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Macro II
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Problem Set 6: Stokey and Lucas, Chapter 2: Problems 2.1, 2.2.

Problem Set 7: Due October 22, 2003.

1. Consider the model in class that was solved using the informal perturbation method. Assume for simplicity that $n = g = \overline{A} = \overline{N} = 0$. Let $V(K_t, A_t)$, denote the value function be the expected present discounted value from the current period forward of lifetime utility of the representative individual as a function of the capital stock and technology.

- Explain intuitively why $V(\cdot)$ must satisfy

  $$V(K_t, A_t) = max_{c_t,l_t}\{[lnc_t + bln(1-l_t)] + e^{-\rho}E_t[V(K_{t+1}, A_{t+1})]\}$$

  where the condition is called Bellman's Equation. Given the log linear structure of the model in class, let us guess that $V(\cdot)$ takes the form $V(K_t, A_t) = \beta_0 + \beta_k lnK_t + \beta_A lnA_t$, where the values of β are to be determined. Substituing this conjectured form and the fact that $K_{t+1} = Y_t - C_t$ and $E_t[lnA_{t+1}] = \rho_A lnA_t$into the Bellman's equation yeilds

  $$V(K_t, A_t) = max_{c_t,l_t}\{[lnc_t + bln(1-l_t)] + e^{-\rho}[\beta_0 + \beta_k ln(Y_t - C_t) + \beta_A \rho_A lnA_t)]\}$$

  Find the first order condition for C_t. Show that it implies that $\frac{C_t}{Y_t}$ does not depend on K_t or A_t.

- Find the first order condition for l_t. Use this condition and the result in the above bullet to show that l_t does not depend on K_t or A_t.

- Substitute the production function and the above bullets for the optimal C_t and l_t into the equation above for $V(\cdot)$ and show that the resulting expression has the form $V(K_t, A_t) = \beta_0' + \beta_K' lnK_t + \beta_A' lnA_t$.

- What must β_K and β_A be so that $\beta_K^{'} = \beta_K$ and $\beta_A^{'} = \beta_A$.
- What are the implied values of $\frac{C}{Y}$ and l ? Are those the same found in class when $n = g = 0$?

2. Consider the model we did in class with the modification that the utility function is

$$u_t = lnc_t + \frac{b(1-l_t)^{1-\gamma}}{1-\gamma}$$

where $b > 0$, and $\gamma > 0$.

- Find the first-order condition that relates current leisure and consumption given the wage.
- With this change in the model, is the saving rate, s, still constant ?
- Is leisure per person $(1 - l)$ still constant ?

3. Define $T : C[0, \frac{1}{2}] \rightarrow C[0, \frac{1}{2}]$ by

$$(Tu)(t) = 1 + \int_0^t u(s)ds$$

for all $u \in C[0, \frac{1}{2}]$. Can you identify a function that is a fixed point of T ?

4. Let y_0 be the constant function 1 on the interval $[0, 1]$; i.e., $y_0 = 1 \forall t \in [0, 1]$. Let the operator be defined as before:

$$(Tu)(t) = 1 + \int_0^t u(s)ds$$

and define $y_1 = Ty_0$, $y_2 = Ty_1$,......, $y_n = Ty_{n-1}$. Construct $y_1(t), y_2(t), y_3(t), y_4(t)$. Can you conjecture what the limit of $y_n(t)$ is as $n \rightarrow \infty$?