Intellectual property rights protection, endogenous growth and unemployment in a north-south model

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Abstract

In this paper we have developed a dynamic general equilibrium model of the international product cycle to study the effects of stronger Intellectual Property Rights (IPR) protection in the South on the rate of growth and the rate of unemployment of the unskilled labour in the South. By introducing efficiency wage consideration into standard Grossman-Helpman(1991b) type North-South framework we have shown that stronger IPR in the South would lead to lower unskilled unemployment in the South when the wage gap between North and South is very high (wide-gap case). The effect on unemployment will be just the opposite if this wage gap is not too high (narrow-gap case). We also have studied the effects of technological improvements in the North and an increase in the supply of the skilled labour of both the region on the rate of unemployment in the South. We find that a unionised economy may grow at a different rate than a non-unionised economy when the unskilled labour market in the South is unionised .

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1 introduction

The purpose of this paper is to investigate the ralationship between growth, unemployment and IPR protection in the South in a North-South product cycle model framework (as developed by Grossman-Helpman(1991b)) where endogenous growth is driven by the introduction of the new differentiated products in the North and unemployment is driven by the existence of efficiency wages in the unskilled labour market in both the North and the South.

There are two branches of the literature to which our paper relates. The first one combines unemployment and growth, as for example, Bean and Pissarides (1993) and Aghion and Howitt(1994). Based on a matching framework these authors finds an ambiguous sign for the trade-off between long-run economic growth and the search unemployment. Palokangas(1996), using a wage-bargaining model with two types of labour, finds that there exist a positive relation between long-run economic growth and unemployment of unskilled labour. Within this branch of the literature a set of authors model growth and unemployment based on the efficiency wages. This line of research has been started by De Groot (1998),Van Schauk and De Groot (1998) and Stadler(1999). In these models, efficiency wage unemployment occurs because effort in one sector of the economy is positively related to the relative wage paid by that sector. Our paper relates most closely to this second line of research.

The second branch of the literature addresses spefically the impact of IPR protection on long run economic growth and welfare in a North-South product cycle framework. Grossman-Helpman(1991b) and Helpman(1993) finds that the protection of IPR in the South would lead the world rate of growth to fall. Lai(1998) finds that relation between the IPR protection and the long-run economic growth crucially depends on the channel of production transfer. If multinationalisation is the channel then IPR protection in the South would lead to an increase the rate of growth and it will fall if imitation is the channel of production transfer. Two recent contributions on this issue are of by Glass and Saggi(2002) and Kaith and Maskus(2001). However they have used a quality ladder framework to explore the relation between growth and IPR protection.

This paper departs from these two approaches in the following sense. We have used a North-South product cycle model to explore the relation between growth, unemployment and IPR protection. In the first approach growth and unemployment was modelled mostly in closed economy framework(one recent exception is Jurgen Meckl(2004)).In the second approach all the existing models are of full employment in nature. Arnold(2002) is the only exception in this class. He adopts the framework of Helpman(1993) but introduces labour market rigidities in the North. He has shown that the growth effect of intensified trade (in other sense, less protection of the IPR) with South crucially depends on the North's labour market flexibility. If it is highly flexible then Helpman's result is obtained ensuring positive relation between growth and trade intensification. But this relation may go other way round if the labour market in the North is sufficiently inflexible. While Arnold focus on the North and his unemployment is caused by the labour market rigidities we have focus both on the North and on the South and our unemployment caused by the efficiency wages. Another very closely related paper to our's is that of Lai(1995). However he focus on the relative wage between the North and the South by introducing two types of labour (skilled and unskilled in both the North and the South) in the basic Grossman-Helpman(1991b)(hereafter GH) model, we here focus on the unemployment of unskilled workers due to IPR protection.

In this paper we have extended the GH model by introducing unemployment. This framework allows us to find the interaction between growth, unemployment and IPR protection in a unified framework. We have shown that the movement of growth and unemployment due to IPR protection may not be unidirectional depends on the North-South initial wage gap. In both the wide gap and the narrow gap case, stronger IPR protection would decrease the rate of growth and the level of unemployment in the North. But in the South the level of unemployment will increase in narrow gap case while it will decrease in the wide gap case. In the wide gap case, aggregate employment in the research sector of the South would fall and hence more skilled labour will be employed in the production sector of the south. Since the relative employment of the skilled labour in the production sector of the South is fixed by the constancy of the relative wages demand for the unskilled labour in the South would grow up reducing unemployment. In the narrow gap case, due to stronger IPR protection, aggregate employment in the research sector of the South would increase and hence less skilled labour is left for the production sector of the south. Constancy of the relative wage in the South would then imply a lower demand for the unskilled labour there increasing the level of unemployment. However in the North aggregate employment in the research sector will fall and hence unemployment of the unskilled labour will fall there. Therefore the effect of IPR protection in the South on the rate of growth and the level of unemployment of the unskilled workers in both the regions crucially depends on the factor cost gap between this to two region. While the positive (or, negative) relation between growth and unemployment and between growth and IPR is not something new in the literature, we contribute to the existing literature by showing the effect of IPR protection on the rate of unemployment and growth in a unified model using the GH framework.

We proceed as follows. Section 2 presents the benchmark product cycle model. Sections 3 presents the reduced form equilibrium conditions and investigates the various comparative static results. Sections 4 extends the basic model of section 2 to incorporate trade union in the unskilled labour market of the South. Section 5 concludes.

2 The Benchmark Model

2.1 General features of the product cycle model

There are two countries in the world, the North and the South that are linked by free trade in differentiated products invented in the North. A Northern firm incurs an upfront innovation cost to invent a product and then earns a stream of monopoly profits from the production of that product until it gets imitated by a potential Southern firm. Patents are perfectly protected in the North and imperfectly protected in the South. Because of lower factor costs a successful imitator from the South earns an infinite stream of positive profits which they balance against the positive imitation costs. This structure of the product cycle model is adapted from GH. However unlike GH we have introduced two types of labour - skilled and unskilled, into the South labour market and assumed that unskilled labour's efficiency depends on the relative wage of unskilled to skill. Thus the endowment of the unskilled labour is endogenous in our model. The introduction of this efficiency function results unemployment in the unskilled labour market in the South.

2.2 The demand for goods

We consider there is a world representative household who maximise the intertemporal utility

$$W = \int_t^\infty e^{-\theta(\tau-t)} \frac{U(\tau)^{(1-\sigma)} - 1}{1-\sigma} d\tau \tag{1}$$

subject to the intertemporal budget constraint:

$$\int_t^\infty e^{-r(\tau-t)} E(\tau) d\tau = \int_t^\infty e^{-r(\tau-t)} I(\tau) d\tau + A(\tau) \forall t \tag{2}$$

where $0 \leq \sigma \leq 1$ and σ is intertemporal elasticity of substitution; θ is the time rate of preference; r is the nominal interest rate; E(τ) is instantaneous expenditure at τ; I(τ) is instataneous income at τ; $A(t)$ is the current value of assets at t. At each date τ, the agent takes A(τ), I(τ), r and prices of goods as given.

The instaneous utility is assumed to have the following form:

$$U(t) = (\int_0^{n(t)} x(z)^\alpha dz)^{\frac{1}{\alpha}} \tag{3}$$

where $0 \leq \alpha \leq 1$; x(z)= quantity of good consumed and n=n(t) is the measure of product invented before time t. We shall refer n as the number of available variety.

It is assumed that the households in the South consists of a large number of household members, so that the proportions of unemployed household members is equal to the aggregate rate of unemployment. As a consequence, households do not face any income uncertainty when making their intertemporal consumption plan. We thus neglect the distributional aspects of unemployment in the South.

The agent can solve this dynamic optimisation problem in two stages. First it can choose the composition of given level of spending to maximise the instantaneous utility (static part). Then it can optimise seperately the time path of spending (dynamic part).The equation for instaneous budget constraint takes the following form:

$$E(t) = \int_0^{n(t)} p(z)x(z)dz \tag{4}$$

In stage 1, the agent's static optimisation exercise is

$$Max \quad U(t) = (\int_0^{n(t)} x(z)^\alpha dz)^{\frac{1}{\alpha}} \tag{5}$$

subject to

$$E(t) = \int_0^{n(t)} p(z)x(z)dz \tag{6}$$

This exercise generates the following demand function for each variety

$$x(z) = E(t)\frac{p(z)^{-\varepsilon}}{\int_0^{n(t)} p(u)^{1-\varepsilon}} \qquad \forall z \in (0, n) \tag{7}$$

This demand function features a constant price elasticity of $\varepsilon = \frac{1}{1-\alpha}$ $(\varepsilon \succ 1)$, and a unitary expenditure elasticity of each product.

In stage 2, we solve the dynamic part of the agent's optimisation exercise. Before we start solving the dynamic part we assume that in North (as well as in South) all brand is produced using the same constant return to scale (CRS) technology. Noting the symmetry of the demand function (7) all produced variety in each country will bear the same price and x(z) is the same for all goods produced in the same country. We denote x_n as the demand for a Northern product not yet imitated and x_s as the demand for a imitated product produced by a Southern firm. Similarly we denote p_n as the price for the Northern based product and p_s as the price for the imitated product. Then substituting (7) into (3) we get [1]

$$U = (n_n {x_n}^\alpha + n_s {x_s}^\alpha)^{\frac{1}{\alpha}} = \frac{E}{P} \tag{8}$$

Where $P^{(1-\varepsilon)} = \int_0^n p(u)^{(1-\varepsilon)}du = n_n p_n^{(1-\varepsilon)} + n_s p_s^{(1-\varepsilon)}$.

[1] the time agrument of the variables are supressed henceforth when doing so does not make any confusion

We assumed on the balanced growth path $\frac{\dot{n_n}}{n_n} = \frac{\dot{n_s}}{n_s} = \frac{\dot{n}}{n} = g$. Also from the R &D sector production function we see the marginal productivity of labour is growing at a rate g. Then normalising the value of a firm as unity we get wage rate in both North and South are growing at a rate g. Therefore prices are all growing at a rate g. Under this conditions we can show that the price index P is growing at a rate $g\frac{2-\varepsilon}{1-\varepsilon}$

Now we write the current value Hamiltonian to the dynamic optimisation problem equation (1) and (2) as:

$$H = \frac{U^{1-\sigma} - 1}{1-\sigma} + m[I(t) - E(t) + rA(t)] \tag{9}$$

where m is the current value Lagrangian multiplier. The First order condition is therefore:

$$H_E = U^{-\sigma}\frac{\delta U}{\delta E} - m = 0 \tag{10}$$

From equation (8) $\frac{\delta U}{\delta E} = \frac{1}{P}$. Therefore

$$m = \frac{U^{-\sigma}}{P} \Rightarrow \frac{\dot{m}}{m} = -\sigma\frac{\dot{U}}{U} - \frac{\dot{P}}{P} \Rightarrow -\sigma\frac{\dot{U}}{U} + (1+\sigma)\frac{\dot{P}}{P} \tag{11}$$

Another first order condition is:

$$\dot{m} = \theta m - H_A = \theta m - rm = (\theta - r)m \Rightarrow \frac{\dot{m}}{m} = \theta - r \tag{12}$$

Equation (11) and (12) together implies that

$$\theta - r = -\sigma g - g\frac{2-\varepsilon}{1-\varepsilon}(1+\sigma) \tag{13}$$

In (13) we have used the normalising condition $\frac{\dot{E}}{E} = \frac{\dot{n}}{n} = g$ and the fact that $\frac{\dot{P}}{P} = g\frac{2-\varepsilon}{1-\varepsilon}$

Then (13) imply

$$r = \theta + [1 - (1-\sigma)\frac{1-\alpha}{\alpha}]g \Rightarrow r = \theta + \phi g \tag{14}$$

where $\phi = [1 - (1-\sigma)\frac{1-\alpha}{\alpha}] \leq 1$.
We assume that $\alpha \geq (1-\sigma)$, so that $0 \leq \phi \leq 1$. This will ensure the stability of the general equilibrium.

2.3 The Steady State

We have $n = n_n + n_s$ where n_n is the number of goods continue to be produced in the North and n_s is the number of imitated goods being produced in the

South. We are assuming steady state in our model and set that on the balanced growth path,

$$\frac{\dot{E}}{E} = \frac{\dot{n}}{n} = \frac{\dot{n_n}}{n_n} = \frac{\dot{n_s}}{n_s} = g \tag{15}$$

We will use the notation x_n and x_s to denote the demand for any good [note symmetry in the demand function (7)] produced by the Northern firm and by the Southern imitator respectively. From (15) we see $\frac{n_n}{n}$ and $\frac{n_s}{n}$ are constant over time. In the next section we will prove that in the steady state instantaneous profit level of all the Northern and the Southern imitator firm [denoted by π_n and π_s respectively] are constant over time.

2.4 The North

We model North in this benchmark case exactly in the same way as in GH. North consists of two sectors - a competetive R &D sector and a monopolistically competetive production sectors. Labour is the only input to both these activities. New goods are invented via R &D and the innovator becomes monopolists in the market for the goods they invent[2]. The production function of the new blueprint takes the following form

$$\dot{n} = \frac{n}{a_n} L^R{}_n.^3 \tag{16}$$

where $L^R{}_n$ is the amount of labour deboted to the R &D sector and a_n is the inverse of the labour productivity parameter in the North. Since R &D sector is competetive labours get the value of marginal product as the wage rate which we denote by w_n and this will grow at the same rate with n. This follows from our normalising condition where we set the value of firm as unity. Assuming that one unit of labour can produce one unit of product of any brand in the North and L_n as the total labour supply there, we have the Labour market clearing conditions in the North as

$$L_n = a_n g + n_n x_n \tag{17}$$

The profit maximisation exercise by the Northern firm results in the monopoly price $p_n = \frac{w_n}{\alpha}$ for all products being produced in the North. With this price the instantaneous profit of a Northern firm is

$$\pi_n = \frac{1-\alpha}{\alpha} w_n x_n \tag{18}$$

[2]For simplicity, though not necessary, we assume that each products are developed by different firms.

[3]Notice the presence of the scale effects and thus this type of production function in the R &D sector is under John's (1995a) critique. For models with R &D technologies without scale effects, see Johns(1995a), Segerstrom(1998), and Arnold(1998).

The Northern 'no arbitrage condition' (or zero profit condition) requires that in equilibrium, the instantaneous profit rate, which is the instantaneous profit divided by the initial investment in developing new blueprient, must be equal to the interest rate plus the risk premium:

$$\frac{\pi_n}{(\frac{a_n}{n}w_n)} = r + \iota \tag{19}$$

where $\iota = \frac{\dot{n_i}}{n_n}$ is called the rate of imitation, which is also the 'hazard rate' at which a Northern product will be imitated at the next instant. It represents the risk premium to be paid by the Northern firm to its shareholder. In the steady state we have $\frac{n_i}{n_n} = \frac{\iota}{g}$.
Using (16), (17) and (18) we get the reduced form Northen equlibrium condition from (19) as

$$\frac{1-\alpha}{\alpha}(\frac{L_n - a_n g}{a_n})(1 + \frac{\iota}{g}) = r + \iota \tag{20}$$

This is the standard NN curve in GH. Now we move to the modelling of the South where we depart from GH.

2.5 The South

As the North, the South does also have a competetive imitative R &D sector and a monopolistically competetive production sector. We introduce two types of labour in the South - skilled (H_s) and unskilled (L_s) where the endowment of the skilled labour is exogenously given. Skilled labours are used both in the R &D and in the production while the unskilled are specialised to production only. We assume each unskilled labour's efficiency depends on the relative wage of unskiled to skill. Thus the aggregate endowment of the unskilled labour is

$$L_s = h(\frac{{w^s}_l}{{w^s}_h})L^s \tag{21}$$

where $h(\frac{{w^s}_l}{{w^s}_h})$ is the efficiency function, $h' \succ 0$, $h'' \prec 0$, $h(0) \prec 0$ and L^s is the number of unskilled labour in the South.

We now consider the imitative activity in the South. A Southern entreprenuer chooses randomly one of the Northern products (not previously imitated) to copy. For this the entreprenuer devotes $\frac{a_s}{n_s}$ units of skilled labours to the task. n_s is the stock of knowledge capital. We assume that $a_s = a_m + \iota_s$ where a_m is the inverse of the productivity parameter of the skilled labour in the South and ι_s is a policy parameter determined by the Southern authority. A higher ι_s reflects a stronger IPR. This means that more resources need to devote for imitating a Northern variety if IPR is strengthened in the South or, in other words, ι_s measures how much of the imitated design must be unique to satisfy the standard[4]. Given a_m a stronger IPR then imply a_s to be higher. We thus measure an increase in a_s as stronger IPR protection in the South in the comparative static part. Under this specification the production function of the imitative R &D sector takes the following form

$$\dot{n}_s = \frac{n_s}{a_s} {H^R}_s \tag{22}$$

where ${H^R}_s$ is the amount of skilled labour used in the R &D sector in the South. Since this sector is competetive the skilled labour's wage $({w^h}_s)$ is its marginal productivity which is growing at a rate g in the steady state.

The production function of each firm in the South takes the following CES form

$$x_s = (\delta {L_s}^{-\rho} + (1-\delta){H^p}_s{}^{-\rho})^{-\frac{1}{\rho}} \tag{23}$$

where ${H^p}_s$ is the amount of skilled labour being used in the production by each firm and ρ is a measure of the elasticity of substitution between skilled and unskilled labour[5]

A typical Southern firm maximises profit

$$Max \;\; \pi_s = p_s x_s - ({w^s}_h {H^p}_s + {w^s}_l \frac{L_s}{h(\frac{{w^s}_l}{{w^s}_h})})$$

subject to the demand function of x_s, with respect to ${w^s}_l$, ${H^p}_s$ and L_s. This yields the following demand functions, the monopoly price and the efficiency wage of unskilled in terms of skilled.

$${H^p}_s = x_s(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1}{\rho}} \tag{24}$$

$$L_s = x_s(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1}{\rho}}.K^{\frac{-1}{1+\rho}} \tag{25}$$

$$p_s = \frac{{w^s}_h}{(1-\delta)\alpha}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1+\rho}{\rho}} \tag{26}$$

$$\frac{h'(.)}{h(.)}\frac{{w^s}_l}{{w^s}_h} = 1 \tag{27}$$

where $K = \frac{1-\delta}{\delta}\frac{{w^s}_l}{h(.){w^s}_h}$ and from (27) the relative wage of unskilled labour is fixed[6]. We assume that there exist unemployment in the unskilled labour market at this relative wage rate[7]. The price given in (26) is the profit maximising monopoly price of the Southern firm. Assuming price competetion

[4]for details of this type of interpretation see Glass and saggi (2002).

[5]Since all firms in the South are homogenous the demand for labour will be the same for all firms and hence to get the aggregate demand for labour we simply multiply the individual demand with the number of firms in the South.

[6]for details of this derivation see Appendix-1

[7]Later we shall derive a condition under which the level of unemployment is positive.

from the Northern firm a Southern imitator's optimal price depends on the wage gap between this two regions. If this gap is large, the Southern firm can charge its monopoly price given in (26). The condition for this to happen is $\frac{w^s{}_h}{(1-\delta)\alpha}(\delta K^{\frac{\rho}{1+\rho}}+1-\delta)^{\frac{1+\rho}{\rho}} \prec w_n$ where w_n is the marginal cost (MC) of Northern production. We shall refer to this case as the *wide gap case* as in GH. If the wage gap is not so large, then a Southern firm will charge the limit price in equilibrium which is its rival MC, w_n. In this *narrorw gap* case $p_s = w_n$. The condition for this to happen is $\frac{w^s{}_h}{(1-\delta)\alpha}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1+\rho}{\rho}} \succ w_n$. In any case we always assume that $\frac{w^s{}_h}{(1-\delta)}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1+\rho}{\rho}} \prec w_n$ where LHS is the MC of Production in the South, i.e., we assume that production is always cheaper in the South than the North. We shall derieve the equlibrium conditions in the South in both the wide gap and the narrow gap cases.

2.5.1 The wide gap case

Under this case the unit price per product in the South is given by (26). Then instantaneous profit per firm is $\pi_s = \frac{1-\alpha}{\alpha}\frac{w^s{}_h}{(1-\delta)}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1+\rho}{\rho}} x_s$. Free entry into the imitative R &D activity in the South imply that in equilibrium the present discounted value of profits per firm equal to the cost of imitation. This imply $\frac{\pi_s}{r} = \frac{a_s}{n_s} w^s{}_h$ or,

$$ra_s \frac{\alpha}{1-\alpha} = \frac{1}{(1-\delta)}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1+\rho}{\rho}} n_s x_s \tag{28}$$

The equilibrium conditions in the skilled and the unskilled labour market takes the following form

$$H_s = a_s g + n_s x_s (\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1}{\rho}} \tag{29}$$

$$L_s - U_s = n_s x_s (\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1}{\rho}} K^{\frac{-1}{1+\rho}} \tag{30}$$

where U_s is the level of unemployment in the South's unskilled labour market.

2.5.2 The narrow gap case

Under this case $p_s = w_n$ and the instantaneous profit per firm is
$\pi_s = (w_n - \frac{w^s{}_h}{(1-\delta)}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1+\rho}{\rho}})x_s$. The free entry condition then imply

$$(w_n - \frac{w^s{}_h}{(1-\delta)}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1+\rho}{\rho}})x_s = (\frac{a_s}{n_s} w^s{}_h)r \tag{31}$$

(29) and (30) of the wide gap case are again the labour market equilibrium condition. We find another equation in the narrow gap case as the relative

demand for the Northern product is constant in this case. We write this as

$$\frac{x_n}{x_s} = \alpha^{\varepsilon} \tag{32}$$

3 The Equilibrium and The Comparative Static Results

3.1 The wide gap case

We write the integrated system of equations in this case as follows

$$\frac{1-\alpha}{\alpha}\left(\frac{L_n - a_n g}{a_n}\right)\left(1 + \frac{\iota}{g}\right) = r + \iota \tag{33}$$

$$r a_s \frac{\alpha}{1-\alpha} = \frac{1}{(1-\delta)}\left(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta\right)^{\frac{1+\rho}{\rho}} n_s x_s \tag{34}$$

$$H_s = a_s g + n_s x_s \left(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta\right)^{\frac{1}{\rho}} \tag{35}$$

$$L_s - U_s = n_s x_s \left(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta\right)^{\frac{1}{\rho}} K^{\frac{-1}{1+\rho}} \tag{36}$$

Equation (33) is the equlibrium condition coming from the Northern free entry condition and the resource constraint. (34) is the free entry condition in the South. (35) and (36) are the skilled and unskilled labour market equilibrium condition in the South respectively. This is a set of 4 equations in 4 unknowns as $n_s x_s$, g, ι and U_s[8]. Note that the skilled-unskilled relative wage in the South is fixed from (27). We solve $n_s x_s$ from (35) and put this value into (34). This gives

$$r a_s \frac{\alpha}{1-\alpha} = \frac{1}{(1-\delta)}(H_s - a_s g)\left(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta\right) \tag{37}$$

From (35) and (36) we get

$$\frac{H_s - a_s g}{L_s - U_s} = K^{\frac{1}{1+\rho}} \tag{38}$$

Now (33), (37) and (38) are three equations in three unknowns as, g, ι and U_s. (37) now gives the SS curve as in GH. We can solve g from (37) and then putting that value in (38) and (33) we can solve U_s and ι respectively. Our solution for g is

$$g = \frac{H_s\left(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta\right) - \frac{\alpha(1-\delta)}{1-\alpha} a_s \theta}{a_s\left(\phi \frac{\alpha(1-\delta)}{1-\alpha} + \delta K^{\frac{\rho}{1+\rho}} + 1 - \delta\right)} \tag{39}$$

(33), (37) and (38) are represented in the figure-1 (both 1.1 and 1.2) as the NN cuve, the SS curve and the UU curve respectively. We find the necessary

[8]Note that (14) gives $r = \theta + \phi g$.

condition for the wide gap equilibrium to exist is $\frac{H_s}{a_s} \succ \frac{L_n}{a_n}$, i.e., the SS curve lie above the g_n ($g_n = \frac{\frac{L_n}{a_n} - \frac{\alpha}{1-\alpha}\rho}{1+\frac{\alpha}{1-\alpha}\phi}$) point in the figure. We also see that if $g \succ g_u$ ($g_u = \frac{(1-\alpha)L_n}{a_n}$) then the SS curve lies everywhere above the NN curve and there can not exist any wide gap equilibrium. Thus there is an upper bound of g in the wide gap case. From (38), positive level of unemployment is guranteed by the assumption that $(\frac{1-\delta}{\delta}\frac{w^s{}_l}{h(.)w^s{}_h})^{\frac{1}{1+\rho}}h(.) \succ \frac{H_s}{L^s}$.

3.1.1 The comparative steady state analysis

In the following analysis we are in the wide gap regime, i.e., $\frac{w^s{}_h}{(1-\delta)\alpha}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1+\rho}{\rho}} \prec w_n$. Any change of the exogenous parameters in our model that causes $\frac{w^s{}_h}{w_n}$ or $\frac{w^s{}_l}{w_n}$ to increase can lead to an violation of the wide gap condition. To the extent that $\frac{w^s{}_h}{w_n}$ or $\frac{w^s{}_l}{w_n}$ increases, we assume in the following analysis that the exogenous changes are sufficiently small such that $\frac{w^s{}_h}{w_n}$ does not exceed $\frac{(1-\delta)\alpha}{(\delta K^{\frac{\rho}{1+\rho}}+1-\delta)^{\frac{1+\rho}{\rho}}}$, or $\frac{w^s{}_l}{w_n}$ does not exceed $\frac{\delta\alpha h(.)}{(\delta+(1-\delta)K^{\frac{-\rho}{1+\rho}})^{\frac{1+\rho}{\rho}}}$, so that we stay in the wide gap regime.

From (39) an increase in the IPR parameter, a_s, will lead to a decrease in g and also a decrease in $a_s g$. From (39), as $a_s g$ decreases, so does U_s and from (33) a decrease in g is always associated with a decrease in ι. Thus protection of stronger IPR will result in lower rate of growth, lower imitation rate and lower level of unskilled unemployment in the South. The intuition of this result is straight forward. An increase in IPR in South leads to an increase in the cost of imitation. Free entry condition then imply profit rate should also increase. But this is possible only when the rate of growth falls. In wide gap case it happens that the demand for skilled labour in the R &D sector, in aggregate, falls. Thus more skilled are employed in the production sector. But from the constancy of the relative wage in the South relative demand for skilled labour is fixed in the production sector of the South. Thus any increase in the skilled labour employment have to be matched by an equal proportionate increase in the unskilled labour demand such that skilled to unskilled ratio is unchanged. This imply a decrease in the level of unemployment in the South unskilled market. Note that the unemployment in the skilled labour can never arise in our model set up due to the technology in the R &D sector. This type of technology imply an infinite demand for skilled labour in the R &D sector.

—— *insert figure-1.1 and figure-1.2 here* ———

We have examined the effect of an technological improvement in the North (decrease in a_n) and the efect of the size of the labour force of both the North and the South on the rate of growth, rate of imitation and on the unemployment of the South. We also have studied the effects of the stronger IPR in the South on the relative wage between the South skilled to that of the North and the South unskilled to that of the North[9]. In the appendix-2 we proved the results and here we are summarising them.

An improvement in the technological progress in the North (captured by a decrease in a_n) will keep the rate of growth unchanged and hence South unskilled unemployment will not changed. This is because in the wide gap case the rate of growth is determined explicitly from the equilibrium conditions in the South. Hence any change in the parameter that affect only Northern equilibrium conditions does not have any effect on the rate of growth and unskilled umployment in the South. However, this will effect the rate of imitation ι. When a_n decreases, more labours are employed in the Northern production sector. This is possible if there are more number of firms producing in the North. Given g this imply the rate of imitation should fall so that $\frac{n_n}{n_s}$ will increase in the steady state.

An increase in the size of the labour force in the North will not have any effect on both the rate of growth and the level of Southern unemployment, however, this will decrease the rate of imitation. The intuition goes in the similar line as the case of decreased a_n which increases the effective labour force in the North.

An increase in the size of the Southern skilled labour will increase the rate of growth, decrease the level of unskilled unemployment in the South and increase the rate of imitation. More supply of the skilled labour in the South will increase both the size of the R &D sector and the production sector in the South. The first one will increase the rate of growth and the second one will increase the demand for unskilled labour in the South and hence reduce the level of unemployment. An increase in the rate of growth will reduce the size of the production sector in the North which is to be matched by the lower share of the North in the aggregate product cycle (i.e.,$\frac{n_n}{n}$ will decrease). This imply that the rate of imitation will increase in the steady state.

[9]see Appendix-2 for relative wages.

—— *insert table-1 here* ———

3.2 The narrow gap case

In the narrow gap case we have the following system of equations of the world economy.

$$\frac{1-\alpha}{\alpha}(\frac{L_n - a_n g}{a_n})(1+\frac{\iota}{g}) = r + \iota \tag{40}$$

$$(w_n - \frac{w^s{}_h}{(1-\delta)}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1+\rho}{\rho}})x_s = (\frac{a_s}{n_s}w^s{}_h)r \tag{41}$$

$$H_s = a_s g + n_s x_s(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1}{\rho}} \tag{42}$$

$$L_s - U_s = n_s x_s(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1}{\rho}} K^{\frac{-1}{1+\rho}} \tag{43}$$

$$\frac{x_n}{x_s} = \alpha^{\varepsilon} \tag{44}$$

(40), (42) and (43) are the equlibrium conditions in the North, the South skilled and the unskilled labour market respectively. (41) is the no arbitrage condition in the South and (44) is the relative demand condition. We solve x_n from the North labour market equilibrium condition as $x_n = \frac{L_n - a_n g}{n_n}$ and x_s from (42) as $x_s = \frac{H_s - a_s g}{n_s(\delta K^{\frac{\rho}{1+\rho}}+1-\delta)^{\frac{1}{\rho}}}$. Substituting these x_n and x_s into (44) and noting that $\frac{n_s}{n_n} = \frac{\iota}{g}$ we get the following

$$\frac{L_n - a_n g}{H_s - a_s g}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1}{\rho}}\frac{\iota}{g} = \alpha^{\varepsilon} \tag{45}$$

Following GH, we depict this relation between ι and g as the XX curve in figure-2 (both 2.1 and 2.2). This gives a positive relation between the rate of growth and the rate of imitation [10]. (42) and (43) could be written together as

$$\frac{H_s - a_s g}{L_s - U_s} = K^{\frac{1}{1+\rho}} \tag{46}$$

[10]The intuition for the positive slope of the SS curve can be found in GH.

Solving $n_s x_s$ from in terms of g from (42) and putting this into (41) we can solve for the relative wage between the North to South skilled as follows

$$\frac{w_n}{{w^s}_h} = \frac{a_s r}{H_s - a_s g}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1}{\rho}} + \frac{1}{1-\delta}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1+\rho}{\rho}} \qquad (47)$$

We now see (40), (45) and (46) are three equations in three unknown as, g, ι and U_s. We can solve g and ι from (40) and (45) and then U_s from (46). Relative wage in the South are given from (27). Once g is solved $\frac{w_n}{{w^s}_h}$ could be obtained from (47). We already noted in the wide gap case that $\frac{H_s}{a_s} \succ \frac{L_n}{a_n}$ is a necessary condition for the wide gap equilibrium to exist. In fact, we see that if this condition is not satisfies then there can be no steady state equilibrium with positive rate of imitation in the South[11]. In such case the South imitates for a while and then produces a fixed set of goods. Thus $\frac{H_s}{a_s} \succ \frac{L_n}{a_n}$ is a necessary condition for any steady state equilibrium (whether wide or narrow gap) to exist with positive ongoing rate of imitation in the South. It can be proved that the XX curve (equation (45)) must be steeper than the NN curve (equation (40)) at any point of intersection and that the curves intersects exactly once. This intersection (at Q, see figure-2) represents the unique narrow-gap equilibrium, provided that the relative wage associated with the point (obtained from (47)) satisfies $\frac{{w^s}_h}{(1-\delta)\alpha}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1+\rho}{\rho}} \succ w_n$[12]. As like wide gap case, the positive level of unemployment is guareented by the assumption that $(\frac{1-\delta}{\delta}\frac{{w^s}_l}{h(.){w^s}_h})^{\frac{1}{1+\rho}} h(.) \succ \frac{H_s}{L^s}$.

—— *insert figure-2.1 and figure-2.2 here* ———

[11] the reason is the maximum possible g coming form North is greater than the maximum possible g coming from the South.

[12] The wide gap case applies whenever point Q lies above the SS curve and the narrow gap case applies when it lies below the SS curve. For details on this issue see GH page 1225 and the reference therein.

3.2.1 The comparative steady state analysis

As like wide gap, we here assume that any changes in the exogenous parameters that causes the relative wage between North and South to change does not violate the narrow gap restriction for the relative wage. To the extent that $\frac{w^s{}_h}{w_n}$ decreases we assume that the exogenous changes are suficiently small such that $\frac{w^s{}_h}{w_n}$ does not fall below $\frac{(1-\delta)\alpha}{(\delta K^{\frac{\rho}{1+\rho}}+1-\delta)^{\frac{1+\rho}{\rho}}}$ and we stay in the norrow gap regime.

Stronger IPR protection in the South will lower the rate of growth and the rate of imitation (as in the wide gap case). However the level of the unskilled unemployment in the South will increase here as opposed to the wide gap case. The intuition goes in the same line as in the wide gap case except the fact that in narrow gap case aggregate level of employment of the skilled labour in the R &D sector in the South ($a_s g$) increases. From figure-2.1 we see that an increase in the value of the IPR protection parameter a_s shifts the XX curve leftward while NN curve remains unchanged. At the new piont of intersection both g and ι decreases. We also have proved in the Appendix-3 that $\frac{\partial g}{\partial a_s} \prec 0$ and $|\frac{\partial g}{\partial a_s}\frac{a_s}{g}| \prec 1$. Thus $a_s g$ increases as a_s increases and hence from (46) U_s increases.

As in the wide gap case, we have examined the effect of an technological improvement in the North (decrease in a_n) and the efect of the size of the labour force of both the North and the South on the rate of growth, rate of imitation and on the unemployment of the South. We also have studied the effects of the stronger IPR on the relative wage between the South skilled to the North and the South unskilled to the North[13]. In the appendix-3 we proved the results and here we are summarising them.

Technological improvement in the North (as captured by a decrease in a_n) will increase the rate of growth and the rate of unskilled unemployment in the South but decrease the rate of imitation. A decrease in a_n will imply the expected present discounted profit flow of a typical Northern firm exceed its cost of the R&D given all other things unchanged. This will increase the rate of innovation in the North. A decrease in a_n will lead to a decrease in ι, given g, for both the NN curve and the XX curve. So both the curve will shift leftward in the figure 2.2. However, in appendix-3 we have shown that for $g \succ \frac{H_s}{2a_s}$ (this is a sufficient condition) , ι will fall in equilibrium. In the South agregate size of the the R&D sector ($a_s g$) will expand leaving few skilled labour for the production. Fixed relative demand of the skilled to the unskilled labour in the South (because of the constancy of the relative wage there) then imply that the rate of unskilled unemployment will increase.

An increase in the Northern labour force will increase the rate of growth, the rate of Southern unskilled unemployment and decrease the rate of imitation

[13]see Appendix-3 for relative wages.

(under the sufficient condition $g \succ \frac{H_s}{2a_s}$) in our model. The intuition goes in the same line as a technological improvement in the North discussed in the last paragraph. This is because a decrease in a_n is directly associated with an increase in the effective labour force in the Northern.

An increase in the size of the South skilled labour will increase the rate of growth and the rate of imitation but decrease the rate of unemployment in the South. More skilled labour in the South will expand the size of both the R&D sector ($a_s g$) and the production sector ($n_s x_s$) in the South. The first one imply the rate of growth will increase and the second one imply that the rate of unskilled unemployment will increase in equilibrium (note that the relative demand of skilled to unskilled in the South is constant). An increase in g will imply that the rate of imitation will increase from the Northern equilibrium condition.

——*insert table-2 here* ———

4 Extension of The Benchmark Model

4.1 Introducing Trade Union

We introduce trade union into the unskilled labour market of the South supposing that all the employed unskilled labourers in the South are unionised which wants to maximise the following objective function

$$TU = (\frac{w^s{}_l}{w^s{}_h})^{\beta}(n_s L^s)^{1-\beta} \tag{48}$$

with respect to $w^s{}_l$ and subject to the aggregate unskilled labour demand function (coming from the profit maximising behaviour of the imitative firm)

$$n_s L^s = n_s x_s[\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta]^{\frac{1}{\rho}} h(.)^{-1} \tag{49}$$

where TU is the trade union's utility function and $n_s L^s$ is the aggregate demand for unskilled labours in the South production sector. So we deviate from the earlier model (section 2.5) in the sense that now the trade union determines the relative wage of the unskilled labour and firm behaves as a wage taker and determines the level of employment according to its unskilled labour demand curve which is generated by maximising its profit function with respect to the skilled and unskilled labour. In section 2.5 we maximised the profit function of the firm with respect to the skilled and unskilled labour and the unskilled wage rate. Thus firm was the wage setter there. In this extension we intend to compare the rate of growth, level of unemployment and the imitation rate when firm is the wage taker vis a vis these variables when firm is the wage setter (that we have derieved in the earlier sections).

Maximising the above objective function trade union determines the relative wage of the unskilled labour as a solution of the following equation

$$\frac{h'(.)}{h(.)}\frac{w^s{}_l}{w^s{}_h} = 1 - \frac{(2\beta-1)}{(1-\beta)(\frac{\varepsilon\delta K^{\frac{\rho}{1+\rho}}+\frac{1-\delta}{1+\rho}}{\delta K^{\frac{\rho}{1+\rho}}+1-\delta}-1)} \tag{50}$$

In deriving (50) we assumed the *wide gap* case (The narrow gap case will be discussed in the next section). To get an easy solution from (50) we assume the following

$$\rho = -\alpha$$

and

$$\frac{1}{\beta} \succ 2-\alpha$$

The first assumption imply that the elasticity of substitution between the skilled and the unskilled labour is positive (since the elasticity of substitution in the CES case is given by $\frac{1}{1+\rho}$) and the RHS of (50) becomes $1-\frac{2\beta-1}{(1-\beta)\frac{\alpha}{1-\alpha}}$. The second assumption guareentee that the RHS of (50) is always positive. Now for $\beta \succ \frac{1}{2}$ (and satisfying the second assumption by the choice of α) the RHS of (50) is always less than 1 which imply that the relative wage fixed by the trade union is higher than that of set up by the firm[14]. For $\beta = \frac{1}{2}$ the relative wage fixed by the trade union is equal to that of set up by the firm and for $\beta \prec \frac{1}{2}$ trade union set up relative wage of the unskilled labour at a lower level than that of set up by the firm. We can write the above fact in the following way

$$(\frac{w^s{}_l}{w^s{}_h})^T = t(\beta), \quad t'(.) \succ 0$$

and

$$\begin{aligned}
\beta &> \frac{1}{2} \Rightarrow (\frac{w^s{}_l}{w^s{}_h})^T > (\frac{w^s{}_l}{w^s{}_h})^F \\
\beta &= \frac{1}{2} \Rightarrow (\frac{w^s{}_l}{w^s{}_h})^T = (\frac{w^s{}_l}{w^s{}_h})^F
\end{aligned}$$

[14]compare with (27)

$$\beta < \frac{1}{2} \Rightarrow (\frac{w^s{}_l}{w^s{}_h})^T < (\frac{w^s{}_l}{w^s{}_h})^F \tag{51}$$

where $(\frac{w^s{}_l}{w^s{}_h})^T$ is the relative wage of the unskilled labour set by the trade union and $(\frac{w^s{}_l}{w^s{}_h})^F$ is the relative wage of the unskilled labour set by the firm.

In the wide gap case the set of equations of our model is given by (33), (34), (35), and (36) except for the fact that now the value of K is fixed by the relative wage set by the trade union. For different values of β the value of K will change (note that K is a positive function of $(\frac{w^s{}_l}{w^s{}_h})^T$ and hence a positive function of β.) From (39)

$$H_s - a_s g = \frac{\frac{\alpha(1-\delta)}{1-\alpha}(H_s\phi + a_s\theta)}{\phi\frac{\alpha(1-\delta)}{1-\alpha} + (\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)} \tag{52}$$

For $\beta \succ \frac{1}{2}$ the value K is higher when trade union set the relative wage. Hence the rate of growth will be higher for the relative wage set by the trade union when $\beta \succ \frac{1}{2}$. Similarly it follows that the growth rate under trade union will be equal to or less than the growth rate under the firm for the values of β either equal to or less than $\frac{1}{2}$ respectively. To restore the positive level of unemployment we need some restriction on β and the quantity of unskilled labour[15]. We can write the above facts in the following way

$$\begin{aligned}\beta &> \frac{1}{2} \Rightarrow (g)^T > (g)^F \\ \beta &= \frac{1}{2} \Rightarrow (g)^T = (g)^F \\ \beta &< \frac{1}{2} \Rightarrow (g)^T < (g)^F \end{aligned} \tag{53}$$

where $(g)^T$ is the rate of growth under the trade union and $(g)^F$ is the rate of growth under the firm.

From (38) we can solve for the level of unemployment as follows

$$U_s = L_s - (H_s - a_s g)K^{-\frac{1}{1+\rho}} \tag{54}$$

For $\beta \succ .5$ we see $(H_s - a_s g)^T \prec (H_s - a_s g)^F$ (see (52)) and $(K)^T > (K)^F$.[16] We also have $\frac{1}{1+\rho} \succ 0$. From $L_s = L^s h(\frac{w^s{}_l}{w^s{}_h})$ we see L_s rises for $\beta \succ .5$ when trade union sets the relative wage. Thus $U_s{}^T \succ U_s{}^F$ for $\beta \succ .5$. Similarly we

[15]This is $L^s h(.)K^{\frac{1}{1+\rho}} \succ H^s$. Since both $h(.)$ and K depend on β this restriction is saying that positive level of unemployment may not be restored given H_s and L^s and if we go on reducing β

[16]upperscript T or F imply that the relevant expression is when trade union sets the relative wage or firm sets the relative wage respectively. This notation is relevant everywhere in rest of our paper.

can compare for $\beta \prec .5$ and $\beta = .5$. We are writing this fact in the following way

$$\begin{aligned} \beta &> \frac{1}{2} \Rightarrow {U_s}^T > {U_s}^F \\ \beta &= \frac{1}{2} \Rightarrow {U_s}^T = {U_s}^F \\ \beta &< \frac{1}{2} \Rightarrow {U_s}^T < {U_s}^F \end{aligned} \tag{55}$$

4.2 the narrow gap case

In the narrow gap case the relative demand of the Northern product to the Southern product is constant and the demand for the Southern product does not depend on the wage rate of the South (since the price of the Southern product is w_n). Under this case maximising their objective function (as set up in the above section) the trade union determines the relative wage of the unskilled to skill as the solution to the following equation

$$\frac{h'(.)}{h(.)}\frac{{w^s}_l}{{w^s}_h} = 1 - \frac{(2\beta-1)}{(1-\beta)(\frac{1-\delta}{(1+\rho)(\delta K^{\frac{\rho}{1+\rho}}+1-\delta)} - 1)} \tag{56}$$

This equation could be obtained from (50) simply putting $\varepsilon = 0$ there. We assume $\rho \succ 0$. Then the sigh of $\frac{(2\beta-1)}{(1-\beta)(\frac{1-\delta}{(1+\rho)(\delta K^{\frac{\rho}{1+\rho}}+1-\delta)}-1)}$ in equation (56) depends on the value of β. For $\beta \succ .5$ this is negative, for $\beta \prec .5$ this is positive and for $\beta = .5$ this is zero. Note that in the wide gap case we restricted $\rho = -\alpha$. Here we are assuming that $\rho \succ 0$. Thus in both the wide and the narrow gap case the elasticity of substitution between the skilled and the unskilled is positive, however it is greater than one in the wide gap case while less than one in the narrow gap case. For $\beta \succ .5$ the LHS of (56) is greater than one and then comparing this with the equation (27) we see that the relative wage set by the trade union is lower than that set by the firm[17]. For $\beta \prec .5$ the LHS of (56) is less than one and then again comparing this with the equation (27) we see that the relative wage set by the trade union is higher than that set by the firm and for $\beta = .5$ (56) and (27) are identical and the relative wage under the unionised economy is the the same as that under the firm. These facts are written as follows

$$(\frac{{w^s}_l}{{w^s}_h})^T = v(\beta), \quad v'(.) \prec 0$$

[17]Note that $\frac{h'(.)}{h(.)}\frac{{w^s}_l}{{w^s}_h}$ is negative function of $\frac{{w^s}_l}{{w^s}_h}$.

and

$$
\begin{aligned}
\beta &> \frac{1}{2} \Rightarrow (\frac{w^s{}_l}{w^s{}_h})^T < (\frac{w^s{}_l}{w^s{}_h})^F \\
\beta &= \frac{1}{2} \Rightarrow (\frac{w^s{}_l}{w^s{}_h})^T = (\frac{w^s{}_l}{w^s{}_h})^F \\
\beta &< \frac{1}{2} \Rightarrow (\frac{w^s{}_l}{w^s{}_h})^T > (\frac{w^s{}_l}{w^s{}_h})^F \qquad (57)
\end{aligned}
$$

We now can see the contrast between (51) and (57). Though we can not compare between (51) and (57) since they are derieved under different assumptions on ρ, putting relatively more weight to the relative wage in the trade union's utility function increases the equilibrium relative wage of the unskilled labour in the South under the wide gap case and it is decreased under the narrow gap case. This statement is conditional on the value of ρ.

Now we compare the growth rate of our global economy when the trade union sets the relative wage with that of under the firm in this narrow gap case. The growth rate and the imitation rate under the narrow gap is solved from (40) and (45). However when trade union sets the wage rate the XX curve (represented by (45)) shifts rightward for $\beta \succ .5$ and the NN curve (represented by (40)) remains unchanged. Hence both the growth rate and the imitation rate increases. The opposite is true for $\beta \prec .5$ and for $\beta = .5$ the rate of growth and the imitation rate is independent on who sets the wage rate, the firm or the union. We are writing these facts as follows

$$
\begin{aligned}
\beta &> \frac{1}{2} \Rightarrow (g)^T > (g)^F \\
\beta &= \frac{1}{2} \Rightarrow (g)^T = (g)^F \\
\beta &< \frac{1}{2} \Rightarrow (g)^T < (g)^F \qquad (58)
\end{aligned}
$$

We do not find any clear result for the movement of the rate of unemployment for different values of β in this narrow-gap regime. The relation between the relative wage of the unskilled labour and their unemployment is ambiguous in this case.

5 Conclusion

Issues on the intellectual property rights protection have gained lots of attention in the recent past. Earlier studies on this issue (using Grossman-Helpman (1991) framework) have raised several crucial questions like how IPR policy of the developing country will affect the inward foreign direct investment, how will it affect the global rate of growth, welfare, income distribution between the developed and developing nations, etc.. However, there are few studies that ask the question: how will stronger IPR protection in the South will affect the unskilled unemployment there? Or, is there any policy option left to the South government that could possibly mitigate the bad outcome, if any, of stronger IPR protection there? In this paper we tried to find the answers of these questions.

We develop a simple dynamic general-equilibrium version of the variety based product-cycle model to study the effects of stronger IPR protection in the South on the rate of unemployment of the unskilled labour in the South. We have got different impact on the rate of unemployment depending on the initial factor cost gap between North and South. When the wage gap between this two region is wide (wide gap case) we see that the rate of unemployment will fall in the South due to stronger IPR protection there while it will rise in the narrow gap regime. Furthermore, introducing trade union in the South unskilled labour market we find that the growth rate and the unemployment rate can be very different in the unionized economy than in the non-unionized economy. Then we say that if the Southern authority have the power to control over the trade union (or control over in our model, or have the ability to set domestic relative wage of skilled to unskilled) then it can possibly overcome the bad effect of IPR policy by choosing appropriate .

From the welfare point of view, we argue in this paper that South should better protect its IPR strongly being in the narrow gap regime than in the wide gap regime. Though the rate of growth falls in both the regime it decreases at a higher rate in the wide gap case than in the narrow gap case (and it is the rate of growth that only matters in the welfare function in our model) . This is because, in the wide gap case the aggregate size of the Southern imitative R & D sector shrinks while in the narrow gap case it expands. This is the most interesting result of this paper, as we believe. South can effectively introduce sales tax over the products that it produces to switch from the wide gap to the narrow gap regime. In an extended version of this paper we solved for this tax rates.

Although our analysis has the main focus on the effects of IPR policy of the South, we also tried to find the effects of technological improvement of the North and the increase in the labour force of both the region on the rate of growth, rate of imitation and the rate of unemployment in this paper.

There are some important caveats in our model. First, we have completely abstracted from the distributional aspect of unemployment in the South in our model. Second, the way we introduce trade union in our model is crude. We have not taken into account of the fact that trade union can have dynamic utility function rather than the static one as employed in the model. Third, we have concentrated on the stead state only in this paper. The welfare results can be misleading if we don't pay attention to the out-of-steady state situation.

Nevertheless, the model constructed here sheds new light on the complex inter-relationships between innovation, imitation and unskilled unemployment in the South.

6 Appendix

Appendix-1

$$Max \;\; \pi_s = p_s x_s - ({w^s}_h {H^p}_s + {w^s}_l L^s)$$

, with respect to ${H^p}_s$, L^s and ${w^s}_l$ imply the following first order conditions

$$\begin{aligned} p_s \frac{\partial x_s}{\partial {H^p}_s} &= \frac{{w^s}_h}{\alpha} \\ p_s \frac{\partial x_s}{\partial L^s} &= \frac{{w^s}_l}{\alpha} \\ p_s \frac{\partial x_s}{\partial {w^s}_l} &= \frac{L^s}{\alpha}(h + h' \frac{{w^s}_l}{{w^s}_h}) \end{aligned} \tag{59}$$

respectively. Also from the production function of x_s we get

$$\begin{aligned} \frac{\partial x_s}{\partial {H^p}_s} &= \frac{x_s}{\delta {L_s}^{-\rho} + (1-\delta){{H^p}_s}^{-\rho}}(1-\delta)({H^p}_s)^{-\rho-1} \\ \frac{\partial x_s}{\partial L^s} &= \frac{x_s}{\delta {L_s}^{-\rho} + (1-\delta){{H^p}_s}^{-\rho}}\delta(L^s)^{-\rho-1}h^{-\rho} \\ \frac{\partial x_s}{\partial {w^s}_l} &= \frac{x_s}{\delta {L_s}^{-\rho} + (1-\delta){{H^p}_s}^{-\rho}}\delta(L^s)^{-\rho}h^{(-\rho-1)}\frac{h'}{{w^s}_h} \end{aligned} \tag{60}$$

From (59) and (60) we can get the conditions

$$\begin{aligned} {H^p}_s &= x_s(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1}{\rho}} \\ L_s &= x_s(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1}{\rho}}.K^{\frac{-1}{1+\rho}} \\ p_s &= \frac{{w^s}_h}{(1-\delta)\alpha}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1+\rho}{\rho}} \\ \frac{h'(.)}{h(.)}\frac{{w^s}_l}{{w^s}_h} &= 1 \end{aligned}$$

where $K = \frac{1-\delta}{\delta}\frac{{w^s}_l}{h(.){w^s}_h}$.

Appendix-2

In this section we prove the various comparative static results for the wide gape case provided in section 3.1.1. For that we will use the system of equations from the extended model. The equations are

$$\frac{1-\alpha}{\alpha}\left(\frac{L_n - a_n g}{a_n}\right)\left(1+\frac{\iota}{g}\right) = r + \iota \tag{61}$$

$$r a_s \frac{\alpha}{1-\alpha} = \frac{1}{(1-\delta)}\left(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta\right)^{\frac{1+\rho}{\rho}} n_s x_s \tag{62}$$

$$\frac{H_s - a_s g}{L_s - U_s} = K^{\frac{1}{1+\rho}} \tag{63}$$

From (62) we solve g as ($n_s x_s$ has been replaced from the skilled labour market clearing condition of the South)

$$g = \frac{H_s(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta) - \frac{\alpha(1-\delta)}{1-\alpha} a_s \theta}{a_s(\phi \frac{\alpha(1-\delta)}{1-\alpha} + \delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)} \tag{64}$$

and

$$H_s - a_s g = \frac{\frac{\alpha(1-\delta)}{1-\alpha}(H_s \phi + a_s \theta)}{\phi \frac{\alpha(1-\delta)}{1-\alpha} + (\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)} \tag{65}$$

From (64) and (65) $\frac{\partial g}{\partial a_s} \prec 0$ and $|\frac{\partial g}{\partial a_s}\frac{a_s}{g}| \succ 1$. Thus $a_s g$ falls as a_s increases and from (63) U_s falls . From (61) $\frac{\partial g}{\partial \iota} \succ 0$. Hence,
$a_s \uparrow \Rightarrow g \downarrow, \iota \downarrow, U_s \downarrow.$

From (64) a decrease in a_n (or an increase in L_n) does not affect g and hence U_s. But they do affect ι. From (61) a decrease in a_n (or an increase in L_n) decreases ι. An increase in H_s will increase g from (64) and $(H_s - a_s g)$ (from (65)) which causes U_s to fall from (63).

The relative wage between this two region can be solved from the following two equations

$$\frac{p_n}{p_s} = \frac{w_n}{{w^s}_h}\left(\frac{1}{\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta}\right)^{\frac{1+\rho}{\rho}} \tag{66}$$

and

$$\frac{p_n}{p_s} = \left(\frac{x_n}{x_s}\right)^{\frac{-1}{\varepsilon}} = \left\{\frac{a_n}{H_s - a_s g}\left[\frac{\alpha}{1-\alpha}(r+\iota) + g - \frac{L_n}{a_n}\right]\left[\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta\right]^{\frac{1}{\rho}}\right\}^{\frac{-1}{\varepsilon}} \tag{67}$$

RHS of (67) increases when a_s falls (since g, r, $a_s g$, ι falls). Hence comparing (66) and (67), $a_s \uparrow \Rightarrow \frac{w_n}{{w^s}_h} \uparrow, \frac{{w^s}_l}{{w^n}_l} \downarrow$. Relative wage of the North skilled to the South skilled rises due to IPR protection in the South. Since relative wage in both this region are constant (due to efficiency wages) relative wage of the South unskilled to the North unskilled falls. In the comparative static section for the wide gap case (section 3.1.1) we mentained that the changes in the exogenous parameters are sufficiently small so that the relative wages between the North and the South does not decrease much (if so) and we are in the wide gap regime. Here we see that there does not exist any potential danger of carrying out the comparative static in section 3.1.1 . since the relative wage between the North to the South (skilled and unskilled) increases.

Appendix-3

In this section we prove the various comparative static results for the narrow gape case provided in section 3.2.1 . For that we will use the system of equations from the extended model. The equations are

$$\frac{1-\alpha}{\alpha}\frac{1}{1-\delta}(\frac{L_n-a_n g}{a_n})(1+\frac{\iota}{g}) = r+\iota \tag{68}$$

$$(\frac{L_n-a_n g}{H_s-a_s g})\frac{\iota}{g}(\delta K^{\frac{\rho}{1+\rho}}+1-\delta)^{\frac{1}{\rho}} = \alpha^{\varepsilon} \tag{69}$$

$$\frac{H_s-a_s g}{L_s-U_s} = K^{\frac{1}{1+\rho}} \tag{70}$$

Taking log on both sides of (68) and (69) and taking total differential we have

$$[\frac{-a_n}{L_n-a_n g}-\frac{\iota}{g(g+\iota)}-\frac{\phi}{r+\iota}]dg+[\frac{1}{g+\iota}-\frac{1}{r+\iota}]d\iota = [\frac{1}{a_n}+\frac{g}{L_n-a_n g}]da_n-[\frac{1}{L_n-a_n g}]dL_n$$

$$[\frac{-a_n}{L_n-a_n g}-\frac{1}{g}+\frac{a_s}{H_s-a_s g}]dg+[\frac{1}{\iota}]d\iota = [\frac{-g}{H_s-a_s g}]da_s+[\frac{g}{L_n-a_n g}]da_n-[\frac{1}{L_n-a_n g}]dL_n+[\frac{1}{H_s-a_s g}]dH_s$$

In matrix form,

$$[\begin{array}{cc} A & B \\ C & D \end{array}][\begin{array}{c} dg \\ d\iota \end{array}] = [\begin{array}{c} E \\ F \end{array}]$$

where $A = [\frac{-a_n}{L_n-a_n g} - \frac{\iota}{g(g+\iota)} - \frac{\phi}{r+\iota}] \prec 0$, $B = [\frac{1}{g+\iota} - \frac{1}{r+\iota}] \succ 0$, $C = [\frac{-a_n}{L_n-a_n g} - \frac{1}{g} + \frac{a_s}{H_s-a_s g}] \prec 0$, $D = [\frac{1}{\iota}] \succ 0$, $E = [\frac{1}{a_n} + \frac{g}{L_n-a_n g}]da_n - [\frac{1}{L_n-a_n g}]dL_n$ and $F = [\frac{-g}{H_s-a_s g}]da_s + [\frac{g}{L_n-a_n g}]da_n - [\frac{1}{L_n-a_n g}]dL_n + [\frac{1}{H_s-a_s g}]dH_s$.

We also have,
$AD-CB = [\frac{-a_n}{L_n-a_n g} - \frac{\iota}{g(g+\iota)} - \frac{\phi}{r+\iota}].[\frac{1}{\iota}] - [\frac{1}{g+\iota} - \frac{1}{r+\iota}].[\frac{-a_n}{L_n-a_n g} - \frac{1}{g} + \frac{a_s}{H_s-a_s g}] =$

$\frac{a_n}{L_n-a_ng}[\frac{-1}{\iota}+\frac{1}{g+\iota}-\frac{1}{r+\iota}]+\frac{1}{g}[\frac{1}{g+\iota}-\frac{1}{r+\iota}-\frac{1}{g+\iota}]-\frac{a_s}{H_s-a_sg}[\frac{1}{g+\iota}-\frac{1}{r+\iota}]-\frac{\phi}{(r+\iota)\iota}]\prec 0.$
Hence, when only a_s changes,

$$0\prec-\frac{\partial g}{\partial a_s}\frac{a_s}{g}=\frac{-\frac{a_s}{H_s-a_sg}(\frac{1}{g+\iota}-\frac{1}{r+\iota})}{AD-CB}\prec 1$$

$\Rightarrow$ as $(a_s\uparrow)\rightarrow(a_sg\uparrow)\rightarrow(U_s\uparrow)$ (from (70)).

$$\frac{\partial\iota}{\partial a_s}=\frac{\frac{-g}{H_s-a_sg}(\frac{-a_n}{L_n-a_ng}-\frac{\iota}{g(g+\iota)}-\frac{\phi}{r+\iota})}{AD-BC}=\frac{+ve}{-ve}\prec 0$$

This relation can alternatively be proved as follows. From (68) $\frac{\partial\iota}{\partial g}\succ 0$. Hence, $\frac{\partial\iota}{\partial a_s}=\frac{\partial g}{\partial a_s}\frac{\partial\iota}{\partial g}=(-ve).(+ve)\prec 0$.

When only a_n changes,

$$\frac{\partial g}{\partial a_n}=\frac{\frac{1}{\iota}(\frac{1}{a_n}+\frac{g}{L_n-a_ng})-\frac{g}{L_n-a_ng}(\frac{1}{g+\iota}-\frac{1}{r+\iota})}{AD-BC}=\frac{\frac{1}{\iota a_n}+\frac{g}{L_n-a_ng}(\frac{1}{\iota}-\frac{1}{g+\iota}+\frac{1}{r+\iota})}{AD-BC}\prec 0$$

and

$$0\prec-\frac{\partial g}{\partial a_n}\frac{a_n}{g}=\frac{\frac{1}{g\iota}+\frac{a_n}{L_n-a_ng}(\frac{1}{\iota}-\frac{1}{g+\iota}+\frac{1}{r+\iota})}{AD-BC}\prec 1$$

. This relation depends on the asumption that the discount rate, θ, is close to zero. In that case compare the term $\frac{1}{g\iota}$ of the numerator with the term $\frac{1}{g}(\frac{1}{r+\iota})+\frac{\phi}{(r+\iota)\iota}$ of the denomenator. Thus when a_n decreases (a_ng) falls and from (70) U_s increases. We have got $\frac{\partial\iota}{\partial a_n}\succ 0$ under the sufficient condition $2a_sg\succ H_s$. If this does not hold then the sign of $\frac{\partial\iota}{\partial a_n}$ is ambiguous.

When only L_n changes,

$$\frac{\partial g}{\partial L_n}=\frac{\frac{-1}{L_n-a_ng}(\frac{1}{\iota}-\frac{1}{\iota+g}+\frac{1}{\iota+r})}{AD-BC}=\frac{-ve}{-ve}\succ 0$$

.
We have got $\frac{\partial\iota}{\partial L_n}\prec 0$ under the sufficient condition $2a_sg\succ H_s$. If this does not hold then the sign of $\frac{\partial\iota}{\partial L_n}$ is ambiguous. Now

from (69) $\frac{\iota}{g}$ can be written as

$$\frac{\iota}{g}=(\frac{H_s-a_sg}{L_n-a_ng})\frac{1}{(\delta K^{\frac{\rho}{1+\rho}}+1-\delta)^{\frac{1}{\rho}}}\alpha^{\varepsilon}$$

Taking differentiation with respect to L_n on both side

$$\frac{\partial(\frac{\iota}{g})}{\partial L_n}=\frac{\frac{\partial g}{\partial L_n}(\frac{H_s}{a_s}-\frac{L_n}{a_n})a_na_s}{(L_n-a_ng)^2}\frac{1}{(\delta K^{\frac{\rho}{1+\rho}}+1-\delta)^{\frac{1}{\rho}}}\alpha^{\varepsilon}\succ 0$$

Hence, we see, as L_n increases, $\frac{\iota}{g}$ increases but $(H_s - a_s g)$ decreases which imply U_s increases.

When only H_s changes,

$$\frac{\partial g}{\partial H_s} = \frac{\frac{-1}{H_s - a_s g}(\frac{1}{\iota+g} - \frac{1}{\iota+r})}{AD - BC} = \frac{-ve}{-ve} \succ 0$$

and $\frac{\partial \iota}{\partial H_s} = \frac{\partial \iota}{\partial g}.\frac{\partial g}{\partial H_s} = (+ve).(+ve) \quad \succ 0$. (68) can be written as

$$(L_n - a_n g)(\frac{\iota}{g}) = (r + \iota)\frac{\alpha a_n}{1-\alpha} - L_n + a_n g$$

. The LHS of the above equation increases due to increase in H_s (since g increases). Inserting this into (69) we find $(H_s - a_s g)$ has to increase and this imply (from (70)) that U_s has to fall.

Relative wage between the North and the South in the narrow gap case can be found from the South free entry codition

$$(w_n - \frac{w^s{}_h}{(1-\delta)}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1+\rho}{\rho}})x_s = (\frac{a_s}{n_s} w^s{}_h)r$$

$\Rightarrow \frac{w_n}{w^s{}_h} = \frac{a_s r}{H_s - a_s g}.(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1}{\rho}} + \frac{1}{1-\delta}(\delta K^{\frac{\rho}{1+\rho}} + 1 - \delta)^{\frac{1+\rho}{\rho}}$.
We have already shown that $a_s \uparrow \rightarrow (a_s g) \downarrow$ and due to efficiency wages, K is constant. Hence form the above equation, as $a_s \uparrow \rightarrow (\frac{w_n}{w^s{}_h}) \uparrow$. Protecting stronger IPR in the South raises the relative wage of the North to that of the South (for both skilled and unskilled). We note the potential danger of carrying out the comparative static in section 3.2.1 . since the relative wage between the North to the South (skilled and unskilled) increases and we may violate the condition for the narrow gap case. However there will not be any problem if we assume that the exogenous changes are sufficiently small.

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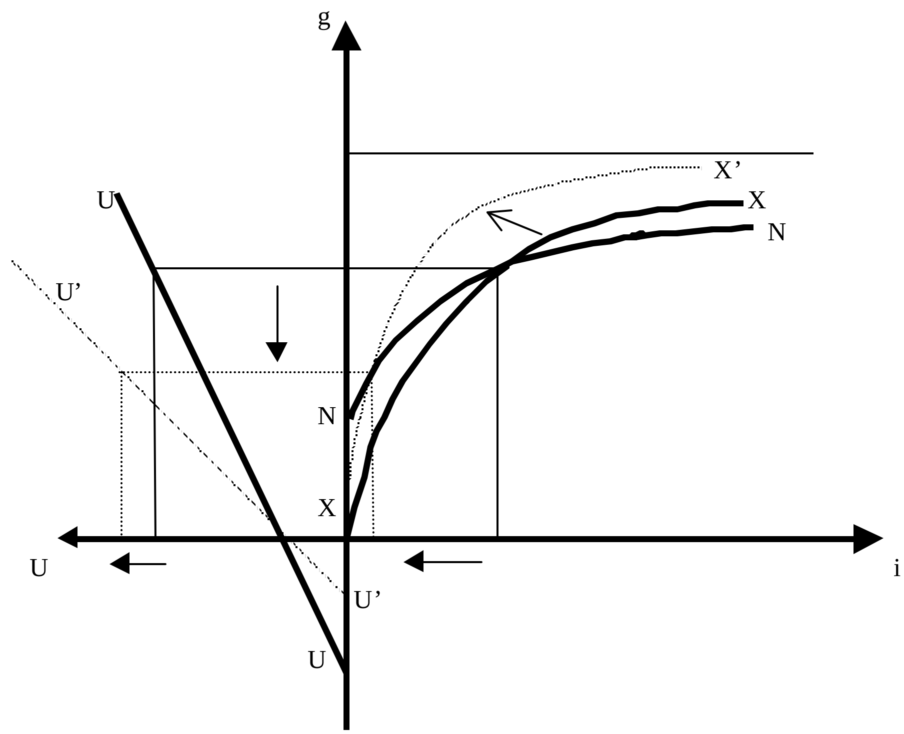

Figure-2.1: Effects of stronger IPR protection (or, increase in a_S) in the South
(Narrow gap)

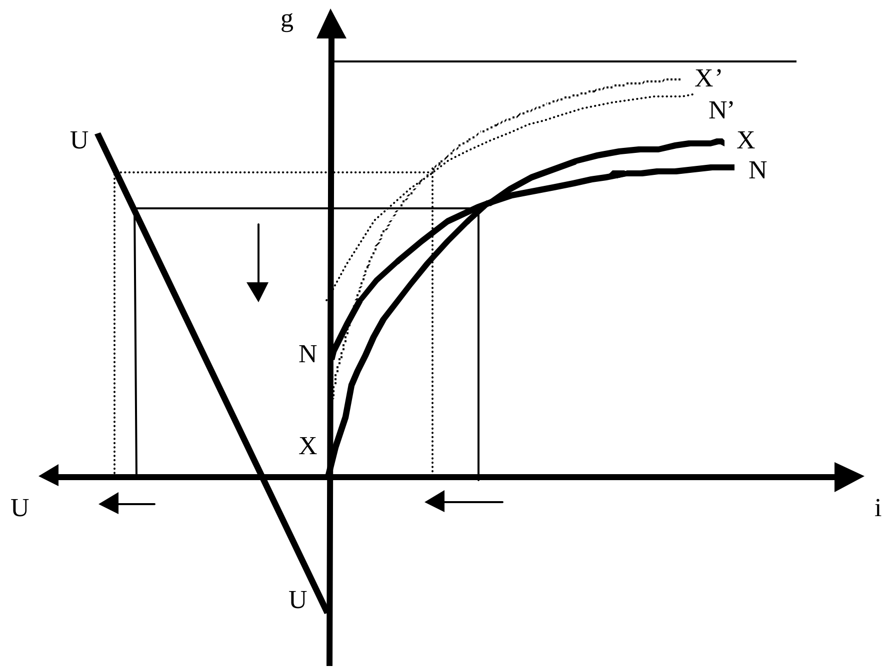

Figure- 2.2 : Effect of technological improvement (or, size of labour force) in North (decrease in a_N or, increase in L_N)

(Narrow gap)

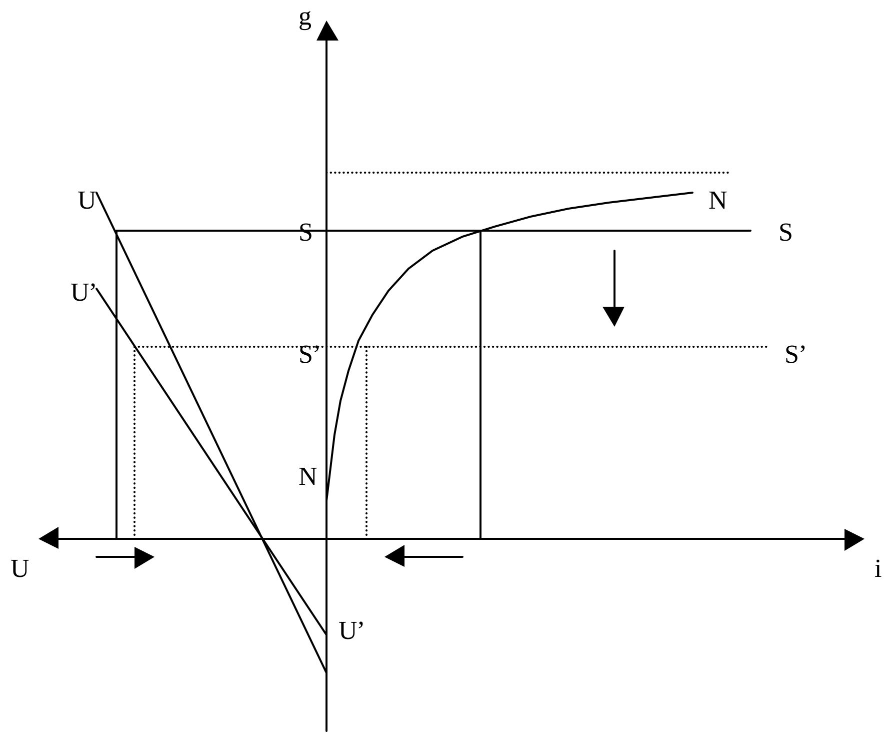

Figure-1.1: Effects of stronger IPR protection (or, increase in a_S) in the South.

(Wide gap)

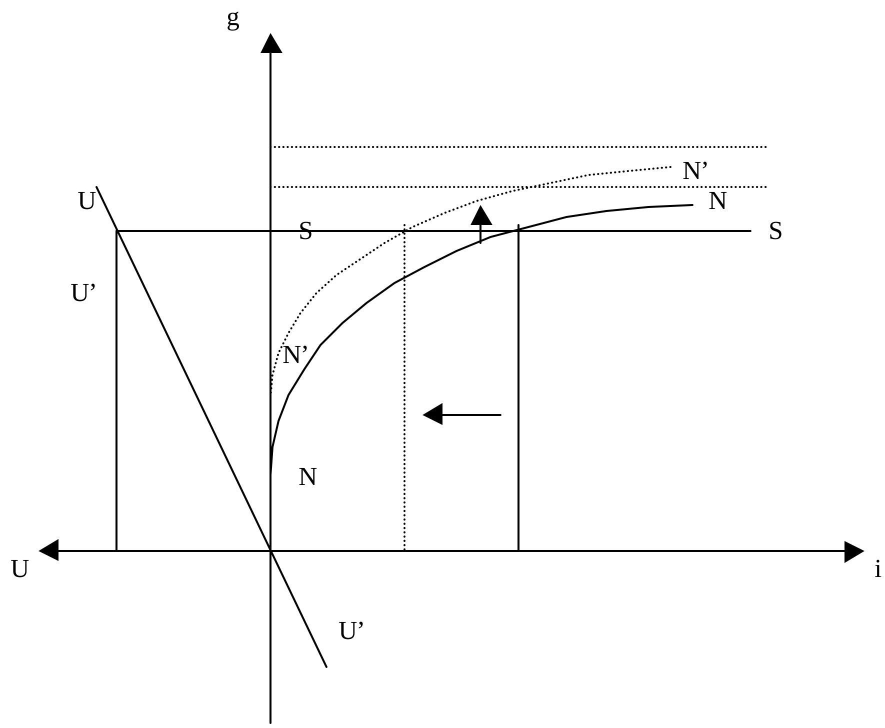

Figure-1.2: Effects of technological improvements in the North or, increase in size of the labour force there (decrease in a_N or, increase in L_N).

(Wide gap)

Table-1: Summary of the comparative static results in the wide gap case:

	g	i	U_S
$a_S \uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
$a_N \downarrow$	NC	$\downarrow$	NC
$H_S \uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
$L_N \uparrow$	NC	$\downarrow$	NC

NC represents the corresponding variable does Not Change.

Table-2: Summary of the comparative static results in the narrow gap case :

	g	i	U_S
$a_S \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
$a_N \downarrow$	$\uparrow$	* $\downarrow$	$\uparrow$
$H_S \uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
$L_N \uparrow$	$\uparrow$	* $\downarrow$	$\uparrow$

* in cell represents this result is under the sufficient assumption

$2a_S\, g > H_S$.