for a survey) and social return to education show that they are very close to each other. As Lange and Topel [30] nicely argue, while there is no evidence that private returns are higher than the social return (and hence negative externality), the evidence that the social returns exceed the private return (and hence positive externality) is also very weak. Among others, Rudd [38], Ciccone and Peri [17], Yamarik [44] (who built on the work of Turner et al [43]) also share the same view that external return to human capital is negligible. Acemoglu and Angrist [2] too reach the same conclusion, however, somewhat arbitrarily argue that if human capital externality exists, a}
how can economists rationalize such ubiquity and importance of government intervention in education when direct spillovers of education are supposedly mild?³

In this paper, I argue that in an economy where no intra and intergenerational human capital externalities are present, externalities in consumption may help make the case for public involvement in education. To that end I restrict my focus within the context of a neoclassical overlapping generations (OG) model. The idea of a consumption externality has long been studied and documented, and involves the notion that people care not just about their own absolute level of consumption but also about how it compares to those around them. The influence that peers, friends and neighbors exercise on an individual’s behavior through social interaction cannot be denied. As noted by Becker and Murphy [12], activities that are most subject to social pressures from others are those that take place publicly. Since majority consumption is easily observable, it is highly susceptible to outside influence. Recently the effect of consumption externalities on optimal physical capital production has been widely discussed in the literature. But surprisingly, in my knowledge, there has been no attempt to investigate the link between consumption externalities and optimal human capital production. This paper tries to fill in this gap.

This notion that an individual’s sense of well being depends on how she compares her own consumption in relation to what she observes around her has been explored in many important papers in the literature. In Duesenberry [23], an agent uses the average level of consumption in the society as a benchmark in determining her utility from her own consumption. Easterlin [24, 25] uses a similar concept in his explanation of the Baby Boom.⁴ This paper also explores consumption externalities that are similar in spirit to the aforementioned papers. Here agents, both middle aged and old alike are influenced by the consumption levels of the rest of the population that is consuming at that time. Thus the externalities considered here are both intra and intergenerational in nature. The effect of one’s "peer group" on magnitude of 1-3% external return is sufficient to justify public subsidies in education. As is common in growth models, I focus on human capital which is measured by education.

³Other popular justifications for strong government involvement in education include the following: a) credit market imperfections (see Becker [9], Schultz [39] and Kodde and Ritzen [29]); b) production of social capital (see Putnam et. al. [35], Durlauf and Fafchamps [22]); c) creating a civic society of knowledgeable voters (see Dee [19]).

⁴In the Easterlin model [24], [25], young agents formed their expectations about the minimal standard of living on the basis of what they experienced while growing up in their parents’ household. If these minimum expectations are not fulfilled, young couples defer births. Thus children growing up during the Depression became accustomed to a low level of consumption. This consumption level of their parents that the agents observed as children provided a reference level for their own consumption in the next period. When they, as adults (after World War II) earned much more than their Depression parents, they consumed some of this unexpected wealth in the form of larger families. When the Baby Boom children, raised in affluent times entered the labor force in a large cohort, driving down their wages, they had to dramatically reduce their family size.
an individual’s consumption behavior is a well accepted social reality. However, it is also not that uncommon to observe that an individual’s consumption choices are also influenced by the consumption patterns of generations other than her own. An agent’s minimum consumption standards are often determined by the consumption levels of her parents’ generation. In fact it gives an individual a sense of pride if she can beat her parent’s consumption for that would mean she has achieved a lot more in a lot less time. Similarly, one can also observe the old generation getting influenced by the young in their consumption choices. The existence of "adults dedicated to indulging their inner child" is quite widespread and they are popularly known in the literature as "rejuveniles". In my knowledge, this paper is the first attempt of its kind to find a link between the existence of these aforementioned consumption externalities and optimal level of both human and physical capital accumulations in an unified framework. An important contribution of the paper is that in a sharp contrast to the endogenous growth models that distinguish three possible regimes of capital accumulation, in this analysis, only two possible regimes are observed; competitive equilibrium either underaccumulates or overaccumulates both types of capital compared to the planner’s solution. Interestingly, the presence of consumption externality guarantees that there will never be a situation where the levels of accumulation of two types of capital (human and physical) differ from the social optimum in opposite directions - thus either underaccumulation occurs in both physical and human capital or there is overaccumulation in both. More importantly, though in our model the type of the consumption externality determines the region where over or underaccumulation takes place, this result holds irrespective of the type of consumption externality present in the economy. Hence a need for government intervention arises.

If we restrict ourselves only to intergenerational arrangements, namely, education and public pension, some important policy prescriptions are worth noting. In line with Docquier et al. [21], if a planner assigns sufficiently high weights to the future generations, the effect of consumption externality is similar to that of a human capital externality in that the justification behind implementing the standard PAYG like structure becomes weak. However, a significantly large regime of capital accumulation exists where implementing standard public pensions (PAYG) through the intergenerational arrangement may be justified. But interestingly, whenever education subsidy is justified, providing public pension benefit (PAYG) is not at all desirable. Thus contrary to the prescriptions of the existing literature, implementation of both education subsidy and standard public pension benefit (PAYG) at

\footnote{For an informal discussion see Noxon [34].}

\footnote{Linking public education and public pension benefit has long been studied and documented. For an informative discussion see Boldrin and Montes [14].}
the same time in order to achieve efficiency is never recommendable. Thus, there can be a role for either subsidizing both human and physical capital or for taxing the savings of both. Interestingly, these policies can be argued independent of more standard production or investment externalities.

The rest of the paper is organized as follows. Under section 2, I present the basic framework of the economy in subsection 2.1 and subsequently in subsection 2.2, I present the competitive equilibrium, while in subsection 2.3, a planner’s first best solution in the presence of consumption externality has been characterized. I present the main results of this paper in section 3. In section 4, I present the results for a representative economy followed by the numerical results. While section 5 concludes, Appendix contains the proofs of all the lemmas, propositions and corollaries.

2 The Model

2.1 Primitives

I consider an economy consisting of an infinite sequence of overlapping generations, an initial old generation, and an infinitely-lived government. As in De la Croix and Michel [18], Boldrin and Montes [14] and more recently in Docquier et al. [21], in this model too, agents live for three periods. In the first period of life (i.e., when they are young), agents borrow to invest in their education. They work and pay off their loans in the second period, and retire in the third period. Let \( t = 1, 2, \ldots \) index time. The generation that works during period \( t \) is indexed by \( t \) and its population size is denoted by \( N_t \). At any date \( t \), the total population consists of old, middle-aged and young agents and its size is given by \( N_{t-1} + N_t + N_{t+1} \). The population is assumed to grow at a gross rate of \( N \) or a net rate \( n \) i.e., \( N = 1 + n \). There is a single final good and it can either be consumed in the same period it is produced, or it can be stored to yield capital in the following period. Let \( K_t \) and \( H_t \) denote the aggregate levels of physical and human capital respectively at date \( t \), and let \( h_t \equiv H_t / N_t \). The aggregate production function is given by \( Y_t = F(K_t, H_t) \) where \( F \) is assumed to be homogeneous of degree 1. This assumption allows us to write \( Y_t = H_t f(k_t) \), where, \( k_t \equiv \frac{k_t}{h_t} \) and \( k_t = \frac{K_t}{N_t} \). The function \( f \) is assumed to be positive, strictly increasing and strictly concave in its argument, i.e., \( f > 0, f' > 0 \) and \( f'' < 0 \). For reasons of analytical tractability, capital is assumed to depreciate fully between periods.

An agent belonging to generation-\( t \) borrows an amount \( e_{t-1} \) from a perfect capital market and invests in education in period \( t - 1 \). The education she acquires in \( t - 1 \) translates into
human capital in period $t$ as described by the process

$$h_t = \phi(e_{t-1}),$$

where $\phi(\cdot)$ is a strictly increasing and a strictly concave function that satisfies the Inada conditions, i.e., $\phi'(\cdot) > 0$, $\phi''(\cdot) < 0$ with $\phi'(0) = \infty$, $\phi''(\infty) = 0$. As already discussed in the introduction, the construction of $h_t$ in this paper ensures that the production of human capital is free from the effect of any kind of externality. Thus I drop all assumptions concerning human capital externality including the standard assumption that agents are born with some human capital (for externality, see for e.g., Tamura [40, 41, 42], Zhang [45], Boldrin and Montes [14], Docquier et al. [21]). Galor and Moav [26] have also worked with a similar production function, albeit in a different context, where human capital production depends only on the amount of government subsidy available. I provide a perfect capital market to guarantee that any deviation from optimal human capital production is not due to the absence of a perfect source of borrowings.

If borrowing for education is difficult and costly, agents who inherit significant initial wealth and do not need to borrow have better means to invest in human capital (see Atkinson [5], Becker [9]). Majority parents exhibit altruism towards their children by leaving inheritance, frequently make sacrifices for them, even by reducing their own consumption. Altruistic parents who plan on leaving bequests can avoid this trade-off by using bequest to finance their investment in children (see Becker [10], Becker and Murphy [11]). Parents finance the bulk of their children’s education not because of a credit market imperfection but mostly out of altruism. Children rarely pay their parents back in any meaningful way in rich countries. Since by investing in one’s children’s education, parents actually influence their future earnings and their economic welfare (see Becker and Tomes [13]), this further strengthens an imagination that parent’s altruism links individuals to benchmark their consumption against those of their parents, similar to what we have done in our model. However, since direct altruism through parental funding in education may constrain the amount of funds available for investing, for the present purpose, I assume a perfect capital market, refraining from modelling bequest as the only or a partial source of funding in education.

It is clear from the previous discussion that preferences of agents play an important role in the ensuing analysis. For simplicity, I assume the young at any date do not consume. Let $c_t$ denote the middle-age consumption of a generation-$t$ individual and let $d_{t+1}$ denote her old-age consumption. An agent compares her own consumption with a reference point while
determining her. More precisely, the lifetime utility of a generation-\( t \) agent is given by

\[ u_t = u \left( \widehat{c}_t, \widehat{d}_{t+1} \right) \]

where \( \widehat{c}_t \) and \( \widehat{d}_{t+1} \) denote the effective levels of consumption when the agent is middle-aged and old respectively. This utility function is strictly increasing, strictly concave, and satisfies Inada conditions for both of its arguments. The effective level of consumption of a generation-\( t \) agent is given by

\[ \widehat{c}_t = \widehat{c}_t \left( c_t, \sigma \left( c_t, d_t \right) \right) \]

and

\[ \widehat{d}_{t+1} = \widehat{d}_{t+1} \left( d_{t+1}, \varphi \left( c_{t+1}, d_{t+1} \right) \right) \]

where \( \sigma \) and \( \varphi \) are two negatively valued functions which represent the externality induced by others on an agent’s own consumption when she is middle-aged and old respectively. Thus \( \sigma \) and \( \varphi \) are interpretable as the reference consumption levels of an agent in the relevant period. That \( \sigma \) and \( \varphi \) are negatively valued essentially implies that an agent’s utility when she is both middle aged and old, is derived from the difference between her absolute and a reference level of consumption. This assumption that an agent derives utility from subtracting the benchmark level from the actual level of consumption has been explored in many studies (see for e.g., Akerlof [3], Ljungqvist and Uhlig [32], Bowles and Park [15], Alonso-Carrera et al. [6], Aronsson and Johansson-Stenman [4] among others). The reference level can be modeled in many different ways. In this study, the reference level of consumption in each period is determined by the average consumption of the two generations consuming at that period. The general form of the consumption floor used in this paper not only allows us to capture all possible influences of every consuming generation, but also gives us the freedom to choose the degree or extent to which an agent gets affected in her consumption choices by her peers and the other consuming generation. Thus it is able to capture the possibility that an agent may compare herself more with some than with others. The construction of the reference consumption level in such a way that it allows an agent to get influenced in her consumption decision by a generation other than her own is a natural extension of the keeping-up-with-the-Joneses phenomenon. Note that since there is no heterogeneity among agents belonging to the same cohort, the average consumption level of a particular generation at the equilibrium is just the consumption of an agent, that is \( \sigma \left( \overline{c}_t, \overline{d}_t \right) = \sigma \left( c_t, d_t \right) \) and \( \varphi \left( \overline{c}_{t+1}, \overline{d}_{t+1} \right) = \varphi \left( c_{t+1}, d_{t+1} \right) \), where \( \overline{c}_t \), \( \overline{d}_t \), \( \overline{c}_{t+1} \) and \( \overline{d}_{t+1} \) represent the average level of consumption of the relevant generation (see for e.g., Abel [1]). Here, for simplicity, I impose the condition of linearity in arguments on \( \sigma \) and \( \varphi \); that is both \( \sigma \) and \( \varphi \) are linear functions.
with respect to their arguments. The specification of the utility function in this paper is more general than the one used in Alonso-Carrera et al. [6]. A close version of these preferences have also been used by Abel [1] who uses the same variables to construct the benchmark level. However in his model, not only do the variables appear multiplicatively in the consumption floor, the reference consumption level also influences an agent’s consumption multiplicatively (also see Alonso-Carrera et al. [7] in this regard). The partial derivative \( |\sigma_c| (|\phi_{t+1}|) \) represents the degree to which a middle-aged (old) agent keeps up with the others in her own generation. On the other hand, \( |\sigma_d| \) and \( |\phi_{t+1}| \) represent the degree to which a middle aged and an old agent respectively keeps up with the other generation. While this specification is fairly general, Barnett and Bhattacharya [8] associate \( |\phi_{t+1}| > 0 \) with ‘rejuvenile’ behavior of the old trying to keep up with their children. These notions can also be extended based on the values of \( |\sigma_c|, |\sigma_d|, |\phi_{t+1}| \) and \( |\phi_{t+1}| \). Relatively high values both \( |\sigma_c| \) and \( |\phi_{t+1}| \) imply that the weight that is given to the level of consumption of the middle-aged (the working class) in the construction of the consumption reference is high. I call this a middle-aged driven externality \([EM]\). Similarly, when both \( |\sigma_d| \) and \( |\phi_{t+1}| \) are relatively high, it is an old driven externality \([EO]\). Thus this general expression of utility helps us investigate the many possible ways in which the consumption levels of others affects an agent’s own consumption.

When middle-aged, agents supply their human capital inelastically in competitive labor markets, earning a wage rate, \( w_t \), at time \( t \), where

\[
w_t \equiv w(\bar{k}_t) = f(\bar{k}_t) - \bar{k}_tf'(\bar{k}_t)
\]

and \( w'(\bar{k}_t) > 0 \). In addition, capital is traded in competitive capital markets, and earns a gross real return of \( R_{t+1} \) between \( t \) and \( t+1 \), where

\[
R_{t+1} \equiv R(\bar{k}_{t+1}) = f'(\bar{k}_{t+1})
\]

with \( R'(\bar{k}_{t+1}) < 0 \ \forall t \) by the property of function \( f \). Thus both the factors are paid according to their marginal productivities.

Parents in this economy are selfish and do not care for the education of their children. A generation-\( t \) agent borrows amount \( e_{t-1} \) in period \( t-1 \) at the gross interest rate \( R_t \). The agent pays off this loan with the income earned during period \( t \). The income \( W_t \) of a middle-aged agent in period \( t \) is given by \( W_t \equiv w_th_t - R_te_{t-1} \). Thus a generation-\( t \) agent’s optimization problem can be written as:

\[
\max_{s_t, e_{t-1}} u(c_t, d_{t+1})
\]
subject to
\[ c_t = W_t - s_t, \quad d_{t+1} = R_{t+1}s_t, \]
where \( s_t \geq 0 \) denotes saving.

Assuming interior solutions and using (1), the solution to a generation-\( t \) agent’s problem is characterized by the following optimality conditions
\[ s_t : u_{\hat{c}_t} = u_{\hat{d}_{t+1}}R_{t+1} \Rightarrow \frac{u_{\hat{c}_t}}{u_{\hat{d}_{t+1}}} = R_{t+1} \quad (4) \]
and
\[ e_{t-1} : w_t \phi'(e_{t-1}) = R_t \Rightarrow \phi'(e_{t-1}) = \frac{R_t}{w_t}. \quad (5) \]
Note that \( s_t > 0 \) implies \( W_t > 0 \). The first optimality condition (4) is straightforward. It simply describes the optimum intertemporal consumption-saving decision of the agent. In this condition, as is usual in a competitive setup, the reference externality functions are not affected by the agent’s decision. The second condition (5) represents the agent’s optimum expenditure decision towards education. This equation is very intuitive and can also be looked at from another angle. It guarantees that optimality is indeed reached where there is no incentive for an agent to increase her education marginally, that is, where \( \frac{\partial W_t}{\partial e_{t-1}} = w_t \phi'(e_{t-1}) - R_t \) equals zero. It can also be seen that this second optimality condition is not directly affected by any type of consumption externality. Before I formally define the competitive equilibrium, let me introduce the market clearing condition for this economy. While a part of the collective savings of the middle-aged is used to finance education for the young, the remaining amount becomes capital stock for the next period. Hence the market clearing condition can be written as \( N_t s_t = K_{t+1} + N_{t+1}e_t \), which means
\[ s_t = K_{t+1}(1 + n) + (1 + n)e_t. \quad (6) \]

2.2 Competitive Equilibrium (CE)

I begin this subsection with the formal definition of a competitive equilibrium.

**Definition 1** A competitive equilibrium for the economy described above is a sequence of household decisions involving consumption allocations \( \{c_t, d_t\}_{t=0}^{\infty} \), allocations of saving, capital and education expenses, \( \{s_t, k_t, e_t\}_{t=0}^{\infty} \), and factor prices \( \{w_t, R_t\}_{t=0}^{\infty} \) such that the agents solve their optimization problem at each date \( t \), the market-clearing condition (6) is satisfied, and the factor prices satisfy (2)-(3).
Henceforth, I use the superscript $CE$ to denote the competitive equilibrium outcomes. Using (2)-(3), I can rewrite (4)-(5) as

\[
\frac{u_{Ct}^{CE}}{u_{d_{t+1}}^{CE}} = f'(k_t^{CE}) ,
\]

and

\[
\phi'(e_{t-1}^{CE}) = \frac{f'(k_t^{CE})}{f(k_t^{CE}) k_t^{CE} f'(k_t^{CE})} ,
\]

which, in turn, implies

\[
e_{t-1}^{CE} = (\phi')^{-1} \left( \frac{R(k_t^{CE})}{w(k_t^{CE})} \right) .
\]

I now proceed to derive the condition from which $k_t$ is determined in the competitive equilibrium. The income $W_t$ can be re-written as

\[
W_t = w_t h_t - R_t e_t - w_t \phi(e_{t-1}) - w_t \phi'(e_{t-1}) e_{t-1}
\]

\[
= w_t (k_t) \phi(e_{t-1}(k_t)) [1 - \eta_{h,e}(k_t)] ,
\]

where $\eta_{h,e}$ is the elasticity of $\phi$ with respect to $e$. Per capita saving at $t$ by the working middle-aged agent is given by

\[
s_t = W_t - c_t = s_t(W_t, d_t) = s_t(w_t(k_t) \phi(e_{t-1}(k_t)) [1 - \eta_{h,e}(k_t)], R_t(k_t) s_{t-1}) .
\]

Using (11) and (6) I get the following equilibrium law of motion for the physical to human capital ratio for the economy:

\[
s_t \left( w_t(k_t) \phi(e_{t-1}(k_t)) [1 - \eta_{h,e}(k_t)], R_t(k_t) (1 + n) \phi(e_{t-1}(k_t)) + (1 + n) e_{t-1}(k_t) \right)
\]

\[
= k_t^{CE} (1 + n) \phi(e_{t+1}(k_t)) + (1 + n) e_t(k_t^{CE}) .
\]

All competitive equilibrium sequences $\{k_t\}$ and $\{h_t\}$ must satisfy (12). A steady state equilibrium is a time-invariant sequence of $c_t$, $d_t$, $s_t$ and $e_t$. In particular, in a steady state,

\footnote{In section 4, we present our results numerically where this calculation is required. In order to make the calculations manageable, I assume a logarithmic utility function where $\varphi = 0$, i.e., the consumption of an old agent is not affected by any kind of externality. However, no such restriction has been imposed on $\varphi$ while presenting the main result. This assumption has been made only while finding the law of motion of physical to human capital ratio.}
a time invariant $k^{CE}$ satisfies (12).

### 2.3 Social Planner’s (SP) allocations

This section deals with a planner’s problem. A planner takes into account the consumption externalities ignored by individual agents. It is easy to verify that the resource constraint for a planner can be written as:

$$h_t f(k_t) = c_t + \frac{d_t}{1+n} + (1+n)(e_t + k_{t+1}).$$

A planner maximizes the sum of lifetime utilities of all the generations over the infinite horizon subject to the above resource constraint. I assume that generational utility is discounted by a factor $\lambda \in (0,1)$. I refer to $\lambda$ as the social weight that a planner attaches to the future generations. A high $\lambda$ implies that the social discount rate at which a planner devalues the future generations is low. In the ensuing analysis, a single planner with different social weights $\lambda$ can also be interpreted as different planners each being indexed by her own $\lambda$. The Lagrangian\(^8\) for a generic planner’s problem is as follows:

$$\mathcal{L} \equiv \sum_{t=0}^{\infty} \lambda^t \left\{ u(\hat{c}_t, \hat{d}_{t+1}) + q_t [h_t f(k_t) - c_t - \frac{d_t}{1+n} - (1+n)(e_t + k_{t+1})] + p_t [\phi(e_t) - h_{t+1}] \right\}$$

where $\lambda^t q_t$ and $\lambda^t p_t$ are the multipliers associated with the resource constraint of the economy and human capital formation technology respectively, at date $t$.

The first order conditions with respect to $c_t^{SP}$, $d_t^{SP}$, $e_t^{SP}$, $k_{t+1}^{SP}$ and $h_{t+1}^{SP}$ are given below where the superscript $SP$ indicates the planner’s outcome:

$$c_t^{SP} : \lambda^t \sigma_{\hat{c}_t} + \lambda^{-1} \sigma_{\hat{d}_t} \varphi_{\hat{c}_t} - \lambda^t q_t = 0$$

$$d_t^{SP} : \lambda^t \sigma_{\hat{d}_t} + \lambda^{-1} \sigma_{\hat{d}_t} (1 + \varphi_{\hat{d}_t}) - \lambda^t \frac{q_t}{1+n} = 0$$

$$e_t^{SP} : -\lambda^t (1+n) q_t + \lambda^t p_t \phi'(e_t^{SP}) = 0$$

\(^8\)An alternative Lagrangian of the Planner’s problem can be written in the following way:

$$\mathcal{L} \equiv \sum_{t=0}^{\infty} \lambda^t \left\{ u(\hat{c}_t, \hat{d}_{t+1}) + q_t [\phi(e_t f(k_t)) - c_t - \frac{d_t}{1+n} - (1+n)(e_t + k_{t+1})] \right\}.$$
\[ k_{t+1}^{SP} : \lambda^{t+1} q_{t+1} h_{t+1}^{SP} f'(\bar{k}_{t+1}^{SP}) \left( \frac{1}{h_{t+1}^{SP}} \right) - \lambda^{t} q_{t}(1+n) = 0 \tag{17} \]

\[ \Rightarrow q_{t}(1+n) = \lambda q_{t+1} f'(\bar{k}_{t+1}^{SP}) \]

\[ h_{t+1}^{SP} : \lambda^{t+1} q_{t+1}[h_{t+1}^{SP} f'(\bar{k}_{t+1}^{SP}) - \frac{k_{t+1}^{SP}}{(h_{t+1}^{SP})^2}] + f(\bar{k}_{t+1}^{SP})] - \lambda^{t} p_{t} = 0 \tag{18} \]

\[ \Rightarrow \lambda^{t+1} q_{t+1}[f(\bar{k}_{t+1}^{SP}) - f'(\bar{k}_{t+1}^{SP})\bar{k}_{t+1}^{SP} - \lambda^{t} p_{t} = 0. \]

From (16), (17) and (18), I have

\[ \phi'(e_{t}^{SP}) = \lambda \frac{f'(\bar{k}_{t+1}^{SP})}{f(\bar{k}_{t+1}^{SP}) - f'(\bar{k}_{t+1}^{SP})\bar{k}_{t+1}^{SP}}. \tag{19} \]

I define the ‘externality factor’ as

\[ \Delta_{t} \equiv \frac{(1 + \varphi_{ct})(1+n) - \varphi_{ct}}{(1+\sigma_{ct}) - \sigma_{ct}(1+n)}. \tag{20} \]

Since I assume that both \( \sigma \) and \( \varphi \) are linear in their arguments, it guarantees that the partial derivatives of \( \sigma \) and \( \varphi \) are time invariant, that is, \( \sigma_{ct} = \sigma_{ct+1} \), \( \sigma_{dt} = \sigma_{dt+1} \) and \( \varphi_{ct} = \varphi_{ct+1} \), \( \varphi_{dt} = \varphi_{dt+1} \) for all \( t \). This makes \( \Delta_{t} \) time invariant even without the assumption of steady state, and therefore, hereafter I denote \( \Delta_{t} \) by only \( \Delta \). Note that from (14) and (15), I have

\[ \frac{u_{ct}^{SP}}{u_{dt}^{SP}} = \frac{1}{\lambda} \Delta, \tag{21} \]

and using (14), (15) and (17), I can derive the following intertemporal relationship:

\[ \frac{u_{ct}^{SP}}{u_{dt+1}^{SP}} = f'(\bar{k}_{t+1}^{SP}) \frac{\Delta}{1+n}. \tag{22} \]

Combining the above two equations, I get

\[ \frac{u_{ct+1}^{SP}}{u_{dt}^{SP}} = \frac{1}{f'(\bar{k}_{t+1}^{SP})\lambda}(1+n). \tag{23} \]

Note that at a steady state,

\[ f'(\bar{k}_{t+1}^{SP}) = \frac{1}{\lambda}(1+n) \tag{24} \]

\[ ^{9} \text{Algebraic calculations are shown in the supplementary materials.} \]
holds. This condition defines the modified golden rule in this model. It is easy to verify that
\( \lambda \) and \( \overline{k}^{SP} \) are positively related, which in turn implies that if a planner assigns high weight
to future generations, there will be an increase in the steady state requirement of golden rule
\( \overline{k}^{SP} \). When there is no externality, at each \( t \), a planner allocates consumption among two
generations in such a way that \( \lambda u_{t}^{SP} = u_{dt}^{SP} \) holds, i.e., marginal utility from consumption
is same for each generation. The term \( \lambda \) appears because in between any two consecutive
generations, the latter generation is devalued at a rate \( \lambda \) by a planner. However, in the
presence of externalities it has to be adjusted by the externality factor \( \Delta \) (see (21)).

3 Planner’s choice and desirability

In this section I explore the conditions under which the market economy would over or
underaccumulate capital relative to what a benevolent social planner would have deemed
correct. The literature goes back to Diamond [20] where he shows that the competitive
solution may be dynamically inefficient. Further, if there are no distortions in the economy
and it is dynamically efficient, there is no feasible way of redistributing resources across
generations in a way that is pareto improving. In case of overaccumulations, intergenerational
arrangement proves useful in increasing welfare of all the generations. However, the present
context is more intricate since the model not only includes human capital as a choice variable
but also involves within and cross-generational consumption externalities. It is clear from
what has been observed in the previous section that a planner’s choice of social weight and
hence her desire to achieve a particular allocation plays a crucial role in determining the
optimal levels of human and physical capital accumulation. Since, for the planner’s part, I
solve an infinite dynamic-planning problem with declining weights on generations to come,
I have to find a way to make the market and the planning solutions directly comparable if I
am to compare the allocations. In order to do so I establish a common point around which
this discussion will be meaningful by devising the notion of a ‘laissez-faire supported’ social
weight which I denote by \( \overline{\lambda} \). By construction, at this specific social weight, if there is no
externality in the economy, planners allocations coincide with that of a laissez-faire economy.
However, in the presence of externality, the market solutions and planning allocations at
this social weight may differ. I mainly compare the optimality conditions in the competitive
equilibrium (7, 8) with the planner’s (19, 22). In comparing the optimality conditions (7)
and (22), one must note that they differ due to the effect of externality. It is only at \( \overline{\lambda} \) that
these optimality conditions become identical and thus the marginal rates of substitution also
coincide. However, in the presence of consumption externality, this does not guarantee that
\( \overline{k}^{SP} \) are \( \overline{k}^{CE} \) are identical at \( \overline{\lambda} \). On the other hand, though (19) varies with \( \lambda \), since human
capital externality is absent, condition (19) is similar to the condition (8). The allocations are same only when $k^{SP}$ equals $k^{CE}$, but, in the presence of externality, it is not necessarily at $\lambda$ except in a very special case of balanced externality which is discussed later. I begin the discussion by studying the effect of consumption externality at this laissez-faire supported social weight. The comparisons then proceed by contrasting competitive allocations with those preferred by utilitarian planners with discount rates different from this weight. Before I present the main results of the paper formally, I will need to establish a few more results.

It can be easily verified that for all $t$, $\frac{\partial k^i_t}{\partial e_{t-1}} > 0$, $i = CE, SP$. Naturally the result also holds in a steady state. If I denote the elasticity of $k_t$ with respect to $e_{t-1}$ as $\eta_{k,e}$, then it is easy to check that for every $k, h, e$ and $t$, $\eta_{k,e} > \eta_{h,e}$ holds, i.e., whenever the investment in education changes, the proportionate change in per capita physical capital dominates the proportionate change in human capital. It can also be verified that both $k^{SP}$ and $e^{SP}$ increase as the social weight increases. Since $\eta_{k,e} > \eta_{h,e}$ holds, an increase in social weight not only increases optimal per capita physical and human capital production but also the ratio of per capita physical to human capital. Thus, I have the following result.

**Lemma 1** $\eta_{k,e} > \eta_{h,e}$ holds for both $CE$ and $SP$. Also, at a steady state, for any $\lambda$, $\frac{d e^{SP}}{d \lambda} > 0$ and $\frac{d k^{SP}}{d \lambda} > 0$.

For a corresponding result of the second part of the above lemma in an endogenous growth framework, see Docquier et al. [21]. Note that as $\lambda \in (0, 1)$, from (24), it is evident that as $\lambda \to 0$, $k^{SP} \to 0$. On the other hand, $k^{SP} \to (f')^{-1}(1 + n)$ when $\lambda \to 1$. Since $\frac{d e^{SP}}{d \lambda} > 0$, at the steady state $k^{SP} \in (0, k^{SP}_{max})$ where $k^{SP}_{max} = (f')^{-1}(1 + n)$. Thus existence of $\lambda$ naturally imposes a restriction that $k^{SP}(\lambda) \in (0, k^{SP}_{max})$ where $k^{SP}(\lambda)$ represents $k^{SP}$ associated with the weight $\lambda$. I show if such a $\lambda = \lambda$ exists then it must be unique. Hence the following lemma.

**Lemma 2** If there exists a laissez-faire supported social weight $\lambda$, then it must be unique.

What Lemma 2 essentially says is that given a competitive equilibrium and its corresponding $k^{CE}$, there is an unique $\lambda$ for each possible $\Delta$. I should mention here that though $\lambda$ depends on $\Delta$, for simplicity of notations, I deliberately use the notation $\lambda$ independent of the values of $\Delta$. There is another important uniqueness result between $k^{SP}$ and $e^{SP}$ which follows directly from (8) and (19) and is stated as Lemma 3 below. The result is quite expected since there is no spillover from human capital.

**Lemma 3** If for any $\Delta$ there exists a $\lambda = \lambda$ so that $k^{SP}(\lambda) = k^{CE}$ holds, then at the same $\lambda$, $e^{SP}(\lambda) = e^{CE}$ and vice-versa.
From the technical point of view, the paper closest in spirit to the current one is Docquier et al. [21], though their setup is different. In their model of endogenous growth through intergenerational human capital externality, the authors consider two ratios, namely, the ratio of physical to human capital, $\bar{k}$, and the ratio of investment in education to the level of human capital, $\bar{e} \equiv e/h$ in comparing the allocations in $CE$ with those under $SP$. Since, for any given value of $\bar{k}$, the presence of human capital externality will always make the laissez-faire level of $\bar{e}$ differ from the social optimum, in their model there does not exist any unique social weight at which the values of the ratios in $CE$ coincide with their respective levels in $SP$. Therefore, they focus on two different social weights for which the above mentioned ratios are equal when compared to the competitive solution. In contrast, my analysis centers around two variables, namely per capita human $h$ and physical capital $k$. More specifically, I concentrate only on the ratio $\bar{k}$. Using the results of Lemma 3 one can check that given feasibility, $h$ and $k$ in $CE$ and $SP$ coincide at a unique social weight. By the virtue of the uniqueness results guaranteed by Lemmas 2 and 3, one then has the freedom to initiate the process of comparison from any particular benchmark social weight where the variables $h$ and $k$ are comparable. Thus I start with the social weight $\bar{\lambda}$ described above. This deliberately chosen starting point itself has a special significance. It specifies the weight where the allocations in $CE$ and $SP$ are identical in an economy without consumption externalities. However, when consumption externalities are present, though marginal rate of substitution for an agent in competitive economy is same as in the planner’s solution at $\lambda = \bar{\lambda}$, the accumulation levels of $h$ and $k$ in the competitive equilibrium differ from the planner’s allocation at this social weight. The rest of the analysis, as will be shown later, is straightforward. I proceed by varying the value of $\lambda$ (and consequently the marginal rate of substitution) till I arrive at that particular social weight where the levels of $h$ and $k$ in the planner’s economy are identical to that in the competitive solution. This particular feature, however, is not available in Docquier et al. [21]. The uniqueness result has a direct consequence in the literature of deviation in efficient and equilibrium allocations.

Now I am in a position to state the main results of the paper. For simplicity, for rest of the analysis, I assume that there is no population growth.

**Proposition 1** At a laissez-faire supported social weight $\bar{\lambda}$, if the competitive allocation differs from the social optimum, then competitive equilibrium either underaccumulates both human and physical capital or overaccumulates both. That is at $\bar{\lambda}$, in the presence of consumption externality there will never arise a situation where the levels of accumulation of the two types of capital, human and physical, differ from the social optimum in opposite directions.
While I ultimately go on to prove that given the type of consumption externality present in the economy, there will never exist a social weight for which the accumulation levels of the two types of capital in the competitive equilibrium differ from the social optimum in opposite directions, I start the analysis from \( \bar{\lambda} \). The above result guarantees that for any given consumption externality and at its corresponding \( \bar{\lambda} \), in a competitive equilibrium the following two outcomes will never be experienced: a) \( k^{CE} > k^{SP}(\bar{\lambda}) \) along with \( h^{CE} < h^{SP}(\bar{\lambda}) \) and b) \( k^{CE} < k^{SP}(\bar{\lambda}) \) along with \( h^{CE} > h^{SP}(\bar{\lambda}) \). More specifically, I present the possible outcomes in the following corollary.

**Corollary 1** At the laissez-faire supported social weight \( \bar{\lambda} \), competitive equilibrium under-accumulates (overaccumulates) both human and physical capital if the externality factor is greater (less) than unity, that is, if \( \Delta > 1 \) (\( \Delta < 1 \)).

Thus according to the above result, if the social weight is \( \bar{\lambda} \) and \( \Delta > 1 \), I have \( k^{CE} < k^{SP} \) and \( h^{CE} < h^{SP} \). However, if \( \Delta < 1 \), then \( k^{CE} > k^{SP} \) and \( h^{CE} > h^{SP} \) hold. Note that \( \Delta \) equals 1 in two different situations, a) when all externalities are absent, and in a very special situation b) when \( \sigma_{ct} + \varphi_{ct} = \sigma_{dt} + \varphi_{dt} \). This latter configuration is termed as ‘balanced’ externality. When externalities are present in both the periods, i.e., when an agent’s consumption is affected by the consumption levels of other agents when she is both middle-aged and old, \( \Delta \geq 1 \) is equivalent to the condition that \( -(\sigma_{ct} + \varphi_{ct}) \geq -(\sigma_{dt} + \varphi_{dt}) \). If an agent’s consumption is unaffected by the consumption levels of others when she is middle-aged, \( \Delta \geq 1 \iff -\varphi_{ct} \geq -\varphi_{dt} \). On the other hand, if there is no consumption externality when an agent is old, \( \Delta \geq 1 \iff -\sigma_{ct} \geq -\sigma_{dt} \). Observe that if an economy is characterized by EM, i.e., the externality in consumption is driven by the middle-aged, \( \Delta \) is always greater than 1. Thus by corollary 1, in this situation, if the planner’s social weight is \( \bar{\lambda} \), there will be an underaccumulation of both types of capital in the competitive equilibrium relative to the planner’s solution. On the other hand, when an economy is classified as EO, \( \Delta \) is always less than 1 and consequently if the social weight assigned by the planner is \( \bar{\lambda} \), competitive equilibrium will overaccumulate both types of capital compared to a planner’s solution. Since in this model a planner’s choice of social weight may differ from \( \bar{\lambda} \), any policy prescription meant to correct the effects of consumption externality will depend crucially on the social weight that the planner assigns to the future generations. For example, when the weights are different from \( \bar{\lambda} \), it is not necessarily the case that an economy characterized by EM will always underaccumulate both types of capital, just as an economy characterized by EO will not necessarily overaccumulate both types of capital. The next proposition deals with a situation when the social weight differs from \( \bar{\lambda} \).
Proposition 2  At any social weight different from the laissez-faire supported social weight \( \bar{\lambda} \), if the competitive allocation differs from the social optimum, competitive equilibrium either underaccumulates both human and physical capital or overaccumulates both. That is, if the planner’s social weight to the future generation differs from \( \bar{\lambda} \), there will never be a situation where the levels of accumulation of the two types of capital, human and physical, differ from the social optimum in opposite directions.

Thus propositions 1 and 2 jointly indicate that for all possible values of the social weight that a planner can assign to the future generations, there can never arise a situation in which the levels of accumulation of both types of capital in the competitive equilibrium differ from the planner’s choice in opposite directions, i.e., there is either underaccumulation or overaccumulation of both human and physical capital. Importantly, the result holds independent of the type of externality present in the economy. The following two corollaries clearly distinguish the different possible regimes of capital accumulation and thereafter I explain how the type of consumption externality present in the economy determines whether the competitive equilibrium will under or overaccumulate both types of capital.

Corollary 2 If the externality factor is less than unity, there exists a social weight \( \lambda_+ > \bar{\lambda} \) such that whenever the planner’s weight exceeds \( \lambda_+ \), competitive equilibrium underaccumulates both types of capital. For any \( \lambda < \lambda_+ \), competitive equilibrium overaccumulates both types of capital. However, if the externality factor is greater than or equal to unity, assigning a weight higher than the corresponding \( \bar{\lambda} \) implies further underaccumulation of both types of capital in the competitive economy.

The above corollary specifies that if \( \Delta \geq 1 \) and \( \lambda > \bar{\lambda} \), I clearly have \( h^{SP}(\lambda) > h^{CE} \) and \( k^{SP}(\lambda) > k^{CE} \). On the other hand if \( \Delta < 1 \), I will have \( h^{SP}(\lambda) < h^{CE} \) and \( k^{SP}(\lambda) < k^{CE} \) whenever the social weight \( \lambda \in [\bar{\lambda}, \lambda_+] \). But when the social weight \( \lambda > \lambda_+ \), \( h^{SP}(\lambda) > h^{CE} \) and \( k^{SP}(\lambda) > k^{CE} \) will always hold. Assigning a high future weight necessarily results in high accumulation levels of both physical and human capital in the planner’s economy. If the externality factor is greater than or equal to one, any weight greater than the laissez-faire supported weight guarantees more accumulation of both types of capital by the planner and the competitive equilibrium then relatively underaccumulates. However, when the externality factor is less than unity, underaccumulation in competitive equilibrium starts from a weight which is higher than the laissez-faire supported weight.

Corollary 3 If the externality factor is greater than unity, there exists a social weight \( \lambda_– < \bar{\lambda} \) such that whenever the planner’s weight is less than \( \lambda_– \), competitive equilibrium
overaccumulates both types of capital. For any $\lambda > \lambda_-$, competitive equilibrium underaccumulates both types of capital. However, if the externality factor is less than or equal to unity, assigning a weight lower than $\bar{\lambda}$ implies further overaccumulation of both types of capital in the competitive economy.

Thus, if $\Delta \leq 1$ and $\lambda < \bar{\lambda}$ holds, where $\bar{\lambda}$ is the ‘laissez-faire supported’ social weight for that particular $\Delta$, there will be overaccumulation of both human and physical capital in the competitive equilibrium, i.e., $h_{SP}^{\lambda} < h_{CE}$ and $k_{SP}^{\lambda} < k_{CE}$. On the other hand, if $\Delta > 1$, $h_{SP}^{\lambda} > h_{CE}$ and $k_{SP}^{\lambda} > k_{CE}$ will hold whenever the social weight $\lambda \in [\lambda_-, \bar{\lambda}]$. However, at that same value of $\Delta$, if $\lambda < \lambda_-$, competitive equilibrium will overaccumulate both types of capital. The above two corollaries guarantee that the presence of externality changes (reduces or increases) the regime of accumulation. When the externality factor equals unity, the competitive equilibrium overaccumulates both human and physical capital for all $\lambda < \bar{\lambda}$. For $\lambda > \bar{\lambda}$, underaccumulation in both is observed in the competitive equilibrium. However, if externality is present and the factor is less than unity, overaccumulation is experienced up to $\lambda_+ > \bar{\lambda}$, which means that notionally the regime of overaccumulation is now extended to $\lambda_+$ from its $\bar{\lambda}$ and consequently the regime of underaccumulation gets reduced. Similarly when the externality factor exceeds unity, the regime of overaccumulation now becomes smaller. In this case, the regime of underaccumulation starts after $\lambda_- < \bar{\lambda}$ which means that the regime of underaccumulation now gets extended. The following diagram which represents the regions of accumulation based on the types of externalities present, makes the above discussion clear.\textsuperscript{10}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Capital accumulation regimes under different externalities.}
\end{figure}

In this paragraph I discuss the intuition behind the above results. All the different types of externalities and their corresponding effects can well be explained by the above results. For example, consider the case when externality is present only when the agent is middle-aged, that is $\varphi = 0$. Here $\Delta > 1$ implies that $-\sigma_{c_t} > -\sigma_{d_t}$ or $|\sigma_{c_t}| > |\sigma_{d_t}|$. In this situation\textsuperscript{10} The positions are hypothetical and are meant just for the purpose of illustration. The weights are not positioned according to any data.

\textsuperscript{10}
there is an underaccumulation of both types of capital at the laissez-faire supported social weight. The reasoning behind this is as follows - since middle-aged consumption has a high benchmark reference level, at a steady state, because of this high consumption, saving falls. This reduction in saving lowers the production of physical capital as a result of which the ratio of physical to human capital also falls at a given level of human capital. This fall in the ratio of physical to human capital makes cost of capital high as well as the wage rate low at a given level of human capital. Thus, a reduction in saving through its effect on the ratio of physical to human capital makes cost of capital high and the wage rate low at a given level of human capital. Furthermore, since this high cost of capital and low wage rate together reduce the incentive for the young generation to borrow, this not only reduces the level of human capital accumulation but also reduces the production of physical capital even further. Therefore the ratio of physical to human capital falls further since $\eta_{k,e} > \eta_{h,e}$ holds (see Lemma 1). Note that the above discussion is consistent with the fact that when $\lambda = \bar{\lambda}$, the ratio of per capita physical to human capital is higher (since interest rate is lower) in the planner’s economy. In fact, since $\Delta$ equals to unity when there is no externality present, the above results hold whenever $\Delta > 1$ is observed at $\lambda = \bar{\lambda}$. However, if the weight assigned by a planner to the future generation is very low, it leads to a lower level of accumulation of both physical and human capital at the social optimum (see Lemma 1). Therefore, in the situation where a planner’s social weight is very low, an $EM$ economy will overaccumulate both types of capital in a laissez-faire setup. Similarly, for example, in the case of $EO$, equilibrium overconsumption by the old generation (under $\lambda = \bar{\lambda}$) leads to an increase in saving which in turn results in a greater production of both physical and human capital. But if the social weight is high, optimal capital production in a planner’s economy increases (see Lemma 1) which in turn raises the possibility that the laissez-faire economy underproduces compared to a planner’s economy. Note that though I had started the discussion by finding out a specific $\lambda = \bar{\lambda}$, it is now clear from the results that one can compare the solutions of competitive equilibrium and the social planner’s for the entire range of the possible values of the social weight. The same explanation at the steady state for different possible values of $\Delta$ can be provided when the types of externalities are different.

Obviously, there are many possible ways to achieve the planner’s solution. However, the more interesting case is where one focuses solely on the intergenerational transfers which take care of optimal physical capital accumulation along with an arrangement for young to borrow funds for education. If the focus is only on the alleviation of the effect of externality, it is clear from the model that similar to endogenous growth models, a transfer between generations can help the production of physical capital stock to reach the optimal level. If a planner assigns very high weights to the future generations, independent of the type of
consumption externalities present, the justification behind implementing the standard public
department benefits where physical capital moves away from the present working class to the
present old becomes weak. However, there are certain types of consumption externalities
for which a significant regime of capital accumulation exists where implementing public
pensions through the standard intergenerational arrangement may be justified. But from the
efficiency point of view, whenever a standard public pension scheme is introduced when there
is overaccumulation of physical capital, an education subsidy becomes unnecessary since an
overaccumulation of physical capital is also accompanied by an overaccumulation in human
capital. Similarly, I can argue that whenever an education subsidy is required because of
underaccumulation in human capital, a standard public pension benefit is not recommended
since an underaccumulation in human capital accompanies an underaccumulation in physical
capital too.

4 An example

In this section I present my claims through an example. Consider the following representation
of human capital and final good production technology respectively:

\[ h_t = \phi(e_{t-1}) = Ae_{t-1}^\alpha, \alpha \in (0, 1), A > 0 \]

(25)

\[ f(k_t) = Bk_t^\beta, \beta \in (0, 1), B > 0. \]

(26)

Here I specify the consumption patterns in the following way so that I can construct an
externality like Alonso - Carrera et al. [6] where the effective levels of consumption of a
generation-t agent is given by

\[ \hat{c}_t = c_t - \gamma v_t^m, \]

and

\[ \hat{d}_{t+1} = d_{t+1} - \delta v_{t+1}^o, \]

where \( \gamma \in [0, 1] \) and \( \delta \in [0, 1] \) measure the intensity of the consumption references, \( v_t^m \) and
\( v_{t+1}^o \), respectively. The consumption benchmarks in any period are assumed to be a weighted
arithmetic average of the per-capita consumption of the two generations consuming in that
period. Specifically,

\[ v_t^m \equiv \frac{N_t c_t + \theta^m N_{t-1} d_t}{N_t + \theta^m N_{t-1}} = \left( \frac{N}{N + \theta^m} \right) c_t + \left( \frac{\theta^m}{N + \theta^m} \right) d_t, \]

\[ \hat{d}_{t+1} = d_{t+1} - \delta v_{t+1}^o, \]
where \( \theta^m \in [0, 1] \) is the weight of a representative old agent’s consumption in the specification of the middle-aged agent’s consumption benchmark. Similarly

\[
v_{t+1}^o = \frac{\theta^o N_{t+1} c_{t+1} + N_t d_{t+1}}{\theta^o N_{t+1} + N_t} = \left( \frac{\theta^o N}{\theta^o N + 1} \right) c_{t+1} + \left( \frac{1}{\theta^o N + 1} \right) d_{t+1},
\]

where \( \theta^o \in [0, 1] \) is the weight of a representative middle-aged agent’s consumption in the specification of an old agent’s consumption reference. Denoting \( N_{t+1}^m \equiv \varepsilon^m \) and \( N_{t+1}^o \equiv \varepsilon^o \) gives \( v_t^m = \varepsilon^m c_t + (1 - \varepsilon^m) d_t \) and \( v_{t+1}^o = \varepsilon^o c_{t+1} + (1 - \varepsilon^o) d_{t+1} \). This specification implies that \( \sigma \equiv -\gamma [\varepsilon^m c_t + (1 - \varepsilon^m) d_t] \) and \( \varphi \equiv -\delta [\varepsilon^o c_{t+1} + (1 - \varepsilon^o) d_{t+1}] \). We specify lifetime utility as a log-linear function of consumption as shown below

\[ u_t(c_t, d_{t+1}) = \log(c_t - \gamma v_t^m) + \rho \log(d_{t+1} - \delta v_{t+1}^o), \]

where \( \rho \in (0, 1) \) is the intertemporal discount factor.

The strength of these influences depend on the deeper parameters \( \theta^m \) and \( \theta^o \). In this specification, while \( \varepsilon^m ((1 - \varepsilon^o)) \) represents the degree to which a middle-aged (old) agent keeps up with the others in her own generation, \( (1 - \varepsilon^m) \) and \( \varepsilon^o \) represents the degree to which the agent keeps up with the other generation. When both \( \varepsilon^m \) and \( \varepsilon^o \) are high, the weight that is given to the level of consumption of the middle-aged (the working class) in the construction of the consumption reference is high, I call it a middle-aged driven externality \([EM]\). Similarly, when both \( \varepsilon^m \) and \( \varepsilon^o \) are low, it is called an old driven externality \([EO]\). More specifically, in this particular example, I use the term ‘high’ when the coefficients exceed the number \( \frac{1}{2} \).

Now I start with the competitive equilibrium. Using (2), (3) and (9) I get

\[
(\phi^i)(e_{t-1}) = \frac{R_t}{w_t} = \frac{\beta B k_t^{\beta-1}}{(1 - \beta) B k_t^{\beta}} = \frac{\beta}{(1 - \beta) k_t},
\]

and from (4) I have

\[
\frac{(d_{t+1} - \delta v_{t+1}^o)}{(c_t - \gamma v_t^m)} = \rho R_{t+1}
\]

along with

\[ c_t = w_t h_t - R_t e_{t-1} - s_t, d_{t+1} = R_{t+1} s_t. \]

As is shown in (12), the relationship between \( k_{t+1} \) and \( k_t \) in this laissez-faire economy is established and shown later. The steady state level of \( k \), namely \( k^{CE} \), is determined from this equation.

Similarly, the planner’s problem can be formulated for this particular economy. As has
been shown in subsection 2.3, the following relationships can be derived from the first order conditions of the planner’s problem:

\[
\phi'(e_{t}^{SP}) = \frac{f'(k_{t+1}^{SP})}{f(k_{t}^{SP}) - f'(k_{t+1}^{SP})k_{t+1}^{SP}}
\]  
(29)

and

\[
\frac{u_{SP}^{t}}{u_{SP}^{t+1}} = f'(k_{t+1}^{SP}) \frac{\Delta}{1 + n}
\]  
(30)

where the externality factor \(\Delta\) in this example turns out to be

\[
\Delta \equiv \frac{\delta \varepsilon^o + \{1 - \delta (1 - \varepsilon^o)\} (1 + n)}{(1 - \gamma \varepsilon^m) + (1 - \varepsilon^m) \gamma (1 + n)}.
\]  
(31)

Further, as is shown earlier, at a steady state,

\[
f'(k_{SP}^{t}) = \frac{1}{\lambda} (1 + n).
\]  
(32)

For simplicity, for rest of the analysis, I assume that the net population growth rate \(n = 0\). From the above analysis, I can compute \(\bar{\lambda}\) by equating \(f'(k_{CE}^{t}) = \frac{1}{\lambda}\). Consequently \(k_{SP}^{t}\) can be calculated by \(k_{SP}^{t} = \left(\frac{1}{\lambda}\right)^{\frac{1}{\lambda}}\) where in this example, \(k_{SP}^{t} = \left(\bar{\lambda} \beta B\right)^{\frac{1}{\lambda}}\) at \(\bar{\lambda}\).

It is worthwhile to note that in this example, \(\Delta = 1\) occurs in two different situations, a) when there are no externalities present and b) when \(\frac{\delta}{\delta + \gamma} \varepsilon^o + \frac{\gamma}{\delta + \gamma} \varepsilon^m = \frac{1}{2}\) or \(\frac{\delta}{\delta + \gamma} (1 - \varepsilon^o) + \frac{\gamma}{\delta + \gamma} (1 - \varepsilon^m) = \frac{1}{2}\), \(\delta, \gamma \neq 0\). When externalities are present in both the periods, i.e., when an agent’s consumption is affected by externalities when she is both middle-aged and old, \(\Delta \geq 1 \Rightarrow \frac{\delta}{\delta + \gamma} \varepsilon^o + \frac{\gamma}{\delta + \gamma} \varepsilon^m \geq \frac{1}{2} \Leftrightarrow \frac{\delta}{\delta + \gamma} (1 - \varepsilon^o) + \frac{\gamma}{\delta + \gamma} (1 - \varepsilon^m) \leq \frac{1}{2}\). It is easy to check that if there is no consumption externality when the agent is middle-aged, i.e., when \(\sigma = 0\) (which is obtained by setting \(\gamma = 0\)), \(\Delta \geq 1 \Leftrightarrow \varepsilon^o \geq \frac{1}{2}\). On the other hand, if there is no consumption externality present when the agent is old, i.e., when \(\varphi = 0\) (which is obtained by setting \(\delta = 0\)), we have \(\Delta \geq 1 \Leftrightarrow \varepsilon^m \geq \frac{1}{2}\). Thus \(\Delta > 1\) or \(\Delta < 1\) occurs in many different situations. It can easily be observed that in this example, if the economy is characterized by \(EM\), \(\Delta\) will always be greater than 1. This means that in this situation, if the planner’s social weight is \(\bar{\lambda}\), competitive equilibrium underaccumulates both types of capital as compared to the planner’s solution. On the other hand, when the economy is classified as \(EO\), at \(\bar{\lambda}\), competitive equilibrium overproduces both types of capital. A few other possible scenarios based on the type of externalities can also be explained from here.

As I have already mentioned in subsection 2.2, in order to make the calculations needed to find (12) manageable, I assume that \(\varphi = 0\), i.e., the consumption of an old agent is not
affected by any kind of externality. I show\(^{11}\) that in this laissez-faire setup, the path of \(\{\bar{k}_t\}_{t=0}^\infty\), that is (12) can be written as

\[
\bar{k}_{t+1} = z^{1-\alpha} \bar{k}_t^{\alpha+\beta-\alpha\beta}
\]

(33)

where

\[
z = \frac{\beta B \rho (1 - \gamma \varepsilon^m)}{1 + \rho (1 - \gamma \varepsilon^m)} \left\{ \frac{(1 - \alpha) (1 - \beta)}{\alpha + \beta - \alpha\beta} - \gamma \frac{(1 - \varepsilon^m)}{(1 - \gamma \varepsilon^m)} \right\}.
\]

Given the specification of the economy, since \(\alpha + \beta - \alpha\beta < 1\) always holds, the path of \(\{\bar{k}_t\}_{t=0}^\infty\) is a concave function. The model guarantees a positive savings when \(z > 0\) which implies from the last equation that

\[
\frac{(1 - \gamma \varepsilon^m)}{\gamma (1 - \varepsilon^m)} > \frac{\alpha + \beta - \alpha\beta}{(1 - \alpha) (1 - \beta)}.
\]

Note that the left hand side decreases with \(\gamma\) but increases with \(\varepsilon^m\). Along the steady state, I get

\[
\bar{k}^{CE} = z^{1-\alpha - \beta + \alpha\beta}.
\]

I assume the following parametric specifications: \(\alpha = 0.2, \beta = 0.33, A = 10, B = 10, \rho = 0.9\). Here I fix the parametric value of \(\gamma\) at 0.9 and vary the value of \(\varepsilon^m\) to represent two different types of externalities and show that all the results can well be verified. I compute the steady state values of \(\{\bar{k}^{CE}, k^{CE}, h^{CE}\}\) and \(\{\bar{k}^{SP}, k^{SP}, h^{SP}\}\) that are shown in Table 1 below for two different externalities.

<table>
<thead>
<tr>
<th>Cases</th>
<th>(\varepsilon^m)</th>
<th>(\bar{\lambda})</th>
<th>(\Delta)</th>
<th>({\bar{k}^{CE}, k^{CE}, h^{CE}})</th>
<th>({\bar{k}^{SP}, h^{SP}, k^{SP}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>0.9</td>
<td>0.3554</td>
<td>3.5714</td>
<td>{0.1898, 2.3434, 0.4447}</td>
<td>{1.2687, 3.7682, 4.7807}</td>
</tr>
<tr>
<td>(2)</td>
<td>0.2</td>
<td>0.0764</td>
<td>0.6494</td>
<td>{0.2437, 2.4946, 0.6079}</td>
<td>{0.1279, 2.1234, 0.2716}</td>
</tr>
</tbody>
</table>

Table 1. Equilibrium at laissez-faire vis-a-vis social optimum at \(\bar{\lambda}\).

It has already been established that the choice of \(\bar{\lambda}\) depends on the value of \(\Delta\). I pick case (1) from Table 1 where the value of \(\Delta\) is greater than 1 and \(\bar{\lambda} = 0.3554\). I choose two possible values of \(\lambda\), both being less than \(\bar{\lambda} = 0.3554\). This result is shown as case (1') in Table 2. When \(\lambda = 0.34\), competitive equilibrium underproduces and when \(\lambda = 0.034\), competitive equilibrium overproduces both types of capital. This guarantees that there exists a \(\lambda = \lambda_- = 0.0995\) where \(h^{CE} = h^{SP}\) and \(k^{CE} = k^{SP}\). Similarly, in (2'), where \(\Delta\) is less than 1, I choose two possible values of \(\lambda\) both of which are greater than the corresponding

\(^{11}\)See Supplementary materials.
\( \bar{\lambda} = 0.0764 \) (see case (2) in Table 1). Observe that in one situation \((\lambda = 0.09)\), competitive equilibrium overproduces while in the other \((\lambda = 0.9)\), it underproduces both types of capital compared to the planner’s economy. This then confirms that there exists a \( \lambda = \lambda_+ = 0.1177 \) so that at \( \lambda_+ \), \( h^{CE} = h^{SP} \) and \( k^{CE} = k^{SP} \).

<table>
<thead>
<tr>
<th>Cases</th>
<th>( \varepsilon^m )</th>
<th>( \lambda )</th>
<th>( \Delta )</th>
<th>( {h^{CE}, k^{CE}} )</th>
<th>( {h^{SP}, k^{SP}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1')</td>
<td>0.9</td>
<td>0.3400</td>
<td>0.0995(( \lambda_- )) 0.0340</td>
<td>3.5714</td>
<td>{0.1898, 2.3434, 0.4447} {1.1875, 3.7063, 4.4011} {0.1898, 2.3434, 0.4447} {0.0382, 1.5697, 0.0600}</td>
</tr>
<tr>
<td>(2')</td>
<td>0.2</td>
<td>0.0900</td>
<td>0.1177(( \lambda_+ )) 0.9000</td>
<td>0.6494</td>
<td>{0.2437, 2.4946, 0.6079} {0.1633, 2.2571, 0.3687} {0.2437, 2.4946, 0.6079} {5.0770, 5.3296, 27.0582}</td>
</tr>
</tbody>
</table>

Table 2. Existence of \( \lambda_- \) and \( \lambda_+ \).

The above two cases have been represented below in two diagrams. While the capital accumulations (\( pc \) and \( hc \) stand for physical and human capital respectively in the diagrams) in competitive equilibrium are shown as horizontal straight lines (since it does not vary with the social weight), dotted curves represent the accumulation levels in the planner’s economy. Vertical lines represent the benchmark social weights - while the dotted vertical line represents the associated capital levels at \( \bar{\lambda} \), the solid line represents the associated \( \lambda \) for which the levels of accumulation are same in competitive equilibrium and in the planner’s solution. In Figure 2 the bold vertical line is essentially drawn at \( \lambda_- \) and in Figure 3 it represents \( \lambda_+ \). In case (1'), it can be observed that competitive equilibrium overaccumulates up to \( \lambda_- \) after which it underaccumulates as social weight starts increasing. On the other hand in case (2'), competitive equilibrium overaccumulates till \( \lambda_+ \) after which accumulation in planner’s solution is much higher compared to the competitive equilibrium.
5 Conclusion

Governments commonly intervene in education, typically in the form of education loans and subsidies. The standard rationale for such intervention is a human capital externality: for the same effort, people learn more if they are around smart people. The intergenerational counterpart of this observation is that a smart generation produces a smarter future generation. In this paper the requirement for education subsidy may actually stem from the presence of consumption externality. That is, consumption externality motivates agents to acquire a different level of education than what a planner would have liked thus making the case for government intervention in education even when no human capital externalities are present.12 In my knowledge, this important link between human capital accumulation and consumption externality has been ignored in the literature. Using a neoclassical overlapping generations model I show that in the presence of consumption externality, if the competitive allocation differs from the social optimum, competitive equilibrium either underaccumulates

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12There are alternative ways to achieve efficiency. But as is more common and interesting in this stream of literature, the focus remains mainly on two programs (education subsidy and pension) which are the two biggest spending commitments by any government in most of the countries. I do not claim that provision of direct government involvement in education is the unique efficient way. For example, instead of using generational transfer instruments, a tax or a subsidy (enough to correct the effects of the consumption externality) on capital income, along with a lump-sum transfer back to the same generation can implement the planner’s solution; also see Richter and Braun [36]. While such alternatives somewhat weaken the case for direct government involvement in education, it can be argued that many countries find it considerably difficult to employ instruments such as a capital-income tax, and choose direct government involvement in education as a easier and simpler alternative. Moreover, it is expected that in the absence of a perfect capital market, the need for direct involvement in education is even higher.
both human and physical capital or overaccumulates both. There will never be a situation where competitive allocation differs from the social optimum in opposite directions. As is common in endogenous growth models, in this model too the requirement for standard structure of public pension benefits becomes weak as the social weight to the future generations increases. But more importantly, whenever there is a need for education subsidy, infusing public pension benefits is not recommendable and vice-versa.
Appendix

Proof of Lemma 1: The proof is simple and straightforward. Taking the total derivative of either (8) or (19) gives

\[ \phi''(e_t)[f(\bar{k}_{t+1}) - f'(\bar{k}_{t+1})\bar{k}_{t+1}]de_t + \phi'(e_t)[-\bar{k}_{t+1}f''(\bar{k}_{t+1})]d\bar{k}_{t+1} - f''(\bar{k}_{t+1})d(\bar{k}_{t+1}) = 0 \]

\[ \Rightarrow \frac{d\bar{k}_{t+1}}{de_t} = \frac{\phi''(e_t)[f(\bar{k}_{t+1}) - f'(\bar{k}_{t+1})\bar{k}_{t+1}]}{f''(\bar{k}_{t+1})[1 + \phi'(e_t)\bar{k}_{t+1}]} \tag{A.1} \]

Given the property of \( f \) and \( \phi \), I clearly have \( \frac{d\bar{k}_{t+1}}{de_t} > 0 \), \( \forall t \), for both the CE and SP. Note that since \( \frac{\partial k_t}{\partial e_t} - \frac{\partial h_t}{\partial e_t} k_t \), \( \frac{d\bar{k}_{t}}{de_t-1} > 0 \) \( \Rightarrow \frac{\partial k_t}{\partial e_t-1} h_t - \frac{\partial h_t}{\partial e_t-1} k_t > 0 \) \( \Rightarrow \frac{\partial k_t}{\partial e_t-1} \frac{e_t-1}{h_t} > 0 \) and thus \( \eta_{k,e} > \eta_{h,e} \).

To prove \( \frac{d\bar{k}_{t+1}}{de_t} > 0 \), at the steady state, I take the total derivative of (24) which implies that

\[ f'(\bar{k}^{SP})d\lambda + \lambda f''(\bar{k}^{SP})d\bar{k}^{SP} = 0 \Rightarrow \frac{d\bar{k}^{SP}}{d\lambda} = -\frac{f'(\bar{k}^{SP})}{\lambda f''(\bar{k}^{SP})} > 0. \tag{A.2} \]

Using (A.1) and (A.2) I clearly have \( \frac{de^{SP}}{d\lambda} > 0 \).

Proof of Lemma 2: Suppose not. Let there exist \( \bar{x} \) and \( \tilde{\lambda}, \bar{x} \neq \tilde{\lambda} \) so that \( \frac{u^{CE}_{\bar{x}}}{u^{CE}_{\bar{x}+1}} = \frac{u^{SP}_{\tilde{\lambda}}}{u^{SP}_{\tilde{\lambda}+1}} \)
holds for both \( \bar{x} \) and \( \tilde{\lambda} \). Without loss of generality, I assume that \( \bar{x} > \tilde{\lambda} \). But if \( \bar{x} > \tilde{\lambda} \), using Lemma 1, I have \( \bar{k}^{SP}(\bar{x}) > \bar{k}^{SP}(\tilde{\lambda}) \). But on the other hand, both \( f'(\bar{k}^{CE}) = f'(\bar{k}^{SP}(\bar{x})) \Delta \) and \( f'(\bar{k}^{CE}) = f'(\bar{k}^{SP}(\tilde{\lambda})) \Delta \) imply that \( \bar{k}^{SP}(\bar{x}) = \bar{k}^{SP}(\tilde{\lambda}) \) given \( f' > 0 \). Hence the contradiction.

Proof of Lemma 3: The proof is straightforward. It can directly be constituted from (19). Given \( \bar{k}^{SP}(\tilde{\lambda}) = \bar{k}^{CE} \), I have

\[ \phi'(e^{SP}(\tilde{\lambda})) = \frac{f'(\bar{k}^{SP}(\tilde{\lambda}))}{f(\bar{k}^{SP}(\tilde{\lambda})) - f'(\bar{k}^{SP}(\tilde{\lambda}))\bar{k}^{SP}(\tilde{\lambda})} = \frac{f'(\bar{k}^{CE})}{f(\bar{k}^{CE}) - f'(\bar{k}^{CE})\bar{k}^{CE}} = \phi'(e^{CE}) \]

\[ \Rightarrow e^{SP}(\tilde{\lambda}) = e^{CE} \] by the property of \( \phi \).
Proof of Proposition 1: At any $\lambda$, only three possible situations can be observed: case (a) when $\Delta > 1$, case (b) when $\Delta < 1$ and case (c) when $\Delta = 1$. In case (a), I have $f'(k^{CE}) > f'(k^{SP}(\lambda))$ which implies that $k^{CE} < k^{SP}(\lambda)$. But $k^{CE} < k^{SP}(\lambda)$ implies that $e^{SP} > e^{CE}$ and thus $h^{SP}(\lambda) > h^{CE}$. Therefore $h^{SP}(\lambda) > h^{CE}$ and $k^{CE} < k^{SP}(\lambda)$ gives $k^{SP}(\lambda) > k^{CE}$. Similarly, in case (b), I can show that $k^{CE} > k^{SP}$ and $h^{CE} > h^{SP}$. It is easy to check that in case (c), $k^{CE} = k^{SP}$ and $h^{CE} = h^{SP}$ hold. Therefore, for each $\Delta$ such that at its corresponding $\lambda$, $k^{CE} \neq k^{SP}$ and $h^{CE} \neq h^{SP}$ hold, $k^{CE}$ and $h^{CE}$ differ from $k^{SP}$ and $h^{SP}$ in the same direction; either both are lower, that is, $k^{CE} < k^{SP}$ and $h^{CE} < h^{SP}$ as in case (a) or both are greater, that is, $k^{CE} > k^{SP}$ and $h^{CE} > h^{SP}$ as in case (b). Hence the proof.

Proof of Corollary 1: This follows from the proof of Proposition 1.

Proof of Proposition 2: I establish the result for $\lambda > \lambda$ and claim the opposite result holds for $\lambda < \lambda$. When $\lambda > \lambda$, I must have $k^{SP}(\lambda) > k^{SP}(\lambda)$ since I already have proved $\frac{d(k^{SP})}{d\lambda} > 0$ in Lemma 1. We know that only three situations are possible: case (a) when $\Delta > 1$, case (b) when $\Delta < 1$ and case (c) when $\Delta = 1$. In case (a), I must have $k^{SP}(\lambda) > k^{SP}(\lambda) > k^{CE}$. Since Lemma 1 holds, this in turn implies that $h^{SP}(\lambda) > h^{CE}$ and thus I also have $k^{SP}(\lambda) > k^{CE}$. Note that in case (c), $k^{SP}(\lambda) > k^{SP}(\lambda) = k^{CE}$ holds. This by the virtue of Lemma 1 means that $h^{SP}(\lambda) > h^{CE}$ and therefore $k^{SP}(\lambda) > k^{CE}$. But when case (b) occurs, note that (from Proposition 1) if $\lambda = \lambda$, we have $k^{CE} > k^{SP}(\lambda)$ as well as $h^{CE} > h^{SP}(\lambda)$. Since both $k^{SP}$ and $h^{SP}$ are continuous in $\lambda$, there exists a $\lambda_{+}, 1 > \lambda_{+} > \lambda$, such that $k^{CE} = k^{SP}(\lambda_{+})$ and $h^{CE} = h^{SP}(\lambda_{+})$ hold. Furthermore, this implies that for all $\lambda > \lambda_{+}, k^{CE} < k^{SP}(\lambda_{+})$ and $h^{CE} < h^{SP}(\lambda_{+})$. Hence for $\lambda > \lambda$, I have shown that there does not exist any situation where $h^{CE}$ and $k^{CE}$ differ from respective $h^{SP}$ and $k^{SP}$ in opposite directions. I omit the proof for $\lambda < \lambda$ since it is clear that exactly the opposite results hold in this situation. Hence the proof.

Proof of Corollary 2 and Corollary 3: These follow from the proof of Proposition 2.
References


