Inequality, Politics and Economic Growth

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Abstract

In this paper we study the effect of inequality on the political choice process and its implications for economic growth. In our model, the government plays a crucial role in determining the growth rate by producing infrastructure. Infrastructure like physical capital is a growth inducing instrument in the economy. The government taxes factor incomes to finance production of infrastructure. Household agents differ in terms of their relative endowments of productive factors namely labor and capital. We show that when the only purpose of the fiscal policy is to provide infrastructure there is no conflict between inequality and growth. All agents irrespective of their relative factor endowments vote unanimously on the same tax rate. This tax rate happens to be the same as the welfare maximizing tax rate for the representative agent economy. However, when we introduce a redistributive aspect into the fiscal policy this unanimity breaks down. The relationship between inequality and growth depends on the nature of the political choice process. When there is voting on both the tax rates and the amount of redistribution higher inequality leads to lower growth rate for the economy.

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1 Introduction

Endogenous growth models attempt to explain differences in the rates of growth across different countries. One of the factors that may influence growth rate of an economy is the degree of inequality. Recently there has been a lot of interest in the role played by inequality in distribution of wealth on the growth performance of an economy. The theoretical literature in this area has largely developed along two strands. One class of models study the effect of wealth distribution when capital markets are imperfect. Aghion & Bolton([1]), Banerjee & Newmann([4]), and Galor & Zeirra([9]) are some of the papers with this flavour. The basic idea behind these models is that when credit markets are imperfect the distribution of wealth will determine the proportion of population that is able to engage in productive economic activity. The activity could be a productive investment project or investment in human capital etc. In these models a more equitable distribution of wealth may have positive effects on growth. An equitable distribution implies that the poorer sections of the population are able to meet collateral requirements in the credit market and are able to borrow enough funds to engage in productive economic activities. This has positive effect on the growth rate of the economy as a whole.

Another class of models like Alesina & Rodrik([2]), Bertola([7]), and Persson & Tabellini([11]), study the effect of inequality on the political equilibrium. In these models the growth rate of an economy depends on a certain policy variable chosen by the government. The government is seen to provide a productive input interpreted as infrastructure. The policy variable is the tax rate on capital. If the distribution of capital is very skewed the median voter prefers a more redistributive policy through a high tax rate on capital. However, as capital is the only growth inducing instrument in the economy a high tax rate on capital leads to lower growth rates.

The empirical evidence on the relationship between inequality and growth is somewhat mixed. Alesina & Rodrik([2]) carried out a cross-sectional regression to test their hypothesis. They regressed average growth rates of countries between 1965-1990 on a measure of inequality prior to 1965. They found the coefficient on inequality to be insignificant for the overall sample of countries. Inequality had a negative and significant coefficient when the same regression was carried out for a sub-sample consisting of democratic countries. The transmission mechanism implied by Alesina & Rodrik is that higher capital tax leads lower growth rates. Perotti([12]) found that this transmission mechanism did not hold. In recent empirical studies the relationship between inequality and growth seems to be non monotonic. Barro([6]) used a measure of political stability and civil liberties to proxy for political regime and found that inequality retards growth in poorer countries(with per capita GDP less than $2000). Forbes([8]) found a positive relationship between inequality and growth. However, she carried out a panel data estimation to test this relationship and as such her sample consisted primarily of OECD countries.

From a theoretical standpoint there are a couple of features of the models used by Alesina & Rodrik([2]) and Persson & Tabellini([11]) that are unsatisfactory. In order to incorporate the role of fiscal policy in determining the growth rate of the economy they used a variant of the model due to Barro([5]). As such they treat infrastructure as a flow variable which cannot be accumulated as a stock like physical capital. They also assume that the only possible tax instrument at the disposal of the government is a capital tax. These two assumptions make their models an "AK" type growth model. Hence due to any reason if the tax rate on capital is high it results in lower growth rates.

Lack of adequate infrastructure has been a major impediment to the growth process in developing economies. It has been noted by a large number of economists, that lack of such a critical factor prevented many economies from attaining high growth. This sentiment was echoed in the World Bank Development Report([16]) with reference to China. It was estimated that the annual economic cost of inadequate transport in China is at least 1% of its GNP. The India Infrastructure Report ([14]) reiterated a similar concern and recommended the government of India to take steps to raise the level of investment in infrastructure. It was projected that the amount of investment in infrastructure needs to be at least 7% of the GDP to ensure that the economy does not stagnate. Aschauer([3]) and Sanchez-Robles([13]) also find infrastructure to be positive and significant factor behind growth.

In our paper we interpret infrastructure as a factor which can be accumulated over time. Hence the economy we study has two growth inducing instruments: physical capital and infrastructure. A similar approach to modeling infrastructure was also adopted by Glomm & Ravikumar([10]). The government is seen as the sole producer of all infrastructural stocks. This conforms largely to the reality of developing economies such as India. Alternatively, one can also regard infrastructure to be a non-excludable public good. Government taxes factor incomes to finance production of infrastructure and provides infrastructure free of user cost to all firms. We also allow the government to tax both labor and capital incomes. We show that when the government can only use capital tax to finance production of infrastructure it is equivalent to a special kind of redistributive policy. The main advantage of our approach is that we can clearly separate out the growth aspect of fiscal policy from the redistributive aspect.

We study the political equilibrium in an economy which consists of agents who differ in terms of their endowments of productive factors; labor and capital. We find that when the only purpose of fiscal policy is to produce infrastructure there is no trade-off between inequality and growth. All agents are unanimous in their preferred policy choice. It is the introduction of redistributive aspect to fiscal policy that creates the conflict between inequality and growth. Even then the direction of the relationship depends on the nature of the political choice process.

The paper is organized as follows. The next section describes the model. In section 3 we characterize the competitive equilibrium of the economy along the balanced growth path. We study the preferred policy for every agent when the only objective of the fiscal policy is to provide infrastructure. We introduce redistributive fiscal policy in section 4 and study the political equilibrium when the amount of redistribution is institutionally given. Section 5 studies the implications for growth when the amount of redistribution is determined endogenously. Section 6 gives the concluding comments.

2 The Model

The economy produces a final good which can be consumed as well as accumulated as capital. The production of the final good requires three inputs, labor(L), capital(K) and infrastructure(G). The aggregate production function of the economy in period t is given by

$$Y_t = A{K_t}^a(G_t L)^{1-a},$$

where "Y_t" is the output, "L" and "K_t" are the aggregate employments of labor and capital for the production of Y. "A" is a technological shift parameter and "G_t" is the amount of infrastructure in the economy. We assume labor is supplied inelastically and normalize it's aggregate value to 1. Hence the aggregate production function can be written as

$$Y_t = A{K_t}^a{G_t}^{1-a} \tag{1}$$

There are competitive markets for labor and capital. The wage rate and rental rate are

$$w_t = (1-a)Y_t, \tag{2}$$

and

$$r_t = a\frac{Y_t}{K_t}, \tag{3}$$

respectively.

The economy consists of a large number(N) of infinitely lived agents indexed by i. Household agents, or simply agents, in the economy are identified by their endowments of capital and labor. The ith agent is initially endowed with k_0^i units of capital and l^i units of labor. The agents can accumulate capital over time however their endowment of labor is constant[1]. The agents behave competitively in all markets. We assume that the agents are able to forecast the sequence of wages (w_t) and rental rates (r_t) perfectly.

There is no market for infrastructure so the government has to provide this input[2]. To begin with we are going to assume that the sole purpose of fiscal policy is to provide infrastructure. In period t, the government taxes a part of income of the agents and transforms it into infrastructure in the following period $t+1$. The production of infrastructure is given by

$$G_{t+1} \;= \tau_t[r_t K_t + w_t 1] = \tau_t Y_t, \tag{4}$$

where τ_t is the income tax rate. After production the government provides infrastructure free of user cost to all firms (as in Barro[5]). We assume that all agents are taxed at the same rate. This is a very simple tax scheme which

[1] One can alternatively think of l^i as the innate skill level of the ith agent.

[2] If infrastructure is non-excludable then a private market for this input will not exist. Alternatively, we can think of infrastructure as a non-rival input in the production process which makes public provison of infrastructure more desirable.

has been used in Barro([5]), and Glomm and Ravikumar([10])[3]. Equation (4) also implies that as an economy accumulates more infrastructure production of new infrastructure becomes easier.

In order to understand the political equilibrium we are going to analyze the model in two steps. In our model the fiscal policy is simply the sequence of tax rates $\{\tau_t\}_{t=0}^{\infty}$. For any given fiscal policy we are going to solve for the competitive equilibrium and the path of consumption of each agent. Then, we are going to study which fiscal policy is optimal for an agent given his/her endowment of labor and capital. If agents vote on the choice of the fiscal policy or to elect a political party to form the government they would like the fiscal policy to maximize their welfare.

Each agent maximizes lifetime utility given a sequence of wage rates(w_t) and rental rates(r_t). The ith agent maximizes lifetime utility given by

$$U^i = \sum_{t=0}^{\infty} \beta^t \log c_t^i \ ,$$

subject to period budget constraints

$$c_t^i + k_{t+1}^i \leq (1 - \tau_t)[r_t k_t^i + w_t l^i], \quad \forall t = 0, 1, ...\infty, \tag{5}$$

taking the factor prices (w_t, r_t) as given. $\beta \in (0, 1)$ is the discount factor which is the same for all agents[4].

[3]This is a convenient benchmark tax scheme to serve as starting point in our analysis. Later on we are going to introduce redistributive motive in the government policy.

[4]We assume that the discount factor is same for all agents as we only want to focus on inequality in terms of factor endowments. That is why we assume that agents are identical in all other aspects including their preferences.

3 Balanced Growth

We will restrict our attention to balanced growth paths only. Balanced growth paths have the property that along this path all variables in the economy grow at constant rates. Proposition 1 summarizes the behavior of key variables of the model along the balanced growth path.

Proposition 1 *Along the balanced growth path the income tax rate(τ_t) is constant. An agent's capital stock(k^i), the aggregate capital stock of the economy(K), the stock of infrastructure(G) and the output(Y) grow at the same rate as the agent's rate of growth of consumption(c^i), i.e.,*

$$\alpha_{c^i} = \alpha_{k^i} = \alpha_K = \alpha_G = \alpha_Y,$$

where α's denote the growth rate of these variables.

Proof: See the appendix. ■

Since the income tax rate is bounded between zero and one, along the balanced growth path it will be constant. From now on we are going to drop the time index on the income tax rate. The result derived in proposition 1 is driven by the fact that all agents are competitive in the factor markets. As a result all of them face the same wage and rental rates. As all agents have the same discount factor they all want their consumption to grow at the rate. The growth rate of consumption equals after tax return on capital times the discount factor i.e., $\alpha_{c^i} = \beta(1-\tau)r$. For this to be feasible they must accumulate capital at the same rate as they want their consumption to grow. Rest of the results follow from the properties of the balanced growth paths and the fact that the production function is constant returns to scale.

As all agents accumulate capital at the same rate and the labor endowment is constant every period the relative share of ith agent's income in total income is determined by the initial distribution of capital and labor. Let s^i denote the

ith agent's share in the aggregate capital stock i.e., $s^i \equiv k_0^i/K_0$. The following lemma characterizes consumption of ith agent along the balanced growth path.

Lemma 1 *Along the balanced growth path the consumption of the ith agent is given by*

$$c_t^i = x^i(1-\tau)Y_t,$$

where $x^i = [(1-\beta)as^i + (1-a)l^i]$.

Proof: See the appendix. ■

Notice, given a tax rate τ, $(1-\tau)Y_t$ denotes the after tax output available for consumption every period. Every agent saves a certain proportion of their capital income. x^i is an average of the ith agent's share of capital and share of labor. x^i also represents the ith agent's share in aggregate consumption every period. This share depends not only on the relative endowments of labor and capital but also on the discount factor and the productivity of labor and capital in the production function. In our model the distribution of the parameter x^i gives us a measure of inequality. In a representative agent economy every agent will have the same endowment of labor and capital. As there are N agents in the economy every agent will have $l = 1/N$ units of labor and $lK_0(= K_0/N)$ units of capital. So $x^i = (1-\beta)l$ for every agent in a representative agent economy.

Proposition 2 *The welfare maximizing income tax rate in a representative agent economy is given by $\eta = \beta(1-a)$. The preferred income tax rate in our economy is same for all agents which is also equal to the tax rate η.*

Proof: See the appendix. ■

The proposition above is somewhat surprising. It tells us that if there is voting on the income tax rate to be chosen by the government then all agents irrespective of their factor endowments will prefer the same income tax rate. In addition this income tax rate is the same as the welfare maximizing tax rate in a representative agent economy. This suggests that there is no relationship between inequality and growth. It will turn out in the subsequent sections of this paper that this result follows from the very special assumption that the only purpose of the fiscal policy is to produce infrastructure which is also a growth inducing instrument.

For the time being let us focus on the intuition behind the above result. Notice that when the government is trying to maximize the welfare of any agent it is essentially trying to achieve the highest possible growth rate of output. This comes at the cost of current consumption as a certain proportion of output is taxed to finance infrastructure. This suggests a general result that in the absence of redistributive policy, if one growth inducing instrument i.e., savings is taxed to finance another growth inducing instrument like infrastructure there is no trade-off between inequality and growth.

4 Redistributive Politics

Now we introduce a redistributive aspect to fiscal policy. Suppose the government cannot use the entire tax revenue to produce infrastructure. Let ϕ denote the amount of tax revenue which the government uses for redistributing income. Production of infrastructure is given by

$$G_{t+1} = (\tau - \phi) Y_t. \tag{6}$$

where $\phi \leq \tau$. Suppose ϕ is institutionally given and the government redistributes uniformly i.e., each agent receives $\phi l Y_t$ $(l = 1/N)$ amount of transfers from the government. Alesina & Rodrik ([2]) assumed that the government could only tax capital. It is equivalent to an institutionally given amount of redistribution. In Alesina & Rodrik([2]) all agents had the same labor endowment i.e., $l^i = l$, $\forall i$. If $\phi = \tau(1 - a)l$, it will be equivalent to having a tax on capital income only. The following lemma characterizes the consumption of each agent when they are receiving transfers from the government.

Lemma 2 *Along the balanced growth path the consumption of the ith agent is given by*

$$c_t^i = [x^i(1-\tau) + \phi l]Y_t.$$

Proof: See the appendix. ■

Due to the redistributive policy poorer agents(whose x^i is low) are the net gainers from the government transfers. As a result the preferred tax rate for each agent will now depend on their endowment parameter x^i. Preferences of the agents are single peaked in the policy variable. So we will study the median voter's preferences to capture the policy outcome under majority voting[5]. As in Alesina & Rodrik([2]), the notion of inequality in our model is the endowment parameter of the median voter. In a representative agent economy $x^i = (1-\beta)l$ for every agent. Inequality means that the median voter's share in consumption is below the average agent's share. Higher inequality implies x^m will be smaller. So the difference between the endowment parameter of the median voter(x^m) and $(1-\beta)l$ gives us a measure of inequality in the economy. The next proposition tells us the relationship between the preferred policy of the agents and their endowments.

Proposition 3 *The welfare maximizing income tax rate for the ith agent depends on the agent's endowment parameter. The preferred tax rate for the agent is given by*

$$\tau^i = \eta + \phi[(1-\eta) + \eta\frac{l}{x^i}].$$

[5] When the preferences are single peaked in the policy variable the median voter's preferred policy will be chosen under majority voting. We will also get the same result if there are two political parties and they have to choose a tax rate as their policy platform.

Proof: See the appendix. ■

As we said before the poorer agents gain from the redistributive policy. When the amount of redistribution is institutionally given the only way for the poorer agents to get more government transfers is to vote for a higher income tax rate. The median voter will vote for a higher tax rate if x^m is low. Hence, higher inequality will lead to a higher tax rate and with ϕ given will also lead to higher growth rate. We should point out that it is not clear that growth will be higher in comparison to the case we studied in the previous section when the only purpose of the fiscal policy was to produce infrastructure. It will depend on the degree of inequality and the amount of tax revenue that is used to produce infrastructure.

5 Voting on Redistribution

In this section we allow for the amount of redistribution to be determined endogenously. We can think of the economy voting simultaneously on the income tax rate and the amount of tax revenue to be used for redistribution. So the policy variable is now a pair (τ, ϕ). When there is voting on the policy pair (τ, ϕ) the agents in this economy get polarized into two groups based on their endowment parameter x^i. Poorer agents with $x^i < l$ are net gainers from the government transfers. Hence every agent with $x^i < l$ will prefer to have income tax rate as high as possible and the keep the transfers low enough so that economy invests in infrastructure. Agents with $x^i > l$ tend to lose when the government redistributes income. So they prefer transfers(ϕ) to be zero and the income tax rate just enough to ensure the desired growth rate through investment in infrastructure. The following proposition characterizes preferences of the agents based on their endowments.

Proposition 4 *Let $\overline{\tau}$ denote the maximum feasible income tax rate. The pre-*

ferred policy pair (ϕ, τ) for the ith agent is given by

$$\begin{aligned}\phi^i &= 0,\ \ \tau^i = \eta \text{ if } x^i > l, \\ \phi^i &= (1-\eta)\overline{\tau} - \eta(1-\overline{\tau})\frac{x^i}{l},\ \tau^i = \overline{\tau} \text{ if } x^i < l.\end{aligned}$$

Proof: See the appendix. ∎

Now we study the policy outcome in the economy. Note when voting is on a policy pair the median voter theorem does not apply in all cases. However, in our model because of the particular preference structure of the agents the median voter will turn out to be decisive in determining the policy variables. The next proposition tells us the relationship between the inequality and growth when there is voting on the amount of redistribution.

Proposition 5 *The median voter is decisive when there is majority voting on the policy pair (τ, ϕ). Growth rate is non-decreasing in the median voter's endowment parameter x^m.*

Proof: See the appendix. ∎

When the agents vote on the amount of redistribution as well as the income tax rate the relationship between inequality and growth reverses. Higher inequality means that the median voter is poor in terms of the endowment parameter x^m. The median voter thus votes for a higher amount of redistribution. Given the upper bound on income tax rate $\overline{\tau}$ it results in a lower growth rate.

6 Conclusion

In this paper we demonstrate that the inequality growth trade-off depends on the nature of the political choice process. There is a large empirical literature now on the relationship between inequality and growth. The results in this literature are quite conflicting. The results differ depending on the estimation technique, the variables incorporated in the regressions and the inequality data used. Recent papers in this area by Barro([2]) and Forbes([8]) seem to suggest that inequality has negative effect on growth in poorer countries while for richer countries the relationship becomes positive. In the context of our model we can provide an explanation for this. In a underdeveloped country like India, where a large fraction of the population is living below poverty there is a high demand for government to follow redistributive policies(like government provided free lunches). This forces the government or political parties to opt for greater amount of redistribution leading to less investment in infrastructure.

From a theoretical standpoint we have shown that if the only goal of the fiscal policy is to provide a growth inducing instrument such as infrastructure there is no conflict between inequality and growth. It is the introduction of redistributive aspect to fiscal policy that creates a trade-off between inequality and growth. In this context we separate out the growth inducing and redistributive effects of fiscal policy. In Alesina & Rodrik([2]), and Persson & Tabellini([11]) the fiscal policy has a growth and redistributive effect simultaneously. So it is not clear what creates the inequality growth trade-off. Furthermore, the direction of the trade-off depends on whether the amount of redistribution is institutionally given or is voted on. In our model it is the consumption inequality that is important for political equilibrium. This inequality depends not only on the distribution of factor endowments(i.e., distribution of labor and capital) but also on the productivity of these factors in production. If the share of labor in output is high then it is the distribution of labor that is more important in determining consumption inequality.

We have used the median voter theorem to study the political equilibrium in the economy. The reason for this is to be able to compare our results with other papers in this literature. In future work it will be interesting to study more realistic voting environments. It has been seen that in societies with high inequality of income distribution, the institutions tend to be underdeveloped in terms of both their efficiency as well as accountability. The politics of such countries also come under severe pressure from the rich capitalist lobbies, to adopt policies to suit their vested interests. Also, the poorer section of the population typically tend to be unorganized and uninformed, compared to their capital rich counterparts. This would imply in turn that the model of democracy in such countries do not fall within the purview of the median voter theorem.

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Appendix

Proof of Proposition 1:

The income tax rate is bounded between 0 and 1. So along the balanced growth path τ_t has to be a constant. The Lagrangian for the ith agent's maximization problem is

$$\pounds^i = \sum_{t=0}^{\infty} \{\beta^t \log c_t^i + \lambda_t[(1-\tau_t)(r_t k_t^i + w_t l^i) - c_t^i - k_{t+1}^i]\}.$$

The first order conditions for maximum are given by

$$\frac{\partial \pounds^i}{\partial c_t^i} = \frac{\beta^t}{c_t^i} - \lambda_t = 0, \tag{7}$$

$$\frac{\partial \pounds^i}{\partial k_{t+1}^i} = -\lambda_t + (1-\tau_t) r_{t+1} \lambda_{t+1} = 0, \tag{8}$$

and a transversality condition

$$\lim_{t \to \infty} \lambda_t k_t^i = 0. \tag{9}$$

Substituting equation (7) in (8) we get

$$\frac{c_{t+1}^i}{c_t^i} = \beta(1-\tau) r_{t+1}. \tag{10}$$

Let $\alpha_{c^i} = \frac{c_{t+1}^i}{c_t^i}$, denote the growth rate of consumption of the ith agent. Along the balanced growth path α_{c^i} is constant. From (5) and (9) it follows that the ith agent must accumulate capital at the same rate as growth in consumption i.e., $\alpha_{c^i} = \alpha_{k^i}$. Since α_{c^i} is constant along the balanced growth path equation (10) implies that the rental rate for capital is also constant. Let r denote that rental rate. As all agents face the same rental rate for capital all agents accumulate capital at the rate given by $\beta(1-\tau)r$. Hence, the growth rate of aggregate capital stock must equal $\beta(1-\tau)r$. The production function (1) is constant returns to scale with respect to capital and infrastructure. Constant rental rate for capital along the balanced growth path implies that the capital-infrastructure ratio must be constant as well. Hence, along the balanced growth path the growth rate of infrastructure must be equal to the growth rate of capital. It is easy to show from (1) that growth rate of output(α_Y) is also equal to the growth rate of capital along the balanced growth path. ■

Proof of Lemma 1:

Consider the budget constraint (5). Given proposition 1 we know that along the balanced growth path rental rate for capital and the income tax rate is constant. Hence the budget constraint can be written as

$$\frac{c_t^i}{k_t^i} + \alpha_{k^i} = (1-\tau)[r + w_t \frac{l^i}{k_t^i}]. \tag{11a}$$

Since $\alpha_{k^i} = \beta(1-\tau)r$ we have

$$c_t^i = (1-\tau)[(1-\beta)rk_t^i + w_t l^i].$$

Using (2) and (3) to substitute for wage rates and rental rates, consumption along the balanced growth path is

$$c_t^i = (1-\tau)[(1-\beta)as^i + (1-a)l^i]Y_t.$$

■

Proof of Proposition 2:

In order to prove this proposition we will calculate the income tax rate that will maximize the utility of the ith agent. If the government wants to maximize the utility of the ith agent the government will solve the following problem:

$$\max \sum_{t=0}^{\infty} \beta^t \log x^i (1-\tau) Y_t,$$

subject to the equation governing the accumulation of infrastructure (4). The Lagrangian for this problem is

$$\pounds^i = \sum_{t=0}^{\infty} \{\beta^t \log x^i (1-\tau) Y_t + \mu_t [\tau Y_t - G_{t+1}]\}.$$

The first order conditions for maximum are given by

$$\frac{\partial \pounds^i}{\partial \tau} = -\frac{\beta^t}{(1-\tau)} + \mu_t Y_t = 0, \tag{12}$$

and

$$\frac{\partial \pounds^i}{\partial G_{t+1}} = \frac{\beta^{t+1}(1-a)}{G_{t+1}} + \mu_{t+1} \frac{\tau(1-a)Y_{t+1}}{G_{t+1}} - \mu_t = 0. \tag{13}$$

From (12) we get $\mu_t = \frac{\beta^t}{(1-\tau)Y_t}$. Substituting in (13) we have

$$\beta(1-a) + \beta(1-a)\frac{\tau}{(1-\tau)} = \frac{G_{t+1}}{(1-\tau)Y_t}. \tag{14}$$

Using (4) to substitute for G_{t+1} in equation (14) we get

$$\eta + \eta\frac{\tau}{(1-\tau)} = \frac{\tau}{(1-\tau)}, \tag{15}$$

where $\eta = \beta(1-a)$. Hence the optimal income tax rate that solves (15) is equal to η. Notice the optimal income tax rate is independent of the ith agent's endowment parameter. Hence, this tax rate will maximize the utility of any agent irrespective of his/her relative factor endowments. In particular η will also be the optimal tax rate for the representative agent economy. ■

Proof of Lemma 2:

Now each agent receives a transfer of $\phi l Y_t$ every period. Along the balanced growth path we can write the budget constraint (5) as

$$\frac{c_t^i}{k_t^i} + \alpha_{k^i} = (1-\tau)[r + w_t\frac{l^i}{k_t^i}] + \frac{\phi l Y_t}{k_t^i}. \tag{16a}$$

Using similar manipulation as in the proof of lemma 1 we have

$$c_t^i = [x^i(1-\tau) + \phi l]Y_t.$$

■

Proof of Proposition 3:

From lemma 2 we know that the tax which maximizes the utility of the ith agent will be the solution to the following problem:

$$\max \sum_{t=0}^{\infty} \beta^t \log[x^i(1-\tau) + \phi l]Y_t,$$

subject to the new equation governing the accumulation of infrastructure (6). The Lagrangian for this problem is

$$\pounds^i = \sum_{t=0}^{\infty}\{\beta^t \log[x^i(1-\tau) + \phi l]Y_t + \mu_t[(\tau-\phi)Y_t - G_{t+1}]\}.$$

The first order conditions for maximum are given by

$$\frac{\partial \pounds^i}{\partial \tau} = -\frac{\beta^t x^i}{[x^i(1-\tau)+\phi l]} + \mu_t Y_t = 0, \tag{17}$$

and

$$\frac{\partial \pounds^i}{\partial G_{t+1}} = \frac{\beta^{t+1}(1-a)}{G_{t+1}} + \mu_{t+1}\frac{(\tau-\phi)(1-a)Y_{t+1}}{G_{t+1}} - \mu_t = 0. \tag{18}$$

Let $\eta = \beta(1-a)$ and substituting (17) in (18) we get

$$\eta + \eta\frac{(\tau-\phi)x^i}{[x^i(1-\tau)+\phi l]} = \frac{x^i G_{t+1}}{(1-\tau)Y_t}. \tag{19}$$

Using (4) in (19) we have

$$\eta + \eta\frac{(\tau-\phi)x^i}{[x^i(1-\tau)+\phi l]} = \frac{(\tau-\phi)x^i}{(1-\tau)}. \tag{20}$$

Hence the optimal income tax rate for the ith agent is

$$\tau^i = \eta + \phi[(1-\eta) + \eta\frac{l}{x^i}].$$

Notice the optimal income tax rate increasing in x^i. ■

Proof of Proposition 4:

The maximization problem here is similar to the one described in the proof of proposition 3. However, now both the income tax rate and the amount of redistribution are choice variables. The Lagrangian for the ith agent maximization problem is

$$\pounds^i = \sum_{t=0}^{\infty}\{\beta^t \log[x^i(1-\tau)+\phi l]Y_t + \mu_t[(\tau-\phi)Y_t - G_{t+1}]\}. \tag{21}$$

However now we have to be careful about the first order conditions. The agents in this economy can be divided into two groups based on their endowment parameter x^i. If $x^i < l$ the agent is a net gainer from the government transfers. Hence every agent with $x^i < l$ will prefer to have taxes as high as possible and the amount of transfers just enough so that economy invests in infrastructure. Agents with $x^i > l$ are net losers in terms of government transfers so they prefer transfers(ϕ) as low as possible and the tax rate just enough to ensure the desired growth rate through infrastructure. The first order conditions for maximum are given by

$$\frac{\partial \pounds^i}{\partial \tau} = -\frac{\beta^t x^i}{[x^i(1-\tau)+\phi l]} + \mu_t Y_t = 0, \text{ if } x^i > l, \tag{22}$$

$$\frac{\partial \pounds^i}{\partial \phi} = -\frac{\beta^t l}{[x^i(1-\tau)+\phi l]} + \mu_t Y_t = 0, \text{ if } x^i < l, \tag{23}$$

and

$$\frac{\partial \pounds^i}{\partial G_{t+1}} = \frac{\beta^{t+1}(1-a)}{G_{t+1}} + \mu_{t+1}\frac{(\tau-\phi)(1-a)Y_{t+1}}{G_{t+1}} - \mu_t = 0. \tag{24}$$

The preferred policy of the agents with $x^i > l$ is easy to characterize. They policy pair that will maximize there welfare is $\phi = 0$ and $\tau = \eta$. For agents with $x^i < l$ substitute (23) in (24). We have

$$\eta + \eta\frac{(\tau-\phi)l}{[x^i(1-\tau)+\phi l]} = \frac{lG_{t+1}}{(1-\tau)Y_t}. \tag{25}$$

Using (6) in (25) we have

$$\eta + \eta\frac{(\tau-\phi)l}{[x^i(1-\tau)+\phi l]} = \frac{(\tau-\phi)l}{(1-\tau)}. \tag{26}$$

If the maximum possible income tax rate is $\overline{\tau}$, from (26) it follows that the optimal policy pair for the agent is $\tau = \overline{\tau}$ and

$$\phi^i = (1-\eta)\overline{\tau} - \eta(1-\overline{\tau})\frac{x^i}{l}. \tag{27}$$

■

Proof of Proposition 5:

Suppose we order the agents according to their endowment parameter. Take two agents i and j and let $x^i > x^j$. Consider two policy pairs (τ_1, ϕ_1) and

(τ_2, ϕ_2) such that both agents prefer the first policy to the second one. For the median voter theorem to hold we need to show any agent s with endowment $x^s \in (x^i, x^j)$ will also prefer the first policy to the second one. If we look at the Lagrangian for the ith agent in equation (21) we can see that utility of the agent is monotonic in x^i. Hence the median voter theorem will hold. Let x^m denote the median voter's consumption share in the aggregate output. From proposition 4 it follows that if $x^m > l$ the median voter will always want the growth maximizing policy pair. However, if $x^m < l$ then the median voter will prefer the maximum possible income tax rate $\overline{\tau}$ and transfers given by (27). It is easy to check that the amount of transfers preferred by the median voter ϕ^m is decreasing in x^m. Hence growth rate is non-decreasing in the median voter's endowment parameter. ■