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Macroeconomics II
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Fall 2003

Review Problem Set 3: Please turn in your solutions in pencil.

1. Find the solutions to the following FOLDE with variable coefficients

- $\dot{x} + \frac{2x}{t} = 5t^2$
- $\dot{x} + \frac{1}{t}x = t$

2. Assume that government goods don't interact with goods in the agent's utility function or the production function. Also disregard, for the moment, exogenous growth in population and technology. We want to show that the path of financing government expenditures is irrelevant for consumers, i.e., the time profile of budget deficits and taxes is irrelevant. Indeed only the present value of government spending and initial debt is important. This is the celebrated hypothesis of Ricardian Equivalence.

- Consider the government's budget constraint (BBC)

$$\dot{B} = G_t + r_t B_t - \tau_t$$

where B_t is the stock of government debt in time period t, G_t are government expenditures in time period t, τ_t are government tax revenues in time period t, and r_t the rental rate on capital. If we require that the present value of debt converge, i.e., $lim_{t\to\infty} e^{-R(t)} B_t = 0$ where $R(t) = \int_o^t r_s ds$, show that the BBC can be re-expressed as

$$constant + \int_0^\infty e^{-R(s)} G_s ds = \int_0^\infty e^{-R(s)} \tau_s ds$$

What does the above equation imply in words ?

- Recall that the slope of the consumption path was given by consumer's Euler equation

$$\frac{\dot{c}}{c} = \frac{r_t - \rho}{\sigma}$$

or $g_c\sigma + \rho = r$. Consider the instantaneous consumer's budget constraint of agents

$$\dot{A} = W_t - C_t - \tau_t + r_t A_t$$

where A_t denote assets in time period t, W_t denotes the wage in time period t, and C_t denotes consumption in time period t. Integrate the CB and show that what is important for household's is only the present value of government spending plus initial debt. If taxes don't affect the slope of consumption, is consumption affected by the way of financing government expenditures given the above set-up ? What is this result called ? Can you criticize some of the assumptions made to generate the Ricardian Equivalent result ?

3. Social security in the Diamond Model. Consider a Diamond OLG model like the one discussed in class. Population grows at a constant rate n, and preferences take the form

$$U(c, x) = lnc + \beta lnx$$

with $\beta \in (0, 1)$, x denoting old-age consumption, and c denoting young age consumption. The production function is Cobb Douglas,

$$Y = K^{\alpha} L^{1-\alpha}$$

where $\alpha \in (0, 1)$. We assume that wages are taxed at a proportional rate τ and that proceeds are used to finance a balanced pay as you go social security scheme. Hence, the first period, after tax income for an agent born in time period t is given by

$$y_1 = (1 - \tau) w_t$$

and his second period retirement subsidy is equal to

$$y_2 = \tau(1 + n) w_{t+1}$$

because there are $1 + n$ young agents for each old agent.

- Maximize $U(c, x)$ subject to the appropriate budget constraints, and solve for the agents savings function, $s^* = s(y_1, y_2, R)$ and his indirect utility function $v(w_t, w_{t+1}, R_{t+1}, \tau_t)$. Taking factor prices as given, is the agent's welfare an increasing function of the social security tax rate ?

- Derive the law of motion for the capital/labor ratio, $Z = \frac{K}{L}$, and compute the steady state values of Z and factor prices as a function of τ. Call these functions $\overline{Z} = Z_s(\tau)$, $\overline{w} = w_s(\tau)$, and $\overline{R} = R_s(\tau)$. Under what conditions is it true that $1+n > R_s(0)$?

- What are the effects of an increase in τ on steady state Z and factor prices ? Compute the following derivatives evaluated at $\tau = 0 : \frac{Z_s^{'}(\tau)}{Z_s(\tau)}, \frac{w_s^{'}(\tau)}{w_s(\tau)}, \frac{R_s^{'}(\tau)}{R_s(\tau)},$

- One of the advantages of the working with a model in which individual preferences are clearly specified is that this gives us a natural criterion for evaluating the desirability of possible policy alternatives. Using your previous results, and considering only its effects on steady-state welfare, when will it be a good idea to introduce a social security scheme ? To answer this question, compute the derivative of a representative individuals's (maximized) welfare with respect to τ, taking into account both the direct effects of the tax and its indirect effects through the induced change in steady state factor prices, and evaluate it at $\tau = 0$. (Hint: Leave everything in terms of $z_s^{'}(0)$).

4. Consider the Ramsey-Cass-Koopmans Model outlined in class. Suppose a social planner maximizes the utility of a representative individual

$$\int_0^\infty \frac{C_t^{1-\sigma}}{1-\sigma} e^{-\rho t} dt$$

subject to the resource constraint

$$\dot{K}_t = F(K_t, A_t) - \delta K_t - C_t$$

where $\frac{\dot{A}}{A} = g$. This is called a planning problem. List the necessay conditions for this problem, and show that they reduce to equations $\dot{c} = \phi(c, Z; \tau_r)$ and $\dot{Z} = \varphi(c, Z)$ derived in class when there are no taxes and no subsidies. Hence, under these conditions, the competitive equilibrium is also a social optimum.