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Problem Set 1: Due Jan 21,2004.

1. Consider the following basic real business cycle model

$$MaxE\sum_{t=0}^{\infty}\beta^t[lnc_t + \nu lnh_t]$$

subject to

$$c_t + i_t = w_t(1-\tau_h)h_t + k_t(r_t - \delta)(1-\tau_k) + TR_t$$

$$k_{t+1} = (1-\delta)k_t + i_t$$

$$h_t + l_t = 1$$

$$z_{t+1} = \gamma z_t + \epsilon_{t+1}$$

with $k(0)$ given. The variables $c_t, i_t, k_t, h_t, l_t, w_t$ are the time t values of consumption, investment, the capital stock, labor supply, leisure, and the wage rate, respectively. r_{t+1} denotes the return on capital from t to $t+1$. The household knows the stochastic shock parameter, $0 < \gamma < 1$, the depreciation rate, δ, and the tax rates on the return to capital, τ_k, and labor income, τ_h. The innovation to the technology shock ϵ is $N(0, \sigma_\epsilon^2)$.

There are a continuum of competitive firms, with the total number of firms normalized to unity, each of which produces output by the technology

$$Y_t = e^{z_t}K_t^\alpha H_t^{1-\alpha}.$$

The government in this economy transfers all tax revenue back to households in a lump sum fashion

$$TR_t = \tau_h w_t H_t + \tau_t(r_t - \delta)K_t.$$

Assign the following parameter values: $\beta = .99, \alpha = .36, \gamma = .95, \delta = .025, \nu = 2, \sigma_\epsilon = .007, \tau_h = .23, \tau_k = .5.$

- Program this problem in Gauss using a quadratic approximation of the value function around the steady state. Find the optimal decision variables for H_t and I_t as a function of the state variables, z_t and K_t. Show that the steady state obtained from these decision rules is the same that is found analytically. Before you do this however, derive the analytical expressions for steady state k and h by taking the first order conditions with respect to k_{t+1} and h_t.

- Do you obtain the same decision rules from the previous problem set (Macro II) when $\tau_k = \tau_h = 0$?.

- Start the economy at the steady state and graph the response of H_t, C_t, I_t, to a one standard deviation shock in ϵ_t.

- Starting again from the steady state, simulate the economy for 100 periods and plot the results for H_t, C_t, K_t, Y_t, and I_t.