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Macroeconomics II
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Fall 2003

Review Problem Set 0: Please turn in *all* solutions in pencil.

1. Consider the Solow Model discussed in class. Assume that the aggregate production function is Cobb Douglas with labor augmenting technological progress, $Y = K^{\alpha}(AL)^{1-\alpha}$, where $\frac{\dot{A}}{A} = g$. Write the intensive form production function, $f(Z)$, expressing output per efficiency unit of labor as a function of the capital-labor ratio in efficiency units, $Z = \frac{K}{AL}$. Derive the law of motion of motion for Z under Solow's assumptions, and solve explicitly for the steady state of the system. What factor's determine a country's long run level of income.

2. Suppose the production function is of the form

$$Y = (B_t K_t)^{\alpha}(A_t L_t)^{1-\alpha}$$

with both capital and labor augmenting technical progress rates given by $\frac{\dot{B}}{B} = g_B$. and $\frac{\dot{A}}{A} = g_A$. Derive the law of motion for the capital labor ratio in effective units, $Z = \frac{BK}{AL}$, under the assumptions of the Solow Model. Show that the system has a balanced growth path, i.e., a constant z solution, if and only if $g_B = 0$, or in other words, technical progress is labor augmenting.

3. Homogenous output is produced according to two types of capital, private and public capital, K and P, according to technology of the form

$$Y_t = K_t^{\alpha} P_t^{\beta}$$

where $\alpha + \beta < 1$. Both types of capital depreciate completely upon use. In each period, the government taxes income at the rate of τ_t, and invests the proceeds in public capital for the next period. Agents save a fixed fraction of their income s of their after-tax income and invest it in private capital. Hence,

$$K_{t+1} = s(1-\tau)Y_t$$

and $P_{t+1} = \tau Y_t$. Using the above equations, derive a single difference equation in Y that describes the evolution of income. Call this equation (Δ). Solve for the steady state value of Y and show that the system is stable. How does the steady state income vary with s and τ ? What value of τ should the government chose if it wants to maximize steady state output ?

4. This is a model on learning by doing. Starting from the Solow model with exogenous technological progress, we will develop a simple model of endogenous growth and examine some of it's implications. Assume that the production function is of the form

$$Y = K^\alpha (AL)^{1-\alpha}$$

Instead of assuming that g_A is a given constant, we will assume that the rate of technical progress g_A reflects the accumulation of knowledge with productive experience. In particular, we will assume that the instantaneous increase of A is proportional to output per worker, that is, $\dot{A} = \gamma Q = \gamma A Z^\alpha$, where γ measures the speed of learning.

- Show under these assumptions that the law of motion of the capital-labor ratio is given by $\dot{Z} = (s - \gamma Z)Z^\alpha - (\delta + n)Z$.
- Construct the phase diagram for the system, and discuss the stability of it's steady state. What is the growth of income per worker along the steady state path ?
- Analyze the impact of an increase in the investment rate on the steady state and on the time path of the system. Things are now quite different then the Solow model with exogenous technological progress. In what sense ?
- Consider two countries that are identical except for their investment rates. Discuss the predictions of the current model and the Solow model with exogenous technological progress concerning the evolution of the relative income levels of the two countries.

5. Consider an economy endowed with an aggregate production of the form

$$Y = K^{\alpha} L H^{1-\alpha}$$

where K is the aggregate stock of physical capital, L is employment in goods production, and H is the average stock of human capital. "Pure knowledge" A increases over time at a constant exogenous rate g, that is

$$A_{t+1} = (1+g)A_t$$

Pure knowlege, teachers time, and human capital, are combined to "produce" next generation's human capital according to

$$H_{t+1} = (\tau H)^{\gamma} A_t^{1-\gamma}$$

where τ is the fraction of the population chosen by teachers, a variable chosen by the government.

Suppose that the population is constant and normalized to 1, so that the labor force is $L = (1-\tau)$. Further, suppose that capital depreciates completely upon use and that agents save a constant fraction, s, of their income. Then the law of motion of the capital stock is of the form

$$K_{t+1} = sK_t^{\alpha} H_t^{1-\alpha}(1-\tau)^{1-\alpha}$$

- Define $Z = \frac{K}{A}$ and $E = \frac{H}{A}$. Using the previous expressions, derive a system of difference equations in Z and E that will describe the evolution of the economy.
- Solve for the steady state values of Z and E, and comute the steady state value of $Q = \frac{Y}{A}$.
- Find the value of τ that will maximize steady state Q.
- Let $z = lnZ$ and $e = lnE$. The system derived in the first bullet should be linear in e and z. Working with the system in logs, compute its eigenvalues, and discuss the stability of the steady state.
- Draw the phase diagram of the system.