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Macroeconomics II
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Review Problem Set 2: Please turn in your solutions in pencil.

1.The eigenvalue, λ, of the log-linearized system discussed in class

$$z_{t+h} = \overline{z} + (z_t - \overline{z})e^{-\lambda h}$$

provides a measure of the speed of convergence in an economy towards its steady state. Show that the half life of the system, described by

$$\dot{z} = -\lambda(z - \overline{z})$$

is given by $H = \frac{ln2}{\lambda}$. Note that by definition, $z_H - \overline{z} = \frac{z_0 - \overline{z}}{2}$. H is the time at which half of the original deviation of z from it's steady state has been eliminated. Also, because z in logs, this is approximately the deviation of z from its steady state in percentage terms.

More generally, the half life, t^*, is the solution to $e^{-\lambda t} = 0.5$. Using the values for n, g, and δ for India, find t^*. If the BJP government enacts the Confederation of Indian Industry (CII) proposal to raise the $\frac{I}{GDP}$ ratio from its current value of 24 % to 32 % (corresponding to an approximately 8 % increase in the saving rate), how much is output per person going to increase, relative to it's previous path, after 1 year, half life, asymptotically ?

2. Based on our discussion in class of equation

$$y_{i,t+1} = x_i + (1 - \beta)y_{i,t} + \epsilon_{it}$$

derive a system of equations for $Var(y_{i,t}) = \sigma_t^2$ and $c_t = Cov(x_i, y_{i,t}) = E(x_i, y_{i,t})$. Discuss the stability properties, and evaluate the steady state $(\overline{\sigma^2}, \overline{c})$. Interpret your results in words. (Hint: Take the variance of both sides of the above equation and notice that $Cov(x_i, y_{i,t+1}) = c_{t+1} = E(x_i y_{i,t+1}))$

3. This problem follows are class discussion, and is based on Mankiw, G., Romer, D. and Weil D, 1992, *A Contribution to the*

Empirics of Economic Growth, QJE, Vol. 107(2):407-37. Suppose that the aggregate production function is of the form

$$Y = K^{\alpha} E^{\gamma} (AL)^{1-\alpha-\gamma} = ALZ^{\alpha} H^{\gamma}$$

where K and E are the aggregate stocks of physical and human capital , L is the size of the labor force, and A is a productivity index that summarizes the current state of technological progress. The normalized variables $Z = \frac{K}{AL}$and $H = \frac{E}{AL}$ denote the stocks of physical capital and human capital per efficiency units of labor.

Suppose $\frac{\dot{L}}{L} = n$ and $\frac{\dot{A}}{A} = g$ and assume that the fraction of GDP devoted to investment in physical capital and human capital, (s_k and s_h) respectively,remain constant over time. Under these assumptions, the accumulation of productive factors is described by the system

$$\dot{K} = s_k Y - \delta K$$

and

$$\dot{E} = s_h Y - \delta E$$

where the depreciation rate, δ is assumed to be the same for both types of capital. Using the fact that $\frac{\dot{Z}}{Z} = \frac{\dot{K}}{K} - n - g$ and $\frac{\dot{H}}{H} = \frac{\dot{E}}{E} - n - g$, the laws of motion for the stocks of physical and human capital can be re-written in terms of the normalized variables,

$$\frac{\dot{Z}}{Z} = s_k Z^{\alpha-1} H^{\gamma} - (\delta + g + n)$$

and

$$\frac{\dot{H}}{H} = s_H Z^{\alpha} H^{\gamma-1} - (\delta + g + n)$$

- Find the steady state values of Z, H, and output per efficiency unit of labor, $\frac{P=Y}{AL}$.
- Now construct a log-linear approximation to the system and use it to derive a convergence equation similar to the one obtained in class. In other words, letting $z = lnZ$, and $h = lnH$ (from where $Z = e^z \& H = e^h$), rewrite the system above representing $\frac{\dot{Z}}{Z}$ and $\frac{\dot{H}}{H}$ in terms of z and h. Show that the linear approximation to the transformed system around the steady state is given by

$$\dot{z} = -(1-\alpha)(\delta + g + n)\tilde{z} + \gamma(\delta + g + n)$$

and

$$\dot{h} = \alpha(\delta + g + n)\tilde{z} - (1 - \beta)(\delta + g + n)\tilde{h}$$

where $\tilde{x} = x - \overline{x}$ denotes the current deviation of variable x from its steady state value. Discuss the stability of the system representing $\dot{z}$ and $\dot{h}$ (and hence that of the original system).

- Using the system described in bullet (ii) and the fact that $p = \alpha z + \gamma h$, derive a linear differential equation in p that describes the approximate behavior of this variable, and solve it. Rewriting the solution in terms of output per worker, $q = p + a$, derive a convergence equation of the form

$$\frac{q_{t+d} - q_t}{d} = g + \frac{1 - e^{-\lambda d}}{d}[p_t - (q_t - a_t)]$$

where d is the duration of the period, and $\lambda = (1 - \alpha - \gamma)(\delta + g + n)$.