Human Capital Accumulation, Economic Growth and Optimal Policy in a Dual Economy*

Manash Ranjan Gupta
and
Bidisha Chakraborty†,
Economic Research Unit,
Indian Statistical Institute,
203, B.T. Road,
Kolkata-700108,
WestBengal, India.
E-mail: bidisha_chakraborty@rediffmail.com; bidisha_isi@yahoo.co.in
Telephone no.:+91-33-24067358
Fax Number:+91-33-25778893

Abstract

This paper develops an endogenous growth model with dualism in human capital accumulation of two types of individuals. The government imposes a proportional redistributive tax on the resources of the rich individual to finance the educational subsidy given to the poor individuals. We find out the properties of optimal tax cum educational subsidy policy using Stackelberg differential game framework. The government plays the role of leader and the representative households are followers. The government chooses the tax rate to maximize the welfare of the economy subject to the decentralized competitive equilibrium conditions. Thus, the maximization problem by the government is also constrained by the private agents' optimal decision rules. We also derive the properties of steady-state growth equilibrium and transitional dynamics of this model.

JEL Classification: C61, J24, O15, O41

Keywords: Human Capital, Dualism, Economic growth, Rich, Poor, Competitive Equilibrium, Steady-state growth, Redistributive Taxation, Educational Subsidy, Transitional Dynamics.

---

*This is a part of a chapter of the prospective Ph.D. work of the second author which is in progress with the first author in the Indian Statistical Institute, Kolkata. All the remaining errors are of solely ours.

†Corresponding Author

1 Introduction

With the emergence of the 'new' growth theory, human capital accumulation and its role on economic growth has become a major area of research in macroeconomics. The literature starts with the seminal paper of Lucas (1988) which shows that growth rate of per capita income depends on the growth rate of human capital which again depends on the time allocation of the individuals for acquiring skill. Since then many eminent economists have dealt with the issue of human capital accumulation and growth and the Lucas (1988) model has been extended in various directions.

A subset of that literature is concerned with the effects of taxation on the long-term growth in these Lucas-type models. This includes the works of Jones, Manuelli and Rossi (1993), Stokey and Rebelo (1995) who use numerical simulations of calibrated models, and the works of Chamley (1992), Mino (1996), Uhlig and Yanagawa (1996), Ortigueira (1998), De Hek (2005) who examine the effects of taxation analytically. However all these authors were interested in analysing the effects of exogenous changes in the fiscal instruments and did not design any endogenous fiscal policy introducing endogenous behaviour of the government. Gomez(2003) and Garcia-Castrillo and Sanso(2000) claim to design optimal fiscal policies in the Lucas-type model. However optimality is defined in a limited sense. Fiscal instruments are designed to correct the market failures arising from the external effects of human capital. However they did not adopt the framework of Stackelberg differential games in which government designs the optimal fiscal policy solving an optimal control problem.

These Stackelberg differential games are nowadays widely used to study the dynamic interaction between the government and the private agents. In a Stackelberg differential game, the government naturally plays the role of a leader setting the fiscal policies and the private agents act as followers responding optimally to policy in their decision on consumption, investment, labour supply and so on. The government then takes the private agents' best response into account and forms the optimal policy. Judd (1985,1997), Chamley (1986), Lansing (1999), Guo and Lansing (1999), Mino (2001) etc were interested in the optimality of redistributive taxation in the exogenous growth models using the framework of Stackelberg differential game. Park and Philippopulos (2004), Alonso-Carrera and Freire Seren (2004), and Ben-Gad (2003) analyse the optimal fiscal policies in the endogenously growing economies using the similar framework and also analyse the role of fiscal policy on the indeterminacy problem of transitional and/or steady-state growth path. Only Ben-Gad(2002) considers a Lucas-type of model. However, in his model, tax revenue is not used to subsidize the human capital accumulation sector though Gomez (2003) and Garcia-Castrillo and Sanso (2000) in their models consider tax revenue to be used in subsidizing the education sector.

Lucas (1990) has already drawn our attention to "increased subsidies to schooling, that would.....have potentially large effects on human capital accumulation and long term growth rates.....[It] might well be an interesting subject for future research." Many authors have analysed the issue of education subsidy in recent years. The set of literature includes the works of Zhang (2003), Blankenau and Simpson (2004), Bovenberg and Jacobs (2003, 2005), Boskin (1975),Blankenau (2005), Brett and Weymark (2003) and of many others. Most of them deal with the effects of subsidies and public expenditures on education and growth. However, none of these papers have analysed the optimality of educational subsidy policy using the framework of Stackelberg differential game. In fact Bovenberg and Jacobs (2005) analyse the optimality of tax policy and educational subsidy policy using a framework of leadership game in which the households solve the static optimization problem and the government solves an optimal control problem. However they neither consider a Lucas-type model nor relate these policies to the growth-theoretic issues like steady-state equilibrium and transitional dynamics.

This paper develops a growth model of an economy in which human capital accumulation is viewed as the source of economic growth and in which dualism exists in the mechanism of human capital accumulation of the two types of individuals — the rich and the poor. The rich individuals have a high initial level human capital endowment and an efficient human capital accumulation technology. The poor individuals lag behind both in terms of initial human capital endowment and in terms of the productivity of human capital accumulation technology. We call them rich and poor because human capital is an important determinant of income [1]. Neither Lucas (1988) nor any extension of Lucas (1988) model have considered this dualism. However the poor individuals are benefitted by the teaching of the rich individuals in this model; and redistributive taxes are imposed on the rich individuals to finance the educational subsidy given to the poor individuals. The government taxes a fraction of the resources of the rich individual and spends it to meet the cost of training of the poor individual. Our objective is to analyse the nature of the optimum fiscal policy and to relate this to the growth theoretic issues like steady state equilibrium and indeterminacy of transitional growth path. We do this adopting a framework of Stackelberg differential game. We have assumed the presence of external effect of the human capital on the production technology [2].

[1] The empirical works on the skilled-unskilled wage inequality in different countries, i.e., the works of Robbins(1994a, 1994b), Lachler (2001), Beyer, Rojas and Vergara (1999), Marjit and Acharyya (2003), Wood (1997)etc. have a debate over this hypothesis. Beyer, Rojas and Vergara(1999) have shown that the extent of wage inequality and the proportion of the labour force with college degrees in the post liberalization period in Chile were negatively related. According to the World Development Report (1995), increased educational opportunities exerted downward pressures on wage inequality in Columbia and Costa Rica. Many other works have shown the opposite empirical picture in many other countries.

[2] There exists a large theoretical literature in both urban economics and in macroeconomics that

We derive some interesting results from this model. A tax financed subsidy policy to the education sector of the poor individuals is optimal if the external effect of human capital of the poor (rich) individuals is very high (low) and if the marginal productivity of teaching is very high. Secondly, the economy converges to a unique steady state growth equilibrium in which the inequality in human capital stock between two groups can not be fully eliminated by this tax financed educational subsidy. Thirdly, there exists indeterminacy of transitional dynamics in the case of tax financed educational subsidy policy.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 presents the properties of the optimal policies and of those of steady state equilibrium and transitional dynamics. Concluding remarks are made in Section 4.

2 The basic dual economy model

We consider a closed economy with two types of individuals –the rich and the poor individuals. All workers are employed in a single aggregative sector that produces a single good. By human capital we mean the set of specialized skills or efficiency level of the workers that accumulate over time. Human capital accumulation is a non market activity like that in Lucas (1988). However, the mechanisms of human capital accumulation are different for two types of individuals. There is external effect of human capital on production. Population size of either type of individual is normalised to unity. All individuals belonging to each group are assumed to be identical. There is full employment of both types of labour and the labour market is competitive.

The single production sector is owned by the rich individuals and they employ the poor individuals as wage labourers. Rich individuals and poor individuals form different types of human capital which are imperfectly substitute. The government deducts $(1-x)$ fraction of the labour time of the rich individual for the tarining of the poor individual. Out of the remaining x fraction of time a rich individual allocates 'a' fraction of the total non-leisure time in production and the poor individual spends u fraction of non leisure time for production. Let H_R and H_P be the skill level of the representative rich and poor individual (worker) respectively. We assume that $H_R(0) > H_P(0)$. This means that the poor individuals lag behind the rich indiduals in terms of initial human capital endowment.

has considered external effects emanating from human capital in explaining growth of cities, religions and countries e.g. Glaeser and Mare (1994), Glaeser (1997), Peri (2002), Ciccone and Peri (2002). In some other literature, it is found that education generates very little externalities. e.g Rudd (2000), Acemoglu and Angrist (2000). Moretti (2003) rightly points out that the empirical literature on the subject is still very young and more work is needed before we can draw convincing conclusions about the size of human capital externalities.

The production function takes the form:

$$Y = A(axH_R)^{\alpha}(uH_P)^{1-\alpha}\bar{H_R}^{\epsilon_R}\bar{H_P}^{\epsilon_P} \tag{1}$$

where $0 < \alpha < 1$, $0 < x \leq 1$. Here $\bar{H_R}$ and $\bar{H_P}$ represent the average level of human capital of all the rich individuals and poor individuals respectively. $\epsilon_R > 0$ and $\epsilon_P > 0$ are the parameters representing the magnitude of the external effect of their human capital. Here $\epsilon_R > \epsilon_P$ implies that the external effect of the human capital of the rich individual is stronger than that of the poor individual. Production function satisfies CRS in terms of private inputs but shows social IRS if external effect is taken into consideration. Y stands for the level of output.

The representative rich individual (worker) owns the advanced type of human capital and his income is given by αY. $(1-\alpha)Y$ is the wage income of the poor workers because the labour market is competitive. The rich household and the poor household both consume whatever they earn and hence they do not save (or invest). So there is no accumulation of physical capital in this model and physical capital does not enter as an input in the production function[3]. So, we have

$$C_R = \alpha Y \tag{2}$$

and

$$C_P = (1-\alpha)Y \tag{3}$$

where C_P and C_R are the level of consumption of the representative poor worker and the representative rich worker respectively. The representative rich household (worker) and poor household (worker) maximize their respective discounted present value of utility over the infinite time horizon with respect to labour time allocation variables. The instantaneous utility function of the ith individual is given by

$$U(C_i) = lnC_i, \tag{4}$$

For $i = R, P$.

Difference in the mechanism of human capital accumulation

Mechanism of the human capital accumulation of the rich individual is assumed to be similar to that in Lucas (1988). The rate at which his human capital is formed is proportional to the time or effort devoted to acquire skill. Hence

$$\dot{H_R} = m_R(1-a)xH_R \tag{5}$$

where $(1-a)$ is the fraction of the labour time devoted to acquiring his own skill level and x is the fraction of labour time available to him after tax. Here $0 <$

[3]Though it is assumed for simplicity, it is a serious limitation of the exercise. However, the model becomes highly complicated when physical capital accumulation is introduced.

$a < 1$; and m_R is a positive constant representing the productivity parameter of the human capital formation function of the rich individuals.

However the mechanism of human capital formation for the two classes of individuals are different. The skill formation of a poor person takes place through the training program conducted by the rich individuals. The government taxes $(1 - x)$ fraction of the available labour endowment of the rich individuals and each rich individual must spend this $(1 - x)$ fraction of its available labour endowment in this training programme of the poor individuals. In reality the government taxes not on labour endowment but on the income or on the non labour resources (wealth) of the riches. In this model labour endowment is the only resource and marginal productivity of labour is always positive. So taxation on labour endowment indirectly implies the taxation on income[4]. The poor individual devotes $(1 - u)$ fraction of non-leisure time for learning. The additional skill acquired by the representative poor worker is assumed to take the following form:

$$\dot{H}_P = m_P(1 - u)H_P[q(1 - x) + 1] \tag{6}$$

Here $q \geq 0$ and $m_P > 0$. We assume

$$\begin{aligned} & q > 0 \quad \text{for} \quad \frac{H_R}{H_P} > 1 \\ \text{and} \quad & q = 0 \quad \text{for} \quad \frac{H_R}{H_P} \leq 1 \end{aligned}$$

However, q is independent of the level of (H_R/H_P) when $(H_R/H_P) > 1$. $q > 0$ is treated as a parameter. This implies that the less knowledgeable persons are benefitted by the teaching of the more knowledgeable persons. $q > 0$ can be interpreted as a productivity parameter of teaching. Here the cost of teaching is met by the educational subsidy and these are measured in terms of labour time. We also assume $m_R > m_P$. If $x = 1$ and if $(1 - a) = (1 - u)$, then

$$m_R > m_P \Longrightarrow \frac{\dot{H}_R}{H_R} > \frac{\dot{H}_P}{H_P}$$

This means that the human capital accumulation technology of the rich individuals is more productive than that of the poor individuals in the absence of teaching, i.e., in the absence of tax financed education subsidy policy.

[4]While almost all the authors consider taxation on income, a few authors consider taxation on the stock of productive resources. For example, Park and Philippopoulous (2004), Benhabib et. al (1997) consider proportional taxation on the stock of physical capital.

3 Optimum Growth Path

3.1 The optimization problem of the rich household

The objective of the representative rich individual is to maximize the discounted present value of utility over the infinite time horizon. The objective functional of the rich individual is given by

$$J_H = \int_0^\infty U(C_R)e^{-\rho t}dt$$

This is to be maximized with respect to a subject to the equation of motion given by

$$\dot{H}_R = m_R(1-a)xH_R$$

and given the initial value of H_R. Here $U(C_R)$ is given by equation (4) and Y is given by equation (1). Here ρ is the constant positive discount rate. The control variable is a and $0 \leq a \leq 1$. The state variable is H_R. The current value Hamiltonian is given by

$$H^R = lnC_R + \lambda_R m(1-a)xH_R$$

where λ_R is the co-state variables of H_R representing the shadow price of the human capital of the rich individual. C_R is given by the equation (2).

The first order condition necessary for this optimization problem with respect to the control variable a is given by the following:

$$\frac{\alpha}{a} - \lambda_R m_R H_R x = 0; \tag{7}$$

Time derivative of the co-state variable satisfying the optimum growth path is given by the following:

$$\dot{\lambda_R} = \rho\lambda_R - \frac{\alpha}{H_R} - \lambda_R m_R(1-a)x; \tag{8}$$

Solving the system there will be family of time paths of state and costate variables satisfying the given initial condition. The member of this family that satisfies the transversality condition given by

$$\lim_{t\to\infty} e^{-\rho t}\lambda_R(t)H_R(t) = 0$$

is the optimal path.

Using equations (7) and (8) we have

$$\frac{\dot{\lambda_R}}{\lambda_R} = \rho - m_R x. \tag{9}$$

Differentiating the equation (7) with respect to time, t, and then using equation (8) we have

$$\dot{a} = m_R a^2 x - \rho a. \tag{10}$$

3.2 The optimization problem of the poor household

The objective of the representative poor individual is to maximize the discounted present value of utility over the infinite time horizon. The objective functional is given by

$$J_{H_P} = \int_0^\infty U(C_P)e^{-\rho t}dt$$

This is to be maximized with respect to u subject to the equation of motion given by

$$\dot{H_P} = m_P(1-u)H_P[q(1-x)+1]$$

and given the initial values of H_P. Here $U(C_P)$ is given by equation (4) and Y is given by equation (1). Here ρ is the constant positive discount rate. The control variable is u where $0 \leq u \leq 1$. The state variable is H_P. The current value Hamiltonian is given by

$$H^P = lnC_P + \lambda_P m_P(1-u)H_P[q(1-x)+1]$$

where λ_P is co-state variable representing the shadow price of the human capital of the poor individual. C_P is given by equation (3).

The first order condition necessary for this optimization problem with respect to the control variable u is given by the following:

$$\frac{(1-\alpha)}{u} - \lambda_P m_P H_P[q(1-x)+1] = 0; \tag{11}$$

Time derivative of the co-state variable satisfying the optimum growth path is given by the following:

$$\dot{\lambda_P} = \rho\lambda_P - \frac{(1-\alpha)}{H_P} - \lambda_P m_P(1-u)[q(1-x)+1]; \tag{12}$$

The transversality condition is given by

$$\lim_{t\to\infty} e^{-\rho t}\lambda_P(t)H_P(t) = 0$$

Using equations (11) and (12) we have

$$\frac{\dot{\lambda_P}}{\lambda_P} = \rho - m_P[q(1-x)+1]. \tag{13}$$

Differentiating equation (11) with respect to time, t, and then using equation (12) we have

$$\dot{u} = m_P u^2[q(1-x)+1] - \rho u. \tag{14}$$

Equations (10) and (14) summarize the private agents' decision rules in a decentralized competitive equilibrium.

3.3 The optimization problem of the government

The government chooses the tax rate, x, to maximize the welfare of the economy subject to the decentralized competitive equilibrium conditions. Thus, the maximization problem by the government is also constrained by the private agents' optimal decision rules given by equations (10) and (14). The objective of the government is to maximize the discounted present value of social welfare over the infinite time horizon. Here the social welfare function is defined as follows:

$$W = b\, lnC_R + (1-b)\, lnC_P$$

where b and $(1-b)$ is the weight given to the consumption of the rich household and of the poor household respectively. The objective functional is given by

$$J_H = \int_0^\infty W e^{-\rho t} dt$$

which is to be maximized with respect to x subject to the equations of motion given by

$$\begin{aligned}
\dot{a} &= m_R a^2 x - \rho a; \\
\dot{u} &= m_P u^2[q(1-x)+1] - \rho u; \\
\dot{H}_R &= m_R(1-a)xH_R; \\
\dot{H}_P &= m_P(1-u)H_P[q(1-x)+1];
\end{aligned}$$

and given the initial values of H_R and H_P. Here C_R and C_P are given by the equations (2) and (3). Here ρ is the constant positive discount rate. The control variable is x where $0 \leq x \leq 1$. The current value Hamiltonian is given by

$$H^g = blnC_R(t) + (1-b)lnC_P(t) + \mu_a[m_R a^2 x - \rho a] + \mu_u[m_P u^2[q(1-x)+1] - \rho u]$$
$$+\mu_R[m_R(1-a)xH_R] + \mu_P m_P(1-u)H_P[q(1-x)+1]$$

where μ_a, μ_u, μ_R and μ_P are the co-state variables.

The first order conditions necessary for this optimization problem with respect to the control variable, x, is given by the following:

$$\frac{\partial H^g}{\partial x} = \frac{\alpha}{x} + \mu_a m_R a^2 - \mu_u m_P u^2 q + \mu_R m_R (1-a) H_R - \mu_P m_P (1-u) H_P q = 0; \quad (15)$$

Time derivatives of the co-state variables should satisfy the following differential equations along the optimum growth path.

$$\dot{\mu_a} = 2\rho\mu_a - 2m_R a x \mu_a - \frac{\alpha}{a} + m_R x \mu_R H_R; \quad (16)$$

$$\dot{\mu_u} = 2\rho\mu_u - \frac{(1-\alpha)}{u} - 2u m_P \{q(1-x)+1\}\mu_u + \mu_P m_P H_P \{q(1-x)+1\}; \quad (17)$$

$$\dot{\mu_R} = \rho\mu_R - \frac{(\alpha+\epsilon_R)}{H_R} - m_R(1-a)x\mu_R \quad (18)$$

and

$$\dot{\mu_P} = \rho\mu_P - \frac{(1-\alpha+\epsilon_P)}{H_P} - \mu_P m_P (1-u)[q(1-x)+1] \quad (19)$$

The transversality conditions are given by

$$\lim_{t\to\infty} e^{-\rho t}\mu_R(t)H_R(t) = \lim_{t\to\infty} e^{-\rho t}\mu_P(t)H_P(t) = 0$$

Note that, if $q = 0$ then from equation (15) we have

$$\frac{\partial H^g}{\partial x} = \frac{\alpha}{x} + \mu_a m_R a^2 + \mu_R m_R (1-a) H_R$$

and $\frac{\partial H^g}{\partial x} > 0$ under the assumption that $\mu_a > 0$ and $\mu_R > 0$. In that case, optimal $x = 1$ for all $t \geq 0$. So we have the following proposition.

Proposition 1 *If $q = 0$, then optimum $x = 1$ for all $t \geq 0$.*

So if there is no role of training given by the rich individuals on the human capital formation of the poor individuals then it is optimal for the government not to impose any tax on the rich individual. This is so because this tax is imposed to finance the cost of training the poor individuals by the rich individuals and $q = 0$ implies that the training is unproductive.

Now following usual practice, we reduce the dimension of the system to facilitate analytical tractability. We define

$$\omega = \mu_P H_P \quad \text{and} \quad v = \mu_R H_R$$

Using equations (19) and (6) we have

$$\frac{\dot{\omega}}{\omega} = \rho - \frac{(1-\alpha+\epsilon_P)}{\omega} \tag{20}$$

Using equations (18) and (5) we have

$$\frac{\dot{v}}{v} = \rho - \frac{(\alpha+\epsilon_R)}{v} \tag{21}$$

Differentiating the equation (15) with respect to time t we have

$$-\frac{\alpha}{x}\frac{\dot{x}}{x} = -\mu_a m_R a^2[\frac{\dot{\mu_a}}{\mu_a}+2\frac{\dot{a}}{a}]+\mu_u m_P u^2 q[\frac{\dot{\mu_u}}{\mu_u}+2\frac{\dot{u}}{u}]-\mu_R m_R(1-a)H_R[\frac{\dot{v}}{v}+\frac{(1\dot{-}a)}{(1-a)}]$$

$$+\mu_P m_P(1-u)H_P q[\frac{\dot{\omega}}{\omega}+\frac{(1\dot{-}u)}{(1-u)}]; \tag{22}$$

This equation (22) can be further simplified[5] using equations (10), (14), (16), (17), (20) and (21); and this simplified form is given by the following

$$-\frac{\alpha}{x}\frac{\dot{x}}{x} = m_R a\alpha - {m_R}^2 a^2 xv - m_P uq(1-\alpha) + {m_P}^2 u^2 q\omega\{q(1-x)+1\} - m_R(1-a)\rho v + m_R(1-a)(\alpha+\epsilon_R)$$

$$+m_R va(m_R ax-\rho)+m_P(1-u)q\rho\omega-m_P(1-u)q(1-\alpha+\epsilon_P)-u\omega m_P q\{m_P u\{q(1-x)+1\}-\rho\} \tag{23}$$

The dynamic system is described by the differential equations (10), (14), (20), (21) and (23).

3.4 Steady-State Equilibrium

Along a steady state equilibrium growth path

$$\dot{a} = \dot{u} = \dot{\omega} = \dot{v} = \dot{x} = 0;$$

and their steady state equilibrium values are denoted by a^*, u^*, ω^*, v^* and x^*. First we use the equations (10) and (14) to determine the steady-state values of a and u in terms of x. They are given by the followings.

$$a^* = \frac{\rho}{m_R x};$$

and

$$u^* = \frac{\rho}{m_P[q(1-x)+1]}.$$

[5] see Appendix (A)

Steady-state equilibrium values of ω and v are obtained from the equations (20) and (21). They are given by

$$\omega^* = \frac{(1 - \alpha + \epsilon_P)}{\rho};$$

and

$$v^* = \frac{(\alpha + \epsilon_R)}{\rho}.$$

Using these steady state equilibrium values of the variables and using equations (10), (14), (20) and (21) and substituting $\frac{\dot{x}}{x} = 0$ from equation (22) we get the steady state equilibrium value of x given by

$$1 - x^* = \frac{q\epsilon_P - \epsilon_R}{q(\epsilon_P + \epsilon_R)}. \tag{24}$$

So here the steady-state equilibrium values of the variables are uniquely determined by the parameters. So the steady-state equilibrium point is unique given the values of the parameters. We do not find the problem of multiple steady-state equilibria. Note that this value of x^* crucially depends on the externality parameters present in the production function and on the parameters in the human capital accumulation function in the backward sector. An interior solution of x^* implies that $0 < x^* < 1$ and this is ensured by the following restriction on the parameters:

$$q\epsilon_P > \epsilon_R.$$

If $\epsilon_P = \epsilon_R = 0$, then x^* is indeterminate. If $\epsilon_P = 0$ and $\epsilon_R > 0$ then equation (24) shows that $(1 - x^*) < 0$ which does not make any sense. In this case, $x^* = 1$. If $\epsilon_R = 0$ and $\epsilon_P > 0$ then $(1 - x^*) = 1$. We find $0 < x^* < 1$ only if both ϵ_P and ϵ_R are positive. This leads to the following proposition:

Proposition 2 $0 < x^* < 1$ *is optimal in the steady-state equilibrium if and only if*

$$q\epsilon_P > \epsilon_R$$

with q, ϵ_P, $\epsilon_R > 0$. *If* $q\epsilon_P \leq \epsilon_R$, *then* $x^* = 1$.

This result has an important economic implication. A policy of subsidizing the education of the poor individuals financed by taxing the rich individuals appears to be optimum only if the external effect of human capital of the poor (rich) individual is very high (low) and if the productivity parameter of teaching takes a high value. This is sensible because the poor individuals are receiving subsidy financed by taxing the rich individuals and the subsidies are used to meet the cost of training programme. A subsidy must be justified when its receivers generate external economies at a higher rate than the bearers of the tax burden and when the subsidy financed programme is highly productive. Our result is similar to that found in Gomez(2004) and Garcia-Castrillo and Sanso (2000) who also analysed the optimality of a tax financed education subsidy policy in a Lucas (1988) model. However, we have made our analysis using a more general framework endogenizing the government's optimizing behaviour on the one hand and allowing dualism in the human capital accumulation of two groups of individuals on the other hand. Those two authors did not consider redistributive taxes like ours because they assumed all individuals to be identical.

There is a literature initiated by Judd (1985), Chamley (1986) etc. dealing with the optimality of redistributive taxes from capitalists to workers. This literature analyses the validity of the Judd- Chamley proposition that, in the steady-state equilibrium, a pure redistributive tax on capital income is not optimal. In this paper we consider a different type of redistributive tax- a tax designed to reduce the inequality in the stock of human capital between two groups of individuals. Inequality in the distribution of human capital is an important determinant of income inequality. Optimality of such a tax is justified in a world where human capital generates externalities and the external effect is stronger in the case of poor individuals.

There are some other interesting results here. In the steady-state equilibrium, $\dot{\omega} = \dot{v} = 0$. However, $\frac{\dot{H}_R}{H_R}$ and $\frac{\dot{H}_P}{H_P}$ may not be so. The steady-state equilibrium rate of growth of H_R obtained from equation (5), is given by

$$\frac{\dot{H}_R}{H_R} = m_R(1 - a^*)x^*;$$

and then substituting the values of a^* we have

$$\frac{\dot{H}_R}{H_R} = m_R x^* - \rho.$$

This rate of human capital accumulation is comparable to that in Lucas (1988) where

$$\frac{\dot{H}_R}{H_R} = m_R - \rho.$$

In Lucas (1988), no tax is imposed on the labour time endowment and so $x^* = 1$ in his model. The steady-state equilibrium rate of growth of H_P is given by

$$\frac{\dot{H}_P}{H_P} = m_P(1 - u^*)[q(1 - x^*) + 1];$$

and then substituting the value of u^* we have

$$\frac{\dot{H}_P}{H_P} = m_P[q(1 - x^*) + 1] - \rho.$$

Since x^* is uniquely determined by the parameters as given by the equation (24), there is no guarantee that H_R and H_P would grow at equal rate in the steady-state equilibrium. Here

$$\frac{\dot{H}_R}{H_R} = \frac{\dot{H}_P}{H_P} \Longrightarrow x^* = \frac{1+q}{\frac{m_R}{m_P}+1}.$$

Now looking at the equation (24), we can establish the following proposition.

Proposition 3 *H_R and H_P grow at equal rate in the steady state equilibrium only if $\frac{m_R}{m_P}+1 = \frac{q(\epsilon_P+\epsilon_R)}{\epsilon_R}$*

So far we have analysed the case with $q > 0$. What happens if $q = 0$? In this case, $x^* = 1$ and the equations of motion are given by the differential equations (20), (21),

$$\frac{\dot{a}}{a} = m_R a - \rho, \tag{10A}$$

and

$$\frac{\dot{u}}{u} = m_P u - \rho. \tag{14A}$$

Solving them we can get a steady-state equilibrium point given by

$$a^* = \frac{\rho}{m_R}$$

$$u^* = \frac{\rho}{m_P}$$

and by ω^* and v^* as obtained in the case with $q > 0$. However the system can not remain in the steady state for ever. With $q = 0$

$$\frac{\dot{H}_R}{H_R} = m_R - \rho$$

and

$$\frac{\dot{H}_P}{H_P} = m_P - \rho$$

So $\frac{\dot{H}_R}{H_R} > \frac{\dot{H}_P}{H_P}$ because, by assumption $m_R > m_P$. $\frac{H_R}{H_P}$ grows exponentially over time and hence H_R exceeds H_P after some point of time. So $q = 0$ can not continue for ever. Since $q > 0$ when $H_R > H_P$, we can prove the following proposition

Proposition 4 *There exists only one steady state equilibrium with $H_R > H_P$ and with $0 < x^* < 1$ when $q\epsilon_P > \epsilon_R$.*

This result has an interesting implication. Even if a policy of subsidizing the education of the poor individuals financed by taxing the rich individuals appears to be optimum, this policy will not fully correct the initial gap in the long-run. The inequality in the accumulated stock of human capital will persist in the long run. What is the intuition behind this? If we look at the expressions of a^* and u^* when $q > 0$, we find that a^* varies inversely with x^* while u^* varies directly with that. So the government policy of subsidizing the education of the poor individuals causes a reallocation of their labour time from human capital accumulation to production; and the taxation on the labour endowment of the rich individuals causes a reallocation of their labour time from production to human capital accumulation. So the full benifits of this policy designed to eliminate the inequality in human capital can not be achieved even in the long run.

Note that, when $q\epsilon_P \leq \epsilon_R$, $x^* = 1$ even if $q > 0$. In this case also our equations of motions are (20), (21), (10A) and (14A); and hence the steady state equilibrium point is similar to that in the case where $q = 0$. However, the economy may remain in this steady-state equilibrium for ever because here $q > 0$; and $q > 0$ is not inconsistent with the exponential growth of H_R/H_P

3.5 Transitional dynamic properties

We now turn to analyse the transitional dynamic properties around the steady state equilibrium point. We consider the case where $q > 0$ because with $q = 0$ the system can not remain in the steady state equilibrium for long. We consider two cases $(i) q\epsilon_P > \epsilon_R$ and $(ii) q\epsilon_P \leq \epsilon_R$.

3.5.1 $q\epsilon_P > \epsilon_R$

In this case, the system is described by the differential equations (10), (14), (20), (21) and (23). Note that this is a system of 5 differential equations. Initial values of the variable — H_R and H_P are historically given and of other two co-state variables μ_R and μ_P can be chosen. Hence ω and v are not predetermined variables here. The values of the control variables, i.e, x, a and u can be chosen by the government, the rich individual and the poor individual respectively. Since none of the five variables are pre determined here, we get the unique saddle path converging to the steady state equilibrium point when all latent roots of Jacobian matrix are positive in sign. If at least one latent root is negative in sign, we have the problem of indeterminacy of transitional growth path [6].

Here the Jacobian matrix corresponding to the system of differential equa-

[6] See Park and Philippopoulos(2003, 2004)

tions (20), (21), (10), (14) and (23) is given by the following:

$$J = \begin{bmatrix} \frac{\partial \dot{\omega}}{\partial \omega} & \frac{\partial \dot{\omega}}{\partial v} & \frac{\partial \dot{\omega}}{\partial a} & \frac{\partial \dot{\omega}}{\partial u} & \frac{\partial \dot{\omega}}{\partial x} \\ \frac{\partial \dot{v}}{\partial \omega} & \frac{\partial \dot{v}}{\partial v} & \frac{\partial \dot{v}}{\partial a} & \frac{\partial \dot{v}}{\partial u} & \frac{\partial \dot{v}}{\partial x} \\ \frac{\partial \dot{a}}{\partial \omega} & \frac{\partial \dot{a}}{\partial v} & \frac{\partial \dot{a}}{\partial a} & \frac{\partial \dot{a}}{\partial u} & \frac{\partial \dot{a}}{\partial x} \\ \frac{\partial \dot{x}}{\partial \omega} & \frac{\partial \dot{x}}{\partial v} & \frac{\partial \dot{x}}{\partial a} & \frac{\partial \dot{x}}{\partial u} & \frac{\partial \dot{x}}{\partial x} \end{bmatrix}$$

where, the elements of Jacobian matrix evaluated at the steady-state equilibrium point are given in Appendix (B).

The characteristic equation of the J matrix is given by

$$|J - \lambda I_3| = 0$$

where λ is an eigenvalue of the Jacobian evaluated at steady state. After substituting the elements of the Jacobian matrix $|J - \lambda I_3| = 0$ can be reduced into the following equation

$$(\rho-\lambda)^2[-\lambda^3+2\rho\lambda^2-\lambda\{\rho^2(1-\frac{\epsilon_R}{\alpha})-{m_P}^2q^2\frac{\epsilon_P}{\alpha}x^2u^2\}-\frac{\rho}{\alpha}\{\rho^2\epsilon_R+{m_P}^2\epsilon_P q^2u^2x^2\}] = 0$$

Hence two of the five characteristic roots are equal to ρ which is positive; and the other three characteristic root can be solved by equating the bracketed term to zero. Among those three roots one root is negative and other two roots are positive [7]. So, there is one negative root and four positive roots of the J matrix; and this is the sufficient condition for indeterminacy of trajectories converging to the long-run equilibrium point. We can establish the following proposition:

Proposition 5 *If $q\epsilon_P > \epsilon_R$, then there exists indeterminacy of the transitional growth path converging to the unique steady-state equilibrium point.*

3.5.2 $q\epsilon_P \leq \epsilon_R$

In this case, $x^* = 1$ and hence equation (23) does not make sense. The dynamic system is described by the equations (20), (21), (10) and (14) with $x = 1$. It can be easily checked that now the Jacobian matrix which is of order 4×4, is a diagonal matrix and all the four diagonal terms are positive[8]. So all the roots are positive. So we have the following proposition.

Proposition 6 *If $q\epsilon_P \leq \epsilon_R$, then there exists an unique transitional growth path converging to the steady state equilibrium point.*

So the existence of the indeterminacy of the transitional growth path is related to the relative strength of external effects of these two types of human capital

[7] Derivation of the characteristic equation and the sign of the roots are shown in the Appendix (B).

[8] see Appendix (C)

and to the magnitude of marginal contribution of the training programme. For example, when the human capital of the poor individuals generates a stronger external effect and when the training programme is highly productive, we find indeterminacy of transitional dynamics. There exists a substantial literature which explains indeterminacy of transitional dynamics in the presence of externalities of human capital in Lucas (1988) model. This includes the works of Xie (1994), Benhabib and Perli (1994) and of many others. They show that a high degree of externalities of human capital on production can explain indeterminacy of transitional dynamics. Since they consider all individuals to be identical, the importance of relative strength of externalities of human capital of the two groups of individuals and the role of teaching are not focussed in their analysis.

4 Conclusion

Existing endogeneous growth models dealing with human capital accumulation have not considered dualism in human capital formation among different classes of people. This paper attempts to develop a theoretical model of endogenous growth involving redistributive taxation and educational subsidy to build up human capital of the individuals belonging to the less privileged section of the community. Here we have analyzed the model of an economy with two different classes of individuals in which dualism exists in the nature of human capital accumulation of those two types of individuals. The government imposes a proportional tax on the resources of the rich individuals and uses that in financing the educational subsidy given to the poors. The optimal tax rate which is also equal to the subsidy rate is found out solving a Stackelberg differential game; and it is shown that the properties of optimal tax (educational subsidy) rate crucially depends on the relative strength of the external effect of human capital of the two types of individuals. We also derive some interesting properties of steady- state growth equilibrium and transitional dynamics of the economy.

The model, in this paper, does not consider many important features of less developed countries. Accumulation of physical capital is ruled out and there is no justification of this apart from the weak excuse of technical simplicity. A Cobb-Douglas production function can not relate the change in income distribution to the change in relative human capital endowment in competitive market structure. Once again technical simplicity is the weak excuse for not considering a non Cobb-Douglas world. The present model does not consider many other problems of dual economy e.g. unemployment of labour, market imperfections etc. Our purpose is to focus on the dualism in the human capital accumulation in a less developed economy and to analyse the properties of optimal educational subsidy policy in this context. In order to keep the analysis otherwise simple, we do all kinds of abstraction—a standard practice often followed in the theoretical literature.

References

Acemoglu D and J. Angrist, 2000, How Large are the Social Returns to Education? Evidence from Compulsory Schooling Laws, NBER Working Paper No. 7444, National Bureau of Economic Research.

Alonso-Carrera J. and M. J. Freire- Seren 2004. Multiple equilibria, fiscal policy and human capital accumulation. Journal of Economic Dynamics and Control. 28 : 841-856

Ben Gad, M., 2003, Fiscal Policy and Indeterminacy in Models of Endogenous Growth, Journal of Economic Theory, 108, 322-344.

Benhabib,J. and R. Perli, 1994. Uniqueness and indeterminacy: On the dynamics of endogenous growth. Journal of economic theory, 63,113-142.

Benhabib, J. and A. Rustichini, 1997, Optimal Taxes Without Commitment, Journal of Economic Theory, 77(2), 231-259.

Beyer,H., P. Rojas and R. Vergara, 1999. Trade liberalization and wage inequality Journal of Development Economics, 59 (1), 103-123.

Blankenau, W, 2005, Public schooling, college subsidies and growth, Journal of Economic Dynamics and Control, 29 (3), 487-507.

Blankenau, W. F. and N. B. Simpson, 2004, Public education expenditures and growth, Journal of Development Economics, 73 (2), 583-605.

Boskin, M, 1975. Notes on the Tax Treatment of Human Capital, NBER Working Paper No. 116, National Bureau of Economic Research.

Bovenberg, A. L and B. Jacobs, 2003. On the optimal distribution of education and income, mimeo: University of Amsterdam/ Tilburg.

Bovenberg, A. L and B. Jacobs, 2005. Redistribution and education subsidies are siamese twins, Journal of Public Economics, forthcoming.

Brett, C and J. A. Weymark, 2003, Financing education using optimal redistributive taxation, Journal of Public Economics, 87 (11), 2549-2569.

Chamley, C., 1986. Optimal Taxation of Capital Income in a General Equilibrium Model with Infinite Lives, Econometrica 54(3): 607 622.

Chamley, C., 1992. The Welfare Cost of Taxation and Endogenous Growth, Boston University, IED Discussion Paper No 30.

Ciccone, A. and G. Peri, April 2002. Identifying Human Capital Externalities: Theory with an Application to US Cities; IZA Discussion Paper, 488, Institute for the Study of Labor (IZA).

De Hek, P. A, 2005, On taxation in a two-sector endogenous growth model with endogenous labor supply, Journal of Economic Dynamics and Control, forthcoming.

Garcia-Castrillo, P and M. Sanso, 2000. Human capital and optimal policy in a Lucas-type model, Review of Economic Dynamics, 3, 757-770.

Glaeser, E.L., 1997, Learning in Cities, NBER Working Paper No.6271, National Bureau of Economic Research.

Glaeser, E.L. and D.C.Mare, 1994, Cities and skills, NBER Working Paper No. 4728, National Bureau of Economic Research.

Gomez, M. A., 2003. Optimal fiscal policy in the Uzawa-Lucas model with externalities, Economic Theory, 22, 917-925.

Guo, J and K.J. Lansing, 1999. Optimal taxation of capital income with imperfectly competitive product markets, Journal of Economic Dynamics and Control, 23, 967-995.

Jones, L.E., R.E. Manuelli and P.E. Rossi, 1993. Optimal Taxation in Models of Endogenous Growth, Journal of Political Economy, 101, 485-517.

Judd, K.L.,1985. Redistributive taxation in a simple perfect foresight model. Journal of Public Economics 28, 59-83.

Judd K.L., 1997. The Optimal Tax Rate for Capital Income is Negative, NBER Working Paper No. 6004, National Bureau of Economic Research.

Lachler, U., 2001, Education and earnings inequality in Mexico, World Bank Working Paper.

Lansing, K.L, 1999. Optimal redistributive capital taxation in a neoclassical growth model. Journal of Public Economics, 73, 423-453.

Lucas, R. E, 1988. On the Mechanics of Economic Development, Journal of Monetary Economics, 22, 3-42.

Lucas, R. E, 1990, Supply-Side Economics: An Analytical Review, Oxford Economic Papers, 1990, 42(2), 293-316.

Marjit, S and R. Acharyya, 2003, International Trade, Wage inequality and the Developing Economy, Physica Verlag, New York.

Mino, K, 1996. Analysis of a Two-Sector Model of Endogenous Growth with Capital Income Taxation, International Economic Review, 37, 227-51.

Mino, K, 2001. Optimal taxation in dynamic economies with increasing returns, Japan and the World Economy, 13, 235-253.

Moretti, E, 2003. Human capital externalities in cities, NBER Working Paper No.9641, National Bureau of Economic Research.

Ortigueira S, 1998. Fiscal policy in an endogenous growth model with human capital accumulation - Transitional dynamics with multiple equilibria, 42, 323-355.

Park, H and A. Philippopoulos, 2003. On the dynamics of growth and fiscal policy with redistributive transfers, Journal of Public Economics, 87, 515-538

Park, H and A. Philippopoulos, 2004. Indeterminacy and fiscal policies in a growing economy, Journal of Economic Dynamics and Control, 28. 645-660.

Peri, G, 2002, Young workers, learning and agglomeration, Journal of Urban Economics, 52, 582-607.

Robbins, D., 1994 a, Malaysian wage structure and its causes, Harvard Institute for International Development.

Robbins, D., 1994 b, Philippine wage structure and its causes, Harvard Institute for International Development.

Rudd, J. B., October 2000. Empirical Evidence on Human Capital Spillovers, FEDS Discussion Paper No. 2000-46, Board of Governors of the Federal Reserve - Macroeconomic Analysis Section.

Stokey, N.L. and S.Rebelo 1995. Journal of Political Economy. Growth Effects of Flat-Rate Taxes, 103, 519-50.

Uhlig, H and N. Yanagawa, 1996. Increasing the capital income tax may lead to faster growth, European Economic Review, 40(8), 1521-1540.

Wood, A, 1997. "Openness and wage inequality in developing countries: The Latin American Challenge to East Asian Conventional Wisdom," World Bank Economic Review 11(1):33-57

Xie, D, 1994. Divergence in economic performance: Transitional dynamics with multiple equilibria, Journal of Economic Theory, 63, 97-112.

Zhang, J., 2003. Optimal debt, endogenous fertility, and human capital externalities in a model with altruistic bequests, Journal of Public Economics, 87(7-8), 1825-1835.

Appendix (A)

We consider equation (22) given by

$$-\frac{\alpha}{x}\frac{\dot{x}}{x} = -\mu_a m_R a^2[\frac{\dot{\mu_a}}{\mu_a}+2\frac{\dot{a}}{a}]+\mu_u m_P u^2 q[\frac{\dot{\mu_u}}{\mu_u}+2\frac{\dot{u}}{u}]-\mu_R m_R(1-a)H_R[\frac{\dot{v}}{v}+\frac{\dot{(1-a)}}{(1-a)}]$$

$$+\mu_P m_P(1-u)H_P q[\frac{\dot{\omega}}{\omega}+\frac{\dot{(1-u)}}{(1-u)}]$$

This equation can be further simplified using equations (10), (14), (16), (17), (20) and (21) as follows;

$$-\frac{\alpha}{x}\frac{\dot{x}}{x} = -\mu_a m_R a^2[2\rho - 2m_R ax - \frac{\alpha}{a\mu_a} + \frac{\mu_R}{\mu_a}m_R x H_R + 2m_R ax - 2\rho]+$$

$$\mu_u m_P u^2 q[-\frac{(1-\alpha)}{u\mu_u}+\frac{\mu_P}{\mu_u}m_P H_P\{q(1-x)+1\}]-\mu_R m_R(1-a)H_R[\rho-\frac{(\alpha+\epsilon_R)}{\mu_R H_R}-m_R(1-a)x$$

$$+m_R(1-a)x - \frac{a}{(1-a)}(m_R ax-\rho)] + \mu_P m_P(1-u)H_P q[\rho - \frac{(1-\alpha+\epsilon_P)}{\mu_P H_P}$$

$$-m_P(1-u)\{q(1-x)+1\}+m_P(1-u)\{q(1-x)+1\}-\frac{u}{(1-u)}\{m_P u[q(1-x)+1]-\rho\}]$$

From the above equation it follows that

$$-\frac{\alpha}{x}\frac{\dot{x}}{x} = m_R a\alpha - {m_R}^2 a^2 xv - m_P uq(1-\alpha) + {m_P}^2 u^2 q\omega\{q(1-x)+1\}$$

$$-m_R(1-a)\rho v + m_R(1-a)(\alpha+\epsilon_R) + m_R(1-a)v\frac{a}{(1-a)}(m_R ax-\rho)+$$

$$m_P(1-u)q\rho\omega - m_P(1-u)q(1-\alpha+\epsilon_P)-\frac{u}{(1-u)}\omega m_P(1-u)q\{m_P u\{q(1-x)+1\}-\rho\}$$

The final simplified form is given by the following equation.

$$-\frac{\alpha}{x}\frac{\dot{x}}{x} = m_R a\alpha - {m_R}^2 a^2 xv - m_P uq(1-\alpha)+{m_P}^2 u^2 q\omega\{q(1-x)+1\}-m_R(1-a)\rho v+m_R(1-a)(\alpha+\epsilon_R)$$

$$+m_R va(m_R ax-\rho)+m_P(1-u)q\rho\omega-m_P(1-u)q(1-\alpha+\epsilon_P)-u\omega m_P q\{m_P u\{q(1-x)+1\}-\rho\}$$

Appendix (B)

The elements of the Jacobian is as follows:

$$J_{\omega\omega} = \rho$$

$$J_{\omega v} = J_{\omega a} = J_{\omega u} = J_{\omega x} = 0$$
$$J_{v\omega} = J_{va} = J_{vu} = J_{vx} = 0$$
$$J_{vv} = \rho$$
$$J_{a\omega} = J_{av} = J_{au} = 0$$
$$J_{aa} = m_R a x$$
$$J_{ax} = m_R a^2$$
$$J_{u\omega} = J_{uv} = J_{ua} = 0$$
$$J_{uu} = m_P u[q(1-x)+1]$$
$$J_{ux} = -m_P u^2 q$$
$$J_{xa} = \frac{x^2 m_R \epsilon_R}{\alpha}$$
$$J_{xu} = -\frac{x^2 m_P q \epsilon_P}{\alpha}$$
$$J_{xx} = -\frac{x^2}{\alpha}[-{m_R}^2 a^2 v - {m_P}^2 u^2 \omega q^2 + {m_R}^2 a^2 v - u\omega m_P q(-m_P u q)]$$
$$J_{x\omega} = -\frac{x^{*2}}{\alpha} m_P q \rho$$
$$J_{xv} = \frac{x^{*2}}{\alpha} m_R \rho$$

Now using the steady state values of v, ω, a, u we get $J_{xx} = 0$.
The charateristic equation is $|J - \lambda I_3| = 0$. Or,

$$(\rho-\lambda)^2 \begin{vmatrix} m_R a x - \lambda & 0 & m_R a^2 \\ 0 & J_{uu} - \lambda & -m_P u^2 q \\ J_{xa} & J_{xu} & J_{xx} - \lambda \end{vmatrix} = 0$$

The above equation can be written as

$$(\rho-\lambda)^2[m_R a x - \lambda)\{(J_{uu}-\lambda)(J_{xx}-\lambda) + m_P u^2 q J_{xu}\} - m_R a^2 (J_{uu}-\lambda) J_{xa}] = 0$$

Hence, two of the charateristic roots are equal to ρ. The other three roots can be found out from the following equation.

$$[(m_R a x - \lambda)\{(J_{uu}-\lambda)(J_{xx}-\lambda) + m_P u^2 q J_{xu}\} - m_R a^2 (J_{uu}-\lambda) J_{xa}] = 0$$

Or,

$$-\lambda^3 + \lambda^2(J_{uu} + m_R ax + J_{xx}) - \lambda[J_{uu}m_R ax + J_{xx}J_{uu} + m_R ax J_{xx}$$

$$-m_R a^2 J_{xa} + m_P u^2 q J_{xu}] + [J_{uu}J_{xx}m_R ax - m_R a^2 J_{xa}J_{uu} + m_P u^2 q J_{xu} m_R ax]$$

Now substituting the elements from the Jacobian matrix we have,

$$-\lambda^3+\lambda^2(m_R ax+m_P u\{q(1-x)+1\})-\lambda[m_P u\{q(1-x)+1\}m_R ax-\epsilon_R {m_R}^2 a^2\frac{x^2}{\alpha}$$

$$-{m_P}^2u^2q^2\frac{x^2}{\alpha}\epsilon_P] + [-\epsilon_R {m_R}^2 a^2\frac{x^2}{\alpha}m_P u\{q(1-x)+1\} - {m_P}^2 u^2 q^2\frac{x^2}{\alpha}\epsilon_P m_R ax]$$

Or,

$$[-\lambda^3 + 2\rho\lambda^2 - \lambda\{\rho^2(1-\frac{\epsilon_R}{\alpha}) - {m_P}^2q^2\frac{\epsilon_P}{\alpha}x^2u^2\} - \frac{\rho}{\alpha}\{\rho^2\epsilon_R + {m_P}^2\epsilon_P q^2u^2x^2\}] = 0$$

From the above equation we see that the coefficient of first term is negative, second term is positive, fourth term is negative. So, whatever be the sign of third term, the number of times the signs of coefficient change is two. So, according to Descarte's rule there are two positive roots and one negative root.

Appendix (C)

In this case, the matrix reduces to

$$J = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 \\ 0 & 0 & m_R a & 0 \\ 0 & 0 & 0 & m_P u \end{bmatrix}$$

The charateristic equation is

$$(\rho - \lambda)(\rho - \lambda)(m_R a - \lambda)(m_P u - \lambda) = 0$$

So the charateristic roots are ρ, ρ, $m_R a$, $m_P u$. All of these are positive.